From the WATCHES to the WAVELETS
(through the "DISCRETE FOURIER TRANSFORMATIONS")

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Abstract
From a few interpretations about the behaviour and characteristics of a hands watch, we will show that this usual wide-spread object can constitute a very valuable didactical tool for the study and the illustration of the Fourier transformations and in going further the same way for the discover of the Wavelets methodology for the signals and shapes analysis.
By means of this way of thinking we are trying to elaborate a highly synoptical procedure for exploring the deep stuctures of some transformations from Time domain into Frequency or Spectral domain.
It will become possible to underline the linkages between every transitions of domains and the **"Jacobien's Operators"** and the **"Z Operators"**

Keywords: Wavelets, Fourier Transformations, Convolutions, Shapes- Analysis,Synoptical Procédures

1. DEVELOPMENT of the SUBJECT

1.1.INFORMATIONS delivered by a WATCH

Watch is a measure instrument for the time elapsing. Watch is illustrating the (Time>Frequency)transition.
In order to understand the conception of our existing rotary watch, we have to remind of its ancestor: Sun- dial.

1.1.1 Composition of a sun-dial: (Fig1) A vertically on ground erected rod of which the shadow described a portion of circle implied by the sunny run through the sky.
Already with this very rudimentary equipment we got a motion of rotation to show the time elapsing. The sun-dial already realized a "Jacobien operator" and simultaneously a "Time-->Movement" transition.
This was a great meaningfull invention!!
In the sun-dial,the movable shadow corresponded to the hours hand and approximately materialized a Phasor (= a complex plane set in rotation with a uniform speed: ω)
Limitation of use: only dayly, by sunny weather!

1.1.2. Mathemeticaland Kinematical Ownerships of a Watch

The watch was elaborated for improving and systematizing the sun-dial.
Besides the sun -independent work, watch usually possesses several hands what vectorialized the single movable shadow of the Sun-dial
Composition of a Watch: Dial and Hands

1.2. USEFULNESS of the DIAL: (Fig2)

Because its circular figure equipped with graduations it constitutes a well matched basis for every periodical or bounded phenomenon. We must become convinced of the equivalence between periodicity and bounded range of phenomena about their mathematical treatments.
The circular distribution of every " periodisable" signal seems always advantageous because we can more easily reach the characteritics of their internal structures .

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1.2.1. Dial is a synoptical configuration to translate the partition of the integer numbers into connection with their residues from the division by 12 or on a further way with their residues by any other integer number. The Dial is well adaptable for this topic because the Modulo (q) partitions are always constituted of a bounded number of components;
indeed: $\{0 \leq N(\text{ components}) \leq q-1\}$modulo(q).

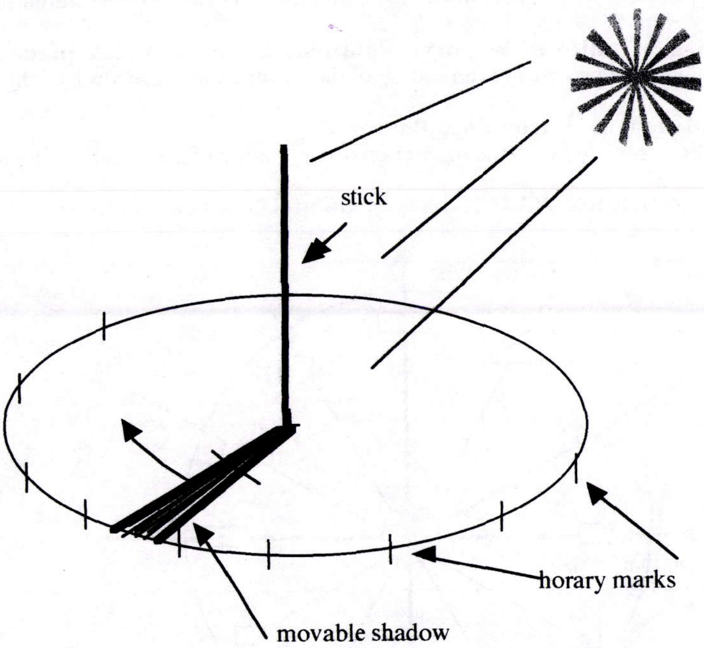

Fig1 Sun-Dial: first rudimentary natural phasor

1.2.2. Dial can also depict the complex plane what is obvious for the polar form of the complex numbers. Real axis is situated on the orientation (**6--->12**).Imaginary axis is situated on the orientation (**3--->9**).
Consequently Dial is the best didactical tool for the localization of the N (Nth) roots of the unit

