An Analysis of Relational Systems

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Abstract
The relational modeling strategy Robert Rosen introduced in chapter 5 of *Life Itself* in order to model biological organization is in many ways very remarkable, not in the least because he is able to give an account of final causation. This article gives an overview of the most characteristic steps of the relational modeling strategy, from the component to augmented abstract block diagrams, putting the emphasis on some epistemological aspects. Robert Rosen's modeling strategy is a formal representation of biological organization, specifically of circular causality. In this regard, the most important contribution of Robert Rosen is the *anschaulichkeit* – the intuitability – of the closure of the organization and of the intertwinement of the different Aristotelian causes, even if they are incommensurable.

Keywords: Robert Rosen, modeling, perspectivism, function, organization.

1. Introduction: Relational Biology

The purpose of relational modeling is to represent a functional organization. It is an inherent part of relational biology, broadly characterized by its adage "throw away the matter and keep the underlying organization" (Rosen 1991; Rosen 1996). More specifically, as Nicolas Rashevsky put it in 1954, relational biology *"must look for a principle which connects the different physical phenomena involved and expresses the biological unity of the organism and of the organic world as a whole"* (Rashevsky 1954). Later on, when Robert Rosen continued the work of Rashevsky, the notion of principle was transformed in the search for the essence of the living system as opposed to its existence (Rosen 2000). The essence of a living system is its functional organization, and the object of study of relational biology is accordingly the study of this organization. Rosen was already in 1958 able to identify a specific kind of organization, which he called an (M,R) system, as a model for a living system, which possessed the necessary condition for a system to be alive (Rosen 1958). His later work consisted, amongst other things, in an investigation of the consequences of the properties of this model. One of the things which he felt to be necessary was an elaboration of the philosophical ramifications, not only of the model itself, but also of the place and meaning of models in science, of science itself and of how the Schrödinger question "What is Life?" can find a place in scientific thinking. This question, so Rosen became convinced of, is not an empirical one, and in the attempt to answer it, a whole new space of thinking has to be constructed. Rosen was able to create this space in order to (i) legitimize the question, and (ii) pinpoint the place where an answer should be looked for. In order to make (ii) possible, Rosen needed to argue for the object itself: How can we study the organization, the essence, of an organism? He needed to make organization into an object, a thing suitable for investigation. Relational modeling is therefore about the specific way organization can be encoded, *i.e.* to make it into an abstract thing.

For a large part his philosophical work is typically transcendental: it is about positing principled boundaries to a certain domain in order to free up another one. In other words, Rosen opens up a space between holism and reductionism, objectivity and subjectivity, mechanism and vitalism, by a careful reasoning about their respective limitations (Rosen 1996).

Specifically, in this article we follow Rosen's line of thinking concerning organization and functionality, two essential characteristics of living systems. To be adequate, a model of a functional organization must be able to account for such elusive notions as ambiguity, plasticity and part-whole dynamics. These are all concepts which are absent in the prevailing Newtonian approach in modeling, based on a state concept and an irreversible time. Rosen, accordingly, abandons the state concept and also time is absent in his relational models. A second difference is the use of the four Aristotelian notions of causality. Or at least, Rosen analyses a mathematical function in terms of four notions of causality which are inspired by Aristotelian causality. Especially final causality is very important to account for the prevalent functionality in living systems. In short, relational models are much richer in entailment structures – they are more generative – than a state based conception.

In the following we stay very close to Robert Rosen's strategy of establishing a relational model. It is based on Chapter 5 (*Entailment Without States: Relational Biology)* of Life Itself.

2. The strategy of relation modeling or how to study functional organization in the abstract

2.1 Components and abstract block diagrams.

In (Van Poucke and Van de Vijver 2008) there was a discussion of Rosen's relational epistemology: the modeling relation and the notion of component as the functional part(icle) of an organization only to be arrived at by an analysis of two interacting systems. Here we want to follow Robert Rosen in his formalisation of the component and its embeddedness in a larger system of components. The formal image we obtain is a relational model of a system of components.

Now, the crucial thing is how we can avoid that in the formal image of a component we don't make it into a static, invariant thing. A component is a functional entity, and as such it should be contingent, not absolute. It is contingent upon the system it belongs to, and that contingency is also responsible for the dynamics and inherent ambiguity about the role it plays in the system. On the other hand, a component should also encompass enough stability to make it meaningful to identify it as a component as such. So, we want to avoid that a component becomes a thing in itself, something that can be isolated and studied apart from the system it is a component of.

