Unbroken Wholeness in Nonlinear Processes

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Abstract

In order to understand the complex behaviors in megaloscopic systems, it is necessary to analyse the detailed mechanisms of non-separability in nonlinear processes. The essential roles of nonlinearity are pursued in the local-global linkage from deterministic points of view carrying out with nonlinear dynamical models; onset of subliminal threshold, self-organization of soft interface, and scaling relation in barrier-free cascade. It is emphasized that the non-separability and fluctuations in soft interfaces are closely associated with emergent behaviors of megaloscopic systems and that the law of unbroken wholeness is described by a relation of relations.

Keywords: Local-Global Linkage, Subliminal Threshold, Soft Interface, Allometry, Many-to-Many Causality

1. Introduction

In nonlinear processes, it is generically impossible to decompose many degrees of freedom into mutually independent variables. Each variable or process couples with others in non-separable manners. As well known in many recent studies of nonlinear dynamics, a number of interesting phenomena are generated owing to the non-separability of coupled nonlinear processes; chaos, turbulence, resonance, bifurcation, synchronization, etc.. In the megaloscopic system which includes a lot of nonlinear processes, it is exceptionally important to analyse the essential mechanisms of the non-separability in nonlinear processes. Nonlinearity often brings about two conflicting effects such as creation and destruction in megalo-systems. In other words, nonlinear processes endow the megalo-system with remarkable coherence as well as disorder. In both cases, however, nonlinearity seems to play some significant roles in integrating all the parts of a megalo-system. What are the essential mechanisms of nonlinearities in megalo-systems? One of the most important points is to develop new ideas and methods necessary for the collective descriptions of the local-global linkage in megalo-systems. The purpose of the present paper is to discuss fundamental features in megalo-systems from the viewpoints of the local-global linkage generated by nonlinear processes.

In this paper, we will study basic problems concerning the local-global linkage or the unbroken wholeness of megalo-systems carrying out with three dynamical-theoretical subjects.

The first point is to show that subliminal thresholds are universally generated in megalo-systems. The subliminal threshold is quite different from other critical points which have been studied so far in relation to bifurcations and phase transitions. The universal structures near the subliminal level, which intervenes between observables and non-observables, are analysed by use of a simple dynamical model called the modified Bernoulli map. The studies of subliminal levels will elucidate the interrelation between chance and necessity, and that it will be stressed that the unbroken wholeness in megalo-systems should be approached by taking a lot of non-observable processes into account.

The second point is to show the significance of the liquid-like soft interface by use of a theoretical model for the cellular slime mold *Dictyostelium discoideum*, where the coupled processes of reaction, diffusion, and motility are studied to describe the local-global linkage of clustering cells (pseudoplasmodium). How does chemical oscillation control the clustering process? What kind of coherent motions can be created in a quasi-cluster? These problems will be analysed by computer simulations of our model. The results seem to be full of suggestions; the phase information in each cellular rhythm plays an essential role in the onset of coherent movements, and that remarkable positional information is associated with the soft interface which surrounds the quasi-cluster. The unbroken wholeness of the quasi-cluster acquires a variety of possibilities by self-organizing the soft interface. It is surmised that the $1/f$ fluctuations, which are universally created in the soft interface, guarantee the high stability and adaptability of the quasi-cluster. Furthermore, the positional information in the soft interface seems to play the same role not only in the onset of coherent motions but also in the differentiation processes from pseudoplasmodium to the fruiting body in the life cycle of *D. discoideum*.

The third point is to show the onset mechanism of global scaling relations in megalo-systems, where we consider the open system for polymerization processes in barrier-free cascade. How is the time-scale self-organized in megalo-systems? How does the nonlinear process control the entropy production in complex chemical reactions? It will be shown that both the time-scale and the bulk-quantities of the megalo-system are self-regulated in the barrier-free cascade to create scaling relations such as "the allometry law" and the Zipf's law. The scaling relation obtained here seems to originate from the hierarchical reaction networks among polymer ensembles. Our results do not show the universality of the scaling indices, but do elucidate a typical mechanism which creates unbroken wholeness in megalo-systems.

Unbroken wholeness is a central concept in studying the local-global linkage in megalo-systems. The viewpoints discussed in the present paper are not enough to understand the wholeness of nonlinearity, but they inspire the internalist's stance toward the theoretical exploration of complex behaviors in megalo-systems. The law of the non-observables, the positional information stored in soft interfaces, and the scaling relations induced by the barrier-free cascade all stem out from the remarkable rules of non-separability in nonlinear processes.

In the last part of the present paper, we discuss a theoretical or axiomatical framework, in which the non-separability of processes is postulated as a first principle. The essence of unbroken wholeness seems to be unreachable from the reductionist approach in modern science, so it is pointed out that the non-separability completely alters the standing of the causality in science, and that the many-to-many causality should take the place of the traditional one-to-one causality in nonlinear megalo-systems.

