Role of Multiplicity for Emergence and Anticipation in Memory Evolutive Systems. An example in Art

by Andrée C. Ehresmann and Jean-Paul Vanbremeersch*
Université de Picardie Jules Verne
* ehres@u-picardie.fr
http://ehres.pagesperso-orange.fr

Abstract
Memory Evolutive Systems (MES) give a frame, based on a 'dynamic' category theory, for studying natural evolutionary systems with an intermingled hierarchy of components varying over time, in which processes of higher complexity, up to intentionality, can emerge. They are also able to act as Dubois' internalist and strongly anticipatory systems [3, 4]. We prove that the possibility of emergence and of anticipation both depend upon a kind of "degeneracy property" (as defined by Edelman [5]) which we call the Multiplicity Principle MP. It says that there are functionally equivalent patterns which are not structurally isomorphic or interconnected by a cluster (more precisely: not isomorphic as Ind-objects [9]). An application is given to the emergence of a new artistic current (*e.g.* Cubism) in the MES representing the Art world.

Key Words: Emergence, Self-organization, Memory, Anticipation, Category

1. Introduction

Our aim is to investigate the following problems raised by natural complex self-organized evolutionary systems, such as biological, neuro-cognitive or social systems:

1. What makes possible the emergence over time of objects and processes of increasing complexity order? Giving a precise definition of this order, we prove that the necessary condition is the *Multiplicity Principle* MP, a kind of 'flexible redundancy' (called "degeneracy" by Edelman [5, 6]) which ensures the existence of *multiform* objects admitting several functionally equivalent, but not isomorphic nor well connected, decompositions in patterns of lower level objects, with possibility of switches between them. If MP is not satisfied, every object is of order ≤ 1 (meaning it binds a pattern of level 0).

2. How to account for their multi-scale self-organization? We explain how the fine dynamics is directed by the cooperation/competition between a net of mutually entailed functionally specialized subsystems, the *Co-Regulators*, with differential accesses to a central flexible *memory* developing over time. Each coregulator operates at its own rhythm, with the partial information it can collect in its landscape. However their logics differing, their operations may conflict, whence the necessity of an 'interplay' among them to which MP confers several freedom degrees.

3. How are intentional and internalist anticipatory processes made possible, so that MES act as strong anticipatory systems (in the sense of Dubois [4])? We show how

MP allows for the development of a subsystem of the memory, the *Archetypal Core* AC, which allows for strong anticipation. AC embodies the "self" of the system, with its variations over time. It is formed by components of higher complexity order, which can self-maintain and diffuse their activation to a large domain. Thus the landscapes of higher coregulators extend and unite into a longer term global landscape on which they develop a two-step process: (i) retrospection to make sense of the recent past; (ii) prospection to conceive more or less innovative long term scenarios.

The above problems are analyzed in the frame of the *Memory Evolutive Systems* which we have been developing since 25 years; we recall the main characteristics, referring to our book [9] for more details. They give a mathematical model (based on category theory) for systems with a tangled hierarchy of interconnected components varying over time, and a multi-scale self-organization allowing for adaptation and internalist anticipation.

An application is given to the emergence of a new artistic current in the Art world, namely Cubism in the early 20th century.

2. The Memory Evolutive Systems SOC and ART

Before recalling the general definition of a Memory Evolutive System, we give a rough description of the MES associated to a society, denoted by SOC, and its subsystem ART associated to the Art world.

2.1. The Hierarchical Structure of SOC and ART

SOC has components of various complexity levels, varying over time: at level 0 we have the members of the society, at higher levels the more or less complex social groups they form, the links between them modelling their interactions. There will be many small groups whose members are highly interrelated, such as familial, social, cultural, professional networks, and larger groups uniting smaller groups. A single individual (or group) can belong to several larger groups. Individuals and groups change gradually over time, some disappear, others are created; a group can keep its complex identity while seeing the number of its members progressively change over time, until it disappears, or it may merge into a larger group.

