Theory of Incursive Synchronization and Application to the Anticipation of a Chaotic Epidemic

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Abstract
This paper deals with a general theory of synchronization of systems coupled by an incursive connection. For systems with a time shift, the slave or driven system anticipates the values of the master or driver system by a future time period giving rise to an anticipatory synchronization. Some extensions show the possibility to enhance the anticipatory synchronization, what we call meta-anticipatory synchronization. An application is shown in the case of an epidemic system represented by a chaotic delayed Pearl-Verhulst map representing the incubation duration of infected susceptibles. A slave model of the infected population is incursively synchronized to the infected population master system, the simulation of which showing that the infected population can be anticipated by a time duration equal to the incubation period.
Keywords: chaos, anticipation, incursion, synchronization, epidemics.

1 Introduction

Some years ago, Dubois and Resconi (1993) proposed an incursive system for synchronizing systems with an application to two chaotic Pearl-Verhulst maps.
It was proposed that two disjoint recursions

$$x(n+1) = \mu x(n)[1 - x(n)] \tag{1a}$$

$$y(n+1) = \mu y(n)[1 - y(n)] \tag{1b}$$

can be connected by the following incursion

$$x(n+1) + D_1(n)[x(n+1) - y(n+1)] = \mu x(n)[1 - x(n)] \tag{2a}$$

$$y(n+1) + D_2(n)[y(n+1) - x(n+1)] = \mu y(n)[1 - y(n)] \tag{2b}$$

by which the recursive system is obtained

$$x(n+1) = [(1 + D_2)\mu x(n)[1 - x(n)] + D_1\mu y(n)[1 - y(n)]]/(1 + D_1 + D_2) \tag{3a}$$

$$y(n+1) = [(1 + D_1)\mu y(n)[1 - y(n)] + D_2\mu x(n)[1 - x(n)]]/(1 + D_1 + D_2) \tag{3b}$$

Because the two independent recursions assume values in the interval between zero and one, the two recursions joined by the incursion can be synchronized for adequate values of D_1 and D_2.

This paper will consider an extension of this incursive synchronization for a master (driver) system and a slave (driven) system.

Let us point out that our incursive theory of synchronization is general and it will be shown that some results are similar to synchronization models presented in the scientific literature, as for example by Pyragas (1995) and Voss (2000).

2 Theory of Incursive Synchronization

Let us consider the two general disjoint recursions

$$x(t + \Delta t) = x(t) + \Delta t f(x(t - \tau)) - \Delta t b x(t) \quad (4a)$$

$$y(t + \Delta t) = y(t) + \Delta t f(y(t - \tau)) - \Delta t b y(t) \quad (4b)$$

that are the discrete system of the differential equations

$$dx(t)/dt = f(x(t - \tau)) - bx(t) \quad (5a)$$

$$dy(t)/dt = f(y(t - \tau)) - by(t) \quad (5b)$$

which are retarded differential equations by the time shift τ. When this system is chaotic, the evolution of the two independent variables x(t) and y(t) are not synchronized due to the sensitivity to initial conditions.

The purpose of this paper is to propose an incursive connection of these two systems in view of synchronizing the second equation of y(t), considered as the slave or driven system, with the first equation of x(t), considered as the master or driver system.

For that, let us generalize the incursive connection, given by Dubois and Resconi (1993), with $D_1 = 0$, and $D_2 = D \geq 0$, in the following way

$$x(t + \Delta t) - x(t) + \Delta t b x(t) = \Delta t f(x(t - \tau)) \quad (6a)$$

$$y(t + \Delta t) - y(t) + \Delta t b y(t)$$

$$+ D[(y(t + \Delta t) - y(t) + \Delta t b y(t)) - (x(t + \tau + \Delta t) - x(t + \tau) + \Delta t x(t + \tau)] = \Delta t f(y(t - \tau)) \quad (6b)$$

This incursive system corresponds to the following differential equations

$$dx(t)/dt + bx(t) = f(x(t - \tau) \quad (7a)$$

$$dy(t)/dt + by(t) + D[(dy(t)/dt + by(t)) - (dx(t + \tau)/dt + bx(t + \tau))] = f(y(t - \tau) \quad (7b)$$

The connection of the slave equation of x(t) to the master equation of y(t), is incursive because, in the factor depending of D, the future value of $y(t + \Delta t)$ depends of itself at the future time $t + \Delta t$ and of $x(t + \tau + \Delta t)$ at the future time $t + \tau + \Delta t$. Thus the connection is anticipatory.

The second differential equation 7b can be transformed in the following way:

$$dy(t)/dt + Ddy(t)/dt = f(y(t - \tau)) - by(t) + D[- by(t)) + (dx(t + \tau)/dt + bx(t + \tau))]$$

and with eq. 7a, $dx(t)/dt + bx(t) = f(x(t - \tau)$, we obtain

$dy(t)/dt = [f(y(t - \tau)) - by(t) + D[- by(t)) + f(x(t))]]/(1+D)$

or

$dy(t)/dt = [(1 + D)f(y(t - \tau)) - (1 + D)by(t) + D[f(x(t)) - f(y(t - \tau)]]/(1+D)$

so the two equations system is

$$\mathbf{dx(t)/dt = f(x(t - \tau)) - bx(t)} \tag{8a}$$

$$\mathbf{dy(t)/dt = f(y(t - \tau)) - by(t) + K[f(x(t)) - f(y(t - \tau))]} \tag{8b}$$

where $K = D/(1 + D)$, is the coupling factor.
When $\tau = 0$, this synchronization is similar to the weak ($K = 1/3$) and strong ($K = 1/2$) synchronization presented by Pyragas (1995) for the chaos map $f(x) = 4x(1 - x)$.
As $D \geq 0$, we have the interval of values for K, $0 \geq K \geq 1$, so

