Collective Behavior of the Metric-Topological Interaction Model by Using Internal Fluctuations in the Flock

Takayuki Niizato[1], and Yukio-Pegio Gunji[1,2]
1. Graduate School of Science, Kobe University, Japan
2. Faculty of Science, Kobe University, Japan
e-mail: t_niizato@yahoo.co.jp

Abstract

There are various views on collective animal behavior such as birds flocking, fish schooling, and insects marching. Despite neither brain nor central control, they behave as a whole. The relation between parts and whole is needed to attack this problem because the collective behavior emerges through the interplay between individuals and the whole collective. However, the bridge between individuals and whole collective remains an open question. Recently, more accurate analysis of flocking behavior has become possible. Cavagna and others found that fluctuations in a flock show the sizes of spatial correlations scales linearly with the size of the flock. They called this scale-invariant coherence of fluctuations "scale-free correlation." We suggest, in this paper, that scale-free correlation fills the gap between individuals' and whole flocks' movement. In a previous study, our model (metric-topological interaction model) succeeded in explaining "scale-free correlation." In this study, the MTI model shows the self-similarity of internal fluctuations and $1/f$ fluctuation. We also show that these critical fluctuations in MTI flocks lead to flock dividing. From these results, we discuss the possibility of internal fluctuations that contribute to smooth movement as a whole flock, despite having the risk of collapsing itself.

Keywords : Scale-free correlation; Self-organized Criticality; Collective Behavior; Parts and Whole; Flocking Model

1 Introduction

It is important to understand a living system in the context of the relation between "parts and whole". A living system makes itself by itself and maintains its systemic property by using its parts. Obviously, we distinguish between "a system" and "making a system" as distinct concepts. A "system", revealing its own entity, has to be distinguished from other systems. On the other hand, "making a system" is very different from "a system"; the notion of "making a system" implies a process of individualizing a system. Obvious individuality of a system is lost in "making a system". "Making a system" has an upper level of the logical status for "system". Thus, to understand a living system, an interface between "system" and "making a system" must be constructed.

Autopoiesis proposed by Maturana and Varela is an important concept to comprehend a living system (Maturana and Varela, 1972, 1980, 1992; Varela, 1979). They define autopoiesis as the interplay between "a system" and "making a system". As we discussed previously, "a system" and "making a system" are logically different concepts. If we try to implement the interaction between "a system" and "making a system", it is inevitable that we undertake a mixture of logically different things. Varela (Varela, 1979), in his earlier research, tried to overcome this problem by constructing a one-to-one correspondence between "a system" and "making a system". Self-organized criticality (SOC) is another example of the interface between parts and whole. Bak and other researchers showed examples of systems that are spontaneously headed to a critical state (Alstrøm and Stassinopoulos, 1995; Bak and Sneppen, 1993; Bak et al., 1987; Stassinopoulos and Bak, 1995). This critical state emerges through local interactions (parts) and global estimations (whole) (Bak and Sneppen, 1993). However, the question of how we should construct the interface between parts and whole is still an open problem.

Collective behavior would be a good example to consider the relationship between parts and whole (Vicsek et al., 1995; Vicsek, 2001). Although there is no obvious central control such as a brain in the aggregation, we can acknowledge that the aggregation moves as though it has one body or one mind (Couzin et al., 2002; Grégoire et al., 2003). In fact, birds flocking, fish schooling and other collective phenomena are sometimes discussed in the context of self-organization (Sumpter, 2006, 2010). Global properties emerge only from local interactions of each individual. However, what is the interface between the global phenomena and local units? To understand collective phenomena, we must construct the interface between parts and whole. Some flocking models are proposed, but they fail to explain the empirical data that we discuss in the next paragraph.

Recently, a more accurate analysis of flocking behavior can be achieved than was formerly possible (Ballerini et al., 2008a,b; Cavagna et al., 2010). Cavagna et al. measured velocity fluctuations of real birds (a precise definition will be discussed later) and found that the range of the spatial correlation does not have a constant value, but it scales with the linear size of the flock (Cavagna et al., 2010). The size of correlation domain is obviously larger than the interaction range of each bird. Birds share more information than they can interact. Cavagna and others called this phenomenon "scale-free correlation". This is a very suggestive result because scale-free correlation indicates that a flock cannot divide into independent subparts. If one individual in a flock changes his direction, its influence would spread to all individuals.

