Hierarchy of events in analysis and forecasting of discrete event processes.

Jiří Pik

Institute of Information Theory and Automation, Academy of Sciences,
Pod vodárenskou věží 4, 182 08 Prague 8, Czech Republic,
fax: (+420) (2) 68 97 008,
pik@utia.cas.cz

Abstract
Analysis and forecasting of the behaviour of a complex natural system represented by a non-numerical time series of the system's atomic activities is considered within an objective framework of discrete event systems and processes. The presented approach is based on a hierarchy of events introduced through context-dependent event-to-event operations and to illustrate it, a simple example is included.
Keywords: discrete event process, analysis, forecasting, hierarchy of events, meteorology.

1 Introduction

A broad research of discrete event systems reflects the different aspects of behaviour of these systems whose terms correspond to logical or symbolic rather than numerical values. A number of approaches have been introduced to the analysis, modelling, and control of such systems and processes, (Cassandras, 1993; Varaiya and Kurzhanski, 1988).

Let us consider some discrete event process over an alphabet of events describing the behaviour in elementary time intervals. A typical example is the meteorological time series containing GWL standardization of daily synoptical weather patterns over the Atlantic-Europe region, (Hess and Brezowsky, 1969). In the literature, two different approaches have been proposed for processing of this time series to analyse and forecast it. In Mares and Mares (1982), the Markov chain theory is taken into account. States of the Markov chain are labelled using the GWL symbols and the corresponding transition matrix is computed. The obtained results are very briefly outlined and it is concluded that the formalism is considered as a first good approximation. A method of analysis based on a transformation of the original time series is proposed in Pik (1994). The method is utilized in the decision support system for analysis and modelling of the atmospherical circulation for the long-range weather forecasting in the Czech Institute of Hydrometeorology and the experience in this application has led to development of new methods.

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In the paper, the analysis and forecasting of the behaviour of a complex natural system represented by a non-numerical time series of the system's atomic activities is considered within an objective framework of discrete event systems and processes. The presented approach is based on a hierarchy of events introduced through context-dependent event-to-event operations. The formalism of formal languages is utilized and a comparison between the formalism of the approach and that of the developmental systems or languages (L-systems) is included.

To characterize a general conception of the proposed approach, a concept of pragmatic model of analogical thinking and reasoning can been adopted from psychology.

2 BASIC CONCEPTS

An alphabet is a finite nonempty set the elements of which we call *event symbols*. If E is an alphabet, then E^* denotes the set of all sequences or strings or words of finite length composed of the event symbols from the alphabet E including the string ϵ consisting of no symbols. The set $E^* - \{\epsilon\}$ is denoted by E^+.
The length of a string X , written $|X|$, means the number of symbols in X when each symbol is counted as many times as it occurs, $|\epsilon| = 0$. The set of all strings X over E with $|X| \leq m$, $m > 0$, is denoted by E^{*m}. The set $E^{*m} - \{\epsilon\}$ is denoted by E^{+m}.
A string X is a substring of a string Y iff there are strings X_1 and X_2 such that $Y = X_1 X X_2$, where $X_1 X X_2$ denotes the concatenation of the strings X_1, X, and X_2. The string ϵ is an identity element in E^*, thus $\epsilon X = X\epsilon = X$ for all $X \in E^*$.

Let φ be an equivalence relation over a set T. Then for every $t \in T$, the set $B_\varphi(t) = \{u : t\varphi u\}$ is an equivalence class. A partition π on T is a collection of disjoint subsets of T, called blocks of π, whose set union is T.

To introduce *a distance of the strings of events*, the Levenshtein distance and the corresponding weighted distance are considered in what follows, (Levenshtein, 1966). Using a-priori defined event-to-event operations, the strings of the event symbols are transformed into new ones.

Let E be an alphabet, an operation over E is an ordered pair $s = (a, b)$ such that $a, b \in E \cup \{\epsilon\}$ and $s \neq (\epsilon, \epsilon)$. The operation $s = (a, b)$ is called (i) deletion if $b = \epsilon$, (ii) insertion if $a = \epsilon$, and (iii) substitution otherwise.