SOC has a hierarchical subsystem *Memory* modelling the knowledge of any nature of the society. It is divided into: an empirical memory consisting of documents (writings, artefacts, artwork, ...) and of individual and collective memories of political, economic, cultural, scientific, religious... events; a procedural memory storing procedures and rules based on knowledge of all kinds (scientific, technical, medical, economic, legal, political, ...); a semantic memory where the various memories are classified in more abstract concepts (perceptual invariants, symbols, values, ideologies, schools of thought ...) translated by words of the language; finally a subsystem AC consisting of integrated memories, which reflects the essence of the society, its main values and culture.

The Art world is represented by a hierarchical subsystem of SOC. It consists of components of SOC involved in the production, organization and consumption of art, in particular groups consisting of artists, professors, art critics, gallery owners, art dealers, patrons, art lovers, museum directors, art administrative instances... These groups are themselves divided in smaller groups: for instance the group of painters in France in the early 20^{th} century consisted of groups of academic painters, of the Impressionists, the Fauvists,... Between 1900 and 1909, the Impressionist group is reduced while maintaining a certain identity and the Cubist group appears.

2.2. The Multi-scale Dynamics

The fine dynamics of the society, and of the Art world, depend on the cooperation/competition among different groups playing the role of coregulators. Such a group appears in two forms in SOC or in ART: as the subsystem G formed by its members and their relationships within the group, and as a higher component G* which binds G and represents the group as such. G operates stepwise at its own rhythm. At a given time t, G collects more or less partial information about its current situation, constraints imposed by other groups and/or the natural environment; this information is collected in the 'landscape' of G at t. Based on these data and results of recorded similar previous events, G will respond by choosing a mode of action, represented by the choice of a procedure (or strategy) on its landscape.

In the landscape of G, we distinguish a subset A_G, called the *artistic landscape* of G; it consists of all the artistic information received by G: artistic skills of its members, their opinions and feelings about art, their cultural references. Thanks to its artistic landscape, the group forms its own idea of 'contemporary art', modelled by the formation of its *art concept* binding together this information in the semantic memory,

The higher the group level, the more its actions require a chain of transmissions to make a decision and implement it by passing commands through levels. When the procedures used by different coregulators are conflicting, the final choice will result from a balancing process between these procedures, the *interplay among coregulators*, benefiting from the fact that the procedures can be implemented under different forms (thanks to MP). It may modify or delete some of them, causing 'fractures' to the corresponding group. For instance, a small group of closely related artists influence each other and may slowly introduce new artistic ideas, thus modifying their concept of art CG. However for creating a new artistic current (such as Cubism in the 20^{th} century), their artworks must be accepted by the critics, the art dealers and later the general public; that is not always the case, or at least may take a long time (cf. Section 5.3).

3. The Hierarchical Structure of MES

The Memory Evolutive Systems are based on a 'dynamic' category theory, integrating time. For the main notions of category theory we refer to the Mac Lane's book [12].

3.1. Categories and Evolutive Systems

Category theory has been introduced by Eilenberg and Mac Lane [10] in the early forties to relate topological and algebraic constructs, but has later acquired a foundational role in mathematics. It will give us tools for studying the binding and emergence problems. Let us recall the following

Definitions. 1. A *category* K is an oriented (multi-)graph with an internal (partial) composition which maps a path (f, g) from A to B on an arrow fg from A to B; this composition is associative and each object has an identity.

2. A *functor* F from K to a category K' maps an object A of K to an object FA of K', an arrow (or 'link') from A to B on a link from FA to FB, and preserves the composition and the identities.

3. A *pattern* (or *diagram*) P in K consists of a family of objects (P_i) of K and distinguished links between them in K. A *collective link* (or cone) from P to an object A of K is a family (s_i) of links s_i from each P_i to A such that:

$$s_i f = s_j \quad \text{if } f \text{ is a distinguished link from } P_i \text{ to } P_j \text{ in P.}$$

4. If P is a pattern in K, an object cP of K is called a *colimit* of P if there is a collective link (c_i) from P to cP satisfying the universal property: for each collective link (s_i) from P to an A there is a unique link s from cP to A such that $c_i s = s_i$.