Scale-free correlation suggests that a flock is constituted by the sum of its parts. We consider that scale-free correlation is the appropriate phenomenon for the interface between parts and whole of a flock. Cavagna et al. (2010) suggested that a large correlated domain in a flock is the effective perception range of one individual, making it easier to react to external perturbations such as predator attacks. Here, we point out that the mixture of parts and whole occurs because the perception of one individual connects to the group perception of his effective perception range. Therefore, we can discuss a flock as a living system if a model exists to implement scale-free correlation.

In a previous study, we proposed a flocking model called the "metric-topological interaction (MTI, in short) model" (Niizato and Gunji, 2010). We showed that the MTI model could explain scale-free correlation, and we obtained data in agreement with the empirical result of Cavagna et al. This is an important result that succeeds in explaining scale-free correlation because Cavagna's study only shows scale-free correlation in still images of real flocks. The MTI model has the potential to explain how fluctuations, which emerge as scale-free correlation in flocks, are used and formed dynamically in the flock. In this paper, we will assert that the probability of fluctuations in flocks may play a role in the interface between unit decisions (parts) and the flocking movement (whole) by using the MTI model. Fluctuations in flocks have properties of self-similarity. We also suggest that well-structured fluctuations lead to a flock dividing itself. The oscillation of the absolute value of fluctuation vectors in the MTI model shows well tuning between order and disorder, which is called 1/*f* fluctuation. A fluctuation in an MTI flock always maintains its state on the boundary between order and disorder and positively contributes to flock movement.

2 Result

2.1 Metric-Topological Interaction (MTI) Model

First, we argue a conceptual sketch of our model. The details of our model and its parameters have been previously reported (Niizato and Gunji, 2010). The MTI model is constructed from two different methods of interaction for each agent: a metric interaction and a topological interaction. The agent switches between the metric interaction and the topological interaction. It cannot use both of them simultaneously. If one agent uses the metric interaction, it must not use the topological interaction.

The concept of a metric interaction is based on the self-propelled particle system, proposed by Vicsek et al (1995). Each agent has an interaction radius. If there are agents within this radius, the agent aligns its direction. This interaction range is determined by metric information only. Thus, this interaction is called a “metric interaction”. Couzin et al (2002) added two more layers to the alignment zone (agents adopt the same direction as those that are close by): an attraction zone (agents move away from very nearby neighbors) and a repulsion zone (agents avoid becoming isolated). We follow Couzin’s interaction method when we use “metric interaction”.

On the other hand, recent field studies (Ballerini et al., 2008a,b) reveal that the interaction of each bird is not metric, but topological. Topological interaction means that each bird can interact with its nearest seven neighbors, regardless of distance.

The MTI model is a hybrid model of metric and topological interactions. Under various conditions, each agent of the MTI model switches between metric and topological interactions. The switching condition is determined by the state (agent’s direction) of the agent’s neighbors. To control switching, we set the threshold parameter. For instance, when an agent uses the metric interaction, it always randomly selects two agents within his interaction range and compares the difference in direction between these selected neighbors. If this difference of direction is greater than the threshold parameter, then this agent uses the topological interaction for the next step. On the other hand, when an agent uses the topological interaction, the agent takes the average of his neighbors’ directions within his interaction domain and computes the difference of directions between its direction and this average direction. If this difference is smaller than the threshold parameter, this agent uses the metric interaction for the next step. This switching property comes from each agent comparing the "sameness" of its direction with that of its neighbors. In this sense, using the topological interaction means that an agent tries to behave in the same way as its neighbors. On the other hand, using the metric interaction means that an agent has recognized that its direction is the same as that of its neighbors Thus, it always checks whether it and its neighbors have the same directions. The threshold parameter controls the timing of switching between the metric and the topological interaction for each agent.