A string Y results from the application of the operation $s = (a, b)$ to a string X, written $X \stackrel{s}{\Longrightarrow} Y$, if $X = A_1 a A_2$ and $Y = A_1 b A_2$, where $A_1, A_2 \in E^*$. To transform a string X into Y, a series of operations $S = s_1, s_2, \ldots, s_q$ is needed, $q > 0$, such that $X = X_0 \stackrel{s_1}{\Longrightarrow} X_1$, $X_1 \stackrel{s_2}{\Longrightarrow} X_2, \ldots, X_{q-1} \stackrel{s_q}{\Longrightarrow} X_q = Y$.

To reflect a difference in the application of the operations, a real number $w(s)$ called a weight of the operation $s = (a, b)$ is associated with each operation. The notion of $w(a, b)$ is extended to a series of operations $S = s_1, s_2 \ldots, s_q$ by

$$w(S) = \sum_{i=1}^{q} w\ (s_i) \text{ and by } w(S) = 0 \text{ for } q = 0.$$

The *weighted distance* $d_w(X,Y)$ between X and Y is defined by

$$d_w(X,Y) = \min\{\ w(S) : S \text{ is a series of operations which transforms } X \text{ into } Y\ \}.$$

A *discrete-event system* (DES) is defined as a 3-tuple

$$DES = (S, E, D),$$

where

S is an alphabet of states,
E is an alphabet of events,
D is a transition function,
$D\ :\ S \times E \rightarrow S \cup \{\epsilon\},$

and ϵ is used to indicate an undefined transition.
Such DES is also called the untimed (logical) DES to distinguish it from the system where the event occurrence time is taken into account.

The event string $e_0e_1 \ldots e_i \in E^*$ is called a sample path of an untimed DES if $s_{j+1} = D(s_j, e_j)$ for an initial state s_0 and all j, $0 \leq j \leq i$.

A *discrete-event process* is introduced through a set of the sample paths of the DES.

3 Hierarchy of events

Using the weighted distance based on context-dependent event-to-event operations, an equivalence relation over the set of the strings of the event symbols and an induced partition of this set can be defined. The strings found in the same partition block are represented by a macro-event symbol and a new representation of each string based on these symbols is possible. Following it, the considered equivalence relation can be viewed as a basis of a hierarchical event structure, (Pik, 1993).

3.1 Context-dependent substitution

Let E be an alphabet, let P be a finite set of rules of the form $XaY \rightarrow XZY$, where $a \in E$, $X, Y, Z \subseteq E^*$ and $\rightarrow$ is not in E. Then a *context-dependent substitution* on E^*, (Kac and Reitbort, 1973; Tanaka, 1988), is a mapping T that is defined on E^* by $T(\epsilon) = \{\epsilon\}$ and

$$T(w) = T(a_1 \cdots a_n) =$$
$$\{v_1 \cdots v_n :\ X_ia_iY_i \rightarrow X_iZ_iY_i \in P,$$

such that

$$a_1 \cdots a_{i-1} \in E^* X_i, \; a_{i+1} \cdots a_n \in Y_i E^*, \; v_i \in Z_i$$

$$\text{for } i = 1, \ldots, n \; \}.$$

A context-dependent substitution is called the substitution with finite (regular) context if for every $XaY \to XZY \in P$ the languages X, Y are finite (regular), it is called ϵ-free if ϵ is not in Z for every $XaY \to XZY \in P$.
Hereafter, the notation $(\alpha\underline{\beta}, \alpha\underline{\gamma})$ is used for the operation (β, γ) if a context α is considered, $\alpha \in E^*$, $\beta, \gamma \in E$.

3.2 Hierarchical event structure

A hierarchical event structure, HES, is defined as a 3-tuple $HES = (h, E, F)$, where
h is a number of the levels of hierarchy , $h > 1$,
E is an alphabet of HES, $E = \bigcup_{i=1}^{h} E_i$,
the alphabets E_i, E_j are not necessarily disjoint for $i, j = 1, \ldots, h$,
F is a set of relations $F = \{\varphi_2, \ldots, \varphi_h\}$,
and for each $i = 2, \ldots, h$:

(i) there exist $m_i, n_i > 0$, $n_i < i$, such that the relation φ_i is an equivalence over $E_{n_i}^{*m_i}$,

(ii) there exists a mapping ψ_i from the induced partition $\pi_i = \{B_{n_i,1}, B_{n_i,2}, \ldots\}$ on $E_{n_i}^{*m_i}$ onto $E_i \cup \{\epsilon\}$,

(iii) the symbols of the alphabet E_i are called macro-events (with respect to the n_i-th hierarchical level).

