A Relativistic Model of a Particle-Antiparticle Pair may Break up the E.P.R. Paradox

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Abstract
A special theory of Relativity in the space-like region has been developed by R. DUTHEIL and A. RACHMAN [1,2,3,4] with the tensor formalism and using Tachyonic Referential Frames (TRF). Now two different theories of Relativity built on two different metrics, define two different Lorentz groups which respectively transform two types of referential frames: Ordinary Referential Frames (ORF) for one theory, Tachyonic Referential Frames (TRF) for the other.

The both theories of Relativity may describe the same event in the physical space: so they need to be unified.

In some previous papers R. DUTHEIL and G. NIBART have shown that particles having a superluminal velocity, named *tachyons* [5,6] may exist [7] and do not violate the Causality Principle [8], and according to our *reinterpretation principle*, the Tachyon Bradyon Identity Principle (TBI Principle) [7] they will always be perceived by any natural observer, using Ordinary Referential Frames (ORF), as being antiparticles having a subluminal velocity.

In the present communication we attempt a unification of the time-like region theory and the space-like region theory into a six-dimensional manifold, where the Lorentz transformations are generalized. Here, the *light barrier* appears as a mathematical singularity that can be removed by a pure algebraical method which eliminates time coordinates (time and energy) from both four-dimensional theories.

In this new model, *time* (the function *t*) is not a coordinate, but it is an « observable » and by definition, it is not reversible. The equation of Klein, Gordon, and Fock can be written in a six dimensional manifold. The Dirac equation can also be written in a six dimensional manifold where it has more conveniently no negative energy solutions.

Furthermore, a pair of particles — here a tachyon (i.e. an antiparticle) and a bradyon— can be considered as one unique event in a timeless six-dimensional space. As its wave function may propagate in two different physical ways, we think this new conception of a particle pair will soon break up the E.P.R. paradox.

Keywords: Time, EPR paradox, Tachyon, Relativity, Particle pair

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1 Introduction

In previous papers and communications [1,2,3,4] DUTHEIL and A. RACHMAN have presented the special theory of Relativity in the spacelike region, developed with the tensor formalism currently used in the general theory of Relativity.

Let us recall that there are two complete orthochronous Lorentz groups, which have the same Lie Algebra, and which are isomorphic with one another

$$\mathcal{L}_{+}^{T} \quad \widetilde{\mathcal{L}}_{+}^{T} \tag{1}$$

$\mathcal{L}_{+}^{T}$ is the usual Lorentz group (orthochronous, subluminal) which preserves the real metrics $g_{\mu\nu}$ having the signature $(+---)$ with the usual real coordinates, called subluminal coordinates

$$x^{\mu} \quad (\mu = 0,1,2,3) \tag{2}$$

$\widetilde{\mathcal{L}}_{+}^{T}$ is the superluminal (and orthochronous) Lorentz group which preserves the real metrics $\widetilde{g}_{\mu\nu}$ having the signature $(-+++)$ with the inherent real coordinates, called superluminal coordinates

$$\widetilde{x}^{\mu} \quad (\mu = 0,1,2,3) \tag{3}$$

As a result there are two types of referential frames: Ordinary Referential Frames (ORF), called subluminal, and Tachyonic Referential Frames (TRF), called superluminal, each type being associated to a different transformation group.
A natural observer taking measurements in relation to a subluminal referential frame (ORF) can only find subluminal velocities

$$\left|\frac{dx^{i}}{dx^{0}}\right| < 1 \quad (i = 1,2,3) \tag{4}$$

and

$$|\beta| < 1 \tag{5}$$

and consequently he interprets all particles as being subluminal particles.
Such particles were called « *tardyons* » [5] and later « *bradyons* » [6].
A hypothetical superluminal observer, i.e. an observer made of « tachyonic matter » [3], taking his measurements with a superluminal referential frame (TRF) would only find superluminal velocities

$$\left|\frac{dx^{j}}{dx^{0}}\right| > 1 \quad (j = 1,2,3) \tag{6}$$

and

$$|\beta| > 1 \tag{7}$$

and would perceive all particles as being superluminal particles, or « *tachyons* » [5,6].

In an other communication [7], R. DUTHEIL and G. NIBART have shown there is a possible equivalence between tachyons and bradyons which can be enhanced to identity, and we have expressed it as a principle: the Tachyon-Bradyon Identity Principle (T.B.I. Principle).

According to the T.B.I. Principle, there is only one type of matter, showing up differently, as ordinary matter, anti-matter or « tachyonic matter » in relation to the type of the observer, subluminal (ORF) or superluminal (TRF).

So for instance, an electron with a subluminal velocity is naturally observed as a negatron $\left(e^-\right)$ while, as we have shown it [7] an electron $\left(\tilde{e}^-\right)$ with a superluminal velocity $\tilde{\beta}$ is observed in the same ORF referential frame as a positron $\left(e^+\right)$ with a subluminal velocity β such as

$$\tilde{\beta}\beta = 1 \tag{8}$$

Moreover in the frameworks of quantum mechanics, R. DUTHEIL has introduced annihilation-creation operators in the spacelike region [9] and considering annihilation-creation operators in both regions

$$b, b^+ \text{ and } \tilde{b}, \tilde{b}^+ \tag{9}$$

he has shown the following relations [9]

$$\begin{aligned} \tilde{b}^+ &= b \\ \tilde{b} &= b^+ \end{aligned} \tag{10}$$

R. DUTHEIL and G. NIBART have proposed a new interpretation of FEYNMAN's diagrams, by introducing the *transceic operator* [7], which is the operator that transforms a bradyon into a tachyon, i.e. an operator which annihilates a bradyon and creates a tachyon. The *transceic operator* defines a tunnel effect through the « *light barrier* ».

We have deduced from it that the *light barrier* is a singularity that can be suddenly crossed through by a fermion, with a change of the sign of the electric charge, or equivalently by changing the metrics and inverting its velocity related to *c*, without macrocausality violation [7].