Remarkably, this model never requires any external noise. Despite the lack of noise, an agent under the MTI model shows noisy behavior. By switching between two interactions, the interaction range of each agent becomes different. This difference in neighborhood drives each agent to switch its neighborhood more. As a result, a flock operating under the MTI model can change its direction abruptly (Niizato and Gunji, 2010). In the following sections, we analyze flocking phenomena using the MTI model.

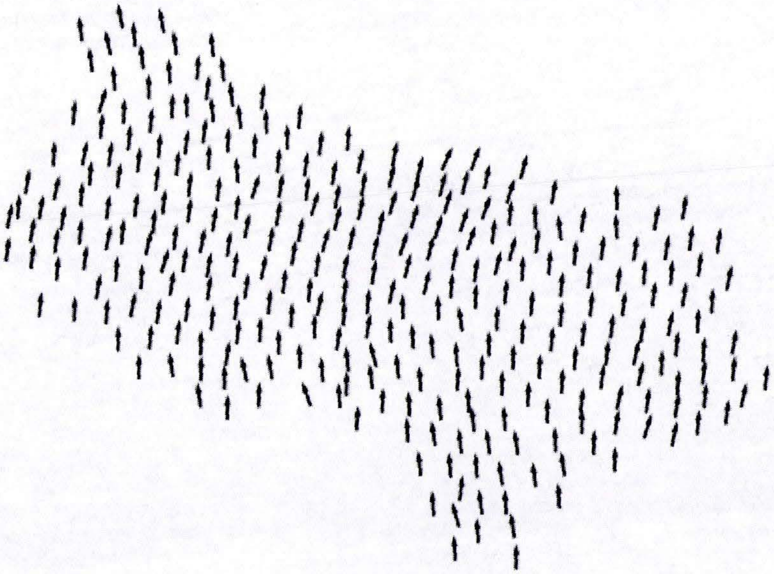
2.2 Renormalization of the Fluctuations

The scale-free correlation suggests that high correlation domains of fluctuations always exist in flocks. Additionally, the range of the correlation domains is not dependent on flock size. In other words, the larger the size of correlation domains, the larger the flock size is. In a previous study, we showed that the MTI model succeeded in explaining the scale-free correlation in two- and three-dimensional simulations. From our simulation, there are always two highly correlated domains in the flocks.

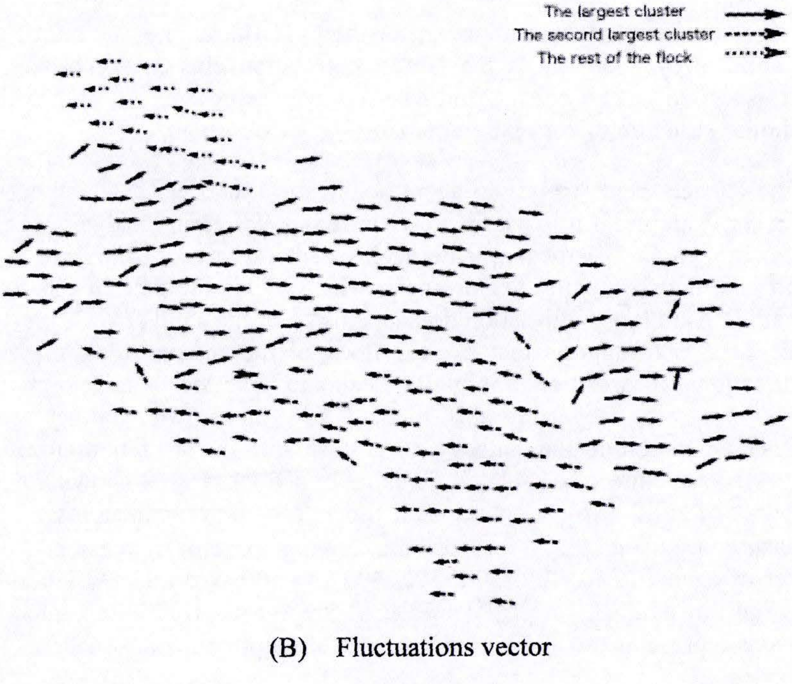
A self-similarity, or fractal, is considered to be an important property when discussing the relation between parts and whole. The self-similarity suggests that a part of the structure contains the information about the whole structure. Therefore, it can sometimes be considered as a bridge between parts and whole. If we confirm that correlation domains in flock have this kind of self-similarity, it could allow us to suggest that the tuning direction that is seemingly random to external observers, has a global meaning in flocks and results in large correlation domains such as scale-free correlation.