1.2.3. Dial's'graduation acts like a discretisation operator of the circle into a dodecagon (= polygon with 12 sides)
This graduation divides the circle into 12 equal parts and consequently the numbers can be considered equivalent to their angular address; ex: 3 --> 3(3π/12)
Different functions of these numbers:
1.2.3.a. azimutal convolution's operators

1.2.3.b.involution's operators of grade 12 -- polygonal numbers what are leading to a powerful presentation of the multidimensional spaces (see quaternions section)

1.3. WORKS of the HANDS (Fig3)

Each hand is accomplishing a rotation at a uniform angular speed and from this fact we can deduce:

1.3.1. Vectorialisation of the Time into hours, minutes and seconds.

It leads to the convenient usual quantification of the time elapsing.

1.3.2. "Time---> Space" Convolution

Indeed the motions of the hands translate the time flow.

1.3.3. Vector of Jacobien's Operators between Time and Motions

This sight is an equivalent way to describe every convolution between both referentials.

1.3.4. Translating of uniform negative Rotations in the complex plane :

e(-jkwt) ; *it is obvious when we are reminding of the complex interpretation of the dial 1.2.2.*

1.3.5. Materialising of Harmonics Phasors.

This logical assertion is related with the inspection of the scaling of the angular velocities of the hands.

To conclude, we underline that WATCH is a very powerful didactical multistage - tool.

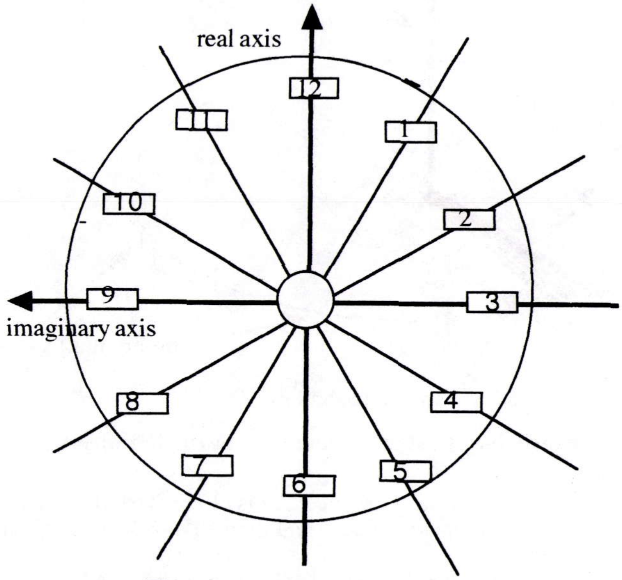

Fig2 Dial like a Complex Plane

2 Connexion between the Watch and the "Discrete Fourier Transformation" (=D.F.T.)

It is obvious that a few characteristics of the (D.F.T.) can be deduced from the careful observation and logical thinking about the work of watches.

For ease's sake we firstly consider a (D.F.T.) of basis =12. related with a particular watch equipped with 12 hands at different speeds. (= successive integer multiples of the slower one.)

These hands are displaying the set of the 12 first harmonics; more accurately

the first one is keeping still on the site "12=0"of the dial and therefore is playing like a sleeping detector for the constant specificities of the transformed signal

the second one is running at the slower speed and therefore is playing like the fundamental one; the followings ones running with speeds which are the integer multiples of the fundamental one translate the serie of the harmonics.

