Information, Bifurcation and Entropy in the Universal Rewrite System

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Abstract

The universal rewrite system, which has previously been applied especially to the generation of the number system and to nilpotent quantum mechanics, not only provides a generalised description for all natural processes but also creates a direct measure of entropy increase and of information transfer, which is especially significant for systems on the edge of chaos. Entropy and information can be shown to have a direct connection with the concept of the generation of the number field at a very fundamental level.

Keywords: universal rewrite system, self-organization, nilpotent quantum mechanics, entropy, chaos, information, bifurcation

1 Introduction

The universal rewrite system, or nilpotent universal computational rewrite system (NUCRS), proposed by Rowlands and Diaz,[1,2] appears to provide a generalised description for all natural processes, based on computational principles. In one application, it provides an origin for the natural number system and for mathematics in general, and in another a foundation for a system of relativistic quantum mechanics and quantum field theory, based on nilpotent wavefunctions, or those which square to zero.[3] The nilpotent structure, applied in quantum mechanics to the fermion and its vacuum, appears to have a more general application to describe all self-organizing systems.[4] The rewrite structure also provides a generic model for information and entropy increase, and for the bifurcation process that occurs in near-chaotic systems.

2 The Universal Rewrite System

We have discussed the universal rewrite system in several publications, including the accompanying paper 'Universal rewrite and self-organization'.[5] Essentially, it provides an infinite series of zero totality 'alphabets' (or unique and unrepeatable descriptions of the universe in which the total sum of the entire information within it must always equal zero), each of which contains all previous alphabets as 'subalphabets' in a succession of zero totality cardinalities. The process, which precedes any mathematical description, is described in a series of 'concatenations' in which subalphabets are checked against their containing alphabets to show that they contain nothing new ('conserve'), while

alphabets are checked against themselves to show that they are not unique and must lead to a further zero totality alphabet ('create'):

conserve: (subalphabet) (alphabet) → (alphabet) *there is nothing new*

create: (alphabet) (alphabet) ⇒ (new alphabet) *a zero totality is not unique*

One way of representing the successive alphabets might be in the form:

Δ_a (R)

Δ_b (R, R^*)

Δ_c $R, R^*, A, A^*)$

Δ_d $(R, R^*, A, A^*, B, B^*, AB, AB^*)$

Δ_e $(R, R^*, A, A^*, B, B^*, AB, AB^*, C, C^*, AC, AC^*, BC, BC^*, ABC, ABC^*)$ ...

generated by the 'create' processes, such as

$$(R, R^*)\,(R, R^*) \Rightarrow (R, R^*, A, A^*)$$

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*) \ldots$$

Here, a term like R^* is some kind of conjugate to R, introduced to ensure the zero totality. A and B require an 'anticommutative' assembly, in which $(AB)(AB) \rightarrow R^*$, rather than a 'commutative' one in which $(AB)(AB) \rightarrow R$, because this is the only way of ensuring that B is distinct from A and that it introduces something new to extend the alphabet. The same will apply to C, D; E, F, and so on. Each term becomes unique because it has a unique partner.

The universal rewrite system is a general description of process which applies to mathematics, computation, physics, chemistry, biology,[3] and any self-organizing system.[4] One mathematical interpretation allows the successive alphabets to generate a series of binary integers. Another creates a Clifford-type algebra, where the alphabets generate commutative sets of quaternion units, $\boldsymbol{i}_1, \boldsymbol{j}_1; \boldsymbol{i}_2, \boldsymbol{j}_2; \ldots$

$$(1, -1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots$$

or combinations of equivalent combinations of quaternion ($\boldsymbol{i}$, $\boldsymbol{j}$); vector ($\mathbf{i}$, $\mathbf{j}$); and complex (i) units:

$$(1, -1)$$
$$(1, -1) \times (1, \boldsymbol{i})$$
$$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j})$$
$$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i})$$
$$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j})$$
$$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \times (1, i) \ldots$$

The sixth alphabet shown in both these cases coincides with the gamma algebra of relativistic quantum mechanics and the Dirac equation, which suggests an application at this level in physics. In physical terms, the first four alphabets suggest the successive emergence of descriptions of the universe in terms of the fundamental parameters mass, time, charge and space, which are described by the algebras created with these alphabets. The simultaneous existence of these independent descriptions of the universe can only be accomplished at the level of the sixth alphabet. This is also the first level at which the processes of conjugation, complexification and dimensionalization, which are successively introduced with the first three alphabets, can be finally incorporated into a repetitive sequence. Mathematically, it represents the commutative combination of the first complete Clifford algebra (that of 3D space) with all its subalgebras. So, in many respects, the sixth alphabet represents a particularly significant stage in the rewrite process, the first at which the repetitive nature of the sequence becomes fully established, and this is, therefore, in effect the rewrite system order code.