$$dx(t)/dt = f(x(t - \tau)) - bx(t) \tag{9a}$$

$$dy(t)/dt = (1 - K)f(y(t - \tau)) - by(t) + Kf(x(t)) \tag{9b}$$

If the values of x(t) and y(t) are in the interval]0,1[in the disjoint systems, the resulting values of y(t) in the coupling will remain in the same interval, due to the fact that K plays the role of a weighting of $f(y(t - \tau)$ and $f(x(t))$.
Let us notice that when $D = 0$, $K = 0$, the original disjoint system 5ab is obtained.
When D is very large, $D >> 1$, K tends to $K = 1$. In this limit case, one obtains

$$\mathbf{dx(t)/dt = f(x(t - \tau) - bx(t)} \tag{10a}$$

$$\mathbf{dy(t)/dt = - by(t) + f(x(t))} \tag{10b}$$

This limit case is similar to the anticipating synchronization presented by Voss (2000).
Let us show that this system is stable by introducing a difference between x and y as

$$z(t) = x(t + \tau) - y(t) \tag{11}$$

The differential equation of z(t) is then

$$dz(t)/dt = f(x(t)) - bx(t + \tau) + by(t) - f(x(t)) = - bz(t) \tag{12}$$

and it is clear that z(t) will tend to zero for $b > 0$. In this case, we obtain the anticipatory relation

$$z(t) = x(t + \tau) - y(t) = 0 \tag{13}$$

or

$$\mathbf{y(t) = x(t + \tau)} \tag{14}$$

This remarkable result means that the slave system y(t) is synchronized to the master system x(t) at the future potential value of $x(t + \tau)$, where τ is the time shift of the master equation of x(t).
More the time shift τ is large, more the slave system anticipates the values of the master system, even if it is chaotic, as we will show in the next section.
When the master system is without time shift, $\tau = 0$, the two systems are synchronized in a normal way.

A first extension to synchronize the two system at any time shift consists in making the following modification of the synchronization system 8ab:

$$\mathbf{dx(t)/dt = f(x(t-\tau)) - bx(t)} \tag{15a}$$

$$\mathbf{dy(t)/dt = f(y(t-\tau)) - by(t) + K[f(x(t-\tau_0)) - f(y(t-\tau)]} \tag{15b}$$

where τ_0 is a time shift the interval of which is given by $\tau \geq \tau_0 \geq 0$.

When $\tau_0 = 0$, we obtain an anticipatory synchronization.

When $\tau_0 = \tau$, the synchronization is normal, similarly to the time-delayed dissipative coupling presented by Voss (2000).

So our general anticipatory synchronization is governed by the anticipatory time shift τ_a

$$\boldsymbol{\tau_a = \tau - \tau_0} \tag{16}$$

The discrete equation system of this differential synchronization is written as

$$\mathbf{x(t+\Delta t) = x(t) + \Delta t f(x(t-\tau)) - \Delta t b x(t)} \tag{17a}$$

$$\mathbf{y(t+\Delta t) = y(t) + \Delta t f(y(t-\tau)) - \Delta t b y(t) + \Delta t K[\, f(x(t-\tau_0)) - f(y(t-\tau))]} \tag{17b}$$

A second extension to the system 15ab is given by

$$\mathbf{dx(t)/dt = f(x(t-\tau)) - bx(t)} \tag{18a}$$

$$\mathbf{dy(t)/dt = f(y(t-\tau)) - by(t) + K[f(x(t-\tau_0)) - f(y(t-\tau-\tau_1)]} \tag{18b}$$

where τ_1 is an additional anticipatory time of synchronization for which an anticipatory synchronization is possible even if $\tau = \tau_0 = 0$.

The discrete equations of this differential synchronization are written as

$$\mathbf{x(t+\Delta t) = x(t) + \Delta t f(x(t-\tau)) - \Delta t b x(t)} \tag{18c}$$

$$\mathbf{y(t+\Delta t) = y(t) + \Delta t f(y(t-\tau)) - \Delta t b y(t) + \Delta t K[\, f(x(t-\tau_0)) - f(y(t-\tau-\tau_1))]} \tag{18d}$$

When the two systems are synchronized, $f(x(t-\tau_0)) = f(y(t-\tau-\tau_1))$, which means that

$$y(t) = x(t+\tau+\tau_1-\tau_0) = x(t+\tau_a) \tag{19}$$

with

$$\boldsymbol{\tau_a = \tau + \tau_1 - \tau_0} \tag{16a}$$

A third extension consists is adding a second slave synchronization system which will synchronize with the slave system in the following way

$$\mathbf{dx(t)/dt = f(x(t-\tau)) - bx(t)} \tag{20a}$$

$$\mathbf{dy(t)/dt = f(y(t-\tau)) - by(t) + K[f(x(t-\tau_0)) - f(y(t-\tau)]} \tag{20b}$$

$$\mathbf{dy_1(t)/dt = f(y_1(t-\tau)) - by_1(t) + K_1[f(y(t-\tau_{01})) - f(y_1(t-\tau)]} \tag{20b}$$

When the three systems are synchronized,

$$y_1(t) = y(t+\tau-\tau_{01}) = x(t+2\tau-\tau_0-\tau_{01}) = x(t+\tau_a) \tag{19a}$$

with

$$\boldsymbol{\tau_a = 2\tau - \tau_0 - \tau_{01}} \tag{16b}$$

so the anticipation is extended to two times the delay τ. I will call this second order anticipatory synchronization, a meta-anticipatory synchronization.