2.1.Adequations between the Elements of the (D.F.T.)-Matrix and the Work of our Special Watch:(Fig3)

2.1.1. The Angular Positions of the 12 Numbers on the dial are pointing out the orientations of the ground vectors of the (D.F.T.) on basis = 12 .
2.1.2. The Discrete Motion of each Hand, flashed in front of each number of the dial,gives the kinematical operation of the components of each correspondent row of the (D.F.T.)--matrix on basis =12 . From this consideration we can always relate each watch's hand with the (D.F.T.)--matrix line of the same frequency in order to show their spectroscopical sweepings over the range of their source vector: (a line of the (D.F.T.)-matrix presents the motion of a harmonic-hand.)

2.1.3. On the other side, the Simutaneous (at the same moment) Observation of the Positions of the Whole of Hands allows us to discover the operational act of a (D.F.T.) -- matrix column. Therefore the operational meaning of each column of the (D.F.T.) -- matrix: an instantaneous sight of the dial's distribution of the set of the hands the address of each column in the (D.F.T.) -- matrix indicates the working time for the column's operators of different frequencies.

Remark: every watch may be considered like a hands- vector or a mechanical show of the (D.F.T.) -- matrix and consequently a didactical display for the Fourier Analysis in 1dimension (time is here a undivided scalar!)

Watch and (D.F.T.) realize both Convolutions
WATCH: (time ---> space) convolution
(D.F.T.): (time ---> frequency) convolution

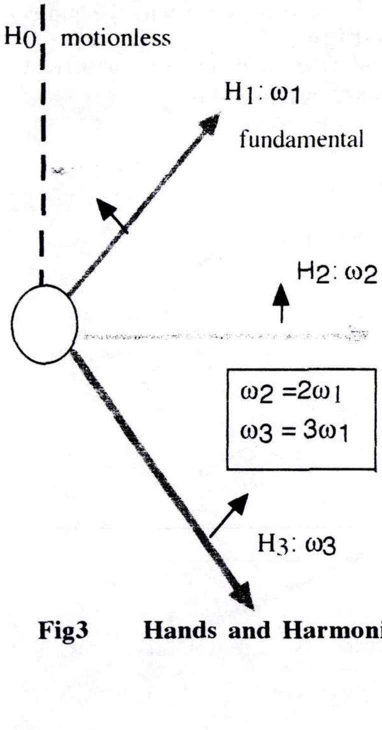

Fig3 Hands and Harmonics

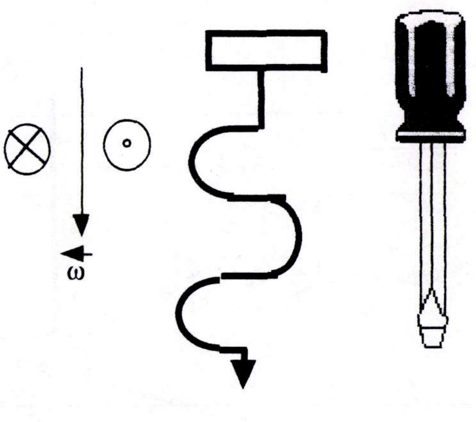

Fig4 Corkscrew and Screwdriver for orienting the POISSON VECTOR

3. Watch, Corkscrew,Screwdriver and "Poisson-Vector"

The corkscrew (or the screwdriver) (**Fig4**) is usually working out helicoîdal motions. When this one is thrusting into the dial, it can pointing out the"Poisson vector" of the move of any hand of the watch. The corkscrew is a strategical tool in mathematics and also in many technical domains because its axial forwarding indicates the orientation of every vectorial product and also of any curl! (when the corkscrew is leading the first vector of the product to the second one by using the shortest way)

4. Quaternions and Three-Ply Watches (Fig5)

The quaternions of Hamilton = (Qt4) are complex numerical operators with 1 real component and 3 imaginary components, which ones can be considered like rotations-axis .
These operators are designed to translate the rotations in a 3dimensional-space.
To the three imaginary components are related the 3 referential axis.
At each axis we may associate the axial forwarding of a corkscrew and the connected watch's dial. From this way we get 3 diversely oriented dials and we technically reach the depiction of threeply- rotations. From this way we are elaborating a well adapted didactical system for depicting the working of (Qt4).
The real component would represent a translation's axis
This is also the key for the easy understanding of the 3dimensional Fourier transformations.
To sum up : for each dimension we need a watch with its hands to show the work of the (D.F.T.) in correlation with the parameter related with this dimension.