Finally the complete description of a relativistic phenomenon involving particles belonging to both regions then comes under two special theories of the Relativity, and implies the simultaneous use of two different metrics

$$g_{\mu\nu} \text{ and } \tilde{g}_{\mu\nu} \tag{11}$$

so the same physical space will be represented by two distinct pseudo-Euclidean manifolds

$$\mathbf{E}_4 \text{ and } \tilde{\mathbf{E}}_4 \tag{12}$$

So we have been led to pair up an ORF referential frame with a TRF referential frame [7] — it is an instantaneous pairing defined for a given relativistic event — we have checked that two Lorentz ORF and TRF transformations such as

$$\tilde{\beta}\beta = 1 \tag{13}$$

preserve the pairing of the two ORF-TRF referentials [10].
Consequently the group velocity of a tachyon is equal to the phase velocity of the antiparticle perceived by a natural observer

$$\widetilde{v}\, v = c^2 \tag{14}$$

In such paired referential frames, the position, the velocity vector and the energy-momentum are represented by pairs of 4 dimensional vectors, which are bound by reciprocal relations.

In the present communication, we show that this formalism using eight components can be simplified, by merging the both Lorentz groups $\mathcal{L}_{+}^{T}$ and $\widetilde{\mathcal{L}}_{+}^{T}$ into one manifold U_6 having only six dimensions and one metrics.

2 The T.B.I. Reciprocity Theorem

The ORF-TRF Reciprocity Relation [ref. 7, p. 313] we have deduced from the T.B.I. Principle for the energy-momentum, has been expressed

$$|\widetilde{p}| = p^0 = \frac{E}{c} \qquad |p| = \widetilde{p}^0 = \frac{\widetilde{E}}{c} \tag{15}$$

So we have stated the following theorem :

The modulus of the time component of the energy-momentum 4-vector is equal to the modulus of the the energy-momentum space 3-vecteur associated in the other region

.

This can be written

$$\begin{aligned} \widetilde{p}^2 &= p^{02} \\ p^2 &= \widetilde{p}^{02} \end{aligned} \tag{16}$$

where the momentum space 3-vector is p with components p^i $(i = 1,2,3)$ in the subluminal referential (ORF) and $\widetilde{p}$ with components $\widetilde{p}^i$ $(i = 1,2,3)$ in the superluminal referential (TRF), and p^0, $\widetilde{p}^0$ are the respective time components.
From the Matter Singleness Principle [7], we postulate that the « rest mass » is equal to the tachyonic mass

$$m_0 = \widetilde{m}_0 \tag{17}$$

thus the well known equation

$$\frac{E^2}{c^2} - p^2 = m_0^2 c^2 \tag{18}$$

with equations (16) leads to the generalized momentum equation

$$\widetilde{p}^2 - p^2 = m_0^2 c^2 \tag{19}$$

<u>Remark</u>: as photons have a null rest mass

$$m_0 = 0 \tag{20}$$

their energy is identical in both regions

$$\widetilde{E} = E = h\nu \tag{21}$$

and their both momentum have the same modulus in both regions

$$|\tilde{p}| = |p| = \frac{h\nu}{c} \tag{22}$$

The both special theories of Relativity describe in the both regions complementary aspects a unique physical reality. It is the reason why the T.B.I. Principle has to be extended to all referential frames (ORF and TRF). The identity

$$ds^2 \equiv d\tilde{s}^2 \tag{23}$$

is an expression of the Physical Space Singleness Principle.
We have already shown [ref. 7, p. 314] that

$$\begin{aligned} ds^2 &= \left(1-\beta^2\right)c^2 dt^2 \\ d\tilde{s}^2 &= \left(\tilde{\beta}^2 - 1\right)c^2 d\tilde{t}^2 \end{aligned} \tag{24}$$

so we may write the squared time ratio as follows

$$\frac{d\tilde{t}^2}{dt^2} = \frac{1-\beta^2}{\tilde{\beta}^2 - 1} \tag{25}$$

and from the T.B.I. principle expressed in the pairing relation

$$\tilde{\beta}\beta = 1 \tag{26}$$

it results that the squared time ratio is as simple as

$$\frac{d\tilde{t}^2}{dt^2} = \beta^2 \tag{27}$$

and we deduce that the tachyonic time may be

$$d\tilde{t} = \pm\beta\, dt \tag{28}$$

Remark: with a good sign convention between an ORF and a TRF referential frame we may write

$$d\tilde{t} = \beta\, dt \tag{29}$$

From equations (24) and (27) we get

$$ds^2 = c^2\left(dt^2 - d\tilde{t}^2\right) \tag{30}$$

so we can deduce the ORF-TRF reciprocity formula of the coordinates

$$\begin{aligned} c^2 dt^2 &= d\tilde{x}^2 \\ c^2 d\tilde{t}^2 &= dx^2 \end{aligned} \tag{31}$$

where the space position 3-vector is x with components x^i $(i = 1,2,3)$ in the subluminal referential (ORF) and $\tilde{x}$ with components $\tilde{x}^i$ $(i = 1,2,3)$ in the superluminal referential (TRF).

Finally we get the generalized expression of the relativist interval

$$ds^2 = d\tilde{x}^2 - dx^2 \tag{32}$$

which defines a unique metrics, function of six independent coordinates, in a six dimensional manifold U_6.

3 The six Dimensional Universe

From our interpretation, the physical universe would be composed of a pair of three-dimensional physical spaces, which are not configuration spaces. So we define a six dimensional manifold U_6 as a the product of two Euclidean three-dimensional spaces

$$\mathbf{U}_6 = \widetilde{\mathbf{E}}_3 \otimes \mathbf{E}_3 \qquad (33)$$

without the time coordinates x^0, $\widetilde{x}^0$. We call it the « six dimensional Universe ».

3.1 Metrics of the six dimensional Universe

With a metrics of signature $(+++---)$ a relation can be established between the six dimensional Universe and a relativistic referential frame, either of the timelike region, having the signature $(+---)$, or the spacelike region, having the signature $(-+++)$.