2. Onset of Subliminal Thresholds

There are two kinds of interfaces universally in megaloscopic systems. One is the material layers between several phases, the other is the threshold in the sense of phenomenology. Both interfaces are always fluctuating and changing under the environment generated in the system itself. Here we consider the interface problems as thresholds, which discriminate the global characteristics into different classes, just like phase transitions and/or bifurcations. Traditional approaches, especially in statistical mechanics which deals with huge ensembles of micro-scopic elements, are more or less based on the decoupling techniques to get the phenomenological descriptions of renormalized fluctuations. However, those decoupling ideas are not adequate in dealing with the complex phenomenon near the thresholds of megalo-systems, which are often composed of huge ensembles of macro-scopic local elements. Local elements themselves are not always passive but active in megalo-systems.

2.1. Observables and Non-observables

There exist a great many non-observable paths as well as observable ones in non-linear processes. From the ergodic-theoretical considerations, two kinds of measures describing each path are universally embedded in a dynamical system, and the totality of such measures defines the dynamial law of the system under consideration; observable paths usually obey the dominant measure which is absolutely continuous with respect to the Lebesgue measure, but non-observable ones belong to the singular class of minor measures.

In comparison with the case of statistical mechanics, the observable phenomenological paths are determined by averaged quantities in a given ensemble, since the micro-scopic elements are more or less stochastic and uncontrollable due to the essential difference in their time scales. But in megalo-systems, this type of coarse-graining is not justified, because the phenomenology in megaloscopy must be understood in terms of macro-scopic behaviors including macro-fluctuations. In other words, both observables and non-observables have the similar scales not only in time but also in space.

The measure-theoretical structure of megalo-systems is always changing by the long time fluctuations of the global state, so that the dominant measure is also changing and alternating. For instance, the most dominant measure changes into the second or third one, and vice versa. This kind of alternation in measure-theoretical order happens frequently in the course of time, and as a result the behaviors of megalo-systems acquire remarkable diversity and complexiy. The most dominant measure describes the observable path, which will be actually realized in the course of time, but the non-observable ones are all hidden behind reality though they have had the possibility to happen with almost the same probability as reality.

The formation of the subliminal level is one of the new features in self-organization processes. A subliminal level sorts out one real event from many other possible events which are all embedded in the law of time evolution; necessity is one realized path selected uniquely from all the possible paths. Therefore the subliminal threshold could be understood as a criterion self-organized in megalo-systems. On the other hand, the remaining paths which have had the possibility to be realized but did not appear, are the latent reality or the chance which lurks behind the reality; strictly speaking, the chance is the realization of the non-observables beneath the subliminal level. To summarize, the threshold imposed by subliminal level not only distinguishes observables from non-observables but also decouples the necessity from chances.

2.2. Non-stationary Chaos Beneath Subliminal Threshold

The onset of the subliminal level can be described by a simple dynamical system which genererates non-stationary chaos. Let us consider the modified Bernoulli map defined in unit interval $x \in [0,1]$.

$$x_{n+1} = \begin{cases} x_n + 2^{B-1}(1-2\epsilon)x_n^B + \epsilon & (0 \le x_n \le 1/2) \\ x_n - 2^{B-1}(1-2\epsilon)(1-x_n)^B - \epsilon & (1/2 < x_n \le 1) \end{cases} \quad (1)$$

Where x_n is the real number at discrete time n. The paramter B changes for $1 \le B < \infty$, and the limit of $\epsilon \to 0$ is studied in what follows. The case for $B = 1$ is the Bernoulli shift well known in ergodic theories.

Here we quickly review the previous results (Aizawa, 1989; Tanaka and Aizawa, 1993), where the time courses generated by modified Bernoulli map are strictly analysed. In the region for $B < 2$, the time series is stationary and ergodic in the ordinary sense; in the Gaussian regime ($1 \le B \le \frac{3}{2}$), the mean value as well as the variance are well defined, but in the non-Gaussian regime ($\frac{3}{2} \le B < 2$), the variance diverges though the mean value converges. Furthermore, the law of large number as well as the law of small number hold in the stationary regime where the large deviation theory is successfully applied. But, on the other hand, in the non-stationary regime ($B \ge 2$), both laws of large number and of small number completely break down. That is to say, there is no dominant measure in the non-stationary regime, but the most dominant measure is uniquely determined in the stationary regime. The transition from observables to non-observables occurs at the threshold point $B = 2$, although the intrinsic non-observability (at $\epsilon = 0$) is suppressed in the asymptotic limit ($\epsilon \to 0$). The observable/non-observable transition can be clearly described by the power spectrum function $S(f) \sim f^{-\nu}$ $(\nu = (2B-3)/(B-1))$, i.e., the $1/f$ spectrum is exactly obtained at the subliminal level.

The observable region ($B < 2$) is characterized by the Kolmogorov-Sinai entropy (a) and the Allan variance's index (b), but the non-observable region ($B > 2$) must be characterized by the Kusinirenko's A-entropy (a') and the stable law's index (b'). These four characteristics completely describe the global aspects in the onset of subliminal threshold (see Fig. 1).