5.Extending of the Manyfold of Watches with a view of representing the Rotations in Many-Dimensional Spaces

It seems straightly logical of increasing the number of watches in accordance with the number of parameters to describe the many-dimensional rotations. (**Fig6**)
It is also ilke an extension of (Qt4) to the "(N+1)nions" = super complex numbers with1real component (translations axis) and N imaginary components (N rotations axis)

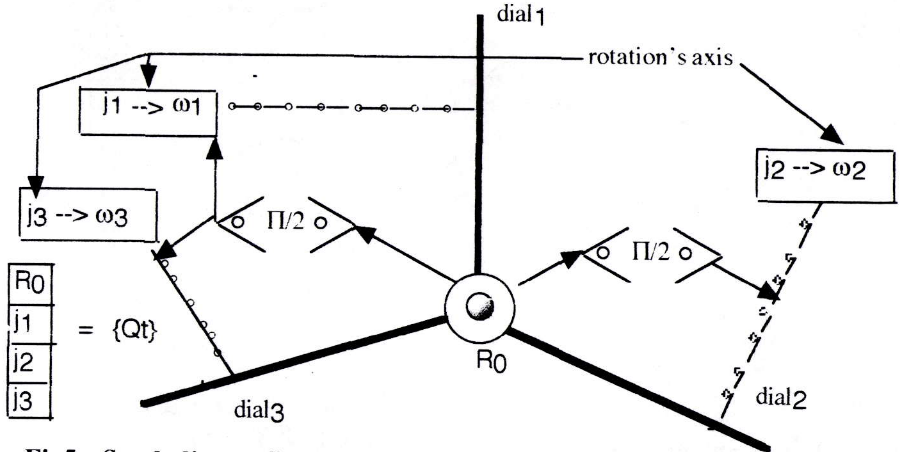

Fig5 Symbolic configuration of Quaternions and associated Dials

6. Use of the Manyfold of Watches for the Visual Description of the Many-Dimensional(D.F.T.)

As a result of deductions from the assertions in 4. & 5. it seems very logical to consider each watch like a"Fourier Analyser" for each dimension. **(Fig6)**
From this view we obtain a watches-vector what is also related with the vectorialisation of time. This last point of view is in accordance with the partition of the macrosystems into a few subsystems of different convolutions- periods.

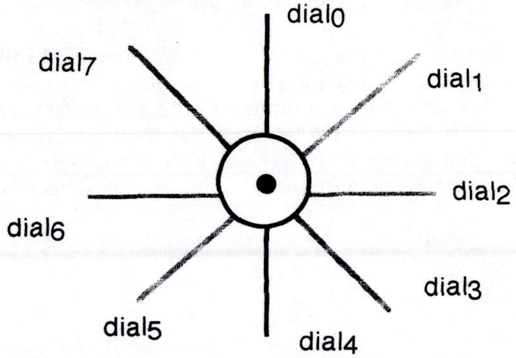

Fig6 Dials vector for multidimensional Fourier transformations
case for 8 dimensions equivalent to a complex number with 8 imaginary components

7. Use of the Manifold of Watches for the Synptical Presentation of the Fast Fourier Transformation (= F.F.T.)

With the view of quickening the automatical Fourier Analysis by computers, the global single transformation is splitted into a sequence of undertransformations in accordance with the partition of the time-addresses of the signal and the frequency-ones of the spectrum. In order to accelerate the computer procedure we divide the total tranformation into a serie of progressive partial undertransformations through mixed transient spaces elaborated by exchanging a time component with a frequency one for each undertransition. We remark that computer is following here an "incursive procedure".
With each undertransition we link a different dimension, what leads to an analogy with the many-dimensional transformations.
This case appears like a vectorialisation of a scalar transformation into a serie of undertransitions which are the components of this new elaborated convolutions- vector;
We also need a manyfold of watches of the same quantitie as the numbers of the partial undertransitions.
(F.F.T) can be presented in the same frame as the one of the many-dimensional (D.F.T.)