Actually, if we define U_6 as

$$ds^2 = G_{\mu\nu} dX^\mu dX^\nu \quad (\mu,\nu=1,2,3,4,5,6) \qquad (34)$$

with

$$X^j = \widetilde{x}^j \quad X_j = \widetilde{x}_j \quad (j = 1,2,3) \qquad (35)$$

$$X^k = x^{k-3} \quad X_k = x_{k-3} \quad (k=4,5,6) \qquad (36)$$

the invariant is identical to

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta \quad (\alpha,\beta=0,1,2,3) \qquad (37)$$

$$ds^2 = \widetilde{g}_{\alpha\beta}\, d\widetilde{x}^\alpha d\widetilde{x}^\beta \quad (\alpha,\beta=0,1,2,3) \qquad (38)$$

and it results in

$$(dx_0)^2 = d\widetilde{x}_j\, d\widetilde{x}^j \quad (j=1,2,3) \qquad (39)$$

$$(d\widetilde{x}_0)^2 = dx_j\, dx^j \quad (j=1,2,3) \qquad (40)$$

So the three-dimensional Euclidean space $\mathbf{E}_3$ associated to the half-signature $(---)$, is perceived by a natural observer as being obviously the physical space, although the Euclidean distance in the other space $\widetilde{\mathbf{E}}_3$ associated to the half-signature $(+++)$, is perceived by him as a linear time flow. This is why the other space is not visible by a natural observer.

Reciprocally, for a hypothetical superluminal observer (i.e. in relation to a TRF referential frame), $\widetilde{\mathbf{E}}_3$ would be visible and $\mathbf{E}_3$ would be invisible.

3.2 Generalized Momentum Equation

We have seen that the equation (18), i.e.

$$p^{0^2} - p^2 = m_0^2 c^2 \qquad (41)$$

with equation (16) leads to the generalized momentum equation in U_6

$$\widetilde{p}^2 - p^2 = m_0^2 c^2 \tag{42}$$

Obviously the 6-vector momentum P defined in U_6 has no time components, because its components are the space components in the both regions $\mathbf{E}_4$ and $\widetilde{\mathbf{E}}_4$

$$P^j = \widetilde{p}^j \quad (j = 1,2,3) \tag{43}$$

$$P^k = p^{k-3} \quad (k=4,5,6) \tag{44}$$

and we may write it with the following symbols

$$P = \{\widetilde{p};\, p\} \tag{45}$$

However we may define energy as

$$E = c\sqrt{\widetilde{p}_1^2 + \widetilde{p}_2^2 + \widetilde{p}_3^2} \quad \Rightarrow \quad E > 0 \tag{46}$$

and as we have already explained it [8], the energy is always positive. So the 6-vector formalism should enable us to build a Dirac equation without negative energy solutions.

3.3 Definition of the Proper Time

Although the six dimensional Universe has no time coordinates, it is possible to define the proper time in the usual way

$$d\tau = \frac{ds}{c} \tag{47}$$

i.e. such as

$$d\tau = dt\sqrt{1-\beta^2} = d\widetilde{t}\,\sqrt{\widetilde{\beta}^2 - 1} \tag{48}$$

Furthermore, the time irreversibility may come from the mathematical definition of the duration measured by the observer. Actually for a natural observer the measured time is

$$dt = \tfrac{1}{c}\sqrt{d\widetilde{x}_1^2 + d\widetilde{x}_2^2 + d\widetilde{x}_3^2} \quad \Rightarrow \quad dt > 0 \tag{49}$$

Such a definition can prohibit time reversal, even in relativist quantum theory (C, P, T transformations) and eliminates definitively the paradox of the positron (e_t^+) being interpreted as an negative electron (e_{-t}^-) running backwards in time [11,12] .

3.4 Expressions of Time Derivatives

3.4.1 Expression of the Usual Time Derivative

Let us consider any 6-vector depending on the 6 coordinates X^μ in U_6.
For a natural observer the usual time t is defined as a function of the 3 first coordinates $\widetilde{x}^i$, so we have

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \widetilde{x}^i}\frac{\partial \widetilde{x}^i}{\partial t} \quad (i=1,2,3) \tag{50}$$

where

$$\frac{\partial \tilde{x}^i}{\partial t} = \frac{\partial \tilde{x}^i}{\partial \tilde{t}} \frac{d\tilde{t}}{dt} \tag{51}$$

uses a space 3-vector velocity defined in the spacelike region as

$$\tilde{v}^i = \frac{\partial \tilde{x}^i}{\partial \tilde{t}} \tag{52}$$

and the time ratio from equation (28) which is equivalent to

$$\frac{d\tilde{t}}{dt} = \pm \frac{1}{\tilde{\beta}} \tag{53}$$

where the sign $\pm$ depends on a sign convention between associated ORF and TRF referential frames. So we get the expression of the time derivative of the timelike region (ORF) as a function of the space coordinates in the spacelike region

$$\frac{\partial}{\partial t} = \pm \frac{\tilde{v}^i}{\tilde{\beta}} \frac{\partial}{\partial \tilde{x}^i} \quad (i=1,2,3) \tag{54}$$

where $\tilde{v}$ is the vector velocity of the associated tachyon in a superluminal referential frame (TRF), and $\tilde{\beta} c$ the scalar velocity of the superluminal referential frame.
Thus the time coordinate derivative may be expressed as

$$\frac{\partial}{\partial x^0} = \frac{\partial}{c\partial t} = \tilde{r}^i \frac{\partial}{\partial \tilde{x}^i} \quad (i=1,2,3) \tag{55}$$

with the notation

$$\tilde{r}^i = \pm \frac{\tilde{v}^i}{\tilde{\beta} c} \quad (i=1,2,3) \tag{56}$$

Remark: the sign $\pm$ depends on a sign convention between associated ORF and TRF referential frames.

3.4.2 Expression of the Tachyonic Time Derivative

A similar calculus from the following definition

$$\frac{\partial}{\partial \tilde{t}} = \frac{\partial}{\partial x^i} \frac{\partial x^i}{\partial \tilde{t}} \quad (i=1,2,3) \tag{57}$$

gives the tachyonic time derivative of the spacelike region (TRF) as a function of the space coordinates in the timelike region

$$\frac{\partial}{\partial \tilde{t}} = \pm \frac{v^i}{\beta} \frac{\partial}{\partial x^i} \quad (i=1,2,3) \tag{58}$$

where v is the vector velocity of the associated subluminal particle in an ordinary referential frame (ORF), and βc the scalar velocity of the ordinary referential frame.
The tachyonic time coordinate derivative may be expressed as

$$\frac{\partial}{\partial \tilde{x}^0} = \frac{\partial}{c\partial \tilde{t}} = r^j \frac{\partial}{\partial x^i} \quad (i=1,2,3) \tag{59}$$

with the notation

$$r^i = \pm \frac{v^i}{\beta c} \quad (i=1,2,3) \tag{60}$$

Remark: the sign ± depends on a sign convention between associated ORF and TRF referential frames.