2.3. Bergson Diagram -complexity and chance-

Non-stationary chaos is not an exceptional singular phenomenon in large nonlinear systems, e.g., non-stationary wandering motions are generically created in non-hyperbolic Hamiltonian dynamics and also in dissipative dynamics (Aizawa, 1995; Mannerville, 1990). Therefore, the onset of subliminal threshold is also universal in megalo-systems. The results mentioned above will be summarized in Fig. 1. It is the essential difference between observable and non-observable regimes that the probablistic description does not hold in the non-observable regime, and that all motions in the non-observable case appear as transient variations or remittent processes. For instance, a non-observable path comes up accidentally into the observable region beyond the subliminal level, but it passes off again into the non-observable region. It is shown in Fig. 1, such paths always pass by the subliminal level, so that the $1/f$ spectrum is universally observed in the transition between chance and necessity. These kinds of dynamical phenomena are often called "self-organized criticality". Furthermore, Fig. 1 seems to explain the dynamical mechanism of the creative roles of chance that was discussed by H. L. Bergson (Bergson, 1907).

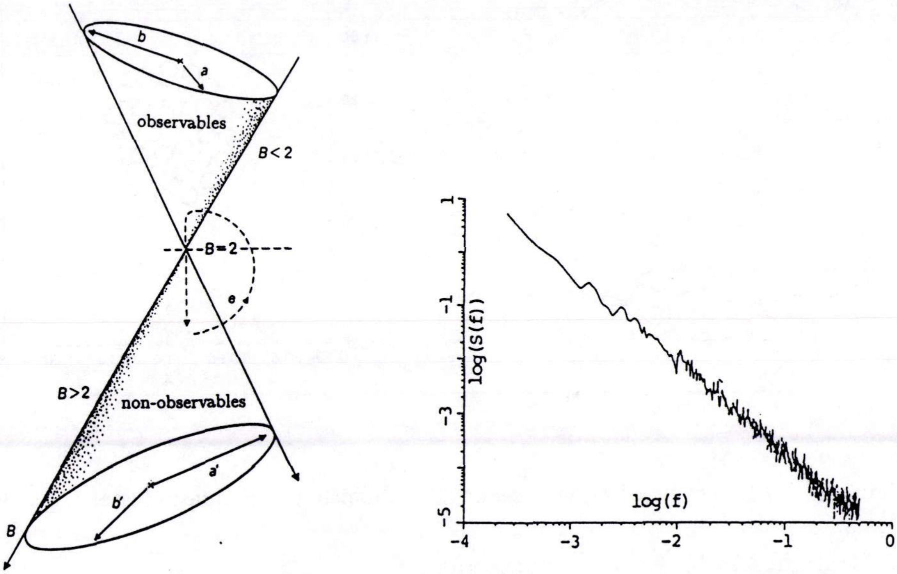

Figure 1: (a) Universal Diagram near Subliminal Threshold. (b) $f^{-\nu}$ spectrum in non-stationary regime ($B = 3$).

3. Self-organization of Soft Interface

Next problems are to discuss the roles of the material boundary layers which are separating many different local phases. The formation of such boundaries contributes to the global functions of megalo-systems in two different manners; one is the unification of the total system, and another is the functional differentitation of each local elements.

3.1. Pseudoplasmodium of *Dictyostelium discoideum*

Here we consider the clustering motion of the cellular slimemold *Dictyostelium discoideum*, where the self-organization of soft interfaces plays significant roles not only in the differentiation processes of each cell but also in the global behaviors of the quasi-cluster. Fig. 2-(a) shows the life cycle of *D. discoideum*. In a less nutritious environment, each amoeba cell emits the attractant (cAMP) which induces motions of other cells, and all cells begin to aggregate into a big migratory cluster called pseudoplasmodium (or slug), of which motion is rather more coherent than that of a single amoeba cell. After sometime each cell in the slug proceeds with its own differentiation process toward the fruiting body, where some of them become stalk and others spores. In the life cycle mentioned above, there are many nonlinear processes in the inter-cellular levels as well as in the intra-cellular ones, but the present paper only studies the clustering motion based on the chemotaxis of cells. Thus our minimal model should include three basic processes: chemical oscillation, diffusion, and motility.

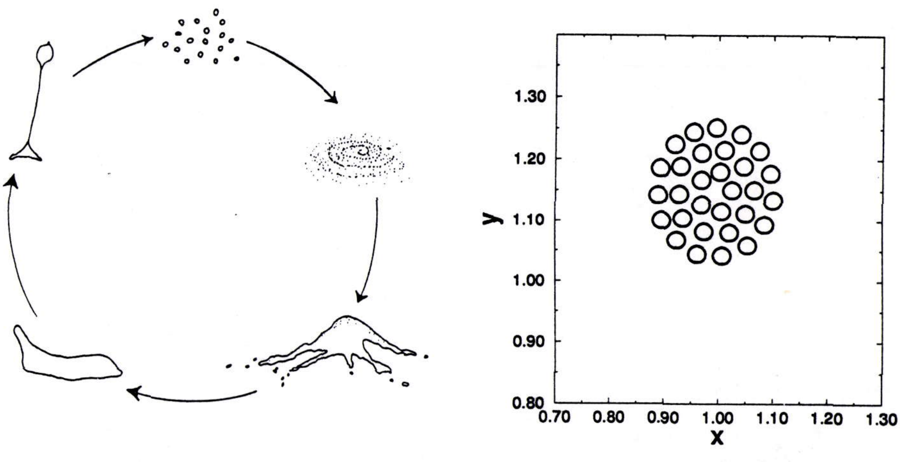

Figure 2: (a) Life cycle of *D. discoideum.* (b) Snapshot of a Quasi-cluster ($N = 30$).