To define *a hierarchy of the strings*, let $HES = (h, E, F)$ be a hierarchical event structure, let for some $i = 2, \ldots, h$ there exist $X \in E_{n_i}^+$ and $Y \in E_i^+$. Then define Y to be a direct representation of X, written $X \overset{HES}{\Longrightarrow} Y$, if the following conditions are satisfied:
$X = X_1 \ldots X_k \ldots X_l$, and $Y = y_1 \ldots y_k \ldots y_l$, $l > 0$, where $X_k \in E_{n_i}^{*m_i}, y_k \in E_i \cup \{\epsilon\}$, such that for each $k = 1, \ldots, l$ there exists a block $B_{n_i,j}$ of π_i on $E_{n_i}^{*m_i}$ for that $X_k \in B_{n_i,j}$ and $y_k = \psi_i(B_{n_i,j})$.

Let $X \in E_j^+$ and $Y \in E_i^+$ for $1 \le j < i \le h$. Then Y is called *a representation* of X (on the i-th hierarchical level) if there exist $z > 0$ and a string $Z_0, \ldots, Z_z$, such that $Z_0 = X$, $Z_z = Y$, and $Z_{t-1} \overset{HES}{\Longrightarrow} Z_t$ for each $t = 1, \ldots, z$.

3.3 Hierarchy of events and L-systems

The formalism of the developmental systems and languages (L-systems), e.g. Herman and Rozenberg, (1974), is very closely related to the presented approach.

While in a 0L-system the rewriting of a symbol does not depend on its neighbours, in a 1L-system it depends on one of the neighbours, either always on the left or always on the right one. In 2L-system, the rewriting of a symbol depends on both neighbours.

Another generalization of 0L-system is based on the production set divided into subsets called tables. In a T0L-system, only productions belonging to the same table can be used at each step of rewriting.

To compare the formalism of the L-systems with that of the considered approach, the main common features of the rewriting process and the application of the event operations are summarized as follows:

(*i*) a possibility of consideration of the neighbours of the processed symbols,

(*ii*) a simultaneous application of the productions to all symbols under scan in L-systems and a possible parallel manner of application of event operations, (Chang et al., 1987),

(*iii*) a use of the tables in L-systems and the distinguished sets of the weights associated with the corresponding operations.

On the other hand, while the productions in L-systems are repeatedly applied to generate new words, the event operations are used to transform one string into the other. Although a hierarchical structure can be introduced to represent the transformed strings, a finite number of the corresponding levels is considered.

4 Application

Consider a natural system of atmospherical circulation over the Atlantic-Europe region affected by endogenous and exogenous activities. Depending on the insight into the problem, suppose a set of twenty-nine well defined non-numerical types of the daily configurations of the pressure fields and another type added to represent an exceptional configuration. To analyse and forecast the system's behaviour, the distinguished configurations are interpreted as discrete events and the corresponding event string is viewed as a sample path of a discrete event system. The considered updated meteorological time series of the pressure field standardization, (Hess and Brezowsky, 1969), begins at 1881 and contains about 43,000 GWL event symbols from the alphabet
$E_{GWL} = \{c, s, w, a, b, h, v, x, z, y, t, r, j, i, f, e, m, o, n, d, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \cup \{u\}$.

The set of the strings of the limited length over the alphabet E_{GWL} is reduced using a hierarchical event structure. From the meteorological point of view, the considered hierarchical event structure introduces a new standardization derived from the original one and based on the macro-event symbols. To get the considered set of the strings, the substrings with properties reflecting some meteorological parameters (e.g., the length, time location, trends, etc.) are extracted from the given sample path.