In a MES, to account for the possible variation of components over time, the system is not modelled by one category, but by an *Evolutive System*, that is a family (K_t) of categories indexed by time, with partial "transition" functors between them; these transitions satisfy a transitivity axiom, so that a component of the system corresponds to a maximal family of objects in the K_t (its successive states) related by transitions. A *link* between components similarly consists of arrows related by transitions.

The category K_t figures the configuration of the system around time t; its objects represent the state of the components of the system existing at t and the arrows model channels through which information or constraints can be transferred between them around t, weighted by their *propagation delay* and *strength* (both are positive numbers) which may vary over time. The transition from t to $t' > t$ models the global change in the configuration, reflecting the possible loss, addition or binding of some components; it singles out 'what' has become of the components still existing, but not 'how' these changes depend on the fine dynamics of the system.

3.2. Binding Process. TheTangled Hierarchy of Components

The systems we consider (such as SOC or ART) have an intermingled hierarchical structure, with their components divided into different complexity levels, so that a component A of a given level admits a decomposition into a pattern P of 'simpler' components of lower levels through which it can operate. In the categorical frame, A will be modelled by the *colimit* (also called *binding*) of P.

The tangled hierarchy in a MES is then defined as follows:

Definition. A category is *hierarchical* if its objects are partitioned into *complexity levels* (numbered from 0 to m) so that an object A of level n+1 is the colimit of at least one pattern P contained in the levels $\leq n$. A *hierarchical Evolutive System* is an ES such that the configuration categories are hierarchical and the transitions respect the level.

Let us remark that an object A has a double face ('Janus'): it is 'simple' if looked at as a component of a higher level object, but 'complex' if we compare it with one of its lower level decompositions P.

An object A of level n+1 has *ramifications* down to level 0, obtained by taking a decomposition P of levels $\leq n$ of A, then a decomposition of lower levels of each component P_i of P and so on, down to patterns in the level 0. Now A may have different ramifications of different lengths. We define the *complexity order* of A as the length of its shortest ramification (it is $\leq n$+1). It measures the smallest number of steps necessary to construct A from level 0 up by successive binding processes.

Remark. The configuration categories admit only colimits of some particular patterns, (in particular they are not toposes), so that new complex objects can emerge over time through a binding process.

3.3. The Simple Links

We know that each component has a decomposition in lower level components. Is it the same for the links between them? No; only some of the links bind clusters of lower level links; we are going to define them first, and later we'll indicate how more 'complex' links can emerge at each level.

Definitions. (i) If P and P' are 2 patterns, a *cluster* from P to P' is a maximal set G of links from components P_i of P to components P'_k of P' satisfying the axioms: (i) For each P_i there is at least one link in G toward some P'_k, and if there are several such links they are correlated by a zig-zag of distinguished links of P' (cf. Figure 1); (ii) G is closed by composition with a distinguished link of P on the left and a distinguished link of P' on the right.

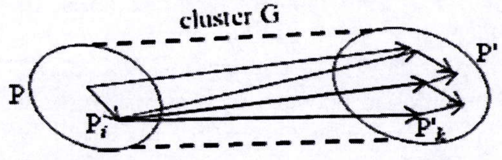

Figure 1: Cluster G from P to P'

(ii) If P and P' have colimits C and A, a cluster G from P to P' binds into a link cG from C to A, called a (P, P')-*simple* link (Figure 2), or just an *n-simple link* if P and P' are patterns contained in the levels $\leq n$.

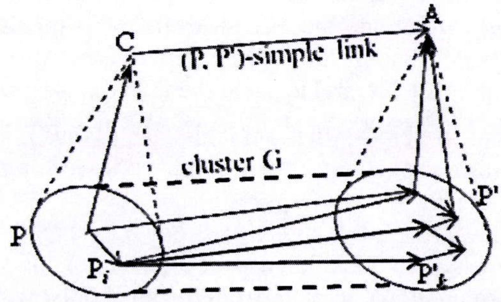

Figure 2: A (P, P')-simple link binding the cluster G

An *n*-simple link represents the cluster of lower levels links it binds as an entity at the higher level, thus translates properties already directly observable through the lower components of C and A. The composite of simple links binding adjacent clusters is still simple. However 'more complex' links may emerge, as we are going to show.

