The Theory of Something: A Theorem Supporting the Conditions for Existence of a Physical Universe, from the Empty Set to the Biological Self

Michel Bounias

INRA (D.S.P.E.), University of Avignon (France) and The Alexandria Institute, New York (USA), Biomathematics Unit, Chemin du Petit Bosquet, 84390 Saint-Christol d'Albion, France (Fax 00(33)Ø 4 90 75 08 88)

Abstract

The theorem stating that "the conditions for existence of a physical universe including conscious perception are provided by the empty set", is based on a sequence of proved lemmas. The space composed of the empty set (as constitutive member), and the set theory and general topology (as logical reasoning system), provides existence to connected topological n-spaces. Then, an observable universe can exist as a sequence of intersections (shown to be Poincaré sections) of such topological spaces owning nonequal dimensions. The Jordan-Veblen theorem states that pathes connecting the respective interiors of two closed spaces own nonempty intersections with their frontiers. Mappings of members from one into another section provide order relations in the sequences of Sections, thus generating a physical arrow of time. Any two sections are connected by a momentum-type structure. Provided the Jordan's points of one closed space are the preimage of a sequence of mappings, since the interior of such spaces is compact and connected, sequences converge to fixed points which account for mental images. The set of fixed points of such a closed also contains: (i) a Brouwer's type fixed points accounting for self-identification of the closed ; (ii) a fractal component. The whole provides the corresponding closed with characteristics of a conscious self, with fractalaided retrieval of stored information, that is non-localized memory.

This completes the proof.

Consequently, also the Planetary ecosystem, whose components are living and nonliving structures involved in a set of functions, owns the structure of a mathematical space demonstrably provided with topologies and shown to be compact, complete, and connected, except if discontinuities are artificially provoked. The Weierstrass theorem states that it can reach its supremum, with maximization of richness and complementarity in the distribution of species and habitat, while the Bolzano theorem requires continuity to be fulfilled for this evolution. Finally, the Heine-Borel-Lebesgue property of compact spaces needs that mutualism is fulfilled as a necessary ecological rule.

Finally: (i) Life is supported by converging sequences of closed topologies fulfilling mathematical conditions. The corresponding mappings own at least a surjective step on the way to fixed points of neuronal chainings, but not in the reciprocal direction: therefore, these sequences are not symmetric since the fixed points would not be the same in both directions. This provides the biological arrow of time with the property of irreversibility, with respect to the conscious self; (ii) The mathematical conditions for functionality of biological beings and of their ecosystems may appear as a purely conceptual "driving pulse" which has sometimes been interpreted in terms of a "project" or a "vitalistic" force, but which is more clearly accounting for an anticipatory process. **Keywords**. Existence Proof; Life; Physical World; Self-Conscious Perception; Topological Foundations.

International Journal of Computing Anticipatory Systems, Volume 5, 2000 Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-9600179-7-8

1 Introduction

What is Universe and what is life remain two largely unsolved questions. On one hand, the major two approaches of modelization of the observable Universe, namely relativity and quantum mechanics still lack consistency with one another and confront a number of difficulties (see for instance: Arunasalam, 1997; Driscoll, 1997; Hannon, 1998; Körtvélyessy, 1999; Krasnoholovets, 1997; Kubel, 1997; Lester, 1998; Meno, 1997-1998; Mitchell, 1997; Rothwarf, 1998; Verozub, 1995; Watson, 1998; Wesley, 1994-1995, 1998). On the other hand, no satisfactory definition of Life has yet been provided (see Bounias, 1990; Loewenstein, 1999), and one can hardly believe that Life and related phenomena like consciousness and abstract thoughts, do not belong to our Universe. Therefore, more fundamental knowledge of what Universe could be is needed before Life is better understood.

Observational facts can be flawed by misperception and by errors in measurements : thus, the first goal of the Global Project, started in the 1990s (Bounias et al., 1999) was to address the following questions, regardless of observation :

(i) What are the primary conditions that would allow a physical Universe to exist?

(ii) What Life could basically represent within this system ?

