Chaotic Firings and Delayed Feedback Control of Rose-Hindmarsh Neuronal Model

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Abstract
Rose-Hindmarsh model is a simple and typical system for describing neuronal firing activities. This paper studies its chaotic firing phenomena, identification of unstable periodic orbits and chaos control in certain parameter regime. Firstly, irregular firing behaviors of the model are proved to be chaotic by numerically calculating the attractor's Lyapunov exponents and fractal dimension. Secondly, low order unstable periodic orbits embedded in the chaotic attractor are identified by simply analyzing interspike interval time series in their return maps. Finally, chaotic firings are stabilized to the period one and period two firing patterns respectively by delayed feedback control. Our preliminary work shows that the method of identification of unstable periodic orbits combined with delayed feedback control can effectively suppress irregular chaotic firings for the model. The technique presented in this paper is in accordance with those features of neuronal systems and may be a simple and actuated scheme for controlling chaotic firings of real neurons in physiological conditions.
Keywords: Rose-Hindmarsh model, chaotic firings, interspike interval, time series, delayed feedback control

1 Introduction

Rose-Hindmarsh (RH) model is a simple and typical system for describing neuronal firing activities (Hindmarsh and Rose, 1984). It will exhibit abundant nonlinear phenomena such as bifurcation and chaos, when parameters of the system are varied. Thus it has aroused much interest in literatures recently. However, the problem of controlling chaotic firings for the system still remains unexplored.

Because chaotic firings are complicated, traditional control methods can not be applied to them directly. Ott, Grebogi and York (Ott et al, 1990) (OGY) have suggested an efficient method of chaos control that can eliminate chaos in 1990. Since then, the studies on chaos control have lead to more extending interests. Taking OGY method as starting point, many new revised methods are induced. At the same time, many new chaos control methods are brought out and applied also. The chaos control techniques are applied in different systems, and good results are obtained. Taking bio-medical engineering for example, Garfinkel et al. applies PPF (Proportional Perturbation

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Feedback) chaos control method in cardiac chaos control firstly (Garfinkel et al, 1992), then, Schiff et al. applies OGY chaos control method on controlling chaos in brain (Schiff et al, 1994). The chaos control methods win important studying values in the field of bio-medical engineering.

The paper is organized as follows: we will briefly introduce RH model in Section 2. Chaotic firing behaviors of the model will be investigated in Section 3. The identification of unstable periodic orbits and chaos control will be given in Section 4 and 5 respectively. Discussions and Conclusions will be drawn in the last two parts respectively.

2 Rose-Hindmarsh Model

The model is presented to describe neuronal firings by Rose and Hindmarsh. It is governed by the following three dimensional nonlinear differential equations:

$$\begin{aligned} dx/dt &= y - ax^3 + bx^2 + I - z \\ dy/dt &= c - dx^2 - y \\ dz/dt &= r(s(x - x_1) - z) \end{aligned} \tag{1}$$

in which x represents the membrane potential. y is the recovery variable. z is an adaptation current. r, a, b, c, d, s, x_1 are constants. I is the applied current. This typical model has received great interests because of its simple form. Recently, Holden and Fan studied the nonlinear dynamic behaviors of this model under different parameters. They found the conditions and forms for this system to be bifurcation and chaotic firings and explained their mechanism (Holden and Fan, 1992).

3 Chaotic Firings

In order to investigate the irregular firings of RH model, fourth order Runge-Kutta method is applied to integrate equations (1) with the time step 0.05. The parameters are set as follows: $a=1.0$, $b=3.0$, $c=1.0$, $d=5.0$, $x_1=-1.6$, $I=3.1$, $r=0.014$. After jumping off transient process, the output time history of variable x and the projection of strange attractor in x-z phase plane are shown in Fig.1 (a)(b). At this time, it can be seen that the neuronal firing pattern is irregular. Results of numerical calculating the system's Lyapunov exponents are given below (Logarithm takes 2 as base) (Wolf et al, 1985): $\lambda_1 = 0.0120469 > 0$, $\lambda_2 = -0.0000600373 \approx 0$, $\lambda_3 = -12.72806 < 0$. From the fact that the first Lyapunov exponent λ_1 is larger than zero, we can conclude that the irregular firing activities of this neuron is chaotic. According to the Kaplan-Yorke hypothesis (Kaplan and Yorke, 1979), we can get the dimension for the attractor: $D_L = 2 + 0.0120469/12.72806 = 2.000946$. Since it is a fraction, the attractor in phase plane is a strange one.

