How Many Dimensions are Required in Physics?

Dieter Gernert
Technische Universität München
Arcisstr. 21, D-80333 München, Germany
Email: t4141ax@mail.lrz-muenchen.de

Abstract
The different concepts of "space" are contrasted which have been developed in mathematics and in physics. Early proposals of extra dimensions are briefly reviewed. The main thesis claims that in physics not the dimension number of the underlying space is essential; rather, there are strong reasons to use vector spaces over the field of complex numbers, not strictly excluding other structures. Reasons for this come both from mathematical rigour and from human cognition. Applications to physics concern nonlocal processes, conditions for quantum entanglement, and a proposal of hidden organizing structures. An experimental test for nonlocal hidden structures is proposed.
Keywords: Complex spacetime, extra dimensions, nonlocality, entanglement, nonlocal hidden structures.

1 A Long-Standing Problem and an Unexpected Answer

The question of the appropriate number of dimensions, suitable for an adequate description of our world in physics, is a long-standing problem. In a historical tradition, a series of proposals has been made to introduce "extra dimensions", with a variety of motivations behind and intentions pursued.

But, by way of contrast, a quite different statement will be advanced here: more important than the frequently discussed dimension number of space is the underlying *algebraic structure.* It will be demonstrated that in special contexts it becomes inevitable to use a vector space on the field of complex numbers. It is true that, in a historical development, some tricks became widespread that permit to circumvent complex numbers; but these tricks may bring about cognitive blockades. (The utility of other algebraic fields, like quaternions, in special applications is not excluded; see Section 6).

On this basis, a special kind of metrics is proposed, which fit for complex vector spaces. These metrics are applied to nonlocal processes, and an experimental test is proposed which may check the possibility of nonlocal hidden structures.

2 The Disparate Concepts of Space in Mathematics and in Physics

Although mathematics and physics jointly evolved in a positive feedback loop, they tacitly developed different definitions for the concepts of space. This fact has been

scarcely noticed, and so it will be adequate to begin with some general remarks on the concept of space in the two disciplines.

The term "space" as such is not defined in mathematics. Generally, mathematical dictionaries only give references to more specific terms which are eponyms or formed by a diacritical adjective, like "Banach space" or "affine space". A mathematical dictionary (Naas and Schmid, 1961, vol. II: 455) presents about 40 such references to other lexicon entries. Another "encyclopaedic dictionary of mathematics" supplies a circumscription which can be hardly considered a definition: "The term space is used in mathematics for any set when certain types of properties are to be discussed or when it is intended to use some sort of geometrical terminology." (Sneddon, 1976: 616)

In physics the concept of space started with the 3-dimensional space from everyday experience. Space is a fundamental notion to characterize the arrangements of bodies or fields with respect to their relative positions and distances. In spite of all modifications meanwhile proposed, we can state that the finite-dimensional vector space still remains the fundamental underlying structure, as long as we want to model "the world we live in". Other versions, like Hilbert space or phase spaces, which are all included in the mathematical definition of space as above and undoubtedly indispensable in their fields of application, are beyond the scope of this paper.

The physical spaces to be studied in the sequel may be viewed as extensions of our familiar 3-dimensional space: we are going to consider finite-dimensional vector spaces, where the crucial question of the appropriate algebraic fields underlying their definitions is to be discussed later.

3 Historical Sketch: Early Proposals of Extra Dimensions

Although it is an empirical fact that the space of our everyday experience requires the space $\mathbb{R}^3$ (the 3-dimensional space over the field $\mathbb{R}$ of real numbers) for an adequate description, philosophers like to ask the question whether the number of dimensions must necessarily take on that value. Several reasons have been put forward constraining the possible number of dimensions (Büchel, 1965: 151-156); only two of them can be summed up here:

1. Optical and acoustical signal transmission can occur without trouble only if both reverberation and distortion are excluded. Reverberation means that a spoken word reaches the listener's ear repeatedly, and that the listener simultaneously hears several words which were spoken at different times. Distortion means that different pitches are transmitted at different intensities.
2. If our space would have more than three real dimensions, then neither planetary systems nor atoms could exist because all orbits would become unstable.