2 Preliminaries

2.1 Conditions for Statement of Scientific Truth

Proposition 2.1. Our reasoning system will be founded on the following principle for the fulfilment of criteria for "scientific truth" : a proposition P is considered true if it is possible to identify a space $\{X, \bot\}$, i.e. a set (X) and combination rules (\bot), in which its validity can be formally demonstrated through a defined logics (Bounias, 1997).

Our demonstrations will be based on the theory of sets and general topology (Bourbaki, $1989-1990_{a,b}$; Choquet, 1984; Schwartz, 1991, 1993) as the logical background and source of combination rules. Then, our demonstrated propositions will be considered true for the identified spaces through this logical background.

2.2 Preliminary Conjectures

Physical conjecture 2.2.1. No preexistence of sets or spaces (e.g. natural numbers and others) nor of structures or quantities (e.g. forces, or energy) is required as a postulate.

Physical conjecture 2.2.2. (Bonaly, 1993: personal communication): (i) An object can be considered physical if it can be in some way observed and can in some way interact with other objects. (ii) A structure can be observed on the condition that it is topologically closed. In effect, a open set does not own a frontier and therefore does not allow a particular function to map into a specific shape of an observed object.

3 Main Results

3.1 Existence of Embedding Spaces

Lemma 3.1.1. The existence of the empty set is a necessary and sufficient condition for existence of a topological n-space.

Proofs have been given (Bounias and Bonaly, 1997a) that :

(i) the empty set owns the properties of a nonwellfounded set, or hyperset (Aczel, 1987; Barwise, 1991), which in addition solves antinomies previously remaining about its properties;

(ii) the empty set owns the main properties of fractal sets, since it is identical to its unions and intersections, and exhibits self-similarity; and (iii) that this gives raise to existence of sets equipotent to natural, rational and finally real numbers, thus providing sets endowed with the power of continuum. These sets provided with the set of their parts give raise to spaces W : then, ε -covers of (W $\cup \infty$) spaces are of infinite order and spaces W_n are endowed with a number n of topological dimensions as great as needed. Thus, condition ($\exists \emptyset$) is sufficient.

Nonexistence of the empty set does not allow the set theory to be built, and therefore, the condition is necessary.

Lemma 3.1.2. A sufficient condition for a subspace S to be closed is that it is the intersection of two topological n-spaces provided with non-equal dimensions.

Proof. Any intersection of spaces W_n and W_p , with $m \neq p$ finite dimensions, contains all its accumulation points (or cluster points), and therefore it is topologically closed (Bounias and Bonaly, 1994).

Corollary 3.1.1. Sections $Si \in \{W_m \cap W_p\}_{m \neq p}$ exhibit properties of arcwise connected Poincaré sections, with fractal features (Bonaly and Bounias, 1995). Proof. Subspaces Si are states of the system analogous to phase-trajectories, with fractal-like scaling, and their space of orbits exclude isolated points.



Fig. 1: The intersections of spaces (Wn, Wm) with nonequal dimensions (m≠n) provide topologically closed spaces (Si). The resulting system allows a function f to take values from 0 to 1, depending on the location of a point x, according to the Urysohn's theorem : therefore a probabilistic-like function is associated to determined structures (see text: Lemma 3.1.1-3.1.2, and corollary 3.2.3).

Lemma 3.1.3. Any of spaces Wn is at least a metric-like space.

Proof. Let Wi a nonempty space taken in non-metric setting. The symmetric difference (Δ) between any two or more parts (A, B, ...) of Wi owns the properties of a mathematical distance (Bounias and Bonaly, 1996), thus providing a simple form to previously examined nonmetric distances (Bonaly and Bounias, 1994).

Since topologies are conserved, and filtering properties hold for (Δ) , this will be called a "topological distance" (Fig. 2).

Corollary 3.1.2. The topological distance, as a nonmetric distance, allows no discrepancy to subsist between microscopic (subparticular) and macroscopic (cosmic) scales.



Fig. 2: The set-difference as a topological distance. The classical symmetric difference (upper part) is generalized into a distance between several sets ($\Delta(A,B,C,D,E,...)$) and its complementary called the 'instans' (m<{A,B,C,D,E,...}>) (lower part).