3.2. Minimal Model for Cell-clustering

The dynamics of cell movement have been theoretically studied by Aizawa-Kohyama model following,

$$\frac{\mathrm{d}W_\mathrm{i}(t)}{\mathrm{d}t} = i\omega_\mathrm{i}W_\mathrm{i}(t) + (1-|W_\mathrm{i}(t)|^2)W_\mathrm{i}(t) + \frac{\epsilon}{\mathrm{N}-1}\sum_{\mathrm{n}\neq\mathrm{i}}^{\mathrm{N}}\int_{t-\Delta t}^{t}\Phi(|\boldsymbol{r}_\mathrm{i}(t)-\boldsymbol{r}_\mathrm{n}(\tau)|,t-\tau)W_\mathrm{n}(\tau)e^{i\delta_1}\mathrm{d}\tau \tag{2}$$

$$\frac{\mathrm{d}\boldsymbol{r}_\mathrm{i}}{\mathrm{d}t} = \frac{\alpha}{\mathrm{N}-1}\nabla\Big(\sum_{\mathrm{n}\neq\mathrm{i}}^{\mathrm{N}}\mathrm{Re}\int_{t-\Delta t}^{t}\Phi(|\boldsymbol{r}(t)-\boldsymbol{r}_\mathrm{n}(\tau)|,t-\tau)(W_\mathrm{n}(\tau)+C)e^{i\delta_2}\mathrm{d}\tau\Big)\Big|_{\boldsymbol{r}=\boldsymbol{r}_\mathrm{i}} \tag{3}$$

$$\Phi(r,t-\tau) = \frac{1}{\sqrt{4\pi D(t-\tau)^3}}\sqrt{rr_0}\exp\Big\{-\frac{r^2}{4D(t-\tau)}\Big\} \tag{4}$$

where we describe the state of the i_{th}-cell by two variables W_i and $\boldsymbol{r}_\mathrm{i}$. The complex variable W_i stands for the concentration of a certain kind of chemical substances which are essentially determined by the metabolic process in the cell. For the sake of simplicity, we assume that the real part of W_i (Re W_i) *corresponds to the concentration of cAMP* in what follows. $\boldsymbol{r}_\mathrm{i}$ indicates the position of the i_{th}-cell. The cells obey the gradient field of the attractant (cAMP). In order to describe the oscillatory emission of cAMP, we adopt the TDGL type equations which are the universal normal form near the super critical Hopf bifurcation. The last term of Eq.(2) expresses the interaction among cells, where the interaction kernel Φ is theoretically determined by solving the 2-dimensional linear diffusion equation for cAMP. Δt is the memory time due to cell membrane, D is the diffusion constant and r_0 is the effective radius of each cell.

In Eqs.(2)$\sim$(4) there are two types of parameters; intensity parameters (C,α and ϵ), and phase parameters ($\delta_{1,2}$ and Δt) that control the inter-cellular communications. The phase parameters are especially important in unifying the phase information of the cells whose natural frequencies are denoted by ω_i. In order to describe the total life cycle of *D. discoideum*, many other nonlinear processes such as the detection of cAMP, cell contact, cell sorting, synergism, etc., must be taken into consideration. Here, however, we only discuss the dynamical aspects of the quasi-cluster (pseudoplasmodium) based on the minimal model of Eqs. (2)~(4).

Fig. 2-(b) shows a snap shot pattern of the quasi-cluster in the late stage, where in the early stage all cells are dispersed very sparsely over a large space. In this clustering process, the chemical oscillations W_i in each cell begins to synchronize with others, and a phase-locking state is gradually established among a large part of the clustering cells. The final stage of the synchronization depends on the parameters of our model and also the distribution of natural frequencies ω_i.

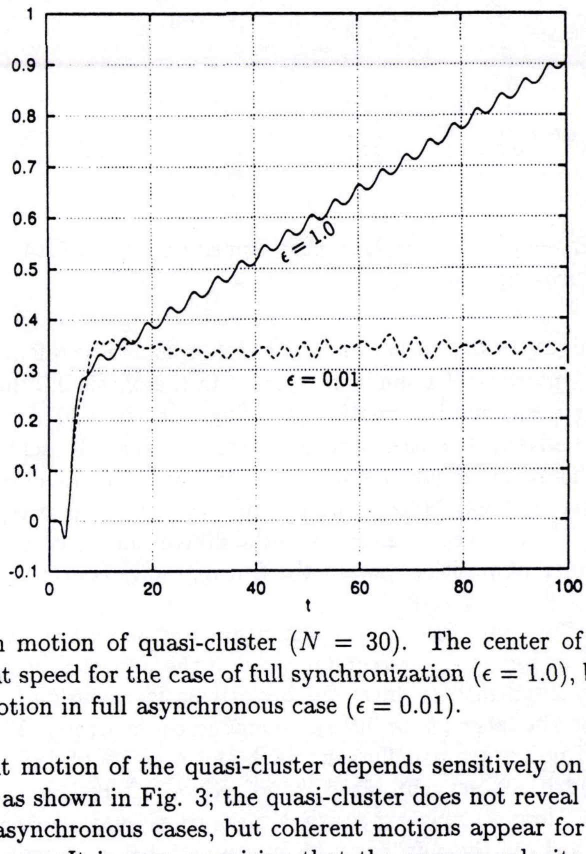

Figure 3: Mean motion of quasi-cluster ($N = 30$). The center of gravity moves in a alomost constant speed for the case of full synchronization ($\epsilon = 1.0$), but it does not show any coherent motion in full asynchronous case ($\epsilon = 0.01$).