A component is characterized by an intrinsic asymmetry: it consists of an input side (representing the influence of the environment, including other components) and an output side (the effect of a component on the environment and other components). A component has thus the ability to be affected and to affect itself. These input and output, or afferent and efferent parts of a component are the contingent part of the component. The component itself, its identity, is then the intrinsic part. (Rosen, 1991, 121)

The formal[1] image should be able to represent both parts of the component: the contingent part and the intrinsic part. Rosen suggest a mapping[2] as the formal image of a component,

$$f: A \rightarrow B \qquad (1)$$

where the asymmetrical structure is represented by the domain A and its range B. The duality between the identity of the component and its contingent part is, respectively, the mapping f itself and the specific arguments which are present in A.

With this unit, this formal representation of a component, it is possible to start composing synthetic systems using common category theoretical operations such as the Cartesian product, the composition or intersection of two mappings. For example:

$$A \overset{f}{\rightarrow} (B \cap C) \overset{g}{\rightarrow} D \qquad (2)$$

The result of these operations is always a bigger relational system and Rosen calls these *"abstract block diagrams*". The notion of organization means precisely this: "*organization is that attribute of a natural system which codes into the form of an abstract block diagram.*"(Rosen, 1991, p. 126). If the component could be seen as an atom of organization, the abstract block diagram is a molecule.

Note, however, that these entailments are only able to entail elements of sets, not mappings. That is why Rosen calls these *inner entailments*[3]. Besides inner entailments, we can also use *outer entailments*. With them, it is also possible to entail mappings from

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[1] For the construction of a relational model, Rosen uses basic forms of category theory. It is however not necessary to go into that part of mathematics to understand those properties of relational modeling on which we focus in this article. Suffice it to say that category theory is a more abstract version of set theory.

[2] Rosen uses the term mapping instead of the mathematical concept of function to avoid ambiguities with the biological notion of function.

[3] The difference between outer and inner entailments lies not only in what they are able to entail. They are different kinds of operations in Category Theory. Inner entailments are only situated in the category itself, here the category of mappings and sets. Outer entailments originate in the defining properties of the category, they are global inferential rules (See Rosen, 1991, p. 128, 129,131)

given mappings, and sets from sets. In this way, we can build constructions of mappings, for example:

$$A \overset{f}{\rightarrow} B \overset{g}{\rightarrow} C \tag{3}$$

And this becomes, if we apply the inferential rules $f \Rightarrow (a \Rightarrow f(a))$, for all a in A, and $g \Rightarrow (b \Rightarrow g(b)$, for all b in B, and if we consider $b = f(a)$:

$$g \Rightarrow (f(a) \Rightarrow g(f(a))) \tag{4}$$

Now, we want to change the expression $g(f(a))$ by $(gf)(a)$ so that we are able to effectively construct a new mapping out of two mappings. We need something of the following form: $(gf) : A \rightarrow C$. We can do that by using the category axiom that allows composition of mappings that gives us the inferential rule: $F\ (f,g) = gf$, so we get

$$F \Rightarrow ((f,g) \Rightarrow gf) \tag{5}$$

2.2 Augmented abstract block diagrams

Until this point we did not combine mappings and sets but in category theory this is possible. If A en B are sets, then the totality of both $H\ (A,B)$ of all the mappings between A and B is again a set. It is possible to entail $H(A,B)$ out of the mappings of the category. The difference is that previously it was only possible for mappings to be given or to be entailed by outer entailments, but now we can use inner entailments to get mappings to cause an effect. Now a mapping itself can be an effect, it can be entailed[4]. The form this takes is the following:

$$\varphi : X \rightarrow (H(Y,Z)) \tag{6}$$

H(Y,Z) encodes components (the mappings between the sets Y and Z) en φ is also a component for which every x in X has an image in the set H(Y,Z). Because φ(x) is at its turn also a mapping, a new element is entailed, namely (φ(x))(y) in Z.
We can represent this situation as follows:

[4] In an augmented abstract block diagram, Rosen leaves the strict separation between sets and mappings he supposed in abstract block diagrams. Abstract block can also be regarded, no matter how much we enlarge them, as components, as long as we hold on to the strict dualism between sets and mappings. The result was that we only could entail elements, no sets and mappings. The move towards augmented block diagrams is precisely to *entail* other mappings, not construct them.

$$X \xrightarrow{\varphi} H(Y,Z)$$

$$\downarrow$$

$$Y \xrightarrow{\varphi(x)} Z \qquad (7)$$

This is what Rosen calls an 'augmented block diagram'. Important to note is that the dotted line is not a composition as met before, where the output of a component is the input of another component (represented by a solid arrow), but something else. The dotted arrow indicates that a component is an output of another, but it does not indicate composition. There are different ways to achieve this, for example:

Ψ: $(H(X,Y) \rightarrow Z$, where a component is an input for another component and,
θ: $H(X,Y) \rightarrow H(U,V)$, where a mapping is both an input and an output for other mappings.