3.2 Towards Physical Features: Instans and J-Momentums

Definition 3.2.1. Let {A} a collection of sets. Then, we call an 'instans' denoted $m < \{A\}$, the union of paired intersections of members of {A}: $m < \{Ai\}_{(i=1 \rightarrow n)} > = \cup_{(i, k=1 \rightarrow n), (i \neq k)} \{Ai \cap Ak\}$ (1)

The complementary of $m < \{A\} > in \{A\}$ is called the generalized (or multiple-) set distance (Bounias, 1997) owning the property of a generalized topological distance, denoted $\Delta(A)$. Figure 2 illustrates the concepts of generalized set-distances and their complementary: the 'instans'. An 'instans' is a irreducible part of a time-like sequence.

Theorem 3.2.1. Parts of the set of closed parts of a topologically closed compact complete space are allowed to provide a physical world. Proof. (i) The Jordan-Veblen theorem states that any path (Γ) connecting the interior of a closed set A to the interior of a closed set B owns nonempty intersections with the frontiers of A and B.

These intersections, noted <u>a</u> in A and $\Gamma(\underline{a})$ in B account for physical-like interactions between A and B, though occurring between abstract objects in mathematical subspaces. Since the embedding space W_n is arcwise connected and filters its subspaces, and that the structure of the founding empty hyperset provides convergence property to its Cauchy sequences, then the theorem holds and a physical world is allowed to exist from abstract mathematical spaces (Fig. 3).



Fig. 3: The Jordan's points allow a system of closed spaces to be connected from each other's frontiers. They may conjecturally be constituted of fuzzy sets.

(ii) The Brouwer's theorem states that in a closed space, a continued function owns at least one fixed point. Therefore, a closed exhibits at least one stable part belonging to the orbits of its members by the set of its functions. This stands for perennial structures, which completes the proof.

Lemma 3.2.2. Any sequence of mappings of members of a Poincaré section Si into a Poincaré section S_j provides a collection of primarily unordered sections (S) with a order relation.

Corollary 3.21. Ordered sequences of such closed subspaces provide the system with an orientation which corresponds to the physical arrow of time. In contrast, the fundamental embedding space W_n and any of its parts Si are timeless physical spaces.

Proposition 3.21. The quantum-like or differential-like structures joining two consecutive sections within a sequence own some characteristics of a momentum. Proof (Bounias, 1997) : let $1_A(x)$ stand for $1_{MA}(x)$ and $1_{\Delta A}(x)$ the indicator functions of m < A> and $\Delta(A)$, such that $1_A(x)=0$ if $x \notin A$, and $1_A(x)=1$ if $x \in A$. Then, let the extended indicator function $f_{\Lambda,B}(x)$ for two sections A, B, such that $1_A(x)=1_B(x) \Rightarrow f_{\Lambda,B}(x)=1$ and $1_A(x)\neq 1_B(x) \Rightarrow f_{\Lambda,B}(x)=0$ (Fig. 4).

Topologies on sections SA and SB exhibit defined homeomorphic properties.

Let A, B \in { α }, and { α_A } be the preimage of the function. Then, since 2^{α} is the cardinal of the set of parts of { α }, one has from the Urysohn's theorem: $f^{\alpha}_{A,B}(\alpha)=(2^{-\alpha}).f_{A,B}(\alpha) \Rightarrow 0 \leq f^{\alpha}_{A,B}(\alpha) \leq 1$ (2)

Then, two kinds of 'moments of junction' (denoted by M-J, or J-momentums) are defined:

$$MJm_{A,B} = m_A < \{\alpha\} > \perp f^{\alpha}_{A,B}(\alpha)$$
(3a)
$$MJ\Delta_{A,B} = \Delta_A \{\alpha\} \perp f^{\alpha}_{A,B}(\alpha)$$
(3b)

The structure of both M-J species is therefore analogous with that of a momentgenerating function of variable (u): $M(u) = \Sigma(u.F(u))$ where F(u) is a distribution function : in effect, m_A and $\Delta_A \{ \alpha \}$ stand for (u), and $f^{\alpha}_{A,B}(\alpha)$ stands for F(u).