The coherent motion of the quasi-cluster depends sensitively on the number of synchronizing cells as shown in Fig. 3; the quasi-cluster does not reveal any regular motions for completely asynchronous cases, but coherent motions appear for the full (or partial) synchronizing cases. It is very surprising that the average velocity of the quasi-cluster takes a maximum value at the critical point between synchronization and asynchronization (see Fig. 4).

3.3. Emergence and Positional Information Controlled by Soft Interface

The pseudoplasmodium of *D. discoideum* are surmised to be surrounded by soft inter-

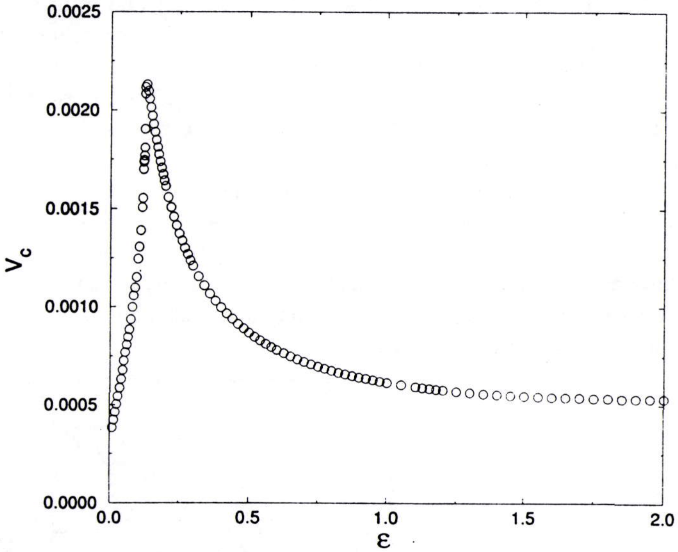

Figure 4: The mean velocity v_c reveals a sharp transition at $\epsilon_c \cong 0.14$ in the case $N = 2$. Rhythms do not synchronize for $\epsilon < \epsilon_c$.

faces composed of high motility cells. One of the most remarkable differences between the central core and the interface of a slug is recognized in the phasic information of each cell which generates the positional information in a slug. The cellular rhythms in the slug-head have relatively advanced phases, but in the slug-tail the cell oscillation exhibits the phase-retardation. These are quite consistent with the experimental observations (Weijer, McDonald and Durston, 1984). The positional information act as triggers to open some new chemical networks for morphogenesis, and the differentiation of *D. discoideum* is accelerated in each inherent position toward the fruiting body (Early, Abe and Williams, 1995).

The formation of soft interfaces is also important for the unification of the total system. As was shown in Fig. 4 the maximum speed of the pseudoplasmodium is obtained near the edge of synchronization, where the big driving forces to push the quasi-cluster are generated due to the large phase difference among contact cells. Then the long time fluctuations like 1/f noises are usually generated, since the mutual phases are intermittently unlocked, and pseudo-synchronized states recover after a long itinerancy (Sawai and Aizawa, 1997). Here we may expect that the intrinsic motive force that integrates the megalo-system lurks in the soft interfaces of the peripheral region.

4. Universality of Scaling Relation

Unbroken wholeness is a central concept in megaloscopic systems, where the globality and the locality are simultaneously self-organized. The traditional reductionist approaches are based on the reality of local elements, so they must be extended. The individual locality should be self-generated in every part of the whole system and differentiates toward a new stage of the local-global linkage. Such internalist's viewpoints were discussed by D. Bohm in relation to the measurement theory in quantum mechanics of which dynamics is based on the non-locality of wave functions (Bohm, 1976). Recently, the same kind of problems are also pursued in the local-global linkage of morphogenetic processes; for instance in reaction-diffusion systems the onset of the locality is discussed in terms of Turing type instability, and the differentiation is subordinated to micro-scopic reaction networks ignitioned by agonistic and antagonistic molecules (Meinhardt, 1995).

It is evident that nonlinear processes guarantee the establishment of a variety of local phases, since the spatial and temporal scales are associated with the process beforehand, for examples, diffusion lengths, relaxation times, and dispersion relations for unstable modes. To go beyond these trivial well-defined scales, the unbroken wholeness should be explored in the local-global linkage with non-trivial scales. To that end, we consider the origin of scaling laws which are often observed in megaloscopic systems.