As a result of mathematical physics (Courant and Hilbert, 1962, vol. 2: 695-698), signal transmission without reverberation is possible only in a space with an odd dimension number. Together with the second argument there remains only a chance for dimension number 3.

In 1908, Hermann Minkowski set out to formulate Einstein's theory of special relativity more conveniently, and proposed to describe points in space and time by vectors of the form (x_1, x_2, x_3, t). So time is considered the fourth dimension, and the space thus defined is dubbed a 4-dimensional space. In particular, he advocated a unified view of space and time:

> *"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." (Conference lecture 1908, printed 1909)*

As a consequence he proposed to use the new term *spacetime*. Minkowski's contributions triggered an enormous progress in science, and are still generally acknowledged today (for a modern overview see Naber (2006)); but nevertheless, in view of special extensions (see Section 4), they give rise to some comments with respect to notation and terminology.

The first instance of an "extra-dimensional theory" was advanced by Gunnar Nordström (1914) under the title (translated): "On the possibility of a unification of the electromagnetic field and the gravitation field". This title also displays a characteristic feature of a large number of later, similar attempts: the principal motive is the *unification* of physical theories. Nordström's proposal was abandoned some years later since it was not compatible with new empirical findings (deflection of light).

The next significant step was due to Theodor Kaluza (1921), whose model also seeks to unify gravity and electromagnetism. He extended general relativity to a "5-dimensional" spacetime, being fully aware of the difficulties and limitations of such a description on the basis of $\mathbb{R}^5$ (p. 74). Most modern scientists are familiar with Kaluza's work only through its connection with the work of Oskar Klein, who modified the theory to the Kaluza-Klein model of spacetime (Klein, 1926; Beichler, 2007); Klein proposed that the fourth spatial dimension is curled up in a circle of very small radius (cylinder condition), such that a particle moving a short distance along that path would quickly return to its starting-point.

In modern times, the Kaluza-Klein theory has experienced a multitude of modifications and extensions (also to more than five dimensions), and still presents an active domain of research (Appelquist et al., 1987; Wesson, 2006).

4 The Necessity to Use Complex Vector Spaces

4.1 A Mathematically Correct Basis for Spacetime

> *The natural progress of science has brought us back to the point where science again needs to consider the physical reality of a higher-dimensional space. (Beichler, 2007: 523)*

This admonishment immediately leads us to our central question: Which mathematical structure is appropriate or required in order to describe and to analyse spacetime correctly?

In the literature, unfortunately we find some non-uniformity in definitions and terminology. Some mathematical tricks have been developed, and even grown up to a certain tradition, which may be "locally acceptable": derivations and results are correct as long as one is constrained within the boundaries underlying the tradition, but there is a risk that steps beyond those boundaries will lead to incorrect conclusions, and that the underlying definition will turn out as a mindblocker.

Therefore, a special, well-known mathematical structure is presented here as the preferred one. In the next section the reasons for this choice will be displayed. Finally, some discussions under the aspects of human cognition are to follow, together with remarks on frequent misunderstandings (Section 4.3).

The preferred mathematical structure is $\mathbb{C}^n$, the n-dimensional vector space over the field $\mathbb{C}$ of complex numbers, where n, the dimension number of the space, is a positive integer. The most important cases are n = 3 and n = 4, but higher values are also possible. In the case n = 3 a vector **x** will take on the shape

$$\mathbf{x} = \begin{pmatrix} a_1 + ib_1 \\ a_2 + ib_2 \\ a_3 + ib_3 \end{pmatrix} \tag{1}$$

As usual, the a_k and b_k are real numbers, and $i = \sqrt{(-1)}$ is the imaginary unit with $i^2 = -1$.