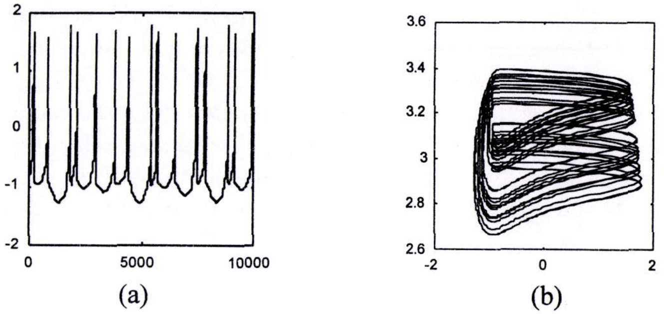

Fig.1 (a): The output time curve of chaotic firings for variable x. The horizontal coordinate is integral steps. The vertical coordinate is amplitude. (b): Strange attractor in x-z phase plane. The horizontal coordinate is x. The vertical coordinate is z

4 Interspike Interval Time Series and Unstable Periodic Obits

Because the neuronal firing behaviors have an all-or-none property, the information conduction is just related to the time intervals of action potential trains. As a result, recording and processing the data formed by continuous pulses is an important technical means in monitoring variations of neuronal firing activities in experiments. Here our numerical process of obtaining time series from the RH model resembles to that of real physiological experiments. First, we record the time beings of occurrence of action potentials, t_1, t_2, ..., according to the output of membrane potential x. Then, we calculate their time intervals between two adjacent pulses, $\Delta t_i = t_{i+1} - t_i$, which are called interspike intervals (ISI). At last, the ISI data are arranged according to their sequences. Substantially, the process of obtaining actional potential trains matches with the Poincare section method which is widely used in the numerical investigation of nonlinear dynamical systems (Gong et al, 1998).

As shown in Fig.2(a), a smooth curve is obtained by plotting the first return map from the ISI time series of chaotic firings. This indicates that the irregular firings are indeed determined by a deterministic function rather than a stochastic process. Because the smooth curve has only one maximum, it is called one hump map. It must be noted that although one hump map is simple in its form, it is the most convincing evidence to judge the existence of deterministic chaos. To our knowledge, it is also the simplest form of chaos ever recorded in real neuronal firings in electrophysiological conditions (Xu et al, 1997). Numerical analysis shows that the largest Lyapunov exponent of the ISI time series determined by the one hump map is (Logarithm takes e as base) (Gong, 1998) $\lambda_{\max} = 0.2639 > 0$. It is larger than zero which also indicates that at this time the irregular firing activities of the neuron is chaotic.

Since infinite unstable periodic orbits make up a chaotic attractor's frame, and chaos is mainly determined by low order unstable periodic orbits and their neighboring points, we should first determine the desired periodic orbits in order to control irregular chaotic firings. In fact, one can get the periodic orbits with ease by just analyzing the ISI time series. As shown in Fig.2(a), curve 1 intersects with 45-degree line at point A. It is just the period one orbit with a period of *41.65*. In Fig.2(b), curve 2 intersects 45-degree line at point B and point C. They are just the period two orbits. Their values are *17.4* and *48.55* respectively, so the period is the sum of these two, i.e., *65.95*.