The discrete equation system of this differential synchronization is written as

$$\mathbf{x(t + \Delta t) = x(t) + \Delta t f(x(t - \tau)) - \Delta t b x(t)} \quad \mathbf{(21a)}$$

$$\mathbf{y(t + \Delta t) = y(t) + \Delta t f(y(t - \tau)) - \Delta t b y(t) + \Delta t K[f(x(t - \tau_0)) - f(y(t - \tau))]} \quad \mathbf{(21b)}$$

$$\mathbf{y_1(t + \Delta t) = y_1(t) + \Delta t f(y_1(t - \tau)) - \Delta t b y_1(t) + \Delta t K_1[f(y(t - \tau_{01})) - f(y_1(t - \tau))]} \quad \mathbf{(21c)}$$

Further extensions can be made in adding more slave systems $y_2(t)$, $y_3(t)$, ... $y_n(t)$ in cascade, giving a meta-anticipation of $\tau_a = (n + 1)\tau$.

3 Application of Incursive Synchronization to a Chaos Epidemic

An application of the incursive synchronization is now considered for an epidemic system. In a first subsection, we will build a model of epidemic as a delayed Pearl-Verhulst system. In a second subsection, some properties of such a delayed Pearl-Verhulst system will be pointed out by numerical simulations. In a third subsection, the theory of incursive synchronization will be applied to such a delayed Pearl-Verhulst system in view of simulating the anticipatory evolution of the epidemic.

3.1 A Mathematical Model of Epidemic as a Delayed Pearl-Verhulst System

A rather simple model of epidemic is given by

$$dS(t)/dt = -aS(t)I(t) + bI(t) \quad (22a)$$

$$dI(t)/dt = +aS(t)I(t) - bI(t) \quad (22b)$$

where $S(t)$ is the susceptible population and $I(t)$ is the infectious population. The parameter a is the contact rate between the susceptible and infectious populations and b is the rate of decrease of the infected population which recovers as susceptible. In this model, the total population $S(t) + I(t)$ is constant. Indeed

$$d(S(t)/dt + dI(t)/dt = 0 \quad (23)$$

so

$$S(t) + I(t) = C \quad (24)$$

and the two equations reduce to the following equation

$$dI(t)/dt = +aI(t)[C - I(t)] - bI(t) \quad (25)$$

This equation is similar to the Pearl-Verhulst equation. The discrete form of this equation is the well-known chaos map.

As pointed out by Dubois and Sabatier (1998), a time delay of the susceptible population to become an infectious population is more adequate: it is not instantaneously that the susceptibles become infected.

In general, susceptibles become infected after a certain time period τ of incubation. In taking into account such a time shift, the equation system 22ab can be generalized as

$$dS(t)/dt = -aS(t-\tau)I(t-\tau) + bI(t) \quad (26a)$$

$$dI(t)/dt = +aS(t-\tau)I(t-\tau) - bI(t) \quad (26b)$$

Similarly to the original model, there is also the conservation of susceptible and infected populations

$$dS(t)/dt + dI(t)/dt = 0 \quad (27)$$

so

$$S(t) + I(t) = C \quad (28)$$

and the two eqs. 26ab are reduced to the following equation

$$\mathbf{dI(t)/dt = +aI(t-\tau)[C - I(t-\tau)] - bI(t)} \quad \mathbf{(29)}$$

which is a time delayed Pearl-Verhulst equation.

3.2 Some Simulations of the Delayed Pearl-Verhulst System

For simulating such a delayed Pearl-Verhulst system, eq. 29 is written in the following discrete equation

$$\mathbf{I(t+\Delta t) = I(t) + \Delta t\, aI(t-\tau)[C - I(t-\tau)] - \Delta t bI(t)} \quad \mathbf{(30)}$$

Figure 1 shows the bifurcation diagram of eq. 30, with $\Delta t = 1$, $b = 1$, $\tau = 0$, $C = 1$, and the parameter a varying from 0 to 4: this is the well-known Pearl-Verhulst map.

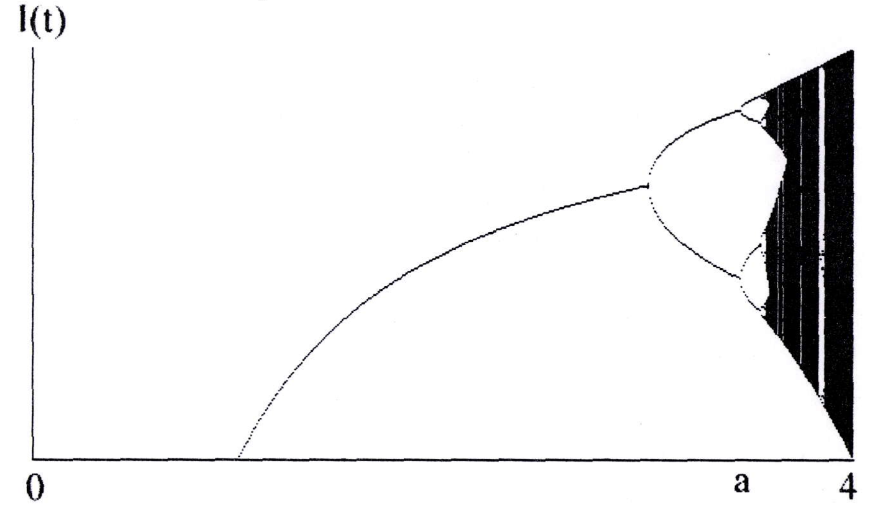

Figure 1: Bifurcation diagram of the Pearl-Verhulst map given by eq. 30.

Figure 2 is the simulation of eq. 30, with $\Delta t = 0.95$, $b = 1$, $\tau = 1$, $C = 1$, and the parameter a varying from 0 to 4.