Remark 3.2.1. This actually represents a part of space where "something is occurring", in the latine sense of the word "momentum". The instans momentum MJm_{AB} is necessarily closed and accounts for matter, while the distance momentum $MJ\Delta_{AB}$ may account for non-material components. Since a non-null J-momentum represents a infinitesimal nonempty space associated with the concept of a motion occurring in our physical spacetime, this in turn provides emergence to the notions of energy and of quantity of motion, i.e. the classical momentum. The J-momentum thus provides a common frame for both moment and momentum, in the physical sense.

Finally, the proposition P=(there exists a physical universe) holds for space $\{X=\emptyset, \bot=C\}$, that is : $P(\emptyset, C)$, where C is the complementarity property defined in set theory (Bounias, 1997).

Corollary 3.2.2. Ordered sequences of sections Si connected by momentums M-J provide existence to a physical spacetime from abstract mathematical spaces as primary foundations. Therefore, upon the sole condition that the empty set exists, there is no need to seek for a creation, a beginning nor an end of universe.



Fig. 4: The structure of the mappings between two sections of closed spaces contains indicative functions which contribute to founding the moment of junction.

Remark 3.2.2. This might shed some light on the meaning of Scriptures about "Verb" as being at the "beginning": here, we demonstrate that existence of Universe could originate in an abstract concept: the set which contains no members, thus accounting for the "Verb".

Corollary 3.23. Topologically-determined phenomena are alternatively describable in terms of probabilities.

Proof (Bonaly and Bounias, 1995). Let $Si \in (Sm \subset Wm)$ a Poincaré section in W_n . The complementaries of Si in Sm and W_n are open in W_n and denoted $\{P_m\}=\mathbb{C}_{Sm,Wn}(Si)$. Then, $\{P_m\}$ and Si are disjoint, and the theorem of Urysohn states that there exists a continued function f of domain in W and values ranging in the real interval [0,1], such that (see Fig. 1):

 $(\forall x \in Si), f(x) = 1$ $(\forall x \in Pm), f(x) = 0$ $0 \le f(x) \le 1$ otherwise

(4)

Section Si is countable and at least locally compact, and there exists in a cover of Si a system of such functions f_j ($0 < f_j < 1$) such that any point x of Si owns a neighborhood in which only a finite number of f_j are not identically null. This system is locally finite and its sum in Si is $\Sigma_{(j \in J)} f_j \equiv 1$. These properties (further adding to values of $f^{\alpha}_{A,B}(\alpha)$) meet the characteristics of a probability function, which completes the proof that deterministic-like events can be described in probabilistic-like terms.

Remark. 3.2.3. The kind of Universe described by ordered sections of closed spaces simultaneously contains the whole of all possible events, whatever their practical "realization", upon appropriate consideration of the sets of functions involved in the mappings. Thus, neither causality nor "chance" need to be involved.

Remark. 3.2.4 Stable parts of the described spacetime-like Universe are provided by two ways: (i) intrinsically invariants, such as properties of topological ultrafilters; (ii) stable parts resulting from fixed points of the Banach-Cacciopoli contraction type, in sequences {Si}: the latter case needs spaces to be compact and complete. The set of such mathematical constraints also holds for biological characteristics, as summarized below.

3.3 The Biological Self and the Function of Perception

Biological condition 3.3.1. The concept of a "self" is amenable to the commonly reported fact that for any individual, all perceptions collected from the outside world are driven to one single inside entity. Accordingly, perceptions are converted into mental constructions (images and thoughts) if the flow of input signals is driven to stable states.

This condition is mathematically fulfilled by the existence of fixed points or fixed parts in the sequences of neuronal chainings, as independently pointed by Bounias and Bonaly (1994) and Carmesin (1995).

Lemma 3.3.1. Perception function readily infers from topological properties of closed sets.