To evaluate this idea, we analyzed a flock with the MTI model by using a fluctuation vector. The fluctuation vector can be obtained by subtracting the average velocity vector from each individual velocity vector. In this section, we only consider the directional variance inside the flock. In other words, we only consider the direction of the fluctuation relative to the motion of the flock. This discrete information, left or right, is not an unnatural definition. Analogously, critical phenomena of a spin glass are constituted from up and down spin. In fact, collective phenomena are discussed in the context of statistical dynamics.

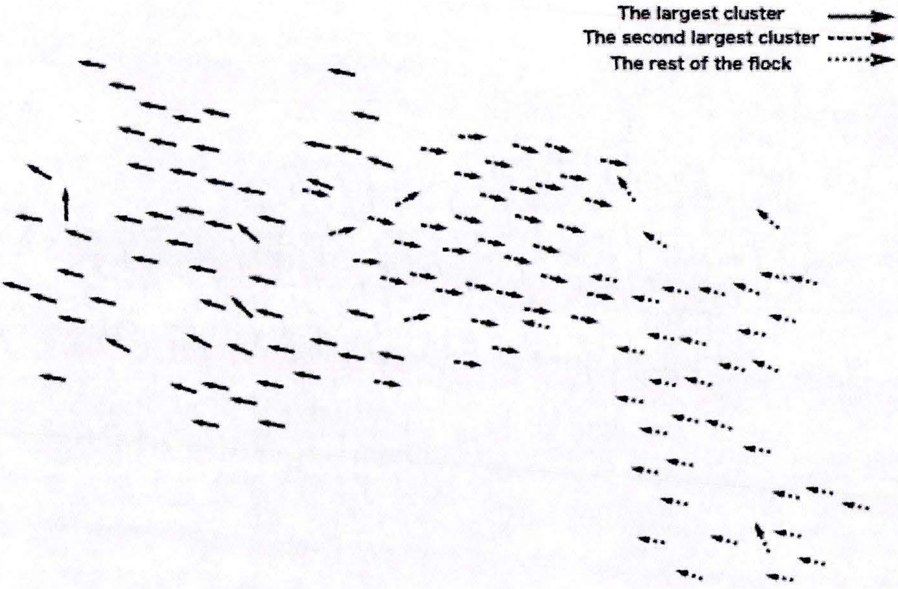
Fig. 1A, 1B and 1C shows an example of renormalization for a flock under the MTI model. We apply renormalization five times for this flock. Fig. 1 (A) is represented by a velocity vector. The number of agents is 300. The size of space is 2000×2000. The Unit length of this space is equal to a unit vector of an agent. We take the unit fluctuation vector (Fig. 1B) and divide this flock into three kinds: solid, broken and dashed arrows. Slid one represents the largest correlated sub-domain in the flock. Broken one represents the second largest sub-domain in the flock. Dashed one corresponds to the rest of the flock.



(A) Velocity vector



(B) Fluctuations vector



(C) Second renormalization of the fluctuation vector

Figure 1: Successional renormalization for the MTI flock. The number of agents is 300. The solid arrow's cluster is the largest correlation cluster; the broken arrow's cluster is the second largest cluster, and dashed arrow represents the rest of the flock. The self-similar structure of correlation domains can be observed.