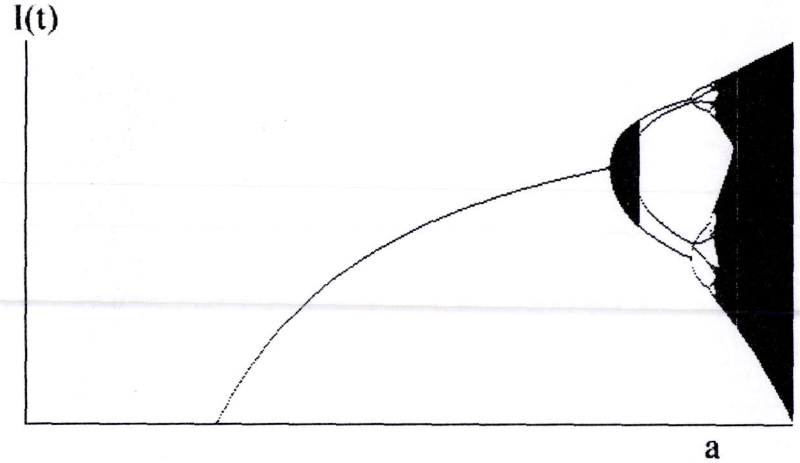

Figure 2: Bifurcation diagram of the delayed Pearl-Verhulst map, with a time shift of $\tau = 1$.

Figure 3 is the simulation of eq. 30, with $\Delta t = 0.95$, $b = 1$, $\tau = 2$, $C = 1$, and the parameter a varying from 0 to 4.

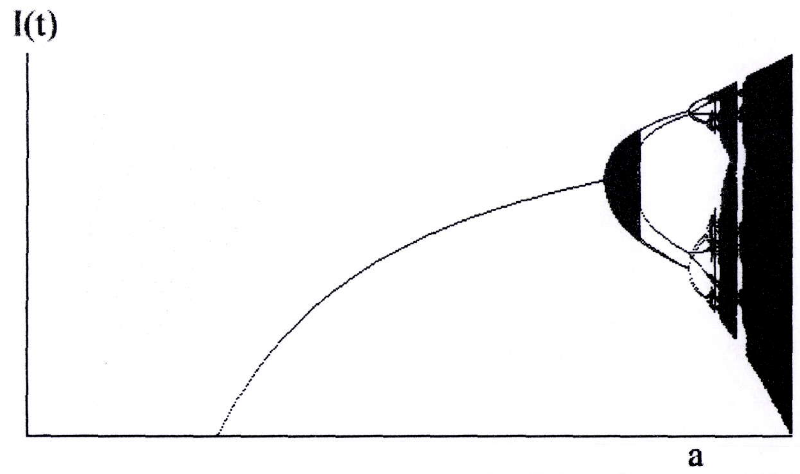

Figure 3: Bifurcation diagram of the delayed Pearl-Verhulst map, with $\tau = 2$. There are different bifurcation specs, similarly to those in Hénon's strange attractor. So, this bifurcation diagram depends on initial conditions.

Figure 4 is the simulation of eq. 30, with $\Delta t = 0.95$, $b = 1$, $\tau = 50$, $C = 1$, and the parameter a varying from 0 to 4. This is similar to the chaos map.

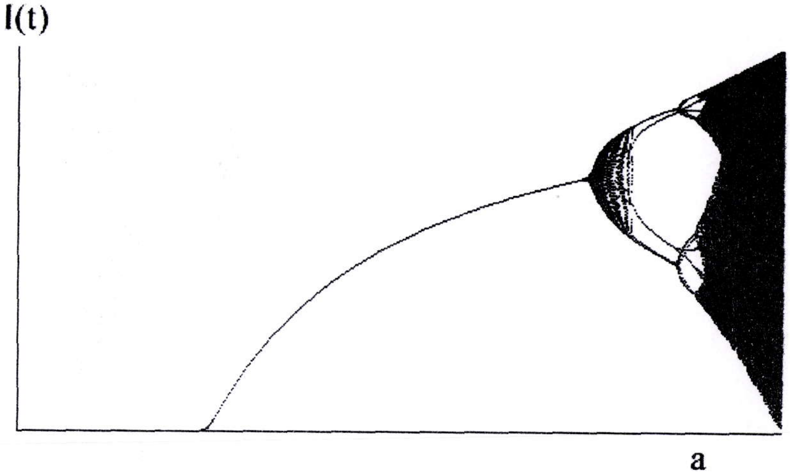

Figure 4: Bifurcation diagram of the delayed Pearl-Verhulst map, with a time shift of $\tau = 50$, and $\Delta t = 0.95$.

Figure 5 is the simulation of eq. 30, with $\Delta t = 0.1$, $b = 1$, $\tau = 50$, $C = 1$, and the parameter a varying from 0 to 4. With such a lower value of $\Delta t = 0.1$, chaos transforms to a strange attractor, as shown in fig. 5a.

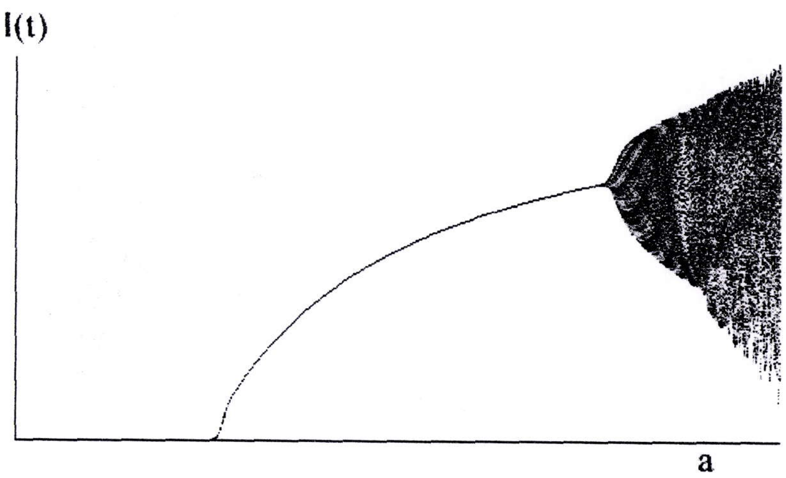

Figure 5: Bifurcation diagram of the delayed Pearl-Verhulst map, with a time shift of $\tau = 50$, and $\Delta t = 0.1$.

