The Maximal Generality Fractal Interpretation Of Information

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Abstract
In real condition, elements form sets whose delimitation is vague, thus defining fuzzy sets in Zadeh's acception or fluids sets according to Gentilhomme's definition.
Any system characterized by uncontrollability and disorder in which the least changed in its status at a certain moment rapidly leads to important changes in the status measured at a later moment can be defined as a chaotic system.
The interdisciplinary study of chaotic system has become a science of complexity or the science of the chaos.
Science has always searched the order in a chaotic universe and the science of the chaos uses a geometry named fractal; fractals are defined as a form at which fractal dimension surpassed its topologic dimension or as any form at which the parts have as many details as the whole or as forms which are strictly self-similar and not statistically self-similar.
The notion of information necessary involves the notion of order and the notion of order involves the rationality of system as through the relation information shows how rationally organized the elements of the system are and what rational functionality they fulfill within the order which defines the system.
By introducing the concept of structural-diachronic cell associate whit the elementary amount of information (the bit) the paper interprets the concept of information in a fractal manner of maximal generality, in the sense that any fraction of the bit is a bit in itself, the structural-diachronic cells being strictly self similar.
The fractal interpretation of the concept of information is the theoretical support of the Basis of Universal Knowledge Similar to the human brain.
Keywords: Fractal, Diachronic, Knowledge, Artificial Inteligence

1. Theoretical Consideration On The Concepts Of Fractals And Information

The theories of the chaos and of complexity, as well as fractal geometry is conceptual tools which have revolutionized matemathics and science in general by ensuring a new way of interpreting reality. (4, Oliver, 1992)
The theory of fractals was set forth by Mandelbrot's book entitled "A theory of fractal series", the first fractal being created as abstract exercises and their significance being unknown to Siepirski, Hilbert, Cantor and Koch.

1.1. The Fractal

The fractal is defined as a form in which the Haussdorff / Besicovitch (fractal) dimension surpassed its topological dimension.
A fractal is any form in which the parts, when magnified, have as many details as the whole.
Devany defines the fractals as form which are self-similar in a strict sense and not in a statistic sense, the Mandelbrot series, according to this definition not being a fractal.

1.2. The Fractal Dimension

By developing Hassdorff's research Beiscovich introduced the notion of a fractional dimension of forms as a measure of the irregular or the rough character of a form.
The fraction of Hassdorff / Beiscovich dimension renamed by Mandelbrot as fractal dimension is defined as the ratio between the logarithm of the number of copies and size of the core corresponding to each copy.

1.3. The Fractal Pattern

The fractal pattern is a geometrical definition of a fractal consisting of the core (parent) form and of copies of this form. The form of the core together with the copies which represent transformations of the form are named parts of the pattern.

1.4. Chaos and Chaotic Symmetry

A chaotic system is a system characterized by uncontrollability and disorder.
The statistic self-similarity within a wide zooming range, represents the chaotic symmetry.
A form characterized by chaotic symmetry does not contain exact copies of itself but copies which are in a non-linear geometric relation with the whole. The non-linear iterated systems such as the Mandelbrot and Julia series are characterized by chaotic symmetries.

1.5. Cellular Automata, Cellular Counting

The cellular automaton represents a computer simulated universe in which a great number of artificial almost identical cells reproduce according to logic or geometric rules.
The counting of cell is a mens of assessing the irregularity or the fractal dimension of a form. The area in which the form is situated is successively divided into square ever larger cells and the number of cells which contain portion of the form is calculated.
The ratio between calculation logarithms is an assessment of the form irregularity. Irregularity can be defined as the roughness which can be qualified as a fractal dimension.

1.6. Level of Detail, Infinite Level

Fractal can be approximated by successively outlining ever more complex level of detail. The form of the core represented by the polygon within the fractal pattern and serving as a parent for all the other parts of the fractal is the first level of detail (level 0) and the rest of the pattern is the next level of detail (level 1).
The fractal itself are an abstraction which exist on the most complex level of detail (level, ∞).

1.7. Reflection, Regular Fractals

A transformed copy of the form represent a reflection. When transformed copies of the whole appear in its means that the form of whole is reflected throughout its parts. If a random factor is not including when fractals are created, the latter are self-similar in a strict sense and they are named regular fractals. The fractals which are only statistically self-similar, such as the Mondelbrot series, are not regular.