We define clusters recursively: An agent belongs to a cluster C, which corresponds solid arrow, for example, if it is within a certain radius (80 in our analysis) of any other agents belonging to C. We focus on the agents of solid arrow, which are the largest cluster, and take the fluctuation vector again, which yields the data in Fig. 1C. It can confirm that there are large correlation domains inside of the solid arrow's cluster of the in Fig. 1B. This fact suggests that the sub-flock of the solid arrow's cluster shares different directional information, although it seems to have the same information for a single renormalization. Repeating this method five times, we can get these high coherent fluctuations. Large coherences never vanish until the last renormalization. This reveals a nested structure of correlated domains in a flock. Fig. 2 shows a decreasing rate of an area of solid arrow's cluster as a function of renormalizations. The flock's area is measured by using 25×25 lattices on a moving space of dimension 2000×2000. The number of agents is 50, 100, 200, 300, 400 and 500. We selected a non-dividing flock and took an average rate of decrease in area, over 1,000 step intervals. The horizontal axis represents the number of instances of renormalization. There is the same tendency for all cases to display a rate of decrease of one half from the 1st to 2nd instances of renormalization, which suddenly decreases to 0.35 at upon a 3rd renormalization. Tails of the graph increase after the 4th renormalization because the relative number of agents becomes large for the last few renormalizations. However, the size of the correlation domains is always greater than one third of the correlation domain before renormalization. Thus, the scale-free correlation is also applicable to correlated sub-flocks. This suggests that there is a self-similar structure of fluctuations in flocks.

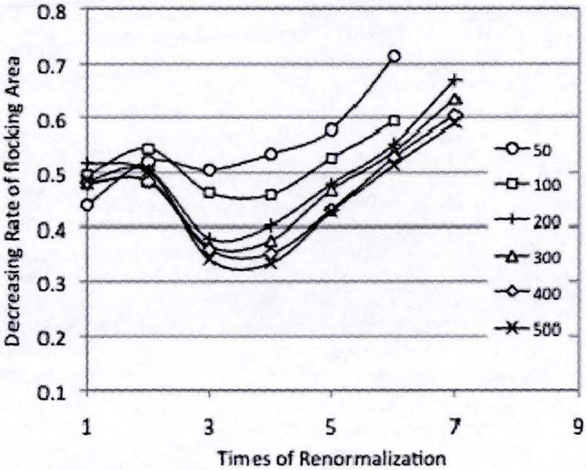


Figure 2: Decreasing rate of flocking as a function of renormalizations. Each shape corresponds to the number of agents (50, 100, 200, 300, 400, 500). For the first and second renormalizations, the rates of decrease show half the degree of flocking present before applying renormalization. For the third and fourth renormalizations, the rate of decrease drops to one-third.

2.3 Fluctuations Induce Split of Flocks

In the previous section, we observed that flocks in the MTI model show a self-similar structure of fluctuations. This self-similarity suggests that directional tuning of each agent who tries to adjust its neighborhood results in global correlation domains such as scale-free correlation. Perpetual local alignments of each agent seem to behave randomly. However, each alignment always connects to the global correlated fluctuation, such as scale-free correlation. In this section, we will discuss the functionality of internal fluctuations contributing to flocking movements. We observed that MTI flocks showed scale-free correlation and that flocks were often divided into two highly correlated parts (colored solid and broken arrow's clusters) as shown in Fig. 1. From many simulations, we observed that when flocks of the MTI model divided into two parts, splitting lines of flocks were mostly on line between two large correlated areas, which correspond solid and broken agents in Fig. 1. In other words, these highly correlated sub-flocks in one flock become individual flocks after the large flock has collapsed. If so, it is possible to consider that these correlated sub-flocks in one flock may have a partial impact on movements as independent flocks.

We investigate the contribution of correlated domains to flock dividing. We report the timing of flock dividing and take snapshots of the 30 steps before flock division. We consider 100 snapshots of MTI flocks. Most of these snapshots show the same pattern as Fig. 3. In other words, the division is located between the two correlation domains which sizes are two largest domains respectively. These two clusters have the opposite direction of one another.