Figure 5a is the simulation of eq. 30, with $\Delta t = 0.1$, $b = 1$, $\tau = 50$, $C = 1$, and $a = 4$, corresponding to the bifurcation diagram of fig. 5. This is the diagram of $I(t + \tau)$ versus $I(t)$, for $t = 0$ to 5000.

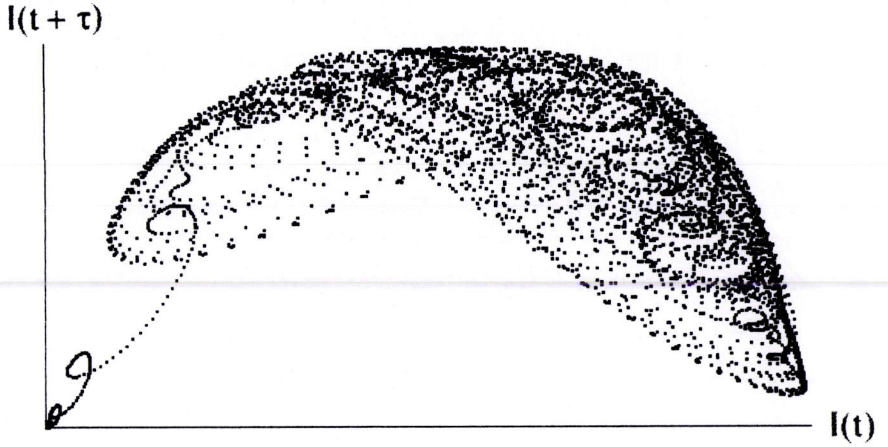

Figure 5a: Diagram of $I(t + \tau)$ versus $I(t)$.

Figure 5b is the simulation of eq. 30, with $\Delta t = 0.062$, $b = 1$, $\tau = 50$, $C = 1$, and $a = 4$. This is the diagram of $I(t + \tau)$ versus $I(t)$. In comparing with fig. 5a, for which $\Delta t = 0.1$, it is pointed out that the system is very sensitive to the value of Δt.

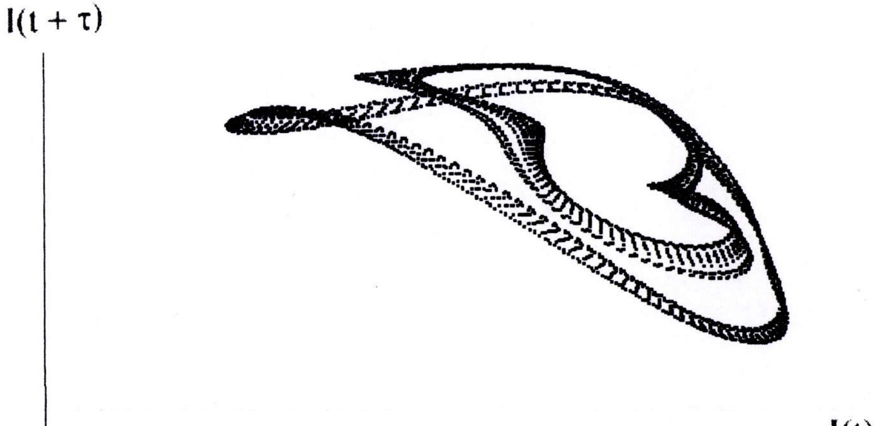

Figure 5b: Diagram of $I(t + \tau)$ versus $I(t)$.

Figure 6a is the diagram of I(t + τ) versus I(t), for eq. 30, with Δt = 0.068, b = 0.97, τ = 64, C = 1, and a = 4, with the initial conditions, I(0) = 0.99 and I(t) = 0 for t = 1 to 64.

This case is interesting because this strange attractor can give rise to a periodic attractor with the different initial conditions, I(0) = 0.99 and I(t) = 0 for t = 1 to 64, as shown in Fig. 6b (due to the value of b = 0.97, a slight negative value of I(t) occurs).

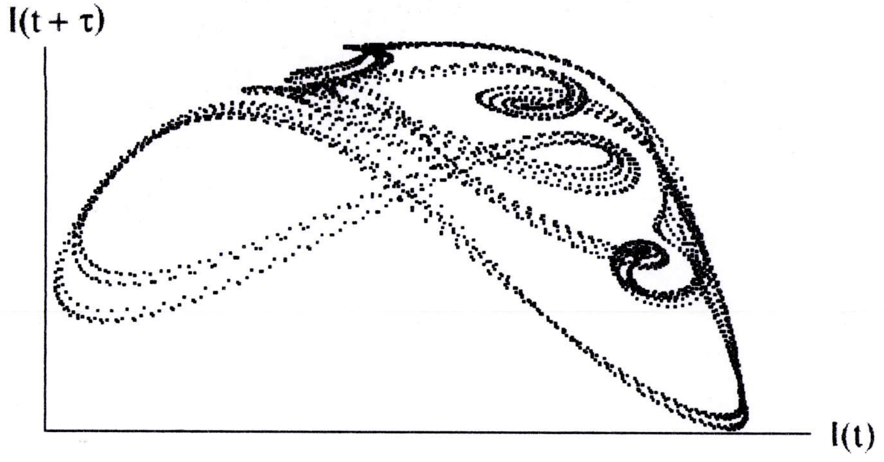

Figure 6a: Diagram of I(t + τ) versus I(t), showing a strange attractor.