8. WAVELETS for a better FLEXIBLE SHAPES - RECORDER

The Wavalets are very short periodical waves equipped with central symmetry,(related to both vertical and horizontal axis) **(Fig 7)**

8.1. Description of the Wavelet Procedure

From an initial wavelet (= mother, primary or source wavelet) we can deduce a family of several daughters or subsequent wavelets (keeping the same profile but oscillating with variable adapted frequencies) in order to investigate the local shape of every signal with flexibility for

saving up large useless frequencies bands. Wavelets behave like systematcally adapting forms-sounds. **(Fig8)**
We have to underline about this shape-detection-procedure that we send, to each fraction of the time-basis of the detected signal, a daughter wavelet which is oscillating in an optimized accordance with the behaviour of this portion of signal. (= adjusting the frequency in correlation with their working - time) As a result we can conclude that only a single wavelet is instantaneous working.

8.2. Operational Translation of the Wavelets

Each wavelet-family belong to a 2-dimensional operational space **(Fig9)** with a time-translations axis and a frequencies (= scaling effect) axis.
From this way we are reminding of the occurencing time of each frequency.
In the watches frame **(Fig10)** we have to link each daughter wavelet with a different hand and subsequently it produces that during every working time of a daughter one we have only to put *in move the hand of the same average frequency. Thus we let successively separately run the* hands of the watch.

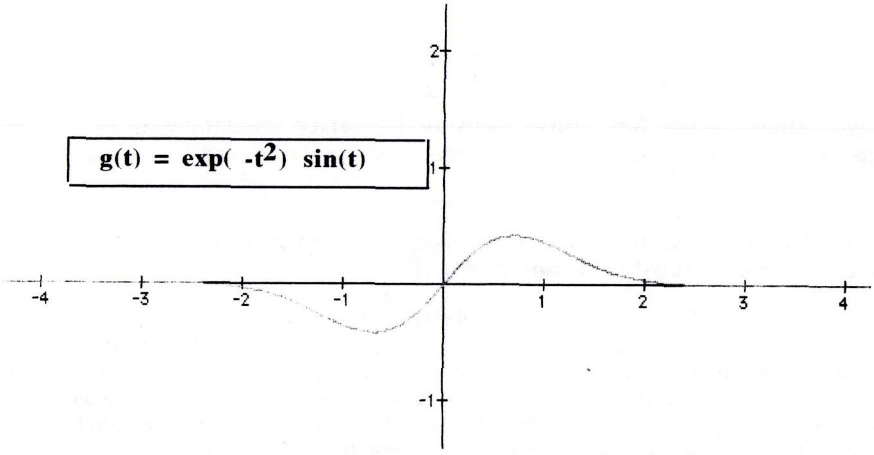

Fig7 Morphology of Wavelet

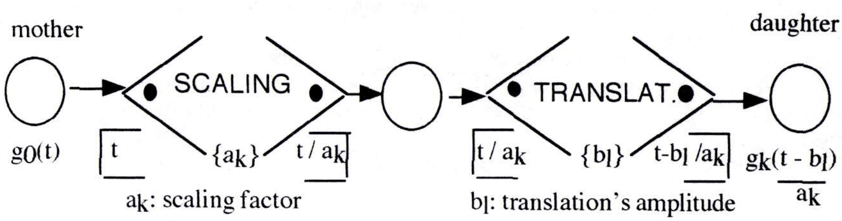

Fig8 Deduction's procedure for the Daughter Wavelets

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9.Use of a Manifold of Dialsin order to Simulate the Many Dimensions Wavelets- Procedure

This extension of the applied method in 8. follows the similar evolution which was already used for the transition from 3. through 4. and 5. to 6. It means that we need a serie of several watches with their axis along the different dimensional directions.
For each dimension the synoptical treatment of the wavelets process implies the succession of the separated runs of the hands over the dial in connection with this particular dimension.