As an example take the classical Minkowski space. If the notation proposed here is accepted then the Minkowski spacetime is represented in $\mathbb{C}^3$, where the x_1, x_2, x_3 correspond to a_1, a_2, a_3 in (1), and b_1 stands for t; here $b_2 = b_3 = 0$. So we have a 3-dimensional space, and a 4-parametric set. (The set of all vectors in $\mathbb{C}^3$ with $b_2 = b_3 = 0$ is a vector space.)

4.2 Reasons for the Consequent Use of Complex Spaces

Why do we need complex numbers, or vector spaces based upon them, in order to study real objects and processes? Complex numbers are well established in down-to-earth disciplines like electrical engineering (analysis of a.c. circuits). In quantum theory there is no chance to circumvent complex numbers; a short argument for this is given by Mohrhoff (2002). Even in textbooks with a high priority to easiness of learning no author thought it advisable to start with a simplified presentation based on real numbers.

Unfortunately, in field and relativity physics there is no such uniform landscape. On the one hand, we find more than 1000 articles, written by physicists and published in refereed journals, dealing with complex (or complexified) Minkowski space; hence the underlying space is the $\mathbb{C}^4$ (or a $\mathbb{C}^n$ with $n > 4$). (Also modern approaches, based upon differentiable manifolds, indeed presuppose a complex space; see, e.g., Flaherty (1976)). On the other hand, there are some "standardized" strategies to avoid the use of complex numbers. Even if it can be acknowledged that both methods and results have

been correct, there remains some concern in view of possible future developments – also in applications of mathematics one should better recoil from "crawling along the margins of legality". This is an additional argument for mathematical rigour, and the following remarks have nothing to do with pedantry or mere mathematical formalism.

Any vector space is defined over a given (algebraic) field F. Frequent instances for F are $\mathbb{R}$ and $\mathbb{C}$, but other types (less important in physics) are admissible by the field axioms, too. Very often, the underlying field is not explicitly named, but it can be taken from the context that it is $\mathbb{R}$ or $\mathbb{C}$.

Each vector, and each component of a vector, may be multiplied with an element of F (often called a "scalar"), but by no means with some number outside F. If an $\mathbb{R}^n$ is underlying, then no imaginary number is allowed to come in. By way of contrast, in Minkowski space a difference element has the form

$$ds^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2dt^2 \qquad (2)$$

(where c is the velocity of light). In this way, the imaginary unit i, which entered through the identity $i^2 = -1$ and is hidden behind the minus sign, is only camouflaged. So the proper imaginary character of icdt, with consequences for its interpretation, is obscured.

Next, the terms "dimension" and "n-dimensional" should be used correctly. The dimension of a vector space is defined through the maximal number of linearly independent vectors, or, equivalently, the cardinality of a basis. The number of variables to which values can be assigned independently can be correctly named a number of *parameters*. So the Minkowski space is 3-dimensional and 4-parametric, whereas, in the general case, a complexified Minkowski space is 4-dimensional and 8-parametric.

4.3 Possible Mindblockers and Frequent Misunderstandings

It is well-known in cognitive science that the kind of problem representation has an essential influence upon the problem-solving process, and hence on its outcome; in many cases, a reorganization of the original problem representation may be a momentous step in the early phase of problem solving (Ziegler, 2000: 310).

There is not only the individual trouble caused by clumsy problem representations. More important is the non-uniformity in the kinds of problem representation, and hence in the cognitive styles, where quantum physicists differ from part of the field theorist (Section 4.2). This divergence is an obstacle on the way to the intended unification, and furthermore it gives rise to misunderstandings. As a striking example, a case of a quantum theorist may be quoted who polemically attacked a field theorist by claiming that the latter "uses a double $\mathbb{R}^4$" – really a $\mathbb{C}^4$ was meant.

A sloppy terminology may trigger a cognitive blockade and imply trouble concerning precise communication. In special cases it must be admitted that there may be a broadly accepted convention, which helps to speed up conversation a little bit, but some reservations about a residual risk remain.