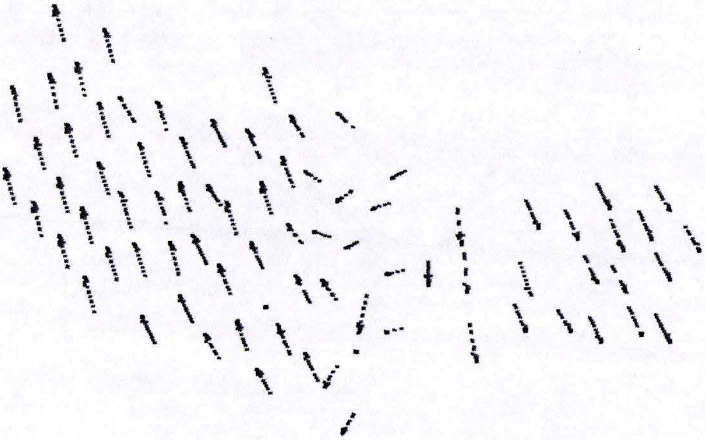


Figure 3: The state of fluctuation vectors for 30 steps before the flock divides.

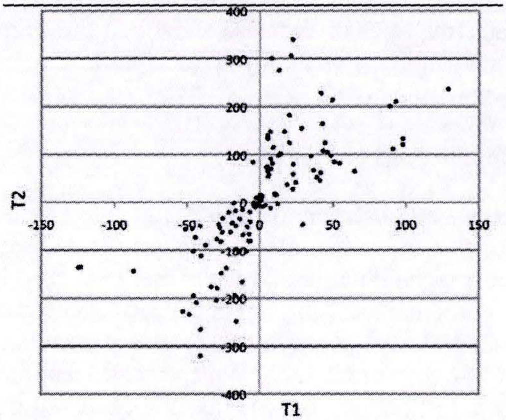


Figure 4: The relation between T_1 and T_2. T_1 is the torque of the largest cluster and T_2 is the second largest cluster. There is a positive correlation between (0.71) T_1 and T_2. This means that each sub flock rotates the same direction

Next, we investigated the torque between two correlated fluctuation vectors. Torque is one of the moments in physics that measures the tendency of a force to rotate an object. The magnitude of a torque is determined by three quantities, the force applied, the length of lever arm that connects an applied force to a basing point, and the angle between two. In this case, the force (F) corresponds to an average of an absolute value of fluctuation vectors in a cluster of interest. A lever arm length (r) is the distance between the center of mass of the whole flock and the cluster of interest. The angle (θ) is determined automatically after these procedures. Then, the torque (T) is

$$T = rFsin(\theta) \tag{1}$$

The value of T indicates the strength of turning, and the sign of T indicates the direction of rotation. We take a torque for two correlated areas whose sizes are the largest and second largest clusters that are indexed 1 and 2. If the signs of T_1 and T_2 are the same, each sub flock rotates in the same direction. Fig. 4 shows the relation between these two quantities (T_1 and T_2). There is a positive correlation (0.71) between T_1 and T_2. This suggests that the coordinated torques of correlated subparts leads to the flock dividing. These subparts become new flocks after flock dividing. This is another aspect of scale-free correlation that is not suggested by Cavagna et al. Scale-free correlation in the flock suggests that a flock continues to coordinate with these correlated sub domains that can be split away.

2.4 Oscillation of the Strength of Fluctuation

In this section, we investigate the amplitude of the fluctuation vector inside of flocks. Fig. 5 shows how the amplitude of the fluctuation vector oscillates with time. We estimate the amplitude of the fluctuation vector as the absolute value of the average of fluctuation vectors that belong to the largest correlated domain. For example, these members correspond to solid arrow's agents in the Fig. 1B. Rate of these members always is above 0.35 (data not shown). One-third of agents have the same direction for their fluctuation vector. The oscillation shown in Fig. 5 shows how these members tune the strength of the fluctuations as a small flock. We compare the oscillation of the largest cluster with the second largest cluster, which is constituted by the broken arrow's agents in Fig. 1B. In the inset of Fig. 5, the gray line corresponds to the oscillation of the second largest cluster. The gray line shows the same shape as the black line. As the amplitude of the black line increases, the amplitude of the gray line also increases. The gray cluster, as we observed in Fig. 1, moves in a direction opposite to the solid arrow's cluster. The second largest cluster compensates for the strength of fluctuation of the largest cluster. This role-sharing tendency is observed throughout our simulation.