The proof can be summarized as follows (Bounias and Bonaly, 1994). Let A and B two closed spaces in space S as defined in Theorem 3.2.1. Then, since such space is arcwise connected (Bonaly and Bounias, 1995), a path Γ connecting a point x in set A to the interior of set B owns intersections $\Gamma(x)=x$ in A and $\Gamma(x)$ in B, with the respective frontiers of sets A and B (Jordan-Veblen theorem). Since B is closed, it owns at least a Brouwer's type fixed point (b_o), which provides its self-identification.

Then, the image of $\Gamma(\underline{x})$ by a sequence of mappings (f) converges to a Banach-Caccioppoli type fixed point : ultimately a(u), upon contraction conditions (Bounias, 1999). Thus, the set of fixed points in B includes $\{a(u), b_0\}$, which associates any perception to a self-identification (Fig. 5). This provides set B with the properties of a conscious observer (Bounias and Bonaly, 1997b).

Proposition 3.3.1. Perception storage is non-localized and retrieval is aided by a fractal component of the neuronal mathematical space.

Proof. A fractal component F is involved in set B due to fractal features of the neuronal chaining (Bonaly, 1994). Such a fractal subspace $F(\Pi, \bot F)$, with (Π) the initiator as member, and ($\bot F$) the generator as combination rule, is Hausdorf separated, topologically convergent to itself, regular, and it is normal since it is provided with the topological distance (Bounias and Bonaly, 1999a). Thus, a space of neurons is called a mixed space: $X = G \cup \{F\}$, with G its nonfractal part. Call go the set of fixed points of G. Then, the fixed point of the mixed space is $FIXm(X) = F \cup \{g_0\}$, and the stable part of the neuronal space of B is $FIX(B) = \cup\{F, a(u), b_0\}$.

Consequently, after a neuronal image has been formed and disconnected, the fractal component is able to drive the reconstruction of a defined neuronal connection, and thus to recall a mental image from any group of neurons : this supports a previous conjecture of Bonaly (1994: personal communication). Therefore there is no need of localization of memory nor to existence of any site of storage of information.

Corollary 3.3.1. The necessary condition for existence of a sequence of mappings of which one should be a contraction, is that space B is provided with a sensorial system driving the signals induced by the Jordan-Veblen's points to the convergence point of the sequence: this precisely is the function which is fulfilled by the neuronal chaining process in the brain of living organisms.



Fig. 5 : Mental images a(u) provided by fixed points of the Banach type in sequences of contraction mappings (f) (materialized by neuronal chainings), are associated with fixed points of the Brouwer's type (bo) (accounting for self-identification). The fractal component (F) contributes to the recall of a mental image from some fragments of it.

Remark 3.3.1. A neuronal chaining sequence can reach a fixed point able to represent the mental image of outside objects, iff the set of connections at fixed point is homeomorphic to a part of the initial set of connections associated with the signal from Jordan-Veblen's points. This implies at least a surjective step, which leads to the demonstration of the specific irreversibility of the biological arrow of time, based on unicity of associated fixed points (Bounias, 1999b).

These last considerations complete the proof of the main theorem, which can be now stated:

3.4 Main Theorem

Theorem 3.4. Existence of a physical-like world including living-like systems as characterized by perception functions associated with self-identification (i.e. consciousness-like properties) is provided by a abstract topological n-manifold originating in a primary space composed with the empty set as the component and the theory of sets and topology as the basic combination rules.

4 Discussion and Conclusion

4.1 About the Fundamental Nature of Life, in Universe

Previous papers (Sharma et al., 1998; Singh et al., 1999; Jain and Shrivastava, 1996), have shown that unicity of fixed points is dependent on surjectivity vs. non surjectivity of sequences of mappings, with or without continuity. Therefore, in a system such as the mammalian brain, exhibiting duplicate structures, unicity of the fixed points, that is unicity of the self, requires compatibility of some functions, which is actually realized in the anatomy of the nervous system by the optic chiasma. Thus:

Proposition 4.1. Life could be characterized as the physical realization, in interacting closed spaces, of mathematical conditions for the functionality of underlying mathematical spaces.

Corollary 4.1. Within the physical world, the presence of sensorial perception systems could be one of the major features differentiating living from nonliving systems.