4. The Multiplicity Principle at the Root of Emergence and Complexity

Most complex systems, such as biological, cognitive or social systems, satisfy a kind of flexible redundancy (or *degeneracy property* in the sense of Edelman [5, 6]): there are patterns which are functionally equivalent though their structures are not isomorphic and they are not connected by a cluster. For instance this degeneracy appears in the neural code, in the genetic regulation, in protein regulation, in behavioural repertories, and so on. We are going to formalize this property and to show that it is at the root of emergence of higher complexity.

4.1. The Multiplicity Principle and the Emergence of Complex Links

A given pattern has at most one colimit (up to an isomorphism). On the other hand, a complex object C can bind quite different patterns; they represent different decompositions of C which, at a time *t,* can be actual or latent, several coexisting, and others disappearing or appearing. These patterns are *functionally equivalent*, meaning that there is a natural isomorphism between their collective links to any object. In [8] we have formalized the above degeneracy property into the following

Multiplicity Principle (MP). There are functionally equivalent patterns P and Q which are *non-connected* in the sense that there is no cluster between them (so that they are not isomorphic as Ind-objects [9]). An object C is *n-multiform* if it is the colimit of two non-connected patterns P and Q of levels < *n*; the passage from P to Q is called a *switch.*

A first consequence of MP is the existence of another kind of links, called *n-complex links*, obtained by composing *n*-simple links binding non-adjacent clusters, for instance a (Q', Q)-simple link and a (P, P')-simple link, where P and Q are non-connected decompositions of an *n*-mu-ltiform object C (cf. Figure 3). These links *emerge* at level n+1 without being generated or directly observable by links between lower components of the extreme objects; they model global properties of the lower levels, only emerging at the level n+1.

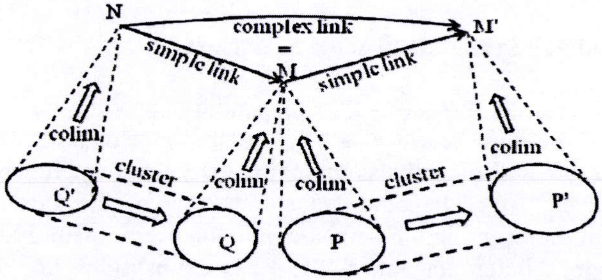

Figure 3: A complex link as a composite of simple links

Complex links play a major role in the proof of the:

Complexity Theorem [9]. *The Multiplicity Principle is a necessary condition for the existence of objects with a complexity order* > 1.

Without MP, all objects could be constructed in one step, as the colimit of a 'large' pattern of level 0; this situation would correspond to a *pure reductionism*.

To avoid this situation, in a MES, we always suppose that MP is satisfied to allow for the existence of components of complexity order > 1. As we are going to show, it will follow that components of increasing complexity orders may emerge over time, and MES resort to an *emergentist reductionism* (in the sense of Bunge [2]).

4.2. Complexification. Emergence Theorem

The coarse dynamic of a MES, reflected by the transitions, depends on the standard changes emphasized by Thom [14]: birth, death, scission, collision. To model such changes, we have explicitly constructed [7, 9] the *complexification* of a category K with respect to a procedure Pr with objectives of the kinds: 'add' external objects, 'suppress' some objects, 'bind' (or respect the binding of) some patterns. It is a category K' which is the universal solution of the problem: find a category K' in which the fixed objectives are realized.

Remark. A procedure Pr can be associated to a *sketch* and the complexification with respect to Pr is then the *prototype* of this sketch, explicitly constructed in [1].

Emergence Theorem [8]. *MP extends to a complexification. Iterated complexifications cannot be reduced to a unique one, and they lead to the emergence over time of an intertwined hierarchy of components of increasing complexity orders, in which the material, formal and efficient causalities are intermingled..*

In a MES, the configuration categories will be obtained from level 0 by successive iteration of this complexification process, leading to the emergence of components of higher order complexity.

The intermingling of causalities means that MES can be classified as *organisms* (and not simple *mechanisms*) in the sense of Rosen [13] in which "causal links cannot be teased apart".