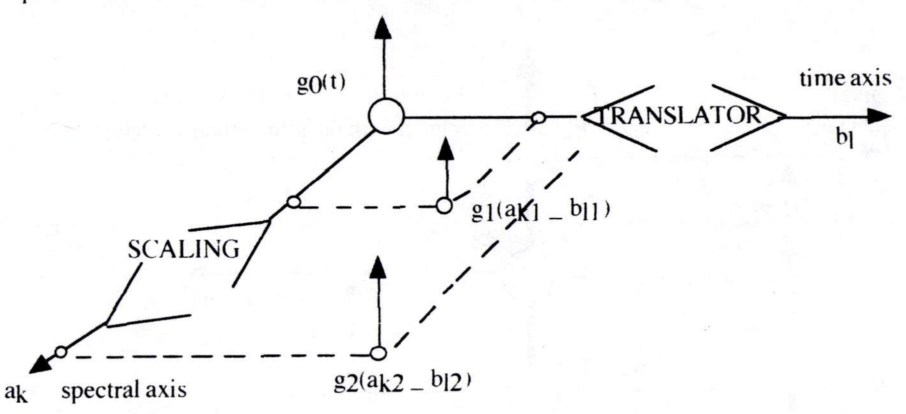

Fig9 Operational space of the Wavelets

Remark: it seems that these wavelets carry fractal ownerships on their ability of frequency scaling related to their translations. Indeed they have the required inward flexible variability for realizing the systematcal building of every elaborate shape by repeating the same basical sheme. (= Fractal generation at the level of the procedure).

10. Conclusions

10.1. Power of the Methobology

With an ordinary usual tool like a watch we have drastically and convivialy explained and illustrated a serie of dynamical algorithms for the signals analysis and the shapes- elaboration. We have followed a nice easy teaching trajectory from the usual rotations in the complex space as far as the fractal approach, in connection with the watches, their dials and hands. We have simultaneously operated a few convolutions from time into space because the hands are acting like the wings of a time-mill which are grinding time into rotations.
With this thinking way we have gained a right understanding insight about the kinematical structures of (D.F.T.) -- (F.F.T.) -- (Wavelets) and their intrinsic ground- correlations .

10.2. Advantages of the Illustrating Processes

These teaching methods are enjoyed with the informations compactness of the pictures. A good picture is able to sum up a whole chapter or sometimes a book! They offer a promotional way to understand and to control the abstracted algorithms. Under this last argument is lying the valuable practical performance of these configurative methods. A finally advice: " **to improve the accuracy and the correctness of your ideas, don't forget to search the right mated picture! "**

10.3. Amticipatory Sight:

This point of view is activated because this procedure is elaborated for increasing and improving the storage memory of autonomous systems) = the readers of this short modest communication). We have linked various methods of signals analysis with the obvious and convivial working of watches. Therefore we hope that this new view contributes to the improvement of our learning processes.

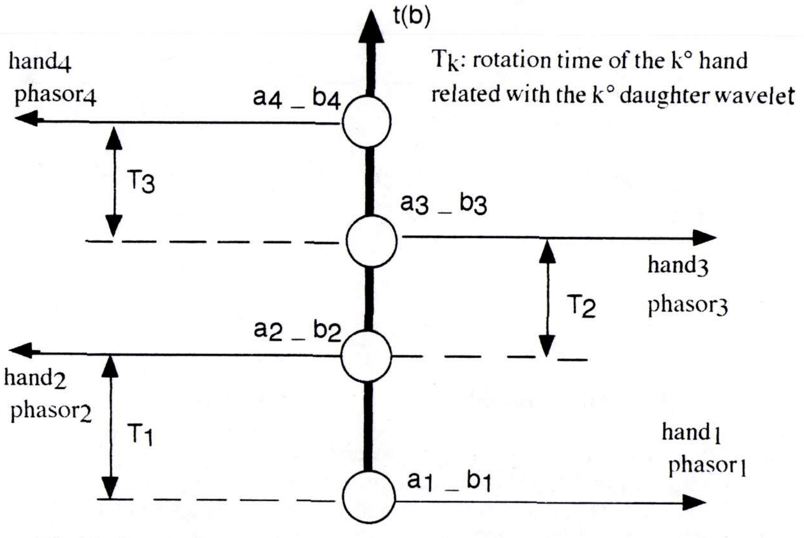

Fig10 Symbolic presentation of the Wavelet activation

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