Since such terms as R, A, B, C, etc., are generated independently of each other by a unique process, no relative numerical 'values' are automatically attached to them, and it is possible, when we use one of the mathematical representations, to choose values for the terms of this alphabet in such a way that its self-product becomes zero and all subsequent alphabets are automatically zeroed as well. This is the nilpotent condition. It can be achieved in an infinite number of ways, each of which is unique, and it appears to apply both to quantum physics and to systems at much higher levels. The nilpotent condition defines its conjugate as a kind of mirror reflection of itself rather than an intrinsically new state, defined new symbol – physically, we call it 'vacuum'. Significantly, the 'conjugate' state to the creation of the initial symbol 1 in binary arithmetic is not defined by a new symbol either, but by a kind of number 'vacuum', ... 111111.

Now, the rewrite process is a fractal one in which alphabets at one level can become the units at another. We can therefore imagine an interpretation of the rewrite structure in which every new symbol, say A, B, C, ... represents the creation of a new nilpotent alphabet. Since the units are now already defined as unique, the connections between them require only the generation of commutative algebraic coefficients, which can then continue to infinity. This suggests the creation of something like a Hilbert space of nilpotent alphabets, defined by a commutative Grassmann algebra.[3] The coefficients increase with every new nilpotent alphabet, but they have no intrinsic significance, serving only to distinguish one nilpotent alphabet from another.

3 Nilpotent Quantum Mechanics

Nilpotent quantum mechanics (NQM) is predicated on the important and critical discovery that Einstein's conservation of energy and momentum equation $E^2 - p^2 - m^2 = 0$ (taking $c = 1$) is factorizable, in the nilpotent form

$$X.X = (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0 \qquad (1)$$

where i is $\sqrt{-1}$, $\mathbf{p}$ is a multivariate vector, and $\boldsymbol{k, i, j}$ are quaternion units, and that each of the solutions / roots (E, $\mathbf{p} = (\mathbf{i}p_x, \mathbf{j}p_y, \mathbf{k}p_z)$, m) is a unique fermion state, together with a dual vacuum state.[3,6,7] The initial zero point therefore defines the totally degenerate physical vacuum state at which none of the solutions / roots can be distinguished from any other, i.e. it can be postulated as specifying the generalised dark energy from which the nondegenerate fermion states specified in terms of the 3D coordinates (x, y, z) may be seen to emerge, with energy E, momentum $\mathbf{p} = (\mathbf{i}p_x, \mathbf{j}p_y, \mathbf{k}p_z)$, and mass m, taking whatever values satisfy the equation. This essential separability of the dynamic spatial quantum degrees of freedom from the conservation equation tells us that the quaternion units (which also correspond to square roots of –1) and i must be those that correspond to the electromagnetic, weak, strong, and gravitational / inertial parameterisations / quantizations of the physics of the elementary particles, which this separability says hold throughout the 3D universe. And evidentially, from the known experimentally determined results of the Standard Model, this is indeed the case, since these quantizations correspond to the symmetries $U(1)$, $SU(2)$, $SU(3)$ and $SU(4)$. Further, the 'hidden' symmetry of a fifth group unit element itself gives us nothing new, but confirms Klein / Kaluza theory, which extends general relativity to a fifth dimension to incorporate the effects of mass or charge, as $SU(5) \approx SU(4) + U(1)$.

The $SU(5)$ / $U(5)$ concept includes gravity / inertia in a grand unified theory, which makes all the forces equal and reduced to $U(1)$ at the Planck mass.[3,8] It relates both to the 5-fold nature of the nilpotent and Kaluza / Klein, where the fifth element is inertial rest mass / electric charge, as in the nilpotent, and, in fact, any $U(1)$ source (each interaction having one), and the total grand unified combination of strong / electric / weak provides the inertial contribution which locally cancels the gravitational (vacuum) negative energy. This seemingly relates to the dark energy, in that this is inertial in origin.[3] Gravity would seem to correspond to a $U(1)$ effect of the dark energy / vacuum, in which, unlike the $U(1)$ of the electromagnetic field, + and – cannot be distinguished. Kaluza and Klein were originally two theories, one representing the origin of mass and the other of charge by a fifth dimension, compactified in a structure equivalent to a $U(1)$ symmetry, which can be equated with the fifth / causality / proper time term in the nilpotent wavefunction.