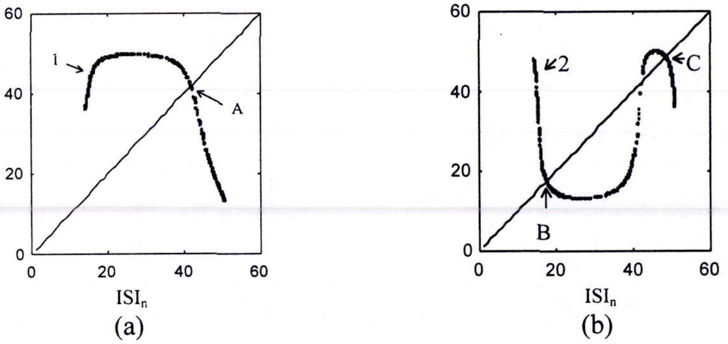

Fig.2 (a): Curve1 is chaotic firings ISI map. The horizontal coordinate is $ISI_{n,}$ the vertical coordinate is $ISI_{n+1.}$ Curve 1 intersects 45-degree line at point A, it is period-one orbit. (b): Curve 2 is chaotic firings ISI map. The horizontal coordinate is $ISI_{n,}$ the vertical coordinate is $ISI_{n+2.}$ Curve 2 intersects 45-degree line at point B and point C, they are period-two orbits

5 Delayed Feedback Chaos Control

After the presentation of the famous OGY method, many chaos control techniques such as the adaptive control, the resonant parametric perturbation, the entrainment control and the migration control are brought forward. Every control method has its own advantages, but it is also constrained for its disadvantages. In 1992, Pyragas presented two methods for controlling continuous dynamical systems (Pyragas, 1992). Both methods are based on the construction of special form of a time-continuous perturbation, which does not change the form of the desired unstable periodic orbits (UPOs), but under certain conditions can stabilize UPOs. Of these two control methods, the delayed feedback control method is particularly convenient for experimental applications. It can be carried out by a simple analogue technique. The feedback is self-controlled and the method does not require an analysis of the system, so it is more practical than the OGY method for experimental realizations.

From eq. 1, one can see that for the RH model, state variable x is the membrane potential which can be recorded in physiological experiments. Variable I is the stimulating current which can be applied and adjusted from outside. Here we try to control neuronal chaotic firings by adopting delayed feedback control.

Because x is a measurable variable of the system output, the controlled perturbation $F(t)$

is applied to the first equation of system (1). The equations added with *F(t)* is showed as eq. 2:

$$
\begin{aligned}
dx/dt &= y - ax^3 + bx^2 + I - z + F(t) \\
dy/dt &= c - dx^2 - y \\
dz/dt &= r(s(x - x_1) - z)
\end{aligned}
\tag{2}
$$

Here， *F(t)* is defined in eq. 3:

$$
F(t) = K[x(t-\tau) - x(t)] = KD(t) \tag{3}
$$

Here, τ is a delay time. *K* is an adjustable weight of the perturbation. If this time coincides with the period of the *ith* UPO $\tau = T_i$ then the perturbation becomes zero for solution of system (1) corresponding to the UPO $x(t-\tau)=x(t)$. This means that perturbation in the form (3) does not change the solution of system (1) corresponding to the *ith* UPO.

To achieve the stabilization of the desired UPO, two parameters τ and *K* should be adjusted in experiment.

According to the previous analysis, the period-one and period-two orbits are controlled respectively. For the period-one orbit, select parameter $K=0.48$, $\tau=41.65$. For period-two orbit $K=0.66$, $\tau=65.95$. The controlled output figures in time domain, excluding the transient process, are shown as Fig.3(a)(b). In order to display the controlled effects more manifestly, the ISI map after controlling the period-one orbits is shown as point D (indicated by dense color) in Fig.4(a). The point D is just dropped on the point of intersection between curve 1 and 45-degree line. This illustrates that the controlled perturbation has not change the quality of neuronal firings themselves. After the period-two orbit being controlled, the variable process of ISI map from chaotic firings to periodic firings is shown as Fig.4(b). The controlling starts at 100000 steps. The chaotic firings distinctly disappear.