In the sequel, some recurrent patterns of misunderstandings and incorrect formulations are listed, together with short comments:

1. The *dimension number*, or *dimension* for short, is an integer which can attributed only to a *space* (exotic cases, like ring dimensions, are outside our scope). E.g., a vector does not have a "dimension", but it is an element a space with specific features.
2. The number of dimensions is no criterion for the capacity and reach of a theory.
3. A subset is not automatically a subspace. It is a subspace if and only if it fulfils the axioms of a vector space itself. (The Minkowski space is a subspace of $\mathbb{C}^3$.)
4. There is no first, second, third, ... dimension. The concept from everyday experience, with length, width, height, cannot be extended to greater dimension numbers or to spaces over $\mathbb{C}$. Any labelling of coordinate axis will be arbitrary and not invariant against coordinate transformations.
5. A finite list of variables does not automatically define a vector space. Either it must be demonstrated that the axioms of a vector space are fulfilled (e.g.: what is the "sum" of two samples?), or it must be declared that we have to deal with a new type of mathematical structure (cf. Section 2), and its rules must be specified.

5 Complex Spaces and Nonlocality

5.1 Specific Metrics Fitting for Complex Spaces

If the underlying space is a $\mathbb{C}^n$, then the possibilities how to define a metric on this space require some deliberation. For any two vectors $\mathbf{x}$ and $\mathbf{y}$, a norm $N(\mathbf{x} - \mathbf{y})$ can be defined, where N is any function defined on $\mathbb{C}^n$ and obeying the axioms of a norm. This $N(\mathbf{x} - \mathbf{y})$ can be understood as the length of the vector $\mathbf{x} - \mathbf{y}$, and, at the same time, as the distance $d(\mathbf{x},\mathbf{y})$ between two points represented by the vectors $\mathbf{x}$ and $\mathbf{y}$. So $d(\mathbf{x},\mathbf{y})$ defines a metric on $\mathbb{C}^n$, which fulfils the usual axioms.

In a real space the distance between two elements of the space corresponds to the length of a shortest trajectory for an object moving from one place to another, but in $\mathbb{C}^n$ the analogous interpretation would have no physical meaning. Here, as a complement to the "classical" metrics as described just before, a different way to define a metric is proposed, which is not based on a function of the two involved vectors, but upon essential properties of the *entities* for which the formal model has been set up – *objects existing in the space and processes running in it.* Along this line we will obtain a metric which immediately permits a physical interpretation, and particularly copes with requirements imposed by specific findings of modern physics. Therefore, before stating the formal definition for this special class of metrics, some remarks on the background are required.

Nonlocality and entanglement belong to the eminent findings of modern quantum theory, experimentally confirmed beyond any reasonable doubt. They are fundamental for the worldwide activities in quantum communication and quantum cryptography. As a highlight only the recent experiment by Anton Zeilinger and his team shall be quoted,

where photons remained entangled even after having travelled 144 km through the atmosphere (Perdigues Armengol et al., 2008).

There are three characteristic features of entanglement which imply specific requirements to be fulfilled by a metric on the set of the involved entities:

1. *Selectivity, exclusivity:* In principle, entanglement concerns a well-defined, finite number of entities (in the most frequent case just two entities, but a larger number is not excluded).
2. *Nonlocality in a dual sense:* Entanglement is insensitive to enormous distances, but conversely, a third party, geometrically very near to one of the two entangled partners, need not be anyway influenced or involved.
3. *Condition-dependence:* Entanglement does not occur at random, but requires a condition to be fulfilled.

With these three features given, the next step will be to develop a mathematical expression for what can be intuitively circumscribed as "affinity" between two partners and the like (for more details on conditions for entanglement to occur see Gernert (2005)). This mathematical tool has the shape of dissimilarity function $d(S_i,S_k)$, which is defined for all pairs $\{S_i,S_k\}$ of elements of the underlying set of structures, and has the usual properties of a metric. A low value of $d(S_i,S_k)$ means that both structures are similar, and high values stand for a marked disparity.