4.3. The Role of MP in the Multi-scale Self-organization

A MES has a multi-scale self-organization depending on the cooperative and/or competitive interactions between a net of specialized functional subsystems, the *coregulators,* each with its own complexity level, its own function, and its discrete timescale extracted from the continuous timescale of the system. The MES has a subsystem Mem representing a central flexible long term memory which develops over time, allowing for a better adaptation. Mem has a subsystem Proc where procedures are memorized with their commands to effectors, and a subsystem Sem representing a *semantic* memory, in which memories are classified into invariance classes called *concepts* (cf. [9] for the construction of the semantic memory).

Each coregulator CR has a differential access to Mem, in particular to Proc to retrieve the procedures associated to its function, and it cooperates to the actualization and development of the memory. It acts stepwise at its own rhythm as a hybrid system:

(i) At each step of its discrete timescale, it forms its *landscape* (modeled by a category L) with the partial information it can access, and selects an adapted procedure Pr in Proc to respond to the situation.

(ii) The commands of Pr are sent to effectors. Their realization during the continuous time of the present step would resort of usual mathematical models, for instance in terms of differential equations implicating the propagation delays and strengths of the links.

(iii) At the beginning of the next step, the result is evaluated by comparing the new landscape L' with the complexification of L with respect to Pr; there is a *fracture* if they are not isomorphic.

The 'local' commands sent to effectors by the various coregulators at a given time may not fit together since the rhythms, functions and logics of these coregulators are different. At the global level, there is need of an equilibration process between these commands, called the *interplay among the coregulators*. It leads to the global operative procedure which may by-pass the procedures of some coregulators and cause dysfunction (temporary '*fracture*' or *dyschrony*) to them. In particular each coregulator has *structural temporal constraints*, and their non-respect can lead to a 'dialectics' between the dynamics of coregulators with heterogeneous complexity levels and rhythms, with cascades of fractures backfiring between them (cf. [9]).

MP gives more flexibility to this interplay, since the commands can be realized through any of their lower order decompositions with possibility of switches between them, and similarly down to level 0. Thus the interplay operates a kind of natural selection between various ramifications of the concerned components. It explains the non determinacy of the system on the long term, and raises the problem of unconventional computation for 'computing' the interplay.

5. Intentional processes and Anticipation in MES

For Rosen [13], an anticipatory system has "a predictive model of itself and/or of its environment". Dubois [3] distinguishes strong and weak anticipation. In what sense can MES act as anticipatory systems?

5.1. The Memory and the Archetypal Core

The long term memory of a MES develops over time by storing past events and the results of tried procedures; its components are connected by links which satisfy a kind of extended *Hebb rule* [11]: the strength of a link from M to M' increases if both M and M' are simultaneously activated. It is not a rigid memory, but a flexible one, which is adaptable to circumstances. Thus by itself it allows for a kind of anticipation with respect to recurring events already met (both locally in the landscapes of coregulators, and globally in their interplay).

Extending to general MES the analysis made in their application MENS to neuro-cognitive systems [9], let us show how MES can develop higher cognitive and intentional processes at the root of strong anticipatory processes. The emergence of components of higher complexity order in the memory (made possible by MP) leads to the development over time of a subsystem of the memory, the *Archetypal Core* AC. It represents an integrative memory acting as an internal reflection of the main characteristics of the system and of its environment. However it is not a rigid model (as in Rosen [13]), but it remains flexible and actualized to account for successive events; it plays an important role for maintaining the identity of the system.

AC consists of higher order memories which integrate various modalities and are often recalled. They are connected by links which, thanks to Hebb rule, become stronger and faster through their constant recall. These links form *archetypal loops* which can diffuse and self-maintain an activation of part of AC. The activation is then propagated to other parts of the system by activating lower level (actual or latent) decompositions of complex components, and resonating between them through switches. Thus a large domain is activated it allows for the formation of a *global landscape* with a longer time-lag which coordinates and extends the landscapes of higher coregulators linked to AC.

5.2. Retrospection and Prospection Processes

Future as a "cognitive expectation" of the present will be imagined in MES through a reflection about the past stored in the memory, in the context of the present situation; thus a MES will act as "a system with multiple potential future states for which the actualisation of one of these potential futures is determined by the events at each current time" (Dubois [3]).