Now, consider the special case where X = Z:

$$Y \xrightarrow{\varphi(z)} Z \xrightarrow{\varphi} H(Y,Z) \qquad (8)$$

There is still one mapping which remains unentailed, namely $\varphi : (Z \rightarrow (H(Y,Z))$.

It is however possible to include it in the organization by the following:

$$Y \xrightarrow{\varphi(z)} Z \xrightarrow{\varphi} H(Y,Z) \xrightarrow{\beta} H(Z, H(Y,Z))\ [5] \qquad (9)$$

Of course, the mapping β is now not entailed in the organization. However, if we repeat this kind of operation, then a similar extension of the diagram is again possible and β will be entailed again. We can infinitely repeat this operation whereby the result will be that the amount of organization will be larger and larger. Every time we enlarge our diagram with another entailment structure, we add new functions to the elements of the system. The limit case of this procedure is a situation where everything in the diagram is entailed.

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[5] By extending the diagram in this fashion there is an increase in organization in the sense that there is a maximal organization when the ratio between unentailed and entailed elements is approaching zero (Rosen, 1991, p.137)

2.3 Aristotelian Causes in Abstract Block Diagrams.

What are then, in these formalizations of components and systems of components, the generative capacities of a relational model? What is possible in a relational environment that is not possible in a state based model? Rosen is going to interpret the formalisations in terms of Aristotelian causes. That is, he is going to ask the "Why?" question to each aspect of a relational system and answer it with one of the appropriate Aristotelian causes.

Let us reconsider our first example of a component:

$$f: A \rightarrow B \qquad (1)$$

This can be written as,

$$f \Rightarrow (a \Rightarrow f(a)) \qquad (10)$$

which can be read as 'f entails 'a entails $f(a)$'. This presentation makes the difference clear between f and a, which are both necessary to generate $f(a)$. These two forms of entailment correspond with two of the Aristotelian 'causes'. If we ask ourselves the question "Why $f(a)$?" and we consider $f(a) = b$, namely as an effect, there are two possible answers[6]:

- $f(a)$ because a (= the input is the material cause of the output $f(a)$)
- $f(a)$ because f (= the mapping is the efficient cause of the output $f(a)$)

With final causes there is of course the peculiar aspect that we have an opposite direction then with the other Aristotelian causes where we ask "Why?" to something which is considered as an effect, and we answer it in terms of the things that entail the effect. A final cause, however, is answered in terms of what the effect entails. This cannot arise in a Newtonian framework because it implies that the future is acting on the present, which is impossible because of the irreversible time. Every notion of finality in a Newtonian framework, can only be answered in terms of intentionality or design. This is again problematic, because it implies a designer, or an external final cause, and that would make an organism into a machine. What we are looking for in biology is immanent, internal final causation.

The relational model of Robert Rosen does not talk about subsequent states in an irreversible time and as such Rosen can interpret the entailment of a component by its function. This is what Robert Rosen calls *functional entailment* as opposed to the inner and outer entailments we have seen before. The final cause is that which is entailed by the effect. If we apply this in our previous case we cannot identify a final cause for $f(a)$,

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[6] It is in this context a bit remarkable that Robert Rosen does not mention the formal cause as a part of the entailment structure of the block diagrams. This is, in our interpretation, because the abstract block diagram is itself the formal cause. The notion is very important in the context of the realisation of a relational system, meaning the step from the essence (the functional organization in the abstract) to the existence (the functional organization realised in a natural, material system). This step in relational modeling is also called by Rosen the 'recapturing of matter from bauplan'.

because it does not entail anything in the diagram. However if we ask "Why?" for *f* or *a*, then the only possible answer is in terms of final causes. For *f* and *a* there are no material or efficient causes to give, but they do entail something else. They have the function to entail *f(a)*. So if we treat the image *f(a)* as an effect, then it has both a material and an efficient cause. If we treat *f* and *a* as effects, then they can only be considered as final causes.
Now, how do we get finality in augmented abstract block diagrams? This is really what makes the relational model of Rosen so peculiar. The answers to the "Why?" question are only possible in a augmented block diagram where you can entail mappings, not in an unaugmented diagram or a Newtonian framework. We quote Rosen almost literally from page 141 and 142 of *Life Itself.*

Let us consider again our special case, the maximally organised diagram:

$$A \xrightarrow{f} B \xrightarrow{\Phi} H(A,B) \qquad (8)$$

Here we are specifically interested in $\Phi \Rightarrow (f(a) \Rightarrow f)$ or, $\Phi(f(a)) = f$; the dotted line is implying that Φ entails the original mapping *f.*
If we again ask the "Why?" question we get the following Aristotelian answers:

1. the mapping Φ, which is itself unentailed. If we ask the question "Why Φ?" Its only answer in the diagram is because *f.* Hence, as with any otherwise unentailed unit of a formalism, the only answer to this question is in terms of function and finality.
2. the original mapping *f*, which in the new situation is now entailed. Thus, if we ask the question, 'why *f*?' we can answer it now as follows:
    a. because *f(a)*: original finalistic answer
    b. because *f(a)*: f is entailed by its value *f(a)*, its material cause
    c. because Φ: *f* is the value of Φ at the argument *f(a)*, Φ is its efficient cause.