Given a finite set of structures $\mathbf{S} = \{S_1, S_2, \ldots, S_m\}$, a dissimilarity function $d(S_i,S_k)$ with the required properties can always be defined (for every pair $\{S_i,S_k\}$, see the Appendix) by means of *graph grammars*. Originally created in 1969 for purposes of theoretical computer science, graph grammars have since demonstrated their efficacy in a great variety of applications. Here their essential properties and a conceptual question will be discussed, while some mathematical details (and references) are placed into the Appendix.

In principle, a graph grammar is a procedure that generates a multitude of graphs by repeated application of a small number of rules. It is always possible to formulate a graph grammar such that *at least* all graphs of the given set **S** are produced, and the dissimilarity between any two graphs from **S** is defined in a natural manner.

"Similarity" and "dissimilarity" are *perspective notions*: the meaning of these terms requires the context to be taken into account – beyond the well-known context-dependence of any word meaning. This requirement finds its correspondence in the mathematical representation. Apart from trivial cases, a graph grammar fulfilling the conditions named above is not uniquely defined. This ambiguity mirrors the character of a perspective notion, and in defining a dissimilarity function there are "degrees of freedom" such that the chosen graph grammar can be adapted to the special situation.

5.2 Applications to Nonlocal Processes

Vector spaces of the form $\mathbb{C}^n$ are indispensable in the study of nonlocal processes, too.

In the course of time, the conditions under which the concept of "hidden variables" could make sense were more and more narrowed down. According to Kochen and Specker (1967) it is impossible to explain quantum indeterminism by an influence of hidden variables. Both local realism and local hidden-variable theories are definitively ruled out by experiments (Freedman and Clauser, 1972; Kwiat et al., 1999). More recently, Leggett (2003) excludes nonlocal hidden-variable theories of a particular class (called "crypto-nonlocal theories"). These contributions from the literature are one if the reasons why in the following the concept of hidden variables will not be used.

As an intermediate summary, we can state that nonlocality and entanglement are secured, whereas hidden variables can be practically excluded. In this situation it is proposed to study whether the effects which are attributed to entanglement – or at least part of them - can be explained on the basis of hidden organizing *structures*. This concept – hypothetical for the moment – deserves a careful study from the beginning.

We are accustomed to saying that some institution is well – or not so well – organized, and we assume that there exists an underlying organizational structure, which is perceived from outside only by its efficiency, the conspicuous system behaviour. In terms of general system theory we have two disjoint, but interacting subsystems (for more details see Gernert (2004)):

- the sphere of observable physical effects, denoted by **O**, and
- the realm of the hidden organizing structure (**H**).

A system of the form {**O**,**H**} can be naturally described by two matrices:

- a transformation X which characterizes the transition from one state of the system to the next one (normal system operation), and
- a second matrix, Y, which represents the influence of **H**.

All matrices are m×m-matrices over $\mathbb{C}$ (they may be time-dependent, but this will not be regarded further). It is presupposed that the system states can be denoted by finite state vectors. Then a transformation step can be written as

$$\mathbf{x}' = YX\mathbf{x} \tag{3}$$

(This equation suggests a deterministic process, but it should be understood as a heuristic cue for the beginning. Later on, stochastic influences can be described, e.g., by modifications of the matrix Y; this is an important point for future research.)

Each of these matrices permits a unique decomposition into a real part and an imaginary part, and the matrices in eq. (3) can be rewritten: $Y = A + iB$ and $X = C + iD$, where A, B, C, D are real. The matrix product in eq. (3) becomes

$$(A + iB)(C + iD) = AC - BD + i(AD + BC) \tag{4}$$

Here the real part, R = AC – BD, represents the observable system behaviour, and at the same time mirrors the effect of **H**. The normal system operation (without an influence from **H**) occurs if Y = A + iB = I (unit matrix). In this case B = O (zero matrix), A = I, and R = C; so eq. (3) is reduced to $\mathbf{x}' = C\mathbf{x}$.