As we discussed previously, fluctuations in the flock had the potential power to split the flock in some cases. Thus, flocks must suppress increases in fluctuation amplitude to prevent division. As a result, the absolute value of the fluctuation vector shows temporal fluctuations. To investigate this detail, we take the power spectrum of the time series in Fig. 5. We define $S(f)$ as a power spectrum for the fluctuating representation in Fig. 5.

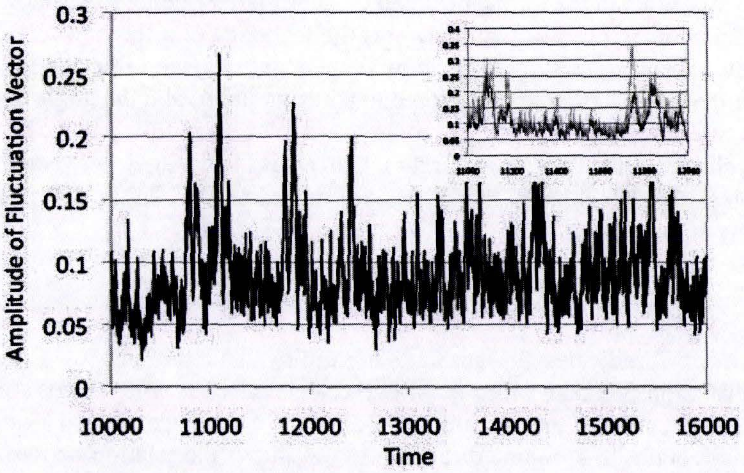


Figure 5: Oscillation of the average absolute value of fluctuation vector in the largest cluster. Inset: the gray line is the oscillation of fluctuation vectors in the second largest cluster. Both lines are highly correlated.

The mathematical expression is;

$$S(f) \sim f^{-\alpha} \qquad (2)$$

Fig. 6 shows the power spectrum of Fig. 5. It shows a 1/*f* power law over a wide rage of time scales with $\alpha \approx 1.5$. Emergence of this kind of power law is considered to be the characteristic property of critical phenomena such as self-organization criticality (SOC).

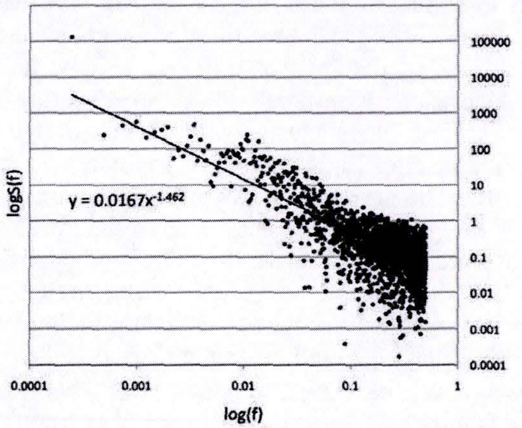


Figure 6: Power spectrum *S*(*f*) for the oscillation in Fig. 5. The graph shows 1/*f* power law.

This fact is consistent with several interpretations, one of which is the collective behavior on critical phenomena. Cavagna and others, who discovered scale-free correlation, suggest that some kind of criticality might in fact be present in starling flocks. Our simulation showed that it is also observed being criticality on a continuous time scale.

3 Discussion

Understanding a living system is a very important problem. Autopoiesis and self-organized criticality (SOC) provide some answers to these questions. Both approaches suggest that interfaces between parts and whole are needed to construct models. Autopoiesis intends to overcome this problem by constructing a one-to-one correspondence between parts and whole(Varela, 1979). SOC intends to overcome this problem by using criticality (Stassinopoulos and Bak, 1995). In this paper, we insisted that understanding collective behavior helps to answer this question because agents behave as one collective without any central control (Couzin et al., 2002; Sumpter, 2006, 2010). Collective behavior emerges only through local interactions. In fact, in order to explain flocking phenomena, the field naturalist Edmund Selous concluded that somehow, a connectivity of individual minds and transference of thoughts must underlie such behavior (Couzin, 2008). We could sometimes admit this kind of mind for a set of individuals. Therefore, the question of "what is a life?" lies closer to the question of "what is a collective behavior?" than we expected.