The nilpotent equation (1) points to the separability of the vacuum into components with an electromagnetic, weak, strong and gravitational phenomenology, where $\boldsymbol{ikj} = -1$ confirms that the *CPT* symmetry holds everywhere, that is, that the laws of physics hold under simultaneous reversal of the signs of space and time coordinates and exchange of particles and antiparticles. This points to a general self-organizational phenomenology where at each stage of the emergent self-organization, the new stages must always be consistent with those that have proceeded them: that is, the new order must incorporate the old pre-existing order within it, and in particular the Standard Model elementary particles as the ultimate basis of any new emergent order. Further, the fundamental condition for finding a particular nilpotent solution of any such self-organization is the assignment of an operator corresponding to the fermion state in question such that this operator leads to a nilpotent solution when the appropriate phase factor is applied.

A significant aspect of nilpotent quantum mechanics, which has been established in

many previous publications,[3,6,7] includes the fact that, because any interaction or change in any part of the system requires correlation with the entire universe, the fermion is necessarily an open system. This is built into the structure of the E and $\mathbf{p}$ terms within the fermion bracket when they are converted into operators, as all possible interaction potentials have to be taken into account. We can thus only ensure conservation of energy over the entire universe; the first law of thermodynamics (or energy conservation) requires supplementing with the second and third to take into account the operations of the entire universe. To describe a reversible or zero-entropy process in thermodynamics we need to be able to define an isolated system, but, in nilpotent quantum mechanics, all processes within a system of any kind are also processes involving the entire universe. Local processes are also nonlocal, and vice versa.[9]

The algebra of the nilpotent structure as a commutative combination of vector, quaternion, and complex units, or a commutative combination of a Clifford algebra of 3D space with all its subalgebras, can also be interpreted as a commutative combination of two Clifford algebras of 3D space.[7] In effect, the first three alphabets in a physical description of the universe, respectively representing mass (scalar), time (complex algebra) and charge (quaternions), combine mathematically to produce the equivalent of a vector or Clifford algebra, identical to that of the fourth alphabet, but commutative with it. In other words the combined algebra is equivalent to that of *two* commutative spaces, only one of which corresponds to real or observed space. This is because the 'space' combining mass, time and charge does not describe a single physical concept but a combination of three independent ones, and only has completion as a mathematical combination of different physical things. Space is the only physical concept that we can actually observe and directly measure, and so the combined 'space' of mass, time and charge can never be directly observed. It is 'real', however, in providing all the known changes that occur in the observed space as a result of the actions of mass, time and charge. In nilpotent quantum mechanics, the two spaces are dual but the unobserved one provides the vacuum or 'rest of the universe' for the observed fermion state. The fact that it is 'real' is evidenced by many well-known vacuum effects, in particular the *zitterbewegung*, which ensures that any known fermion spends half of its existence in this alternative space, and the oscillation between the two 'spaces' provides the fermion's rest mass.

4 Many Worlds or One Fermion

Two interpretations of quantum mechanics which are of particular interest in connection with the universal rewrite system are the many worlds interpretation, first proposed by Hugh Everett in 1957,[10] and the one-fermion theory of the universe, first put forward as the one-electron theory by John Wheeler in the 1940s, and made public by Richard Feynman in his 1965 Nobel lecture.[11] According to the many worlds interpretation, a quantum system in a state of superposition splits into multiple universes, each of which represents one of the superposed states. When a measurement is made by an observer, the states not measured carry on with their own evolution but it is one which will be forever out of reach of the observer. In effect, the universe splits

into two at every measurement, with only one part being observed. An interpretation of this idea in nilpotent terms would involve the dual creation of the nilpotent state and its vacuum partner. A measurement or observation would cause a bifurcation between what becomes part of the real (fermionic) state, and what is consigned to vacuum. The only meaning of 'event', in physical terms, lies in a decision-making of this kind. It is the creation of a nilpotent in a new state, a new ψ in the overall progression determined by Hilbert space, with a doubling of the algebra, which also means creating a new vacuum into which the non-observed states can be consigned.