Remark 4.1. The role of the body must be to provide the organic hardware of the brain with the needed energy and repair supply.

Remark 4.2. Fractal features of the brain functioning and basic properties of the neuronal system involve anticipatory components (Bounias, 1999_b ; Bounias and Bonaly, 1999) which further provides theoretical support to the validity of the hyperincursive models of Dubois (1998).

4.2 About the Evolution of the Planetary Ecosystem

Life takes place in compact complete arcwise connected topological spaces. These mathematical constraints allowed some corollaries to be derived in terms of convergence of ecosystems (Bounias and Bonaly, 1999b) and epistemological reflections.

Corollary 4.2.1. An ecosystem can reach its optimum state.

Proof is given (Bounias and Bonaly, 1999b) by the theorem of Weierstrass, since an ecosystem can be demonstrated to be a topological space discrete and finite, i.e. a compact space. Continuity is provided by the functional structure of the space of orbits of each point by the set of functions.

However, the Bolzano theorem states that discontinuities prevent the convergence of ecosystems evolution towards the optimum stage where the diversity and the complementarity of species and habitat should be maximized.

Corollary 4.2.1. The Heine-Borel-Lebesgue property of compact spaces demands that each habitat or niche and resources of one specie contains a part which is the image of a part of another habitat or niche and resources of another specie. This accounts for ecological "mutualism" as a condition to be fulfilled in addition to minimization of competition.

These results provide theoretical proof to previous ecological conjectures about maximization of habitat and resources and minimization of competition (Moore, 1990).

4.3 About Epistemological Features

Remark 4.3.1. That predetermined mathematical conditions hold for both living and nonliving structures, provides a kind of driving phenomenon, in which features of further states of the systems are anticipatedly defined. Therefore, sequences are converging within the frame of hyperincursive properties, as suggested by Dubois (1998). This driving "impulse" (in an abstract meaning) may account for what has sometimes been called "project", or even "vitalism".

Remark 4.3.2. Upon confrontation of topological invariants of the topological model presented here, and the corresponding structures of the experimentally observable universe, a formal study could be started about which structures of spacetime could actually be constants. This would provide a new basis to further cosmological advances.

Remark 4.3.3. About the concept of velocity of signals, it should be pointed out that our spacetime is necessarily embedded in a topological 4-space (i.e. owning four topological dimensions, regardless of parameter time), since $W_{3,t} \in \bigcup(W4 \cap W3)$: such a space owns the property that each of its points is the center of the whole (Berger, 1990), so that the concept of duration of a traject from one to another part of this space vanishes. This might shed new insights on debates about the velocity of light or non-locality.

Remark 4.3.4. It may be noteworthy that 4-dimension geometrical spaces possess properties that spaces with either lower or higher dimensions do not have (Donaldson, 1983; Freedman and Yau, 1983).

4.4 Conclusion

At the startpoint of this work, in about 1990 (see Bounias, 1999c; Bounias et al., 1999 for review), our aim was twofold: (i) to examine whether or not some major features of the observed spacetime, as including nonliving and living matter, could be predicted from a theoretical development, regardless of experimental observation; (ii) to identify which primary axioms would be needed in support of this theoretical reasoning.

The presented work provides a formal identification of necessary and sufficient conditions needed for existence of a physical universe which would include the concepts of perceiving systems, accounting for living organisms, up to full ecosystems. Within a given frame of reasoning composed of the theory of sets and general topology, and without having to assume even the previous existence of specified mathematical sets and spaces, we have demonstrated that the existence of a universe can be justified from a primary axiom, namely the existence of a founding space composed of the empty set and the property of complementarity. Our demonstration proves that this condition is sufficient, while the use of set theory makes necessary the axiom of existence of the empty set.