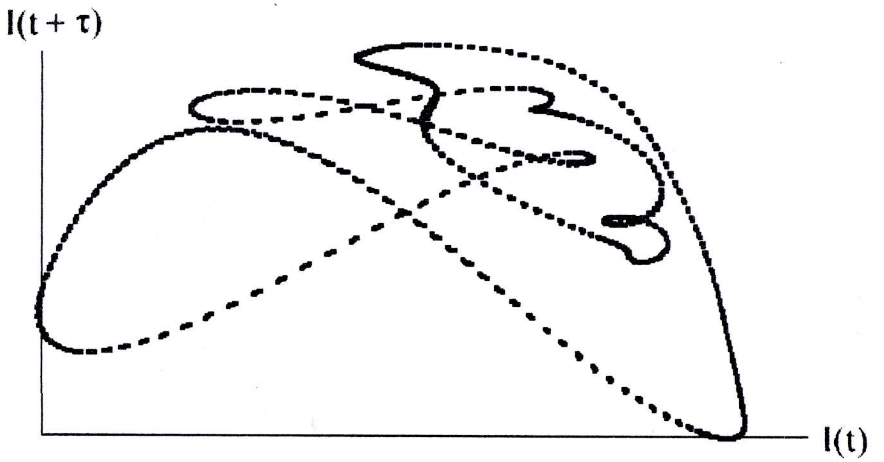

Figure 6b: Diagram of I(t + t) versus I(t), showing a periodic attractor, with different initial conditions from those of Fig. 6a.

3.3 Simulations of Anticipatory Synchronization of Delayed Pearl-Verhulst Map

Let us now apply the incursive synchronization theory to the delayed Pearl-Verhulst system in view of anticipating the evolution of the such a system.

Let us consider the equation 30 as the master or driver system and let us construct a slave or driven system.

In applying eqs. 17ab to the eq. 30 system, the synchronization system is given by

$$I(t + \Delta t) = I(t) + \Delta t a I(t - \tau)[C - I(t - \tau)] - \Delta t b I(t) \quad (31a)$$

$$I^*(t + \Delta t) = I^*(t) + \Delta t a I^*(t - \tau)[C - I^*(t - \tau)] - \Delta t b I^*(t)$$
$$+ \Delta t[D/(1 + D)][a I(t - \tau_0)[C - I(t - \tau_0)] - I^*(t - \tau)[C - I^*(t - \tau)] \quad (31b)$$

where eq. 31a is the master system I(t) and eq. 31b the slave system I*(t).

Figure 7a is the simulation of eq. 31a for $\Delta t = 0.95$, $a = 4$, $C = 1$, $\tau_0 = 0$, $\tau = 50$, and $b = 1$. This gives the time evolution of I(t) as a function of time t = 0 to 5000. This is a typical chaos pattern. The initial conditions of I(t) are generated by a random generator, I(t) = 0.01 + random(98)/100, for t = 0 to 51, and the initial conditions for I*(t) being equal to zero.

Figure 7b is the simulation of eq. 31b, with D = 1. This gives the time evolution of I*(t) as a function of the time t. After a transient time, the two systems I(t) and $I^*(t - \tau)$ are synchronized, meaning that the slave system I*(t) anticipates the master system I(t) in the following way, $I^*(t) = I(t + \tau)$. This means that the slave system I*(t) anticipates the master system I(t) by an anticipatory duration equal to the delay τ.

In view of showing the anticipatory synchronization, figure 7c gives the time evolution of the difference between the values of $I(t + \tau) - I^*(t)$. So, this confirms that chaos is anticipated by a time duration equal to the delay time.

Figures 8ab are the simulation of eqs. 31ab for $\Delta t = 0.1$, $a = 4$, $C = 1$, $\tau_0 = 0$, $\tau = 50$, and $b = 1$. This gives the time evolution of I(t) and I*(t) as a function of time t = 0 to 1000. It is well-seen that I*(t) is synchronized to I(t) with an anticipation $\tau_a = \tau = 50$.

Figures 9ab are the simulation of eqs. 31ab for $\Delta t = 0.1$, $a = 4$, $C = 1$, $\tau_0 = 25$, $\tau = 50$, and $b = 1$. This gives the time evolution of I(t) and I*(t) as a function of time t = 0 to 1000. So, I*(t) is synchronized to I(t) with an anticipation $\tau_a = \tau - \tau_0 = 50$.

In applying eqs. 18cd to the eq. 30, the synchronization system is given by

$$I(t + \Delta t) = I(t) + \Delta t a I(t - \tau)[C - I(t - \tau)] - \Delta t b I(t) \quad (32a)$$

$$I^*(t + \Delta t) = I^*(t) + \Delta t a I^*(t - \tau)[C - I^*(t - \tau)] - \Delta t b I^*(t)$$
$$+ \Delta t[D/(1 + D)][a I(t - \tau_0)[C - I(t - \tau_0)] - I^*(t - \tau - \tau_1)[C - I^*(t - \tau - \tau_1)] \quad (32b)$$

where τ_1 is an additional anticipatory time.

Figures 10ab are the simulation of eqs. 32ab for $\Delta t = 0.1$, $a = 4$, $C = 1$, $\tau_0 = 0$, $\tau_1 = 10$, $\tau = 50$, $b = 1$. This gives the time evolution of I(t) and I*(t) as a function of time t = 0 to 1000. So, I*(t) is synchronized to I(t) with an anticipation $\tau_a = \tau + \tau_1 - \tau_0 = 60$.
For a master equation without delay, the anticipatory synchronization is $\tau_a = \tau_1 - \tau_0$.

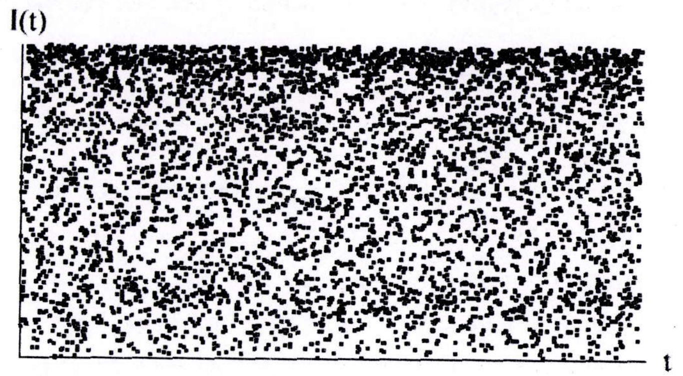

Figure 7a: Delayed chaos map I(t) as a function of time t.