A possibility in line with the nilpotent / self-organization proposed here, and Feynman's sum of (phase) histories description of quantum mechanics, follows from Huygens' principle of secondary sources, where the nilpotent self-organization is the universal source, and galactic structure constitutes the secondary sources of this source, so that in this version of the multiverse, each 'universe / galaxy' is now a real 3D dynamic entity / object, many of which are visible from Earth. Based on the same information process as is evident in quantum mechanics, the continuing self-organization then predicts the birth of new galaxies as a natural consequence of further self-emergence.

An alternative metaphor for quantum mechanics is the 'one electron' theory of the universe, proposed by Wheeler, which can be refined to a 'one fermion' theory. In principle, all events in this theory can be explained by a single fermion moving backwards and forwards in both space and time, and producing the effect of many fermions appearing to act simultaneously. This is because time reversal is equivalent to switching the sign of E or changing fermion to antifermion, so allowing creation and annihilation of many fermions by describing the world line of a single one. It corresponds exactly to a theory in which the rest of the universe or 'vacuum' acts as a mirror image of a single fermion state. Two previous objections to the original one electron theory were the obvious asymmetry of matter and antimatter in the universe, and the possibility that fermions and antifermions would mutually annihilate. In fact, neither objection holds in the nilpotent theory, since this theory, like the fermion wavefunction itself, requires the coexistence at all times of equal numbers of fermion and antifermion states, though the fermions and antifermions *occupy different spaces*, which is why only the fermions (in real space) are observed, the antifermions defining a dual but unobserved vacuum space. It is also why they do not mutually annihilate.[7,12]

A single fermion universe encompassing the entire possible histories of the universe fits perfectly with the representation of NUCRS universe by Marcer and Rowlands as a self-organization described in terms of the automorphisms of the complete symmetry Galois group, where the NUCRS is the unique birth order field automorphism of all such possible histories.[13,14] A significant consequence of the one-fermion representation is that we can model an ensemble of fermions as a single fermion, and so apply a form of the nilpotent condition to larger-scale structures, and, in fact, a form of the Dirac nilpotent equation can be developed which uses commutators rather than differentials, and can be used for classically discrete as well as quantum systems, because the $i\hbar$ quantum condition does not need to be applied to the differential operator.[3] This degree of scale-independence between the single fermion and ensemble of fermions is exactly

what we would expect from a renormalization group procedure. Since an ensemble will be less narrowly localised than a single fermion, its vacuum will, therefore, be correspondingly less local. Defining the whole universe as a fermion suggests that we should view such a universe as being constituted from all the possible space and time conditions, or 'bit flips' in the terminology of Seth Lloyd,[15,16] which is precisely what we previously have called vacuum.

But, if all possible space and time conditions for any single fermion constitute its vacuum, or the 'rest of the universe' as seen by the fermion, including any state to which the fermion might evolve at a future time, then the 'one fermion' contains within its vacuum all the possible histories of the entire universe. In principle, this can be seen as an anticipatory process, and, if written in the discrete form of calculus, we can imagine a connection with the anticipatory mechanisms proposed by Dubois.[17,18] Physical events and fermionic states will occur in a universal birth-ordering, an absolute causal sequence in line with the absolute time sequence determined by the second law of thermodynamics. The vacuum state for any fermion would incorporate the entire possible history or histories of the universe within the constraint of its event horizon. In the language of the many worlds interpretation, this would be all possible universes, though in this interpretation they would be only vacuum universes, and not ones realizable by any observer. Though this might seem deterministic, we, in fact, could only define the entire history of the universe by exactly localizing the fermion in both space and time, and this is prevented by the uncertainty principle. The concept remains only an ideal, a way of defining the nonlocal as the sum of all other potential states, in both space and time, as determined by the real or observed states.

5 Chaos Theory and the Fractal Universe

The growth of a chaotic system, like an event in quantum mechanics, provides a perfect parallel with the universal rewrite system. It has been observed that in a typical situation leading to chaos, say the growth of an animal population, there comes a point at which, when the growth rate increases above a certain value, the equation produces a bifurcation between two possible outcomes. Further increases in the growth rate produce a series of further bifurcations of each bifurcation, at a frequency determined by a single scale factor for preserving self-similarity, the universal Feigenbaum number 4.669 … . This can be seen as a characteristic extension of an alphabet by the creation of a new one, exactly as in the 'create' process involved in universal rewrite. Bifurcations are also important in the related subject of catastrophe theory.