1.8. On the Definition of Information

Information is a scientific concept which is considered in philosophy as having an universal character in existence. Information is related to meaning and meaning involves a phenomenological process.
Information has not been rigorously defined as 'any pieces of news which contains the results of an event, fact or process' or as 'the message which conveys news about facts, events, objects, processes'.
The concept of information involves the concept of order and the concept of order involves the rationality of the system, i.e. how rationally the elements of the system are organized and what type of rational functionality they fulfill inside the order that defines the system. (6, Miriță, 1997).

1.9. The Measurement of Information

Shannon has established the criterion which enables the comparison of the amount of information contained in different signals i.e. the measurement of information.
Information can be conveyed only by an unexpected event chosen and random from a set of possible events.
The notion of information should be related to the probability that an event could have one result or another without being known with certainty.
The main property of random events is the undetermined character of their occurrence.
Information, which is obtained by inducing one of the possible states of the events excludes the undetermined character and that is why the amount of information is numerically equal to the undetermined character contained by an event.
The undetermined character of event X contained by the set is the average value of the

undetermined character of the situation included in the event.

$$H(X) = -\sum_{i=1}^{n} p_i \cdot \log\ p_i \qquad (1)$$

where H(X)- the value of the general undetermined character of event X, named entropy by Shannon.
The amount of information is equal to the entropy and thus it can be inferred from the relation:

$$I = -\sum_{i=1}^{n} p_i \cdot \log\ p_i \qquad (2)$$

where p_i -the probability of including state x_i of event X.
As an information unit the binary information unit or the BIT has been adapted. This unit represents the amount of information received as a result of a unique choice out of to equal - probability possibilities.

$$I = -\log\ p_i = \log\ n_i \qquad (3)$$

In the case of non equal probability events the number of information units can be calculated in the relation:

$$I = -\frac{1}{N} \bullet \sum_{i=1}^{n} n_i \cdot log\ \frac{n_i}{N} \qquad (4)$$

where I - information (in binary units); N - total number of events; n_i - the number of events of p_i probability.

1.10. Abstract Models

Abstract models contain the invariant relation which can explain the architecture and dynamics of system. System been structured, it is impossible to explain the either by starting from the whole to the part or from the part to the part to the whole. An explanation is possible only by starting from the constellation of invariant relation which characterize them.
The concept of structure designated the constellation of necessary relation relations

between the elements of a system. These relations are invariant and independent from the elements of the system and thus they are formalizable involving an topological and relational approach.

1.11. The Relation Between Structure and Genesis

Structural analysis is the starting point for a historical and genetical analysis. The structure itself becomes comprehensive and the dialectical method is a unit of the structural and functional analysis with a historical-genetical analysis. This involves the study of the origin and the evolution of the corresponding structure as historical products an autonomous process of equilibrium, the structural coherence appearing not as a static reality but as a dynamic virtuality.
The structural analysis correlated with the historical-genetical analysis explains the switch from one structure to another.
Each system has a determinated structure which includes within itself the resources of its own surpassing.

2. THE MAXIMAL GENERALITY FRACTAL INTERPRETATION OF INFORMATION

2.1. The Diachronic Space (7, Miriţă, 1994)

This paper introduces an extra coordinate besides space and time, namely the diachronic coordinate, and we shall name diachronic hipercartesian space the limitless set of diachronic levels consisting of a sequence of levels (N_i), each diachronic level corresponding to a "step" in universal becoming.
The reference system of the diachronic space contains the access of diachrony and synchrony.

2.1.1. The Universal Parameters of Diachronic Space

The diachronic space is characterized by the following universal parameters:

1. Diachronic levels (N_i);
2. The amount of information corresponding to the level (I_i);
3. Level probability (P_i);
4. Equivalence or level cardinal number (n_i).

Each diachronic level has a corresponding class of equivalence, therefore a cardinal number n_i.

$$n_i = 2^{N_i};\ p_i = \frac{1}{2^{N_i}};\ I_i = \log_2 2^{N_i} \qquad (5)$$

As cardinal number are classes of equivalence which involve binary equivalence relations defined in Cartesian Space, the diachronic space constitutes a "hyper-Cartesian space".