3.4.3 Expressions of Time Derivatives of the Second Order

From equation (55) we get the subluminal time coordinate derivative operator of the second order

$$\frac{\partial^2}{c^2 \partial t^2} = \tilde{r}^i \tilde{r}^j \frac{\partial}{\partial \tilde{x}^i} \frac{\partial}{\partial \tilde{x}^j} + \sqrt{1-\beta^2} \frac{\partial \tilde{r}^k}{ds} \frac{\partial}{\partial \tilde{x}^k} \quad (i,j,k=1,2,3) \tag{61}$$

and from equation (59) we get the superluminal time coordinate derivative operator of the second order

$$\frac{\partial^2}{c^2 \partial \tilde{t}^2} = r^i r^j \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} + \sqrt{\tilde{\beta}^2 - 1} \frac{\partial r^k}{ds} \frac{\partial}{\partial x^k} \quad (i,j,k=1,2,3) \tag{62}$$

where s represents the curvilinear abscissa in $\mathbf{E_3}$.

In a rectilinear propagation (case of the special theory of Relativity) the time coordinate derivatives can be simplified into

$$\frac{\partial^2}{c^2 \partial t^2} = \tilde{r}^{i2} \frac{\partial^2}{\partial \tilde{x}^{i2}} \quad (i=1,2,3) \tag{63}$$

and

$$\frac{\partial^2}{c^2 \partial \tilde{t}^2} = r^{i2} \frac{\partial^2}{\partial x^{i2}} \quad (i=1,2,3) \tag{64}$$

3.4.4 Expression of Time Derivatives in U_6

From equations (54) and (58) we deduce that the propagation of an « event » in U_6 has two direction vectors (it is a particle pair): $\tilde{r}$ having its space components in the spacelike region and r having its space components in the timelike region.
These two direction vectors may be represented with the 6-vector R defined in U_6 as

$$R^j = \tilde{r}^j \quad (j = 1,2,3) \tag{65}$$

$$R^k = r^{k-3} \quad (k=4,5,6) \tag{66}$$

and we represent this definition with the symbolic notation below

$$R = \{\tilde{r}; r\} \tag{67}$$

i.e. the bivector

$$R = \pm \frac{1}{c} \left\{ \frac{\tilde{v}}{\tilde{\beta}}; \frac{v}{\beta} \right\} \tag{68}$$

where $\widetilde{v}$, v are vectors and $\widetilde{\beta}$, β are scalars, and where the sign ± depends on the sign convention between the associated ORF and TRF referential frames.
So we can rewrite time derivatives in U_6. The subluminal time coordinate derivative operator of the first order can be written in U_6 as

$$\frac{\partial}{c\partial t} = R^i \frac{\partial}{\partial X^i} \quad (i=1,2,3) \tag{69}$$

and the superluminal time coordinate derivative operator of the first order can be written in U_6 as

$$\frac{\partial}{c\partial \widetilde{t}} = R^i \frac{\partial}{\partial X^i} \quad (i=4,5,6) \tag{70}$$

From equation (61) we get the subluminal time coordinate derivative operator of the second order in U_6

$$\frac{\partial^2}{c^2\partial t^2} = R^i R^j \frac{\partial}{\partial X^i}\frac{\partial}{\partial X^j} + \sqrt{1-\beta^2}\frac{\partial R^k}{ds}\frac{\partial}{\partial X^k} \quad (i,j,k=1,2,3) \tag{71}$$

and from equation (62) the superluminal time coordinate derivative operator of the second order in U_6

$$\frac{\partial^2}{c^2\partial \widetilde{t}^2} = R^i R^j \frac{\partial}{\partial X^i}\frac{\partial}{\partial X^j} + \sqrt{\widetilde{\beta}^2 - 1}\frac{\partial R^k}{ds}\frac{\partial}{\partial X^k} \quad (i,j,k=4,5,6) \tag{72}$$

where s represents the curvilinear abscissa in $\mathbf{E_3}$.

In a rectilinear propagation (case of the special theory of Relativity) the time coordinate derivatives can be simplified into

$$\frac{\partial^2}{c^2\partial t^2} = R^{i^2}\frac{\partial^2}{\partial X^{i^2}} \quad (i=1,2,3) \tag{73}$$

and

$$\frac{\partial^2}{c^2\partial \widetilde{t}^2} = R^{i^2}\frac{\partial^2}{\partial X^{i^2}} \quad (i=4,5,6) \tag{74}$$

3.5 Expression of the Dalembertian in U_6

The definition of the Dalembertian operator may be generalized to six components in U_6 from the tensor definition

$$\Omega = G^{\mu\nu}\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial X^\nu} \quad (\mu,\nu=1,2,3,4,5,6) \tag{75}$$

As the metrics tensor is diagonal in the special theory of Relativity, we have more simply

$$\Omega = \partial_i \partial^i + \widetilde{\partial}_j \widetilde{\partial}^j \quad (i,j=1,2,3) \tag{76}$$

and it may be related to the Laplacian operators defined in the manifolds $\mathbf{E}_4$ and $\widetilde{\mathbf{E}}_4$

$$\nabla^2 = \frac{\partial}{\partial x_i}\frac{\partial}{\partial x^i} \quad (i=1,2,3) \tag{77}$$

$$\widetilde{\nabla}^2 = \frac{\partial}{\partial \widetilde{x}_i}\frac{\partial}{\partial \widetilde{x}^i} \quad (i=1,2,3) \tag{78}$$

With the metrics $G^{\mu\nu}$ of signature $(+++---)$ the 6-Dalembertian operator may be expressed with a Dalembertian Laplacian relation in U_6

$$\Omega = \widetilde{\nabla}^2 - \nabla^2 \tag{79}$$

4 Klein, Gordon and Fock Equations

Let us write the equation of KLEIN [13], GORDON [14], and FOCK [15] (called F.G.K. equation) of a subluminal boson in the timelike region, and the F.G.K. equation of a superluminal boson in the spacelike region (without external field).

Considering the generalized momentum equation in U_6 we propose a new F.G.K. equation of a pair of bosons (without external field).