The rewrite system describes the evolution of a *process* rather than a physically-defined system, though the process might itself require a bifurcation in the system. In effect, a near-chaotic system becomes subject to particularly rapid overall change because of its high degree of nonlinearity and interconnectivity, and the bifurcation occurs at the level of the whole system or the process applied to the whole system, rather than in only a part of it. When the process applies collectively, rather than to just part of the system, the expansion of the rewrite structure leads to a complete bifurcation or doubling of the options, and we would expect this to happen repeatedly. A system

operating near chaos, with a high degree of nonlinearity and connectedness of its parts, will have a high efficiency in transferring free energy, and be subject to rapid development. The existence of a universal scale factor in the outcome may be taken as a consequence of the relative holism of a system or process on the edge of chaos.

Significantly, while quantum mechanics uses a commutative (Hilbert) space for the processes of superposition and combination, chaos specifically requires *anticommutativity*, the complex plane and the golden section or number, and the NUCRS combines both aspects. In creating the same 'nilpotent' formulation $X_n^2 = 0$ at all levels of its rewrite structure, the universal rewrite system becomes self-similar of fractal dimension 2 and so embeddable in the complex plane. We have previously shown that this corresponds to the universal fractal attractor of the golden number and relates to the wave behaviour seen at the boundary of the Mandelbrot set.[13]

6 Entropy and Information

A significant fact about the universal rewrite system is that, because every alphabet always includes the previous one, it is intrinsically irreversible. Every new alphabet is always necessarily an extension of the last. And it provides the simplest of all definitions of entropy. The constant bifurcation at every new creation produces 2^n components at the n^{th} stage. If we take the standard definition of entropy, where W is the number of available microstates, $S = k \ln W$, where k is only there because we have historically separated the measurement of temperature from that of energy on the basis of the properties of water. Really, temperature T has no meaning except as kT. We can't use T in any physics without k. So the true measure of standard entropy is $\ln W$, which is just a number. Now, we can simply redefine entropy, say S', as the number n, the order of the alphabet from any given beginning. Then, the previous measure of entropy S becomes $k\, n \ln 2$, just a choice of numerical factor: $S' = S / k \ln 2$. The number of equally probable microstates at the n^{th} stage is 2^n, and this gives a measure of the increasing complexification / disorder. This is true whatever the rewrite process is, and at whatever level – quantum physics, chemistry, biology, the brain, macrosystems of every kind. Because entropy always increases in our experience, we can use this as evidence that the rewrite system is always bifurcating, and the rate of entropy increase will be a measure of the bifurcation that actually occurs, and at the same time a standard clock. We can actually see how rewrite operates in any given system, and we can establish that it is a universal process.

Since digital logic and information in the computational sense may be seen as a specialised result of the universal rewrite procedure, this definition of entropy would accord with Shannon's view of 1948 that at least $kT \ln 2$ must be dissipated per transmitted bit of information, based on the assumption that communication in linear systems is done by waves, and that part of the energy of which the message consists must be dissipated, and so lost from the message.[19] A paper by Rolf Landauer on 'Energy requirement in communication' (1987) begins with this then-accepted view.[20] If the energy in the message is indeed dissipated, this may be a minimum dissipation, and also an information measure. Self-organization allows us to take this loss as in fact

a measure, not simply of the loss, but of the information which enables the self-organization to proceed accordingly, and to treat the self-organization as if it is a complete binary tree of ordinal measure and has a natural canonical power series structure, which the NUCRS construction shows must indeed be the case. At some scale, the measure of information transfer and entropy increase is determined directly by the level of the alphabet reached in the universal rewrite process.

Temperature is, of course, a global parameter, which shows up, for example, in the radiation formula as kT because it concerns the summation over each linearly independent wave phenomenon phase θ across the entire universe, i.e. from zero to infinity, which is how k, $\hbar$ and c become fundamental global scaling constants of the universal rewrite system. Of course, the information in nilpotent quantum mechanics is not lost, but rather accumulated to make the global process irreversible. It passes from nonlocal / indistinguishable to local / distinguishable. Translations from nonlocal to local effects are discussed in the accompanying paper by Rowlands on 'Local and Nonlocal'.[8] Such translations cause time asymmetry because the local requires asymmetric, timelike and consequently irreversible solutions, whereas the nonlocal does not. As the paper suggests, all nonlocal processes also have local manifestations, and this appears to be how the time component of the vacuum manifests itself in local effects. Observability is an indication of an event in the present, for the future remains as part of the unobservable or nonlocal vacuum.