2.2. Generalized Syllogistic Hypergraph

The generalized syllogistic hypergraph represents the model of the most general diachronic structure consisting of structural-diachronic cells; it is elaborated by the superposing of tree arborescent structures: of proprieties P_{ik}, of diachronic $\Re_{iki\text{-}lk}$ and synchronic R_{ikik+1} relation, and of object O_{ik}.

2.3. The Structural-Diachronic Cell

We shall designate as structural-diachronic cell the minimal three-object, three-relation, four-proprety set expressed as a set of three minimal sets.

$\{O_{i\text{-}lk}, O_{ik}, O_{ik+1}\}, \{P_{i\text{-}1k}, P_{ik}, P_{i+1k}, P_{i+1k+1}\}, \{\Re_{iki\text{-}1k}, R_{ikik+1}, \Re_{ik+1i\text{-}1k}\}$

The structural-diachronic cell can be modeled mathematically by three elementary matrices:

1. The property elementary matrix;
2. The object elementary matrix;
3. The relations elementary matrix.

2.4. The Four-Dimension Interpretation of the Bit (6, Miriţă, 1997)

The bit representing the elementary unit for measuring information, we shall associate it as a measure of the structural-diachronic cell. By associating the bit to the structural-diachronic cell it can be noticed that the semantic architecture of the bit involves a configuration of three objects: $O_{i\text{-}1k}$ - precursor,O_{ik}, O_{ik+1} - successors, between which exist two diachronic relation $\Re_{iki\text{-}1k}$, $\Re_{ik+1i\text{-}1k}$ and the synchronic relation of contradiction R_{ikik+1}, the object having the properties $P_{i\text{-}1k}$ --aprioric, P_{ik} in act and P_{i+1k}, P_{i+1k+i} - potential. The configuration of the three objects involves the existence of three levels in the diachronic space: $N_{i\text{-}1}$, N_i, N_{i+1} (precursor, actual, successor).

The four - dimensional interpretation of the bit involves:

1. The vector of diachronic levels;
2. The matrix of properties;
3. The matrix of objects;
4. The matrix of relation: diachronic and synchronic.

The triple arborescence of properties objects and relation is asserted by the arborescence of truth modeled by the matrix and it is represented by a hypergraph.

2.5. The Topological Relativistic Configuration of the BIT in Information, as a Measure of the Tree-matriciality of the Syllogistic Diachronic Structures in Hyper-Cartesian Space

By applying the relation theory, the topological relativistic configuration of the bit in information can be revealed.
The structur-diachronic cell associated to one bit of information represents the elementary structure of a diachronic syllogistic-inferential structure.
The complex diachronic syllogistic structures involved the existence of a pyramidal configuration of structural-diachronic cells on multiple diachronic levels represented by hypergraphs and modeled mathematically by the objectual matrices as well as by the matrices of properties and diachronic-syncronic relations.
Information can by defined as a measure of the three-matriciality of the syllogistic diachronic structures in hyper-Cartesian space.
A certain amount of information corresponds to each syllogistic structure according to the degree of its complexity. By analyzing the complex diachronic syllogistic structure, it can be noticed that the number of structural-diachronic cells increases with the number of bits, each cell occupying a certain diachronic level and having a certain position within that level.
Therefore by the equation of the amount information in the case of equiprobable events we shall not calculate only the number of bits, but also the number of diachronic levels of the diachronic structure corresponding to the given amount of information.
By associating one bit to each structural-diachronic cell in the complex structure it can be noticed that the relative position of the cell inside the structure involves the topological relativistic configuration of the bits in information.(6, Mirițǎ, 1997)

2.6. The FRACTAL Interpretation of the Information Unit (the BIT)

We shall associate the BIT to a structural-diachronic cell and the sturctural-diachronic cell to a FRACTAL, defining the fractal as an abstract model of maximal generality (and not by a geometrical definition) which involves three diachronic levels: N_{i-1}, N_i, N_{i+1}; four properties P_{i-1k}, P_{ik}, P_{i+1k}, P_{i+1k+1}; three objects O_{i-1k}, O_{ik}, O_{ik+1} and three relations $\Re_{iki-1k}$,R_{ikik+1}, $\Re_{ik+1i-1k}$. By interpreting the BIT as a FRACTAL, the fraction of the bit are also bits but on the succesor diachronic levels which will be associated to the detail levels. As the structural-diachronic cell associated to the bit involves a four-dimensional interpretation, the fractal interpretation of information involves:

1. A fractal structure of properties (fig. 1);
2. A fractal structure of objects (fig. 2);
3. A fractal structure of syncronic and diachronic relation (fig. 3);
4. A fractal structure of truth (fig. 4);
5. A fractal diachronic structure of the bits (fig. 5).