4.1 F.G.K. Equation in an Ordinary Referential Frame

The F.G.K. equation of a boson (in the timelike region) is usually written as

$$\Box\psi = \chi^2\psi \tag{80}$$

where ψ is the wave function of a subluminal boson and χ is related to the rest mass with

$$\chi = \frac{m_0 c}{\hbar} \tag{81}$$

and where the Dalembertian operator is defined as

$$\Box = \nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \tag{82}$$

Plane waves ψ which are solutions of the F.G.K. equation are functions of the energy momentum of a boson, such as

$$\psi = e^{i/\hbar (Et - p\cdot x)} \tag{83}$$

and they are related to coordinates in an ordinary referential frame (ORF).

4.2 F.G.K. Equation in a Tachyonic Referential Frame

From the energy momentum equation expressed in the spacelike region [1],

$$\widetilde{E}^2 - \widetilde{p}^2 = -m_0^2 c^4 \tag{84}$$

R. DUTHEIL has deduced the F.G.K. equation of a spinless tachyon [2,3], i.e. the F.G.K. equation of a superluminal boson

$$\left(\widetilde{\nabla}^2 - \frac{\partial^2}{c^2 \partial \widetilde{t}^2} + \frac{{m_0}^2 c^2}{\hbar^2}\right)\widetilde{\psi} = 0 \tag{85}$$

which can be written as

$$\tilde{\Box}\tilde{\psi} = -\chi^2\tilde{\psi} \tag{86}$$

where $\tilde{\psi}$ is the wave function of a superluminal boson having the « rest » mass m_0

$$\chi = \frac{m_0 c}{\hbar} \tag{87}$$

and where the Dalembertian operator in the spacelike region is

$$\tilde{\Box} = \tilde{\nabla}^2 - \frac{\partial^2}{c^2 \partial \tilde{t}^2} \tag{88}$$

The change of sign between the equations (80) and (86) can be related to the change of sign between the signatures of ORF and TRF metrics.

R. DUTHEIL has shown [3] that plane waves $\tilde{\psi}$ which are solutions of the tachyonic F.G.K. equation are functions of the energy momentum of a superluminal boson, such as

$$\tilde{\psi} = e^{i/\hbar\left(\tilde{E}\tilde{t} - \tilde{p}\cdot\tilde{x}\right)} \tag{89}$$

and that they are related to coordinates in tachyonic referential frames (TRF).
If we write the plane wave equation (83) of a subluminal boson as

$$\psi = e^{i/\hbar\left(p^0 x^0 - p\cdot x\right)} \tag{90}$$

and the plane wave equation (89) of a superluminal boson as

$$\tilde{\psi} = e^{i/\hbar\left(\tilde{p}^0 \tilde{x}^0 - \tilde{p}\cdot\tilde{x}\right)} \tag{91}$$

we see that we can find a relation between both wave functions. From equations (16) and (31), i.e.

$$\begin{aligned} {p^0}^2 &= \tilde{p}^2 \\ {\tilde{p}^0}^2 &= p^2 \end{aligned} \tag{92}$$

$$\begin{aligned} {dx^0}^2 &= d\tilde{x}^2 \\ {d\tilde{x}^0}^2 &= dx^2 \end{aligned} \tag{93}$$

we get

$$\begin{aligned} {p^0}^2\, {dx^0}^2 &= \tilde{p}\, d\tilde{x}^2 \\ {\tilde{p}^0}^2\, {d\tilde{x}^0}^2 &= p^2\, dx^2 \end{aligned} \tag{94}$$

and then

$$\begin{aligned} p^0\, dx^0 &= \pm\tilde{p}\cdot d\tilde{x} \\ \tilde{p}^0\, d\tilde{x}^0 &= \pm p\cdot dx \end{aligned} \tag{95}$$

We know that there is no invariant transformation [3] between ORF and TRF frames. Nevertheless an ORF-TRF pairing may be defined with the coincidence of their time and space origins (at the gravity center of the particle pair for example), so we may integrate into

$$\begin{aligned} p^0\, x^0 &= \pm \tilde{p} \cdot \tilde{x} \\ \tilde{p}^0\, \tilde{x}^0 &= \pm p \cdot x \end{aligned} \tag{96}$$

and we may write plane waves equations (83) and (89) as

$$\begin{aligned} \psi &= e^{i/\hbar\left(p^0 \cdot x^0 - p \cdot x\right)} \\ \tilde{\psi} &= e^{\pm i/\hbar\left(p \cdot x - p^0 \cdot x^0\right)} \end{aligned} \tag{97}$$

Thus the plane waves of a subluminal boson and a superluminal boson of the same pair may be *identical* or *complex conjugates*

$$\tilde{\psi} = \psi \quad \text{or} \quad \tilde{\psi} = \psi^* \tag{98}$$

4.3 Generalized F.G.K. Equation in U_6

With the usual method of quantization of relativist mechanics, replacing macroscopic variables with quantum operators, as follows

$$p \to -i\hbar\nabla \quad \tilde{p} \to -i\hbar\tilde{\nabla} \tag{99}$$

the transformation of the momentum equation (19) in U_6

$$\tilde{p}^2 - p^2 = m^2c^2 \tag{100}$$

gives

$$-\hbar^2\tilde{\nabla}^2 + \hbar^2\nabla^2 = m^2c^2 \tag{101}$$

and then

$$\nabla^2 - \tilde{\nabla}^2 = \frac{m^2c^2}{\hbar^2} \tag{102}$$

so it leads to the generalized F.G.K. equation in U_6

$$\Omega\Phi = -\chi^2\Phi \tag{103}$$

where Φ is the wave function of a particle pair, and Ω is the Dalembertian operator expressed from definition (79), i.e.

$$\Omega = \tilde{\nabla}^2 - \nabla^2 \tag{104}$$

and where χ is the squared root of the constant in the F.G.K. equation

$$\chi = \frac{m_0 c}{\hbar} \tag{105}$$

4.4 Correlations in a Pair of Bosons

Two possible relations (98) between bosons wave functions $\tilde{\psi}$ and ψ have been established from the pairing relations of an ORF and a TRF.