Now, referring to the bifurcation process which is the manifestation of the universal rewrite system, we can say that the rate at which it happens must be proportional to the (free) energy involved. The higher the energy, the higher the rate of bifurcation events. Near chaotic systems, involving nonlinearity and high connectivity of the components, transfer energy at near maximum efficiency, and so bifurcate rapidly, generating a correspondingly large measure of entropy. This is especially true of biological systems, which have evolved to be highly organized and composed of many interconnecting parts. Rapid information transfer and states of high entropy become strongly correlated. The process in general acts as a 'clock', with the time interval determined by the rate at which the available options are doubled. The fact that all natural systems are entropic, and irreversible in time, is evidence that all act in terms of the universal rewrite process.

7 Entropy and the Square Roots of –1

The conclusion that number and entropy are synonymous is supported by a number of additional facts. Firstly, Conway's work on the surreal numbers is defined by a rewrite methodology and generates the complete mathematical field of all numbers.[21] A recent paper by Marcer and Rowlands[12] supplements this with the fact that it may also be described by the complete symmetric / Galois group of automorphisms – the class of all possible groups / permutations. Further input from a paper by Clement, Coveney and Marcer,[22] which investigates the concept of Kauffman[23] that the parentheses (,) is a generalised symbolic form of the universal computational gate, as for example in the Dirac notation < | > or Conway's Dedekind cut (|) above. It singles out XOR rather than NAND as an instance of the universal physical / quantum gate, and, as pointed out

by Clement, XOR corresponds to the binary Gray code and Hamiltonian path or circuit defined as a path where each point is visited only once, exactly as in the NUCRS. This property of a Hamiltonian path may be relevant to the Riemann zeta function (discussed in the next section), where Hardy proved that there are an infinite number of zeros, as it would indicate these zeros as roots are all unique.[24]

Secondly, we can consider self-organization in a single heat bath, where Second Law irreversibility and entropy are idealised by the Quantum Carnot Engine of Scully et al.[25] It results in new states of matter as a consequence of quantum coherence parametrised by phase θ. That is, in an increase of hierarchical complexity and not just a decrease as might be supposed in the case of the classical Carnot Engine, and which on Earth is evidently the case. In the Quantum Carnot Engine, the new emergent states of matter concern quantum phase so that phase is implicitly a measure of entropy.

In Wilson's renormalization group methodology[26] applied to Einstein's conservation equation, this relates to the concept of scaling and the general mathematical form for the single part of the free energy near critical points c. This says that, at each value of $\varepsilon = (T - T_c) / T_c$, there is a dominant scale on which the fluctuations are important – the point where the Quantum Carnot Engine comes into play. Wilson's mathematical solution for finding such points, where for example Laudau's classical solution fails and the Ising model succeeds, is to map the space of the Hamiltonian energy onto itself. The repeated mapping then eventually leads to one or more of the stable points, as attractors (with the implication that $\mathcal{H}$ itself may be an unstable fixed point). Wilson also shows that this methodology concerns the concept of universality and of a universality class, where given the dimensionality of the system d and the symmetry of the order parameter p, only a number (usually small) of fixed points are available, i.e. the class depends only on dimensionality d and the symmetry of p.

Suppose now we take i as the order parameter, giving us the fermion states as the universal class $\mathcal{H}$ of solutions X of the Einstein conservation of energy equation, where self-organization necessitates the mapping of $\mathcal{H}$ onto itself. Then for every set of quaternions $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$, there is a local closure because, for each of the elements i, $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$, there is a unit element, from which we might infer that the $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ and i concern some unit and a scaling constant c, such as the velocity of light and special relativity. This gives 4 as the dimensionality (of space-time) where, as Wilson and Fisher found, this concerns a parameter $p = 4 - d$, and taking d with respect to any i as 2 might foot the bill. This definition of i as the order parameter certainty makes sense in the context of exp ($i\theta$) as the wavefunction,

The concept of i as an information measure, where + and – cannot be distinguished, and $i^2 + 1 = 0$ is an information metric, fits perfectly into the NUCRS picture of $\mathcal{H}$ as describing the dark energy and as a nondeterministic (quantum) model of computation, where only the relative quantum coherence phase θ, as for example in geometric / Berry phases, can be measured. Such a concept provides us with an ideal mathematical definition of indeterminacy. Taking $i^2 + 1 = 0$ as the metric in relation to the unit disc, concerns the infinite unit vectors each of unique phase θ, so that this metric is not only a phase information metric but also corresponds to a measure of dissipation or entropy.