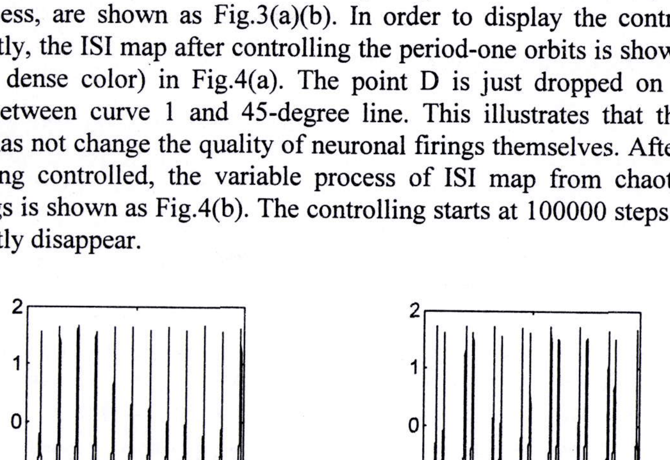

Fig.3 (a): After period-one orbit being controlled, variable x output figure in time domain. (b): After period-two being controlled, variable x output figure in time domain.

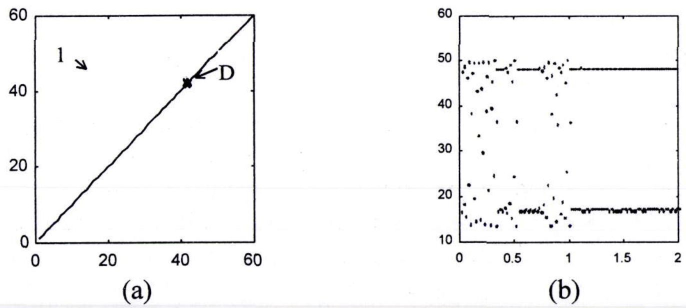

Fig.4(a): After period-one orbit being controlled, the ISI map. The horizontal coordinate is ISI_n, The vertical coordinate is ISI_{n+1}. Point D is dropped on the point of intersection between curve 1 and 45-degree line. (b): After period-two orbit being controlled, the variable process of ISI map from chaotic firings to period-two orbit. The horizontal coordinate is steps, coordinate value multiply 100000. The vertical coordinate is ISI.

6 Discussions

In summary, we numerically prove that the irregular firings of the RH model in certain parameter regime is chaotic. The unstable periodic orbits embedded in the chaotic attractor are identified by analyzing the ISI time series.

The irregular firing behaviors of the neuronal system are stabilized to the period one and period two firing patterns respectively by delayed feedback control. Our scheme for identifying unstable periodic orbits and controlling chaos is in accordance with those features of neuronal systems. It seems very simple in principle and prone to realize in experiment.

First, adopting ISI map can exactly identify the desired controlled periodic orbits. After adopting delayed feedback signal and adjusting the feedback weight properly, RH neuronal chaotic firings can be controlled very well.

Secondly, because the selected output variable is measurable in reality and the perturbation can be import into system in some way, the method can be realized in real experiment. If adopting analogue signal to control the system, only a simple delay line is required. If adopting digital signal, DSP (Digital Signal Processor) can easily realize it.

Finally, it should be pointed out that during the process of coding and conducting information, noise is not only unavoidable but also widely exits for real neurons. Now its possible positive effect called stochastic resonance is receiving great attentions (Wiesenfeld and Moss,1995; Gong et al,1998). But noise will bring much difficulty in identifying periodic orbits from ISI data directly measured in experiments. However, recent studies show that with a simple nonlinear noise reduction technique, one can effectively dispart and exclude noise components from chaotic signals by preprocessing noisy experimental ISI data (Gong et al,1999). Thus, it can be expected to identify periodic orbits from experimental ISI data with accuracy.

7 Conclusions

This paper studies the chaotic firings phenomena and delayed feedback control approach of Rose-Hindmarsh model, which is a simple and typical system for describing neuronal firings activities. It is validated by numerical calculation that the low order unstable periodic orbits emmbedded in the chaotic attractor of this model can be identified by analyzing interspike interval time series in their return map and the chaotic firings can be stabilized by properly applying delayed feedback control approach. The scheme presented in this paper is simple in principle and can be further realized to analyze and control chaotic firings of real neurons in physiological conditions.

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