It is not easy to find a simple relation between the wave functions $\tilde{\psi}$, ψ and the wave function Φ in U_6 just because $\tilde{\psi}$ (or ψ) represents one superluminal (or one subluminal) boson, although Φ represents a pair of two bosons. These wave functions are related to different types of referential frames and they use different configuration spaces, as shown below

Table 1: Different types of descriptions of a quantum

quantum	**wave function**	**referential**	**manifold**	**configuration space**
subluminal boson	ψ	ORF	$\mathbf{E}_4$	p^0, p
superluminal boson	$\widetilde{\psi}$	TRF	$\widetilde{\mathbf{E}}_4$	$\widetilde{p}, \widetilde{p}^0$
pair of bosons	Φ	6-dimensional	U_6	$\widetilde{p}, p$

Nevertheless we may consider together the F.G.K. equations of the two bosons. Using the second order time derivative expression (55) we may write the subluminal F.G.K. equation (80) as

$$\nabla^2\psi - \widetilde{r}^{i\,2}\frac{\partial^2}{\partial\widetilde{x}^{i\,2}}\psi = \chi^2\psi \quad (i=1,2,3) \tag{106}$$

Using the tachyonic time derivative expression (59) we may write the superuminal F.G.K. equation (86) as

$$\widetilde{\nabla}^2\widetilde{\psi} - r^{i\,2}\frac{\partial^2}{\partial x^{i\,2}}\widetilde{\psi} = -\chi^2\widetilde{\psi} \quad (i=1,2,3) \tag{107}$$

As it is shown by the alternative relations (98) two cases may be considered: substituting $\widetilde{\psi}$ by ψ or ψ^* in equation (107) we get

$$\widetilde{\nabla}^2\psi - r^{i\,2}\frac{\partial^2}{\partial x^{i\,2}}\psi = -\chi^2\psi \quad (i=1,2,3) \tag{108}$$

or

$$\widetilde{\nabla}^2\psi^* - r^{i\,2}\frac{\partial^2}{\partial x^{i\,2}}\psi^* = -\chi^2\psi^* \quad (i=1,2,3) \tag{109}$$

but the two resulting equations (108), (109) are complex conjugates of each other. Thus the system of the equations (106), (107) is equivalent to the equations system

$$\begin{aligned} \nabla^2\psi - \widetilde{r}^{i\,2}\frac{\partial^2}{\partial\widetilde{x}^{i\,2}}\psi &= \chi^2\psi \\ \widetilde{\nabla}^2\psi - r^{i\,2}\frac{\partial^2}{\partial x^{i\,2}}\psi &= -\chi^2\psi \end{aligned} \quad (i=1,2,3) \tag{110}$$

Adding equations (110) leads to an equation without mass

$$\left(\widetilde{\nabla}^2 - \widetilde{r}^{i\,2}\frac{\partial^2}{\partial\widetilde{x}^{i\,2}} + \nabla^2 - r^{j\,2}\frac{\partial^2}{\partial x^{j\,2}}\right)\psi = 0 \quad (i,j=1,2,3) \tag{111}$$

which represents E.P.R. correlations between the two photons of a pair, which have propagation direction vectors $\widetilde{r}$ and r. So the behavior of a photons pair may be expressed with the unique equation in U_6

$$\left(1^{\mu} - R^{\mu 2}\right)\frac{\partial^2}{\partial X^{\mu 2}}\psi = 0 \quad (\mu=1,2,3,4,5,6) \tag{112}$$

where 1^i is the unity tensor. Such an equation represents a six-dimensional photon which is equivalent to a photons pair.

Subtracting equations (110) leads to a new equation with mass

$$\left(\widetilde{\nabla}^2 - r^{i\,2}\frac{\partial^2}{\partial x^{i\,2}}\right)\psi - \left(\nabla^2 - \widetilde{r}^{j\,2}\frac{\partial^2}{\partial \widetilde{x}^{j\,2}}\right)\psi = -2\chi^2\psi \quad (i,j=1,2,3) \tag{113}$$

which represents E.P.R. correlations between the two bosons of a pair, which have propagation direction vectors $\widetilde{r}$ and r. So the behavior of a photons pair may be expressed with the unique equation in U_6

$$\left(\widetilde{\nabla}^2 + \widetilde{r}^{i\,2}\frac{\partial^2}{\partial \widetilde{x}^{i\,2}}\right)\psi - \left(\nabla^2 + r^{j\,2}\frac{\partial^2}{\partial x^{j\,2}}\right)\psi = -2\chi^2\psi \quad (i,j=1,2,3) \tag{114}$$

So the relativist behavior of a bosons pair may be expressed with the equation in U_6

$$G^{\mu}\left(1^{\mu} + R^{\mu^2}\right)\frac{\partial}{\partial X^{\mu^2}}\psi = -2\chi^2\psi \quad (\mu,\nu=1,2,3,4,5,6) \tag{115}$$

where

$$G^{\mu} = G^{\mu\mu} \quad (\mu,\nu=1,2,3,4,5,6) \tag{116}$$

is the diagonal of the metrics tensor of U_6 which is equal to the signature $(+++---)$ in the framework of the special theory of Relativity.

Now let us compare the equation (115) that we have obtained with the generalized F.G.K. equation (103) in U_6

$$\Omega\Phi = -\chi^2\Phi \tag{117}$$

Considering that the 6-Dalembertian operator expressed with the relation (79)

$$\Omega = \widetilde{\nabla}^2 - \nabla^2 \tag{118}$$

is equivalent to

$$\Omega = G^{\mu}\frac{\partial}{\partial X^{\mu^2}} \tag{119}$$

we can deduce a relation between the wave function Φ of a bosons pair in U_6 and the wave function ψ of the subluminal boson within the pair

$$\Phi\left(X^{\mu}\right) = \tfrac{1}{2}\left[1^{\mu} + R^{\mu^2}\right]\psi\left(x^j\right) \quad (\mu=1,2,3,4,5,6\ j=1,2,3) \tag{120}$$

where Φ is a function of the six coordinates in U_6 and ψ is a function of the subluminal space coordinates.

5 Dirac Equations

Let us write the Dirac equation of a subluminal fermion in the timelike region, and the Dirac equation of a superluminal fermion in the spacelike region (without external field).

Considering the generalized momentum equation in U_6 we propose a new Dirac equation of a pair of fermions.

5.1 Dirac Equation in an Ordinary Referential Frame

P.A.M. DIRAC [16] has deduced his equation from the relativist Hamiltonian function of an electron in an electromagnetic field. In this paper we examine the case of a fermions pair without external field.