4.1. Polymerization Process in Barrier-free Cascade

Consider an open reactor with constant volume, where monomers are injected from the outside at a constant rate and polymerization processes occur. There are many polymers created by the coagulation and collapsation processes, but let us suppose that polymers of which length n is longer than S $(n \geq S)$ are immediately removed from the reactor. Namely, the parameter S stands for the maximum length (or weight) of a polymer in the open system under consideration. The birth and death processes of polymers obey the following Smoluchowski type equations,

$$\begin{aligned}
\frac{dx_1}{dt} &= 0 \\
\frac{dx_2}{dt} &= ax_1^2 - \{b + 1 + \sum_{i=1}^{S} kx_i + K_S(\sum_{i=1}^{S} ix_i + 2x_2')\}x_2 + x_2^2 x_2' \\
\frac{dx_2'}{dt} &= bx_2 - x_2^2 x_2' - 2K_S x_2 x_2' \qquad (5) \\
\frac{dx_4}{dt} &= \frac{1}{2}\sum_{i+j=4} kx_i x_j - kx_4 \sum_{i}^{S} x_i + 2K_S x_2 x_2' + 2K_S x_2^2 - 4K_S x_4 x_2 \\
\frac{dx_n}{dt} &= \frac{1}{2}\sum_{i+j=n} kx_i x_j - kx_n \sum_{i=1}^{S} x_i + (n-2)K_S x_{n-2} x_2 - nK_S x_n x_2 \\
& \qquad (n = 3, 5 \leq n \leq S)
\end{aligned}$$

Here x_i denotes the number of the i-polymer in the reactor. As we are interested in the self-organization of the time scale, one simple autocatalytic process (so-called Brusselator) is assumed in the dimerization kinetics (x_2, x_2'), and the kinetic coefficients are all fixed constant in this paper; $K_s = 0.001$ and $a = 2.0$.

In the stationary state, variables x_i reveal periodic oscillations with the same period T, that is to say, the attractor in our model is a S-dimensional limit cycle. When we change the details of kinetic processes, it can be surmised that the structure of the attractor will be deformed largely. However, it will be shown next that the global scaling laws hold invariantly in spite of large deformation of the attractor (Aizawa and Suzuki, 1997).

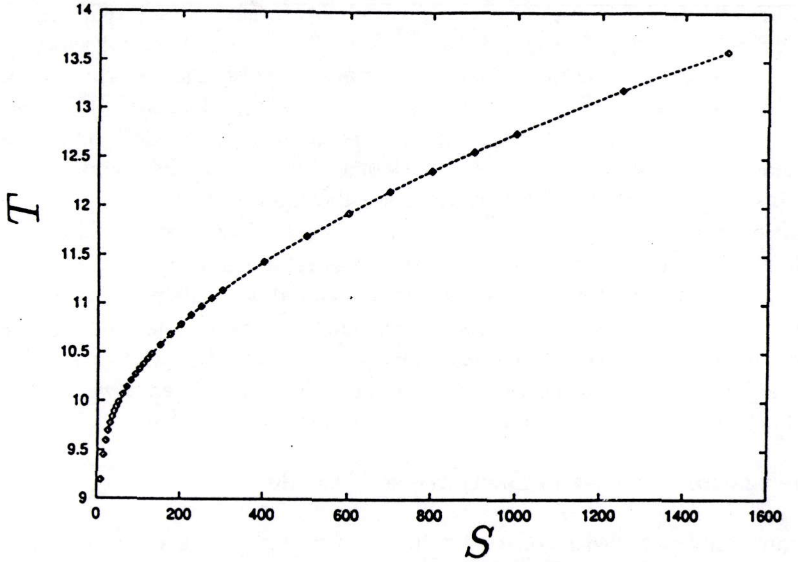

Figure 5: Scaling law of time.

4.2. Allometry Law and Zipf's Law

In our simulations, the parameter S is increased up to several thousands, since the main concerns of the present paper is to study the megaloscopic features of huge systems; mass spectrum $m_n = nx_n$, total mass $M = \sum_{n \leq S} m_n$, efflux $J_s = K_s \sum_{i+j>S}(i+j)x_i x_j$, and entropy production σ.

The entropy production σ (for a period T) is determined by use of the ideal gas chemical potential, i.e., $\sigma/T = \sum_\rho A_\rho V_\rho$, where A_ρ and V_ρ are the affinity and the reaction velocity of the ρ-reaction respectively. First, we calculate those quantities for various values of the parameter S, and then the mutual relations are obtained by eliminating S, e.g., $T - T_0 \sim (S - S_0)^\tau$, $M - M_0 \sim (S - S_0)^\mu$, then $T - T_0 \sim (M - M_0)^{\tau/\mu}$, where these scalings are well confirmed by fitting the adjustable values: S_0, T_0, M_0, etc.. Figs. 5 and 6 show the allometry of our model; the numerical plots and the fitting curves in the case of $b = 7$ give,

$$T - T_0 \sim (M - M_0)^\mu \quad \alpha \cong 0.58 \tag{6}$$

$$\sigma - \sigma_0 \sim (M - M_0)^\beta \quad \beta \cong 0.512 \tag{7}$$

$$J - J_0 \sim (M - M_0)^\gamma \quad \gamma \cong 0.518 \tag{8}$$

The scaling properties obtained here do not sensitively depend on the values of parameters, though the numerical values of the indices α,β, and γ change in a systematic way. It can be surmised that the scaling laws are stably maintained under a certain loose condition. Indeed, the same kind of scaling laws have been obtained even when we adopt the chaotic autocatalytic process such as the Williamowski-Rössler reaction instead of the Brusselator reaction. The essential mechanism leading to the scaling laws might be found in the generic features of barrier-free cascades, one model of which is proposed in the present paper.