More explicitly, a striking or unexpected event activates part of AC, thus leading to the formation of a global landscape GL in which creative and anticipatory processes will develop through a sequence of the following overlapping processes:

(i) First a *retrospection process* in GL allows sensemaking of the present situation by starting a search in the memory to recognize its different aspects with their possible causes and effects, to diagnose new trends and find adequate strategies.

(ii) Then a *prospection* process can be developed in the global landscape, through the formation of virtual landscapes in which different sequences of procedures can be tried without damage for the system, and their risk of dysfunction evaluated. It leads to various scenarios for long term planning and anticipation, some almost embedded in the present and the contextual environment (accounting for present trends), but also some more creative ones, for instance inspired by desired outcomes. The use of switches between actual but also latent decompositions of multiform components increases the freedom degrees in the construction of scenarios.

For instance the scenarios can add new decompositions to a component; they can require the re-organization, the fusion or even the suppression, of some coregulators, and the formation of new ones. By repeating the process, other changes can ensue from them, since the Emergence Theorem shows that iterated complexifications allow for the emergence of new components which cannot be obtained in a single step. Thus scenarios requiring several steps can lead to the emergence of a real novelty, not easily foreseen at first view, thus enlarging the number of possibilities. Once a scenario is selected, a retrospection process allows back-casting to find sequences of strategies to realize it.

5.3. Creation and Emergence of a New Artistic Trend

Let us give an application to the emergence of a new artistic trend in ART. A small group G of closely related artists with common interests discuss and analyze their mutual works. If the works of one of them N resonate enough with their common artistic ideas while adding a certain novelty, they analyze what new procedures are used and they memorize them in the form of procedures to be used in future work. Thus their works move in the same direction, creating a sort of artistic revolution within the group which impacts on each other. Gradually, the group will conceptualize the underlying ideas and bind them into a new artistic trend D, which emerges in the artistic landscape A_G of the group, and is conceptualized in the semantic memory.

For instance, Cubism was created from 1907 to 1914 by a small group G of artists gathered around Picasso and Braque. They were interested by the new pictorial space and forms created by Cezanne, by the discovery of primitive art, by scientific and technical discoveries. Together they developed new procedures such as the deconstruction of objects into fragments and the creation of multiform objects with double reading. In this way, their work anticipated the cubist revolution, though its advent needed the recognition of the Art world.

Indeed, once a new trend has been created by a group G, it will survive only if it is recognized by other groups. Some groups ignore it, some accept it if it resonates with their current ideas, some reject it because it causes a fracture in their own artistic landscape. The final judgment of the Art world and later of the whole society, results from the interplay among the procedures of its different groups. In particular art dealers play an important role: they develop an anticipatory process to make sense of a new trend and prospect its value by buying some artworks, trying to sell them, and by encouraging and materially supporting artists in the future of whom they believe. For instance the art dealer Kahnweiler became the sponsor of the cubists, while the critics were frightened by the audacity of certain works, such as the multiform portraits, and several years were necessary before Cubism was generally accepted.

Finally the new trend will be integrated within the artistic conception of the time; or it can be impeded to spread widely and wither for lack of support, if its value is not great enough, or if it is too ahead of its time: e.g. the new pictorial landscape created by Cézanne became anticipatory only after its recognition by the cubists.

6. Conclusion

The aim of this paper has been to analyze how anticipatory processes can arise in multi-scale complex self-organized systems, such as biological, cognitive or social systems, with an application to the development of a new trend in the Art world. The study has been made in the frame of the Memory Evolutive Systems, a model (based on category theory) which has been developed by the authors in preceding papers, with applications in different domains (cf. http://ehres.pagesperso-orange.fr).

It has led to single out the properties at the basis of the emergence of creative anticipatory processes: the existence of multiform objects (Multiplicity Principle) allowing for the emergence of objects of higher complexity, the development of an Archetypal Core reflecting the Self of the system, and the formation of a global landscape in which anticipation develops through a sequence of overlapping retrospection and prospection processes.

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