We insisted in this study that the phenomenon of scale-free correlation might be a bridge between parts and whole. Flocks always contain scale-independent correlated subparts that are represented by fluctuation vectors. Each correlated sub-domain provides an effective perception for each individual that belongs to the sub domain (Cavagna et al., 2010). This means that a perception of one individual corresponds to the group perception. Cavagna and others, in fact, implied that scale-free correlation (or a criticality of flock) might contribute to the fascinating "collective mind" metaphor at a more quantity level. We observed that the MTI model satisfied various critical properties such as self-similar structure and $1/f$ fluctuation of the oscillation strength of fluctuations. For both cases, flocks in the MTI mode have a certain type of criticality for both extremes, such as spatial and temporal aspects. MTI flocks spontaneously direct and keep their fluctuations on the critical state for spatial and temporal aspects. These results are very consistent with SOC properties (Bak et al., 1987; Bak and Sneppen, 1993; Stassinopoulos and Bak, 1995) and support Cavagna's presumption that flocks are on critical states, for various aspects.

Is fulfilling these critical properties sufficient to understand a life or a mind? Critical properties are indeed very important for flocks to react more sensibly to external perturbations. Flocking behavior may be on the border between order and disorder. This interpretation resembles the idea of the edge of chaos as the metaphor for life (Kauffman and Johnsen, 1991; Langton, 1990). However, we consider that scale-free correlation in real flocks has a more suggestive aspect than the critical state. We recall that correlated domains not only have a role in effective perception that is suggested by Cavagna and others, but also are subparts that have the possibility to behave as an independent flock. This possibility was discussed in the section about splitting flocks. Extending an agent's perception range increases his ability to respond to external perturbation such as a predator attack. There is no benefit for survival, however, if flocks promptly change their directions to another side. In a similar way, there is a difference between "a system" and "making a system".

This is a shortcoming of SOC because most SOC models focus on the problem of "keeping its critical state" (Arcangelis et al., 2006; Bak, 1997; Carlson and Doyle, 2002). In fact, some SOC models are associated with a lack of efficiency of learning, despite showing versatility (Alstrøm and Stassinopoulos, 1995; Stassinopoulos and Bak, 1995). In other words, critical fluctuations never assist with efficient learning, although it shows criticality. If we want to discuss a living system, we must consider both sides, which are "system" and "making a system". "Making a system", in this case, corresponds to "using its critical state" to drive its own system. For example, MTI flocks rapidly change direction without any external noise (Niizato and Gunji, 2010). Noise is inherent. Each internal but scale-invariant fluctuation enables the flock to change direction abruptly. This fact suggests that criticality in MTI flocks never sacrifices their mobility as flocks. Thus, the MTI model holds both conditions, "system" and "making a system".

The general concept of "life on the critical state" is broadly embraced by several researchers (Arcangelis et al., 2006; Bak, 1997; Cavagna et al., 2010; Kauffman and Johnsen, 1991; Stassinopoulos and Bak, 1995; Tagliazucchi and Chialvo, 2011). It is important in flocking cases that the critical state provides high sensibility to external perturbations to extend agents' effective perception range. The critical state is also observed in MTI flocks for spatial and temporal cases. However, we observed that this critical state, which is a correlated sub-domain in this case, also has the potential to split the flock's body. Additionally, these correlated subparts behave as individual flocks after they split. Furthermore, MTI flocks can change their direction abruptly by using internal fluctuations. These facts suggest that internal fluctuations, which are on the critical state, contribute to dynamical flocking motion in reducing the risk of collapse. A system that utilizes critical fluctuations positively has to be distinguished from a system that passively receives critical fluctuations. Flocks maneuver internal fluctuations when they act. It is suggested that critical phenomena play an essential role in driving a living system. This calls attention to another aspect, which is "how to use criticality for a living system". Scale-free correlation in flocks would have both aspects of the role of criticality and properly connect "system" with "making a system".

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