We have used general topology because it represents, in association with set theory extended to nonwellfounded sets, the most abstract and powerful reasoning system so far available to human mind. That some essential features of life and ecosystems are predicted by the model strengthens the validity of the approach, which will be now developped towards particular topics. Among further studies, we plan to revisit some important theorems on nonmetric versus metric spaces. In effect, as far as a space is composed of a set and a combination rule, since a set is necessarily endowed with the set-distance (Δ) holding on its parts, a number of nonmetric condition are now shifted to Δ -metric setting. In particular, the classical meaning of convergence as a nonmetric property contrasting with the Cauchy properties as metric ones should be revisited. In turn, the concept of complete spaces now extends to Δ -complete spaces, and the fixed point theory needs to be revisited within this perspective. These works will hopefully help in clarifying the problem of the nature of self-consciousness in living organisms, and further to formally identify the similarities and differences between mind and matter, without introducing the subjectivity of human feelings. Furthermore, the setdistance may well shed some light on the problem of measuring universe, at cosmic versus subatomic structure scales. How these new concepts could help in shedding some light on foundamental questions still debated in physics remains a goal to our further works.

References.

Aczel Peter (1987). Lectures on Non-Well-Founded Sets. CSLI Lecture-notes 9, Stanford, USA.

Arunasalam V. (1997). Einstein and Minkowski versus Dirac and Wigner: covariance versus invariance. Physics Essays, 10(3), 528-532.

Barwise Jon, Moss, J. (1991). Hypersets. Math. Intelligencer, 13(4), 31-41.

Berger Marcel (1990). Géométrie. Nathan, Paris, vol. 1, 422 pp., vol. 2, 541pp.

Bonaly André (1994). The fractal memory. J. Ultra Scientist of Physical Sciences, 6(1), 133-135.

Bonaly André, Bounias Michel (1994). Theory of existence : dimensional fate and fractality conditions. J. Ultra Scientist of Physical Sciences, 6(1), 104-113.

Bonaly André, Bounias Michel (1995). The trace of time in Poincaré sections of a topological space. Physics Essays, 8(2), 236-244.

Bounias Michel (1990). The creation of life: from matter to spirit. Le Rocher, Paris, 444 pp.

Bounias Michel (1996). Recent advances in the "Global Project" and some sociological implications. Invited Lecture at the Institute of Physics, University of Szczecin, Poland, April 22.

Bounias Michel (1997). Definition and some properties of set-differences, instans and their momentum, in the search for probationary mathematical spaces. J. Ultra Scientist of Physical Sciences, 9(2), 139-145.

Bounias Michel (1999b). A theorem proving the irreversibility of the biological arrow of time, based on fixed points in the brain as a compact topological space. Proceedings of CASYS'99 International Conference, Inst. of mathematics, Liège university, Belgium, 9-14 Aug., to appear in: American Institute of Physics Conference Proceedings (edited by D.M. Dublois). Bounias Michel (founder and coordinator) (1999c). Global Project International and Chart of the Living. International collective work. 'Ecosystem' ed., Paris, First printing, 1-22.

Bounias Michel, Bonaly André (1994). On mathematical links between physical existence, observability and information: towards a "Theorem of Something". J. Ultra Scientist of Physical Sciences, 6(2), 251-259.

Bounias Michel, Bonaly André (1996). On metrics and scaling: physical coordinates in topological spaces. Indian Journal of Theoretical Physics, 44(4), 303-321.

Bounias Michel, Bonaly André (1997a). Some theorems on the empty set as necessary and sufficient for the primary topological axioms of physical existence. Physics Essays, 10(4), 633-643.

Bounias Michel, Bonaly André (1997b). The topology of perceptive functions as a corollary of the theorem of existence in closed spaces. BioSystems, 42, 191-205.

Bounias Michel, Bonaly André (1999a). On fixed parts in the mathematical brain : some topological foundations of learning and memory. Submitted, 17 pp.

Bounias Michel, Bonaly André (1999b). Ecosystems as topological spaces: clues for the future of life on Earth. In : Internat. Symp. U.N., ICSU, UNESCO, on The Future of Universe and The Future of Our civilization". Budapest-Debrecen, 2-7 July 1999, World Scientific, Singapore.