Let us consider a simple example concerning the strings over a subset E'_{GWL} of E_{GWL}, $E'_{GWL} = \{c, s, w, a, b\} \subset E_{GWL}$. The events c, s, w, a, and b are referred in Hess and Brezowsky (1969) to "Westlage, zyklonal", "Südliche Westlage", "Winkelförmige Westlage", "Westlage, antizyklonal", and "Hochdruckbrücke über Mitteleuropa" configurations, respectively. Based on a physical analogy of the corresponding pressure fields, a two-level hierarchy is defined by $HES = (h, E, S)$, where

$$\begin{aligned} h &= 2, \\ E &= E_{GWL} \cup E_M, \\ F &= \{\varphi_2\}. \end{aligned}$$

The limited length of the substrings over G_{GWL} is denoted by m and the alphabet of macro-event symbols by E_M.

The equivalence φ_2 over E^{*m}_{GWL} is defined using the weights $w(s_k)$ of the context-dependent event-to-event operations; the mapping ψ_2 is a correspondence between blocks of the partition on E^{*m}_{GWL} and the event symbols of $E_M \cup \{\epsilon\}$.

To define φ_2, two sets of the weights of the operations are taken into account. The weights $w(s_k) = w(\gamma\underline{\alpha}\delta, \gamma\underline{\beta}\delta)$, $\alpha, \beta \in E_{GWL} \cup \{\epsilon\}$, and $\gamma, \delta \in E^{*}_{GWL}$, of the former set follow a physical analogy of the pressure fields and are as follows

$w(\gamma\underline{\alpha}\delta, \gamma\underline{\alpha}\delta) = 0$ for all $\alpha \in E_{GWL}$,
$w(\gamma\underline{\alpha}\delta, \gamma\underline{\beta}\delta) \leq c_1$ iff $\alpha, \beta \in \sigma_i$ for some $i \in \{1, 2, \ldots, 10\}, \alpha \neq \beta$,
$w(\gamma\underline{\alpha}\delta, \gamma\underline{\beta}\delta) \geq c_2$ iff $\alpha \in \sigma_i$, $\beta \in \sigma_j$, $i, j \in \{0, 1, \ldots, 10\}$ and $i \neq j$, $0 < c_1 < c_2$,
where

$\gamma, \delta \in E^{*}_{GWL}$,
and
$\sigma_0 = \{\epsilon\}$, $\sigma_1 = \{c, s, w\}$, $\sigma_2 = \{a, b\}$, $\sigma_3 = \{h\}$, $\sigma_4 = \{v, x, z, y, t, r\}$,
$\sigma_5 = \{j, i\}$, $\sigma_6 = \{f, e, m, o\}$, $\sigma_7 = \{n, d\}$, $\sigma_8 = \{1, 2, 3, 4, 7, 8, 9\}$, $\sigma_9 = \{5, 6\}$,
$\sigma_{10} = \{u\}$,
and

$E_M = \{\varsigma_1, \ldots, \varsigma_{10}\}$,
$\psi_2(\epsilon) = \epsilon$,
$\psi_2(\sigma_i^{+m}) = \varsigma_i$ for $i = 1, \ldots, 10$.
Using HES, a part of the time series over the reduced alphabet E'_{GWL}

... [a] [b] [c] [w] [b] [s] [c] [s] [s] [w] [a] [b] [a] [c] [c] ...

is represented on the second level by the string of the macro-events

$$\cdots \boxed{\varsigma_2}\boxed{\varsigma_2}\boxed{\varsigma_1}\boxed{\varsigma_1}\boxed{\varsigma_2}\boxed{\varsigma_1}\boxed{\varsigma_1}\boxed{\varsigma_1}\boxed{\varsigma_1}\boxed{\varsigma_1}\boxed{\varsigma_2}\boxed{\varsigma_2}\boxed{\varsigma_2}\boxed{\varsigma_1}\boxed{\varsigma_1}\cdots$$

or

$$\cdots \boxed{\varsigma_2^2}\boxed{\varsigma_1^2}\boxed{\varsigma_2^1}\boxed{\varsigma_1^5}\boxed{\varsigma_2^3}\boxed{\varsigma_1^2}\cdots .$$

A more complex case we introduced represents the weights depending on a correspondence between the weather and the standardization and, in effect, the twelve sets (or alternatively, tables) of the monthly weights are considered.

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