The Dirac equation of a fermion (in the timelike region) is usually written as an energy equation with the Hamiltonian operator

$$i\hbar\frac{\partial}{\partial t}\psi = \mathbf{H}\psi \tag{121}$$

where ψ is the wave function of a subluminal fermion and $\mathbf{H}$ is the Hamiltonian (without external field) of the fermion, which is defined as

$$\mathbf{H} = a\mathbf{p} + bm \tag{122}$$

where the a and b matrix are related to Dirac matrix with

$$a = \gamma^0\gamma \qquad b = \gamma^0 \tag{123}$$

and where $\mathbf{p}$ is the energy momentum 4-vector, which may be represented by the tensor p^μ.

So from the Hamiltonian Dirac equation (121) the following spinor Dirac equation is usually deduced as

$$\left(\gamma^\mu\frac{\partial}{\partial x^\mu} - \chi\right)\psi = 0 \qquad (\mu=0,1,2,3) \tag{124}$$

where χ is related to the rest mass with the definition (81) as

$$\chi = \frac{m_0 c}{\hbar} \tag{125}$$

In usual presentations, most authors explain that the Dirac equation (124) is Lorentz invariant because the Dirac matrix γ^μ satisfy the following relation

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \tag{126}$$

so Dirac matrix have in an ordinary referential frame (ORF), the usual properties shown below

$$\left(\gamma^0\right)^2 = +1 \quad \left(\gamma^1\right)^2 = \left(\gamma^2\right)^2 = \left(\gamma^3\right)^2 = -1 \tag{127}$$

corresponding to the signature $(+---)$ of a subluminal metrics.

5.2 Dirac Equation in a Tachyonic Referential Frame

R. DUTHEIL has already deduced the *tachyonic* Dirac equation [4] from the subluminal Dirac equation, using the light cone coordinates referential, which has been named Infinite Momentum Frame (IMF) when it has been introduced into the partons theory [17].

With the same method, we deduce the Dirac equation of the spacelike region from the usual Dirac equation (124) rewritten for tachyons as

$$\left(\gamma^{\mu}\frac{\partial}{\partial\tilde{x}^{\mu}}-\tilde{\chi}\right)\tilde{\psi}=0 \quad (\mu=0,1,2,3) \tag{128}$$

where the tachyon has an imaginary « rest » mass defined by

$$\tilde{\chi}=i\chi \tag{129}$$

where γ^{μ} are usual Dirac matrix, $\tilde{x}^{\mu}$ is a notation of tachyonic coordinates and $\tilde{\psi}$ is a notation of the tachyonic wave function.

So the spinor Dirac equation (124) applied to tachyons becomes

$$\left(i\gamma^{\mu}\frac{\partial}{\partial\tilde{x}^{\mu}}+\chi\right)\tilde{\psi}=0 \tag{130}$$

Finally we get the following tachyonic spinor Dirac equation

$$\left(\tilde{\gamma}^{\mu}\frac{\partial}{\partial\tilde{x}^{\mu}}+\chi\right)\tilde{\psi}=0 \tag{131}$$

where tachyonic Dirac matrix are defined by

$$\tilde{\gamma}^{\mu}=i\gamma^{\mu} \tag{132}$$

and satisfy the following relation in a Tachyonic Referential Frame (TRF)

$$\tilde{\gamma}^{\mu}\tilde{\gamma}^{\nu}+\tilde{\gamma}^{\nu}\tilde{\gamma}^{\mu}=2\tilde{g}^{\mu\nu} \tag{133}$$

which means they have the spacelike properties shown below

$$\left(\tilde{\gamma}^{0}\right)^{2}=-1 \quad \left(\tilde{\gamma}^{1}\right)^{2}=\left(\tilde{\gamma}^{2}\right)^{2}=\left(\tilde{\gamma}^{3}\right)^{2}=+1 \tag{134}$$

corresponding to the signature $(-+++)$ of superluminal metrics.

5.3 Correlations in a Pair of Fermions

A pair of a subluminal fermion and a superluminal fermion (without external field) may be represented by the following Dirac equations system

$$\begin{aligned}&\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}}-\chi\right)\psi=0\\&\left(i\gamma^{\mu}\frac{\partial}{\partial\tilde{x}^{\mu}}+\chi\right)\tilde{\psi}=0\end{aligned} \quad (\mu=0,1,2,3) \tag{135}$$

Subtracting equations (135) leads to the following equation representing common solutions $\tilde{\psi}=\psi$ with mass

$$\gamma^{\mu}\left(i\frac{\partial}{\partial\widetilde{x}^{\mu}}-\frac{\partial}{\partial x^{\mu}}\right)\psi+2\chi\psi=0 \tag{136}$$

It represents E.P.R. correlations between two fermions of the same pair.
Adding equations (135) leads to the following equation representing common solutions $\widetilde{\psi}=\psi$ without mass

$$\gamma^{\mu}\left(i\frac{\partial}{\partial\widetilde{x}^{\mu}}+\frac{\partial}{\partial x^{\mu}}\right)\psi=0 \tag{137}$$

It represents E.P.R. correlations between the two fermions of the same pair — the pair being equivalent to a null mass particle (photon).

5.4 Dirac Equation of a Pair of Fermions in U_6

Now let us write in U_6 the Dirac equations of a pair of fermions, by eliminating time coordinate derivatives from common solution equations obtained in section 5.3 above.
Separating time components and space components of the common solution equation (136) with mass

$$\gamma^{0}\left(i\frac{\partial}{\partial\widetilde{x}^{0}}-\frac{\partial}{\partial x^{0}}\right)\psi+\gamma^{j}\left(i\frac{\partial}{\partial\widetilde{x}^{j}}-\frac{\partial}{\partial x^{j}}\right)\psi+2\chi\psi=0 \quad (j=1,2,3) \tag{138}$$

and substituting time coordinate derivatives with equations (55) and (59) we get

$$\gamma^{0}\left(ir^{j}\frac{\partial}{\partial x^{j}}-\widetilde{r}^{j}\frac{\partial}{\partial\widetilde{x}^{j}}\right)\psi+\left(i\gamma^{j}\frac{\partial}{\partial\widetilde{x}^{j}}-\gamma^{j}\frac{\partial}{\partial x^{j}}\right)\psi+2\chi\psi=0 \tag{139}$$