The infinite square roots of –1 constitute a representation of the NUCRS universal alphabet,[27] and, as pointed out elsewhere, i defines a metric $i^2 + 1 = 0$, and an equation for chaos, character G, the golden number fractal dimension 2. It also has the desired quantum logic √(NOT = –1 = False), describes Feynman's computer constructor universal computational primitive unit wire, realised for example in terms of the chemical bond, and ensures signals in the plane (y, x) are a Fourier duality pair.[5] These attributes would all seem to point to the fact that, in a self-organized system, entropy and negentropy share the same measure i, so explaining a hypothesis postulated by many different authors, including Schrödinger.[28]

8 The Riemann Zeta Function

The conclusion reached in the previous section fits with the fact that the boundary of Mandlebrot set gives way to Julia set wave behaviour, and that Connes has established that Riemann's famous zeta function can be interpreted to describe a quantum physical phenomenology.[29] We can therefore ask the question whether Rowlands and Diaz's discovery about the infinite square roots of –1 as computer-usable generators of the universal rewrite alphabet,[27] sheds new light on the famous unsolved Riemann hypothesis, that all the zero solutions for the complex zeta function have real part ½. For the infinite rewrite alphabet corresponds to the line $(½ + iy)$ as the location of all the NUCRS unique fermion states, spin ½, that are nilpotent, i.e. the nondegenerate zeros of a quantum mechanical universe. This, as Rowlands points out, corresponds to the quaternion structure, equivalent to that of the Pauli matrices,[3] and provides an analytic algebraic mapping onto the integers. The generation of the unique zeros for the zeta function may thus be connected with the unique zeros in the universal rewrite structure.

We suggest the following as an outline proof of the Riemann and Erlangen programme hypotheses. The complex plane defined by the circle at zero is composed of unique unit vectors, each characterized by a unique phase θ, the mathematical span of which is 2π. θ thus spans the whole of mathematical transcendental number field, that is the whole of mathematics – a single body of knowledge as hypothesized in the Erlangen programme, and so must be expressible as a single analytic function in the complex plane. But such a function the zeta function $\zeta(z = x + iy)$, with parameter $i = \sqrt{-1}$, is well known. Parametrically all the zero solutions or roots of ζ can therefore be expressed by mapping i onto itself so that $i.i = -1$, which provides all the infinite unique distinct unit vectors characterized by θ , Consequently, these must all lie on the single line parameter i in the complex plane $z = x + iy$, i.e. on a single vertical line at x. But, since by empirical calculation it is known that at one least one zero (in fact millions) lies on the line $x = ½$, then all the roots of the zeta function must lie on this line. Conway's surreal generation of all the numbers great and small may be seen as a necessary corollary, along with the fact that zeta is predicated on the parameter i where this concerns the qubit logic √(NOT = –1). It is almost certainly significant in this context that the complex plane provides a natural $U(1)$ symmetry, like that which emerges as a direct consequence of nilpotency and the construction of a fermion singularity with spin ½ from a double 3D space.[11] In effect, the production of new

nilpotent fermion states is intrinsically connected with the generation of number and the entropy produced by the universal rewrite system.

9 Conclusion

According to the NUCRS, all systems have the same overall structure, defined by the need to retain zero totality with respect to the rest of the universe, and seemingly all processes that take place over time follow a route defined by increasing entropy. In our interpretation, entropy is an automatic consequence of the bifurcation process which comes from the key requirement of the NUCRS that any change must create a new zero totality structure which is guaranteed to be unique. Indeed, it becomes a signature of the operation of the rewrite structure, as well as an information measure. We have already associated the rewrite structure with the generation of the entire number field, and here we associate entropy and information directly with the concept of number. A particular interest attaches to systems near the edge of chaos, where the bifurcation occurs quickly over the whole system or process applied to the system (rather than, more restrictively, to some component within it) because of the nonlinearity and the interconnectedness of the parts.

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