Figure 7b: Anticipatory synchronized I*(t) as a function of time t.

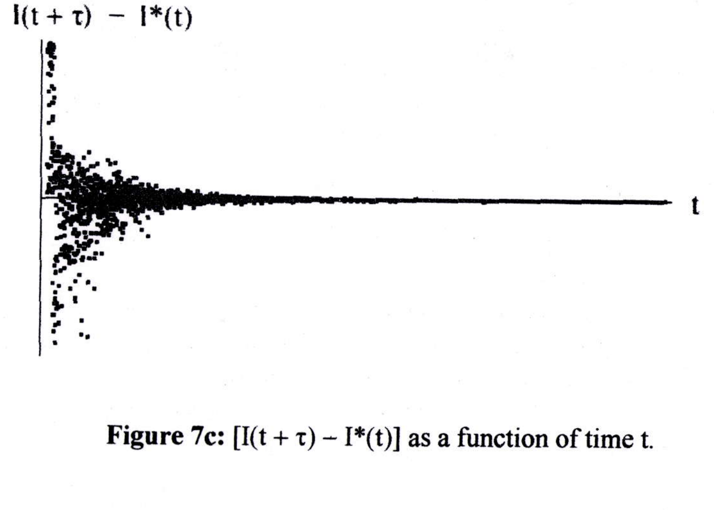

Figure 7c: [I(t + τ) – I*(t)] as a function of time t.

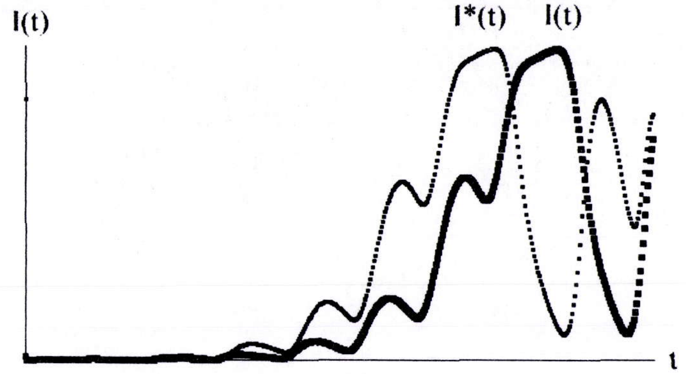

Figure 8a: Anticipatory synchronization of I*(t) and I(t) with $\tau_a = 50$, versus time t.

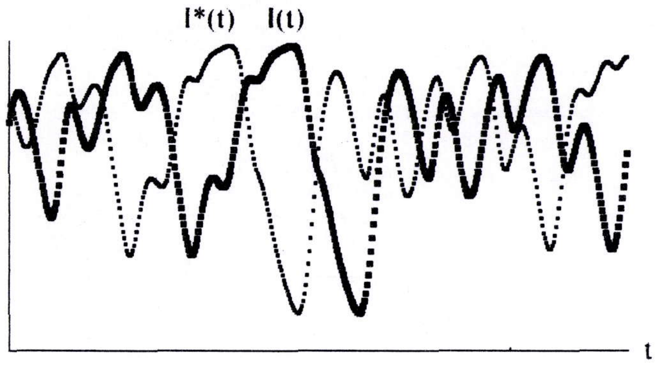

Figure 8b: Continuation of Fig. 8a.

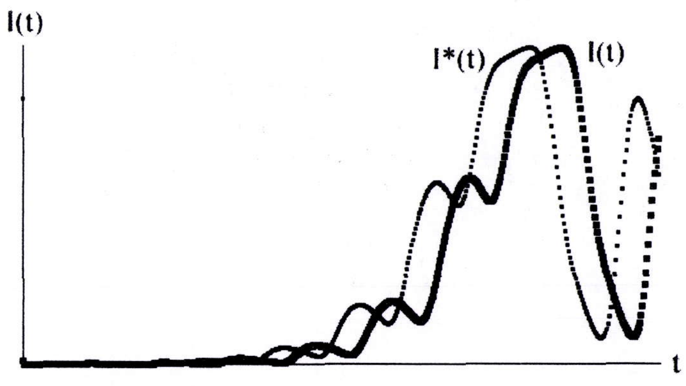

Figure 9a: Anticipatory synchronization of I*(t) and I(t) with $\tau_a = 25$, versus time t.

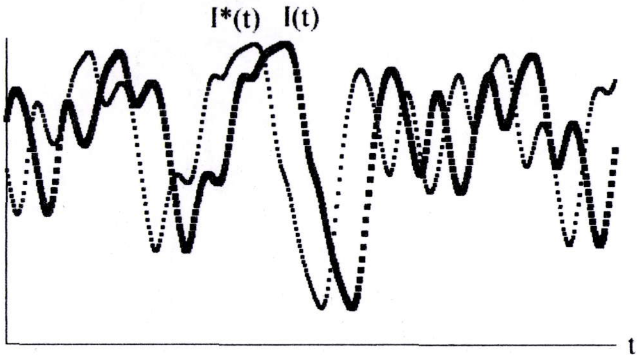

Figure 9b: Continuation of Fig. 9a.

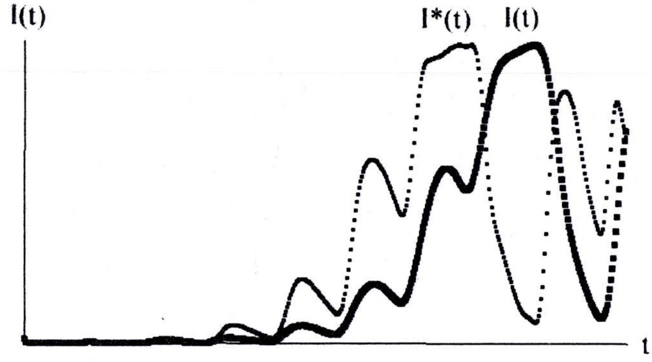

Figure 10a: Anticipatory synchronization of I*(t) and I(t) with $\tau_a = 60$, versus time t.