The remarkable thing to notice here is that the first two answers for "Why *f*" are the same. We refer to the next section for a discussion.

3. Concluding Remarks

We started this paper with the demand that a relational model must be able to show some essential aspects of a living system such a plasticity, ambiguity and part whole dynamics. Now that organization was made into a thing, an abstract block diagram, we can consider some of the characteristics of it.

When considering Rosen's way of relational modeling the first thing to notice is the way he uses a specific language and a way of writing the formalism. This form, or symbolisation, has the purpose to be able to talk about entailments. To write eq. (1) $f: f: A \rightarrow B$ as eq. (10) $f \Rightarrow (a \Rightarrow f(a))$ is first of all meant to be able to consider other ways of entailment then just the efficient cause. Moreover, to ask to each term in the diagram the "Why?" question implies a particular way of perspectivism. It means that there are different ways to analyse an organization depending on the very question you ask. The result is a different entailment structure for each question and for each term. The relationship between those structures are independent, i.e. they cannot be interchanged or reduced to each other.[7] Although the same element can have different meanings according to the question you ask, you cannot lose track of the initial question posed. This constitutes a very specific kind of contextuality: the context of the question – of the subject – lies in the very heart of relational modeling. When we asked "Why f?" in eq. (8), we discovered that its material cause was the same as its final cause. First of all, this situation is only possible when there is a closed loop in an augmented abstract block diagram and this is a consequence of the increase of organization – measured in terms of the ratio between entailed and unentailed elements[8]. Secondly, it means that "*the function of f has now become part of the entailment of f* (…), the unique circumstance that *the function of f is the entailment of f." (Rosen 1991).*

A second thing to note is the use of category theory to break through the dualism between sets and mappings in order to introduce a new way of entailment, functional entailment. It is the functional entailment that opens up the dynamics of the system. By embedding a component in an abstract block diagram we give $f(a)$ new functions while at the same time the other components in the diagram also get new functions. This refers to the plasticity aspect of organisms. For example, if we have a mapping $f: A \rightarrow B$ and a different mapping $g: C \rightarrow B$, and there is an element $g(c) = f(a)$, then the final cause will not be able to discriminate between on the one hand *g* and *f* and on the other hand *c* and *a*, i.e. the answer to the questions "Why *g*" and "Why *c*" will be the same as to the questions "Why *f*" and "Why *a*". In terms of function, of what they entail, "*they are equivalent in terms of function*" (Rosen, 1991, p. 139) although they are not identical. Rosen therefore connects final causation with the idea of possibility, while the other forms of causation are tied to notion of necessity. Another aspect, namely the ambiguity of an identified part of an organism, can be formulated as: The same component is able to fulfil different roles at the same time; and the same function can be manifested in different ways (*ibid.*, p. 140). Formulated in another way: "*There is nothing about a component per se that entails any particular function it may manifest, nor is there anything about a particular organization that entails a specific component"* (*Ibid.*, p. 140). There is no unique way of breaking down, as in a machine, an organism in fixed parts and tying an invariant function to it, and this is precisely what reductionism is. Those specific relations that reductionism discovers between a part and its function are accidental, in the sense that they are the result of a specific realisation of an organization. The particularities have nothing to do, at least not in a relational way, with the organization which is studied in an abstract block diagram; they are fixed instances of it and in that way do not exhibit the essential properties.

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[7] Rosen states: "*The causal categories do not entail each other"* (Rosen, 1991, p.132)

[8] The idea of hierarchy and levels of organization in the context of relational systems is only meaningful in this sense: by the embeddedness of closed loops in augmented block diagrams. Any interpretation of Robert Rosen's work in terms of natural hierarchical levels then becomes very difficult, especially when applied outside the epistemology of the modeling relation, because it is purely an abstract organizational term.

We discussed how Robert Rosen makes the idea of functional organization into something which can be studied in the abstract and by doing that we could point out some peculiar properties, generative aspects, of a relational model. To be clear, we did not at all talk about Rosen's necessary condition for life, the (M, R) system, nor about central concepts in his work such as anticipation, complexity and mechanism. However, Chapter 5 in *Life Itself* is the place to start an investigation of what a relational model means and as such it is in our view the most important chapter in the book.

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