Separating time components and space components of the common solution equation (137) without mass

$$\gamma^{0}\left(i\frac{\partial}{\partial\widetilde{x}^{0}}+\frac{\partial}{\partial x^{0}}\right)\psi+\gamma^{j}\left(i\frac{\partial}{\partial\widetilde{x}^{j}}+\frac{\partial}{\partial x^{j}}\right)\psi=0 \quad (j=1,2,3) \tag{140}$$

and substituting time coordinate derivatives with equations (55) and (59) we get

$$\gamma^{0}\left(ir^{j}\frac{\partial}{\partial x^{j}}+\widetilde{r}^{j}\frac{\partial}{\partial\widetilde{x}^{j}}\right)\psi+\left(i\gamma^{j}\frac{\partial}{\partial\widetilde{x}^{j}}+\gamma^{j}\frac{\partial}{\partial x^{j}}\right)\psi=0 \tag{141}$$

We see that all these expressions use the pairs of Dirac matrix Γ^{μ} defined in U_6 as

$$\Gamma^{\mu}=\left\{\gamma^{\mu};\gamma^{\mu}\right\} \tag{142}$$

i.e.

$$\begin{aligned}\Gamma^{j}&=\gamma^{j} \quad (j=1,2,3)\\ \Gamma^{k}&=\gamma^{k-3} \quad (k=4,5,6)\end{aligned} \tag{143}$$

and it is useful to introduce the « pseudo-metrics » tensor $J^{\mu\nu}$ defined as

$$G^{\mu\nu}=\left(J^{\mu\nu}\right)^{2} \tag{144}$$

from $G^{\mu\nu}$, the metrics tensor of U_6. In the framework of the special theory of Relativity, the tensors $G^{\mu\nu}$ and $J^{\mu\nu}$ evaluate to

$$G^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \qquad J^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & i \end{bmatrix} \tag{145}$$

For common solutions $\widetilde{\psi} = \psi$ with mass, a pair of fermions may be represented by the following equation in U_6

$$\left(-\gamma^0 R^\mu G^{\mu\mu} + i\Gamma^\mu\right) J^{\mu\mu} \frac{\partial}{\partial X^\mu}\psi + 2\chi\psi = 0 \quad (\mu=1,2,3,4,5,6) \tag{146}$$

For common solutions $\widetilde{\psi} = \psi$ without mass, a pair of fermions is equivalent to a photon and may be represented by the following equation in U_6

$$\left(\gamma^0 R^\mu + i\Gamma^\mu G^{\mu\mu}\right) J^{\mu\mu} \frac{\partial}{\partial X^\mu}\psi = 0 \quad (\mu=1,2,3,4,5,6) \tag{147}$$

Finally the behavior of pairs of fermions can be expressed with equations showing two directions of propagation which are represented by the bivector R already introduced (68) as

$$R = \pm\frac{1}{c}\left\{\frac{\widetilde{v}}{\widetilde{\beta}};\frac{v}{\beta}\right\} \tag{148}$$

Let us remark that Dirac equations in U_6 do not have the inconvenience of « negative energy solutions », just because they contains no energy operator.

6 Wave Functions in U_6

6.1 Decomposition of Wave Functions in U_6

From the viewpoint of quantum mechanics, any wave function φ can be decomposed into « stationary states » φ_m .

Usually, the wave function ψ of a boson or a fermion may be considered as a linear combination of plane waves ψ_m and similarly $\widetilde{\psi}$ may also be considered as a linear combination of plane waves $\widetilde{\psi}_n$.

We think that a wave function Φ should be considered a linear combination of stationary states ψ_m and $\widetilde{\psi}_n$ respectively defined in the timelike region and in the spacelike region, but the model we have built up from paired ORF and TRF has imposed a constraint in the energy momentum of the particles within a pair. So the stationary states ψ_m and $\widetilde{\psi}_n$ are not independant.

6.2 Notion of Dihedral Wave

Obviously, the two plane waves ψ_m and $\widetilde{\psi}_n$ do not propagate in the same direction, as it is shown by the expression of time derivatives. We have seen above that the wave function of a particle pair (photons, bosons, fermions) is depending on the 6-vector R defined in U_6 as

$$R = \pm\frac{1}{c}\left\{\frac{\widetilde{v}}{\widetilde{\beta}};\frac{v}{\beta}\right\} \tag{149}$$

Thus such a wave φ which can be observed in both regions $\mathbf{E}_4$ and $\widetilde{\mathbf{E}}_4$ as two plane waves having two different directions, will be named « *Dihedral Wave* ». We would say that the *dihedral wave* φ can be decomposed into stationary *dihedral waves* φ_m. Dirac equations in U_6 (146) and (147) depends on the coordinates of different types ($\widetilde{\mathbf{E}}_3$ and $\mathbf{E}_3$) but it does not refer to time any way. Consequently it represents the correlations of a pair of particles independently of the observer's time.

To understand experiments with particle pairs, we have to consider that two manifolds $\mathbf{E}_4$ and $\widetilde{\mathbf{E}}_4$ are associated to the same unique physical space. The existence of one *dihedral wave* implies two distinct propagation directions that can be observed as if they were separated. However from the viewpoint of quantum mechanics the both particles of the same pair are not separable, and they can be represented by a Dirac equation in U_6 related to six coordinates which are not separable.

7 Conclusion

Modern Physics has met some difficulties that can be related to the relativist singularity (the light barrier). Firstly in the theory of Relativity: the divergence of energy and the existence of two distinct metrics, secondly in quantum theory: time reversibility and negative energy states, make an important problem that can be solved by expressing the laws of Physics in a six dimensional manifold.

An other difficulty which brings the quantum mechanics and the EINSTEIN's Relativity into conflict about the causality principle, is the inconsistency of the mass point concept with the quantum concept. For a pair of two inseparable particles, the quantum is localized in two distinct points in the physical space. It means that the same quantum has two different localizations that are separated by a spacelike interval (E.P.R. paradox).

We think the E.P.R. paradox may be resolved by representing particle pairs as a unique entity in a six dimensional manifold, and we have shown it in the case of bosons (F.G.K. equation) and in the case of fermions (Dirac equation) and for photons.

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