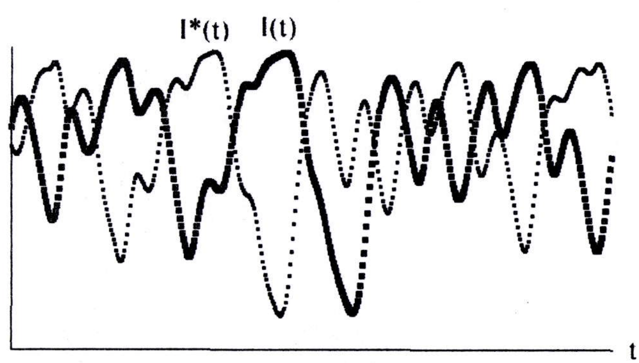

Figure 10b: Continuation of Fig. 10a.

In applying eqs. 21abc to eq. 30, the synchronization system is given by

$$I(t+\Delta t) = I(t) + \Delta t a I(t-\tau)[C - I(t-\tau)] - \Delta t b I(t) \quad (33a)$$

$$I^*(t+\Delta t) = I^*(t) + \Delta t a I^*(t-\tau)[C - I^*(t-\tau)] - \Delta t b I^*(t)$$
$$+ \Delta t[D/(1+D)][aI(t-\tau_0)[C - I(t-\tau_0)] - I^*(t-\tau)[C - I^*(t-\tau)] \quad (33b)$$

$$I_1^*(t+\Delta t) = I_1^*(t) + \Delta t a I_1^*(t-\tau)[C - I_1^*(t-\tau)] - \Delta t b I_1^*(t)$$
$$+ \Delta t[D_1/(1+D_1)][aI^*(t-\tau_{01})[C - I^*(t-\tau_{01})] - I_1^*(t-\tau)[C - I_1^*(t-\tau)] \quad (33c)$$

Figures 11ab are the simulation of eqs. 33abc for $\Delta t = 0.1$, $a = 4$, $C = 1$, $\tau = 50$, $\tau_0 = \tau_{01} = 0$, $b = 1$ and $D = D_1 = 1$. This gives the time evolution of I(t), I*(t) and I_1*(t) as a function of time t = 0 to 1000. So, I_1*(t) is synchronized to I*(t) which is synchronized to I(t) with a meta-anticipation of $\tau_a = 2\tau = 100$.

So, in this meta-anticipatory synchronization, the second order anticipation I_1*(t) is synchronized to the first order anticipation I*(t) of I(t).

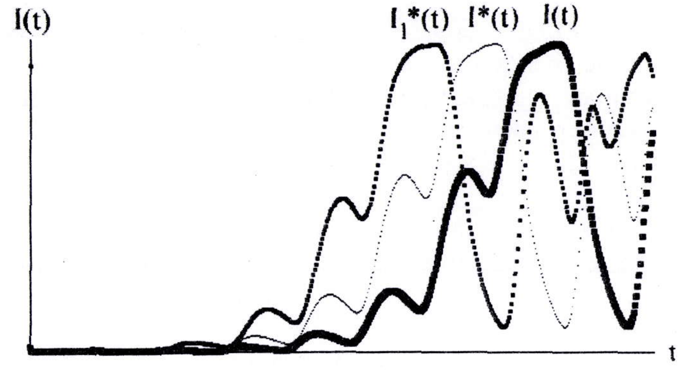

Figure 11a: Meta-anticipatory synchronization of I_1*(t), I*(t), I(t), versus time t.

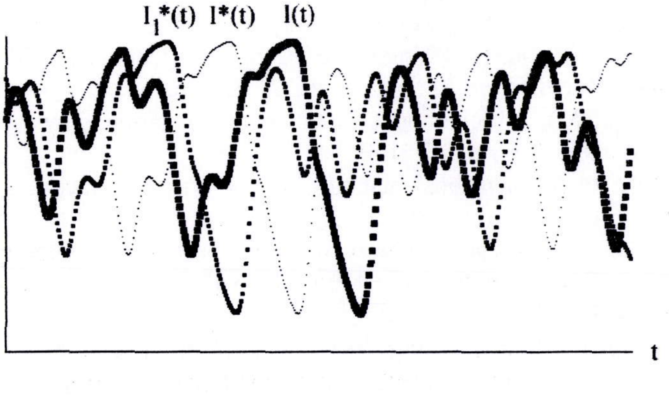

Figure 11b: Continuation of Fig. 11a.

4 Conclusion

This paper presents a new exciting approach to computing anticipatory systems based on anticipatory synchronization of slave or driven systems on master or driver systems. Anticipation of evolution of systems is thus possible by anticipatory synchronization. The theory of incursive synchronization, presented in this paper, is an extension of the incursive synchronization proposed several years ago by Dubois and Resconi (1993). For delayed systems, the anticipation period is the delay time. It was shown that meta-anticipatory synchronization is also possible. In this case, a second order anticipatory synchronization is performed on the first order anticipatory synchronization. An application with numerical simulations is developed for a delayed Pearl-Verhulst system, representing a chaos epidemic. At our knowledge, the delayed Pearl-Verhulst model is original: with the variation of the parameters, chaos transforms to strange attractors. Very interesting generic properties seem to emerge from delayed systems.

This paper is just a first attempt to design computing anticipatory systems based on synchronization. The application to chaos epidemic is just an example to show the power of such an approach. It must be pointed out that synchronization plays a central role in many natural and artificial systems.

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