Bounias Michel, Bonaly André, Santoli Salvatore, Magnon Anne, Ansari Anees, H., Laibow Rima, Stubblebine Albert N., Sharma B.K., Méhois Marie-José (1999). At the least upper bound of accessible knowledge. A global project on whole-scale existence and properties of Universe and its components. J. Ultra Scientist of Physical Sciences, 11(3), 1-16.

Bourbaki Nicolas (1989). General Topology, Chapt. 5-10. Springer-Verlag, Berlin, Heidelberg, New York ..., 363 pp.

Bourbaki Nicolas (1990a). Théorie des ensembles. Masson, Paris, 352 pp.

Bourbaki Nicolas (1990b). Topologie Générale, Chap. I-IV. Masson, Paris, 376

Carmesin Hans Otto (1995). Neurophysics of adaption. Physics Essays, 8(1), 38-51.

Choquet Gustave (1984). Cours de topologie. Masson, Paris, 317 pp.

pp.

Donaldson S.K. (1983). Self-dual connections and the topology of smooth 4manifolds. Bull. Am. Math. Soc., 8(1), 81-83.

Driscoll Robert (1997). A detailed special relativistic analysis of the Michelson-Morley experiment. Physics Essays, 10(3), 394-400.

Dubois Daniel (1998). Hyperincursive methods for generating fractals in automata related to diffusion and wave equations. Int. J. General Systems, 27(1-3), 141-180.

Freedman Michael, Yau Shing-Tung (1983). Homotopically trivial symmetries of Haken manifolds are toral. Topology, 22(2), 179-189.

Hannon R.J. (1998). An alternative explanation of the cosmological redshift. Physics Essays, 11(4), 576-578.

Jain R., Shrivastava A. (1996). A common fixed point theorem for three surjective mappings. J. Ultra Scientist of Physical Sciences, 8(1), 105-106.

Körtvélyessy Laszlo (1999). The electrical universe. Effo Kiado és Nyomda, Budapest, 704 pp.

Krasnoholovets Volodymyr. (1997). Motion of a relativistic particle in the vacuum. Physics Essays, 10(3), 407-416.

Kubel H. (1997). The Lorentz transformation derived from an absolute aether. Physics Essays, 10(3), 510-523.

Lester J. (1998). Does matter matter ? Physics Essays, 11(4), 481-491.

Loewenstein W. (1999). The touchstone of life: molecular information, cell communication, and the foundations of life. Oxford University Press, Oxford, 368pp.

Mitchell William (1995). The cult of the Big-Bang. Cosmic Sense Books, Carson City, USA, 240 pp.

Meno Franck (1997). The photon as an aether wave and its quantized parameters. Physics Essays, 10(2), 304-314.

Meno Franck (1998). A smaller Bang? Physics Essays, 11(2), 307-310.

Moore John A. (1990). Science as a way of knowing. VII A conceptual framework for biology, Part III, Amer. Zool., 30(suppl.), 123 pp. .

Rothwarf A. (1998). An aether model of the universe. Physics Essays, 11(3), 444-466.

Schwartz Laurent (1991). Analyse I: théorie des ensembles et topologie. Hermann, Paris, 404 pp.

Schwartz Laurent (1993). Analyse. Topologie générale et analyse fonctionnelle. Hermann, Paris, 437 pp.

Sharma B.K., Sahu, D.R., Bounias, M., Bonaly, A. (1998). Fixed points for nonsurjective expansion mappings. Internat. J. Math. & Math. Sci., 21(2), 277-288.

Singh Thakur B., Banerjee A., Bounias Michel, Bonaly André (1999). Fixed points in the neuronal brain that do not need continuity in neuronal chaining. J. Ultra Scientist of Physical Sciences, 11(2), 191-195.

Verozub L. (1995). The relativity of spacetime. Physics Essays, 8(4), 518-523.

Watson G. (1998). Bell's theorem refuted: real physics and philosophy for quantum mechanics. Physics Essays, 11(3), 413-421.

Wesley James P. (1994). Experimental results of Aspect et al. confirm classical local causality. Physics Essays, 7, 240, and (1998) 11(4), 610.

Wesley James P. (1995). Classical quantum theory. Apeiron, 2(2), 27-32.