For the sake of simplicity, A = I is supposed, and the important case R = C – BD with BD ≠ 0 must be studied. This BD ≠ 0 characterizes the deviation from normal system operation. Therefore it must be analysed under which circumstances BD ≠ 0 can hold, or, more precisely: when will BD take on not too small numerical values, such that the disturbing influence from **H** cannot be neglected? This is not trivial, because two nonnegative matrices with their nonzero entries in a "normal" numerical range can have rather lessened entries in their product ($0 < a, b < 1$ implies $ab < \min(a,b)$). The question of a non-negligible magnitude of BD suggests the metaphor of "*key-lock mechanism*" (proposed by Emil Fischer (1852-1919) in 1894 within his enzyme studies): if and only if B and D fit together – in a sense still to be made precise – the effect of BD will be significant.

The required mathematical tool is supplied by *dissimilarity functions*, as introduced in Section 5.1; here we need the dissimilarity d(B,D). As explained before, the mathematics for defining dissimilarity functions can be based on graph grammars; but here, in the context of matrices, also hierarchical matrices (block-structured matrices) should be mentioned, which have proved their utility in numerical mathematics (Lütkepohl 1996; Bebendorf 2008). Both styles are essentially equivalent since any quadratic matrix can be rewritten as an edge-labelled graph, and vice versa.

For a system of the form {**O**,**H**} it must be analysed what can in principle be controlled by **H**, and which special qualifications will be required for **H**. An analogy from real life will give a helpful cue.

In a planned economy (centralized-administration economy) "target figures" are determined, fixing in which quantity the diverse commodities must be produced. Take screws as an example. If the total weight of the screws to be produced within a year is fixed, then big screws will be made; if the total number is defined, then an enormous number of small screws will come out. Any more differentiated scheme of goods to be manufactured will require a more detailed plan; or, in other words, with a small number of data only a primitive kind of "control" is possible.

This finally leads to the statement: *Structures can be controlled only by structures.* This is a second argument for not handling one hidden variable, nor a finite set of such, but a hidden organizing structure. (The latter does not permit us to smuggle in variables again. If, e.g., a graph is given, then a lot of graph invariants, like the graph's chromatic number, can be derived, but these are a topic from pure mathematics and have nothing to do with physics.)

The reflections on hidden structures can be traced back to a long history. It belongs to the indispensable core of theoretical physics that the realm of visible phenomena is founded upon and explained by invisible structures (Kanitscheider, 1979: 288). Of course, any formulation of a hidden structure requires an interpretation, that is "a set of normative regulative principles which can neither be deduced nor be refuted on the basis of the mathematical codification" (Primas, 1990: 172f); in other words, we need

an empirical test that gives answer about the efficacy or inefficacy – and hence the existence or non-existence - of the alleged structure.

5.3 Proposed Experimental Test

Such an experimental test certainly will not be easy, but it is not impossible. What we need is a process of structure formation with indefinite outcome, such that changes in the relative frequencies of the different outcomes can be observed. For some time, it is folklore among chemists that newly synthesized compounds with more than one admitted form of crystallization (e.g., stereo-isomers) do crystallize very clumsily in the beginning, whereas later on, after some quantities of the substance had been produced (and already had crystallized), crystallization may run rather quickly.

A systematic study can consist of the following steps: Several novel substances with ambiguous crystal forms are synthesized. For each substance the original speed of crystallization is measured. Then the substances are completely destroyed in order to exclude "classical" entanglement in the following step. In the last step substances of the same kind as before are synthesized anew, and it is checked for each substance whether its crystallization rate exceeds the earlier one. If this should be the case, then we would either have entanglement across time, or the influence of a hidden organizing structure. A vivid discussion will be sure to start.

6 Concluding Remarks and Outlook

There is no exclusive, and not even a preferred number of dimensions. Rather, the characteristic properties of the underlying space depend on the individual problem – e.g., layout problems lead to a 2-dimensional model in real space. At least as important as the number of dimensions is the selection of the adequate algebraic structure. In modern physics there is some preponderance in favour of $\mathbb{C}^n$, where n is specific for the problem.

Other than in everyday language, it does not make sense to give numbers or names to "dimensions", since in mathematics the term "dimension" is defined only for an entire space, and any numbering or labelling of coordinate axes would not be invariant against coordinate transformations. Some formulations found in literature, like "three time-like dimensions" only bring in an unjustified bias with respect to a later interpretation.