The microscopic theory may also be possible for the barrier-free cascade if we describe the Brownian motion for one molecule. Every molecule wanders from one polymer to another due to the coagulation and the dissociation processes. These molecular processes look like fractal Brownian motions in the hierarchical structure composed of a number of polymer ensembles. The detailed analysis of these molecular pictures will be studied in a forthcoming paper, but here we will see an evidence which reveals the hierarchical structure in the barrier-free cascade.

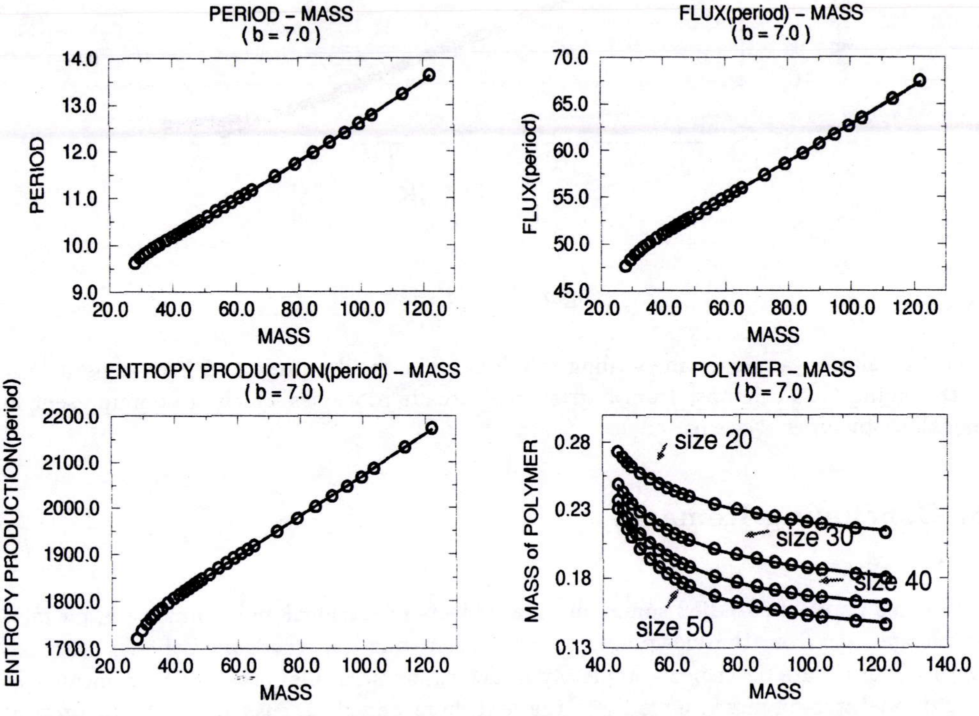

Figure 6: Allometry laws and size-distribution.

Fig. 7 shows the distribution of the polymer-size in our model, which is illustrated in the rank-size relation. The size m_i is measure by the total weight of i-polymers, and the rank n_i is the integer which shows the order of the abundance measured by the weight m_i. The most abundant polymer takes rank 1. By eliminating i in the m_i and n_i, the rank size relation is obtained as is shown in Fig. 7. When the paramter S becomes large, the Zipf's law realizes,

$$size \propto (rank)^{\xi} \tag{9}$$

where the Zipf's index $\xi = -0.45$ for $S = 1500$ and $b = 7$.

The allometry as well as the Zipf's law have been extensively reported in many megaloscopic systems (Huxley, 1932; Zipf, 1949; Schmidt-Nielsen, 1984), but so far there is no clear-cut explanation for the interrelation between them. As seen in Eqs. (6)~(8), they

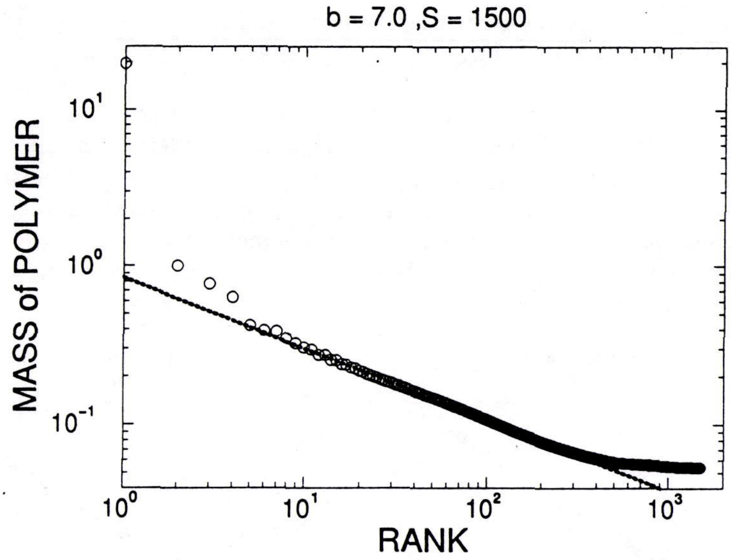

Figure 7: Zipf's law

reveal a relation among some scaling relations. The results presented here suggest that both scaling laws originate from barrier-free cascade processes which are omnipresent in megaloscopic open systems.

5. Concluding Remarks

In this paper we studied some universal aspects of the local-global linkage in megaloscopic systems from the viewpoints of nonlinear dynamics, where the coherence as well as the disorder are described completely in deterministic frameworks. The significance of each model was discussed already in the text, here we will discuss about the theoretical frameworks in which the unbroken wholeness must be studied.