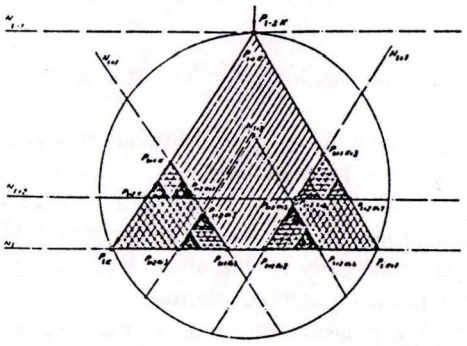

Fig.1. The fractal structure of proprieties

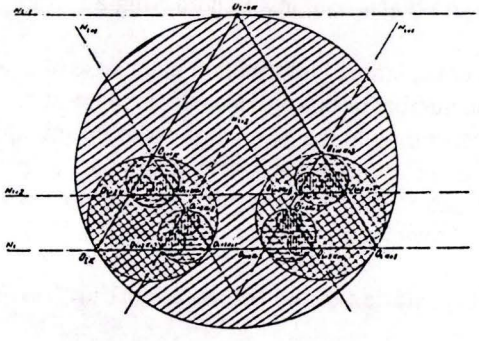

Fig.2. The fractal structure of objects

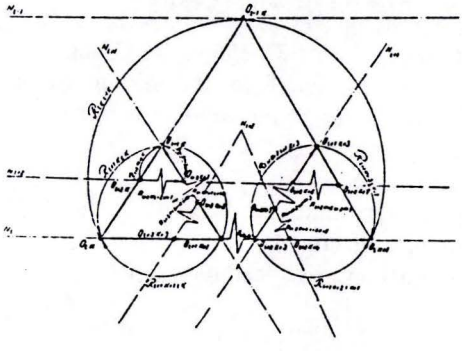

Fig.3. The fractal syncronic-diachronic relation

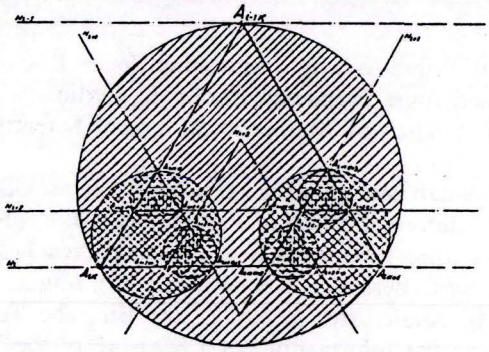

Fig. 4. The fractal interpretation of truth

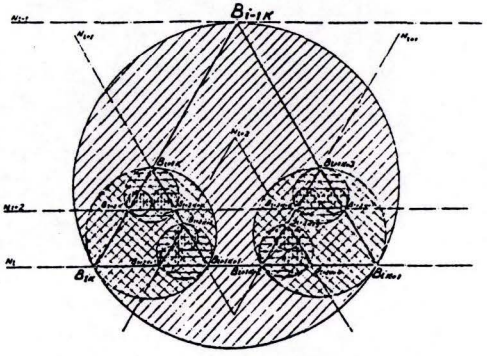

Fig.5. The fractal interpretation of information

3. CONCLUSION

1. By associating the BIT to the structural-dichronic cell and the structural-diachronic cell to a FRACTAL, the geometric definition of the fractal, which consist of a core and of copies of a core, is generalized by an abstract model which refers to a FRACTAL proerty-object-relation structure.
2. The maximal generality fractal interpretation of information demonstrates that the fraction of information units (the fractions of the bits) are also bits on successor levels of detail.
3. The fractal approach to the concept of information allows the achievement of THE BASIS OF UNIVERSAL KNOWLEDGE, THE EXPERT SYSTEMS AND THE ARTIFICIAL INTELLIGENCE.

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