In a similar manner, no separate interpretation of the real parts and the imaginary parts is possible. From time to time, a proposal comes up to build a bridge across the Cartesian cut in such a way that the *res extensa* (matter) is identified with the real parts, whereas the imaginary parts are assigned to the *res cogitans* (mind). Whenever interacting parts must be regarded as a wholeness, such a partitioning of a mathematical model should be regarded with some reserve.

Research on higher-dimensional theories is still strongly proceeding (see, e.g., Wesson (2007)). Finally, it should not be forgotten that there are numerous contributions in the literature demonstrating that not only $\mathbb{C}$, but also two other algebraic fields, quaternions and octonions, are useful - or even inevitable - in special

branches of physics (see, e.g., Baez (2002), Dixon (1994), Okubo (1995)). We may look forward to new branches of research and novel results outside the familiar categories of human thinking.

Appendix: Graph Grammars

Graph grammars are a versatile mathematical tool that has proved its utility in various fields of applications. Here they are relevant as a tool to formulate a dissimilarity function for pairs of structures (Section 5.1). In the easiest case, a finite set of graphs $\mathbf{G} = \{G_1, G_2, \ldots, G_m\}$ is presupposed; possible extensions will be addressed later. A graph grammar is given by a startgraph and a finite number of production rules. Each production rule permits the generation of a new graph from one of the already existing graphs. A production rule is a triple $\{S_l, S_r, E\}$, where S_l is the subgraph to be replaced (left-hand side), S_r is the subgraph to be substituted for it (right-hand side), and E denotes the embedding rule governing the way in which S_r has to be inserted.

For the present application, a graph grammar Γ is required which generates at least all graphs in the given set **G**. For a fixed graph grammar Γ, the requested dissimilarity function is defined by

$$d(G_i, G_k) = \min L(G_i, G_k), \tag{5}$$

where $L(G_i, G_k)$ denotes the length of a path leading from G_i to G_k by following the lines "upward" and "downward" which represent the generation of these two graphs in a "pedigree-like" diagram depicting Γ. Each such step contributes 1 to L. (Low dissimilarity means great similarity, and vice versa; the mathematical transformation is trivial.)

For a given finite set **G** of graphs it is always possible to set up a graph grammar Γ such that at least all graphs in **G** are generated by Γ. But, apart from trivial cases, a graph grammar specified in this way cannot be unique. Rather, there is a multitude of graph grammars which are all suited to represent the desired dissimilarity function on **G**. The reason behind this lies in the fact that distance and dissimilarity are perspective notions, and no quantification is possible without a reference to the purpose (and the entire context) of such an assessment. Dissimilarity between structures is never a property of the structures themselves; rather, it is defined by an observer, and different dissimilarity functions mirror the different subjective views of different observers.

The procedure described above can be generalized to graphs with edge- and/or vertex-labels, and to hierarchical graphs. (For these extensions, and for technical details, diagrams, etc. see Gernert (1997); for the relationships between meaning, dissimilarity and other perspective notions see Gernert (2006); a bibliography on graph grammars is given by Drewes (2009).)

References

Appelquist T, Chodos A, Freund P.G.O. (Eds.) (1987) *Modern Kaluza-Klein Theories.* Menlo Park CA: Addison-Wesley.

Baez J. (2002) The octonions. *Bulletin of the American Mathematical Society* **39**, 145-205.

Bebendorf M (2008) *Hierarchical Matrices*. Berlin: Springer.

Beichler J.E. (2007) Three logical proofs: the five-dimensional reality of space-time. *Journal of Scientific Exploration* **21**, nr. 3, 523-542.

Büchel W (1965) *Philosophische Probleme der Physik*. Freiburg: Herder.

Clauser J.F. (2002) Early history of Bell's theorem. In: R.A. Bertlmann and A. Zeilinger (Eds.), *Quantum [Un]speakables*. Berlin: Springer, pp. 61-98.