We have pointed out that fluctuations are also deterministically generated in soft interfaces, and that soft interfaces play a significant role in self-regulating the unbroken wholeness of megalo-systems. Though the extraordinary functions of soft interfaces have not yet been completely analysed in the present paper, it was shown that the $1/f$ fluctuation characterizes a remarkable feature of soft interfaces. Generally speaking, $1/f$ fluctuations are associated with two emergent behaviors in megalo-systems; one is the preservation of long time memories, and the other is the creation of coherent processes.

$1/f$ fluctuations are analogous to biological evolution in some sense, i.e., the singular point (at the ghost frequency $f = 0$) of the spectral density $S(f) \sim f^{-\nu}$ implies the transitional growth of reference states (Matsumoto and Aizawa, 1997), where time averaged quantities are not fixed, but reveal non-stationary evolution in the course of time. In other words, though $1/f$ fluctuation is stochastic, it can drive the process toward a new regime of local-global linkage.

The case of $\nu = 1$ seems to have a special meaning. The homeostasis of megalo-systems cannot be guaranteed anymore in non-stationary case ($\nu > 1$). But in the opposite case ($\nu < 1$), the homeostasis is too stiff to renew the present regime in unbroken wholeness.

The subliminal threshold can always give rise to an innovation in the local-global linkage, where the behaviors of megalo-systems must be analysed by taking into account communications between observables and non-observables. This point of view postulates a new theoretical framework of the causality in complex behaviors of megalo-systems.

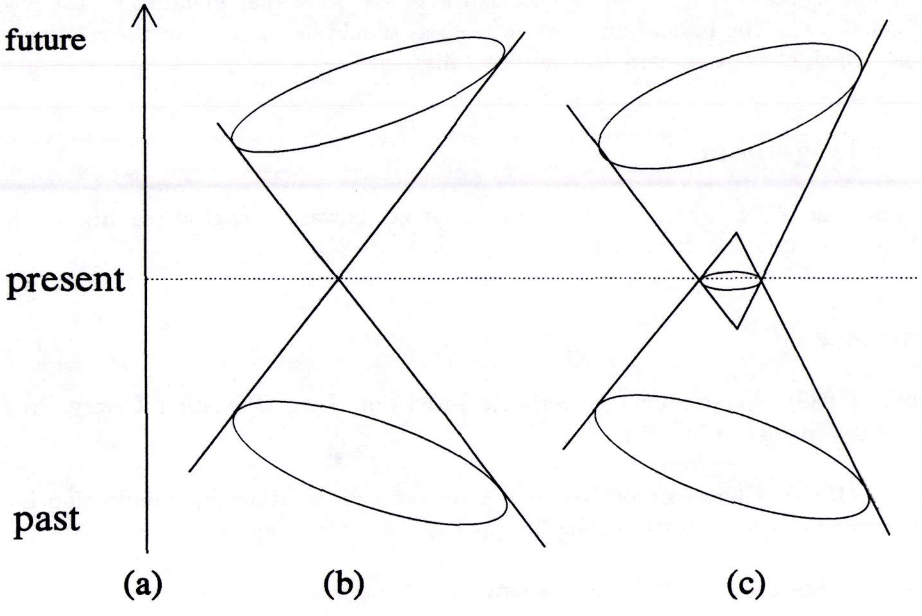

Figure 8: Schematic diagram for many-to-many causality.

In small systems, the arrow of time is clearly assigned by a series of observable events in reality, e.g., an elementary process of a particle which is strongly localized in space is simply described by one-to-one causality as is illustrated in Fig. 8-(a). Furthermore, in the case of a few coupled processes, the arrow of time must be assigned by a bundle of many processes as shown in Fig. 8-(b), where the non-separability of nonlinear processes plays an essential role in integrating processes into each cone. However, in megaloscopic systems, which are usually composed of a huge number of local elements, the arrow of time should be expressed in a completely different way from the previous two cases. This is because the reality at the present time is not the unique cause that generates the state in the future; the omnipresence of a number of subliminal levels induces the latent reality of non-observables, and that the long time memories embedded in $1/f$ fluctuations are transferred into the future. Let us call the arrow of time mentioned above the many-to-many causality, of which conceptional diagram is shown in Fig. 8-(c).

It should be stressed that the many-to-many causality be recognized in the context of subliminal thresholds, and that it is quite different from the traditional one-to-one causality which describes physical realities. The many-to-many causality is nothing but a theoretical postulation we have made to formulate the local-global linkage in megalo-systems by taking into account a number of latent realities beneath subliminal levels. In the traditional framework of causality, the present is imprisoned into a point on the arrow of time. However, in the many-to-many causality, the present is extended in a finite region which intervenes the past and the future. Thus, in the overlapping region, which are ruled over by the many-to-many causality mentioned above, we can expect to find out the law of unbroken wholeness.

Both the allometry law and the Zipf's law could be correctly understood in the framework of the many-to-many causality. In fact, they are not the laws that describe simple relations among some physical quantities, that is to say, what they manifest is "the relation of relations". The laws of unbroken wholeness should be explored in the relation of relations, which obeys the many-to-many causality.

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