Courant R, Hilbert D (1962) *Methods of Mathematical Physics*. New York: Interscience.

Dixon G.M. (1994) *Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics*. Dordrecht: Kluwer.

Drewes F (2009) Graph grammar bibliography. Available at: http://www.cs.umu.se/~drews/gragra. Accessed January 2009.

Flaherty E.J. (1976) *Hermitian and Kaehlerian Geometry in Relativity.* Berlin: Springer.

Freedman S.J., Clauser J.F. (1972) Experimental test of hidden-variable theories. *Physical Review Letters* **28**, 938-941.

Gernert D (1997) Graph grammars as an analytical tool in physics and biology. *BioSystems* **43**, 179-187.

Gernert D (2004) Formal treatment of systems with a hidden organizing structure, with possible applications to physics. *International Journal of Computing Anticipatory Systems* **16**, 114-124.

Gernert D (2005) Conditions for entanglement. *Frontier Perspectives* **14**, no. 2, 8-13.

Gernert D (2006) Pragmatic information: historical exposition and general overview. *Mind and Matter* **4**, nr. 2, 141-167.

Kaluza T (1921) Zum Unitätsproblem der Physik. *Sitzungsberichte der Preussischen Akademie der Wissenschaften (Phys.-Math. Klasse),* pp. 966-972.

Kanitscheider B (1979). *Philosophie und moderne Physik.* Darmstadt: Wissenschaftliche Buchgesellschaft.

Klein O (1926) Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik* **37**, 895-906.

Kochen S, Specker E.P. (1967) The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics* **17**, 59-87.

Kwiat P.G. et al. (1999) Ultrabright source of polarization-entangled photons. *Physical Review A* **60**, R773-R776.

Leggett A.J. (2003) Nonlocal hidden-variable theories and quantum mechanics: an incompatibility theorem. *Foundations of Physics* **33**, 1469-1493.

Lütkepohl H (1996). *Handbook of Matrices*. Chichester: Wiley.

Minkowski H (1909) Raum und Zeit. *Physikalische Zeitschrift* **10**, 104-111.

Mohrhoff U (2002) Why the laws of physics are just so. *Foundations of Physics* **32**, 1313-1324.

Naas J, Schmid H.L. (Eds.) (1961) *Mathematisches Wörterbuch*. Berlin: Akademie-Verlag.

Naber G.L. (2006) Introductory article: Minkowski spacetime and special relativity. In: J.-P. Francoise et al. (Eds.), *Encyclopedia of Mathematical Physics*. Amsterdam: Elsevier, vol. 1, pp. 96-109.

Nordström G (1914) Über die Möglichkeit, das elektromagnetische Feld und das Gravitationsfeld zu vereinigen. *Physikalische Zeitschrift* **15**, 504-506; reprint in Appelquist et al. (1987), pp. 57-60; English version ibid. pp. 50-56.

Okubo S (1995). *Introduction to Octonion and Other Non-associative Algebras in Physics*. Cambridge: Cambridge University Press.

Perdigues Armengol J.M. et al. (2008) Quantum communications at ESA: towards a space experiment on the ISS. *Acta Astronautica* **63**, 165-178.

Primas H (1990) The measurement process in the individual interpretation of quantum mechanics. In: M. Cini and J.-M. Lévy-Leblond (Eds.), *Quantum Theory Without Reduction*. Bristol: Hilger, pp. 49-68.

Sneddon I.N. (Ed.) (1976) *Encyclopedic Dictionary of Mathematics for Engineers and Applied Scientists*. Oxford: Pergamon.

Wesson P.S. (2006) *Five-dimensional Physics. Classical and Quantum Consequences of Kaluza-Klein Cosmology*. Singapore: World Scientific.

Wesson P.S. (2007) *Space - Time - Matter. Modern Higher-dimensional Cosmology*. Singapore: World Scientific (second edition).

Ziegler A (2000). Problemlösen. In: *Lexikon der Psychologie*. Heidelberg: Spektrum, pp. 310-311.