Analog Neural Networks: Non-linear Feedback (Analog Logic & Factorial Switches)

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Abstract

Neuronal schemes realizing operations with continuous quantities are presented. These schemes are based on an analog neuron as a "diode-like" summator of continuous quantities (spike frequencies). Analog (continuous) logic, non-linear feedback, and neuronal structures, which can realize the complex features of information filtration, are discussed. Special attention is paid to factorial switches and synthesis of neuronal structures. Neuronal structures processing **n**-dimensional continuous vectors by non-linear feedbacks can realize the factorial switch, which stores and reproduces information about decreasing order of the components of the vectors.

Keywords: analog neuron, analog logic, non-linear feedback, factorial switch.

1 Introduction

The nervous system of animals and especially of the man is a surprising creation of nature. According to the currently prevailing opinion, the functional organization of the nervous system is based on neural networks, transneuronal connections and specific properties of neurons and their contacts. The paradigm states that the functional role of neural networks is to identify the organism's environment, *i.e.* create and improve an informational model of the environment, make decisions according to the model, and control the organism's action as well as realize goal-oriented programs.

Neurobiological experiments clearly show that neurons as non-linear summates (logical and algebraic) possess certain linear properties (Ratliff, 1965; Granit, 1966). In addition, there is substantial evidence that in biological networks there is strong feedback (by way of axon collaterals, interneurons, *etc.*), both between nearby neurons and between bigger structural units of the nervous system (ganglia, nuclei, cortical fields, *etc.*) (Poliakov, 1965). It has been emphasized that it is these feedback collaterals that grow and form new synaptic contacts during the life of an organism (Hubel, 1963). It has been pointed out that this feedback is non-linear, and its significance for the functional properties of the neuronal network has been considered (Gutman, 1984).

Physicochemical and biological models of dynamical systems, their phase portraits clearly demonstrate the significance of non-linear feedback for a system and even for its functions. It is known that non-linear positive feedback in dynamical systems may induce autogeneration, hystereses, bifurcations. All these effects are observed in neurophysiological experiments and may be explained by some theories (Barlow, 1969).

More interesting and less investigated effects arise from non-linear feedback. The dynamical systems theory shows that positive feedback leads to unnecessary parasitic autogeneration, whereas negative feedback helps to stabilize the system. One of the best - known examples of a system, which uses negative feedback, is the regulator. On the base of the regulator automatic regulation and control theory has been created (Grodins, 1963). Such systems and their functional organization are of great interest to biologists because regulation is one of the most prominent features of living organisms. Non-linear feedback (interaction) is fundamental to some mathematical models Jacob-Monod of cell morphogenesis-differentiation, which explain possible bifurcations in embryogenesis (Rosen, 1972). It has been suggested that the nervous system is the main system, which determines the organism's regulator-like properties. Therefore, it can be assumed that it is the negative neuronal feedback that forms these regulator-like properties.

These properties of non-linear feedback, in the context of the purpose of the nervous system, call attention to a statement by the creator of biometrics, mathematician R. Fischer, in which he proposed that dynamical systems with non-linear feedback might identify an object, or create a model of the object (*according to* Эйкхофф, 1975). This means that non-linear neuronal feedback may be one of the most important mechanisms in brain functional organization and functioning. Therefore, it makes sense to formulate a purpose and find a way to synthesize appropriate functions, i.e., a neuronal structure which would generate the needed phase portrait. One should explore the potential of negative neuronal feedback and create a basic memory-endowed network, a continuous neuronal factorial bifurcate, which would be able to remember the permutation that arranges the positive continuous components of an input vector in increasing order.

There are some neural network models with non-linear feedback, which endows with new specific features: to separate and pass on only the highest value out of several parallel inputs (the maximum filter); to form differentially selective neuronal structures according to their thresholds and neurons selective to the intensity of the input signal (Kirvelis, 1967-1998; Dubois, 1999).

Therefore, understanding of the synthesis of neuronal structures could explain not only neurobiological facts but also would help create more effective technology for information processing.

2 The Functional Characteristics of the Analog Neuron

Neuromorphological studies show that the structural and functional element of the nervous system, the neuron, has a multitude of synaptic contacts with other neurons and one long process, the axon. The axon branches and impinges on neurons and other cells, making its synapses. This is how neuronal structure and neural networks are formed. Neurons come in different shapes but in most cases, they may be divided into "stellate" and "pyramidal" neurons. For the sake of simplicity we assume that our neurons (quasineurons) are summators with many functional inputs and one functional output (Fig. 1).

It is known from neurophysiological studies that neurons generate spikes, thus expressing their level of excitation. A non-excited or inhibited neuron is silent, whereas its excitation makes it generate neuronal spikes of different frequencies, the frequency being indicative of the level of the excitation. Due to the fact that spikes last for certain time and are subject to refractory effects, neurons have their maximal firing frequency X_M. Generally, this frequency does not reach 1000 spikes/sec, although some small interneurons may fire at as many as ~1500 spikes/sec. It has been suggested that the firing frequencies may be summed with a positive (+) and a negative (-) signs, and also with different summations weights. Therefore, the quasineuron is considered to be a summator of continuous (analogous) inputs (spike frequencies). It is able to weigh every synaptic input by a synaptic weight *S*, which may take on any value. Since synapses may be excitatory and inhibitory, the weights of excitatory synapses are often considered positive (+*S*), whereas the weights of inhibitory synapses are negative (-S).

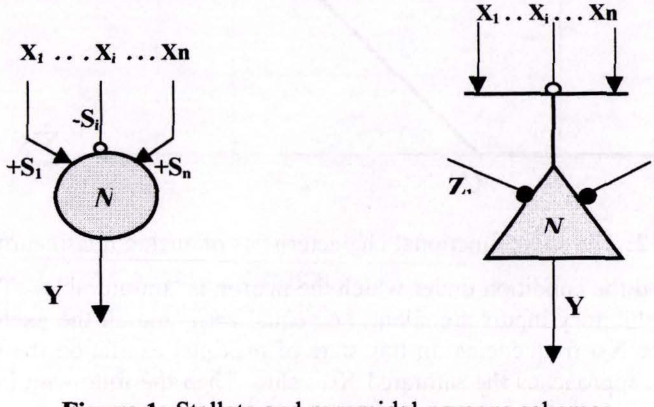


Figure 1: Stellate and pyramidal neurons schemes.

Therefore, the static functional characteristics maybe described by a non linear equations for stellate

$$Y = N\left\{\sum_{i=1}^{n} S_i \cdot X_i\right\} = \begin{cases} \sum_{i=1}^{n} S_i \cdot X_i, ..if.. \sum_{i=1}^{n} S_i \cdot X_i \geq 0, \\ 0, ... if.. \sum_{i=1}^{n} S_i \cdot X_i \leq 0. \end{cases} \tag{1}$$

and pyramidal quasineurons

$$Y = \begin{cases} N\left\{\sum_{i=1}^{n} S_i \cdot X_i\right\}, ...if..Z_1.OR.Z_2 = 0, \\ 0, ... if..Z_1.OR.Z_2 \neq 0. \end{cases} \tag{2}$$

Their graphic representation is depicted in Fig. 2. It is easy to notice that this is a "diode-like" non-linearity with saturation at X_M (in the case when the sum of inputs exceeds the maximal frequency that the neuron may reach).

In some cases it is important to take into account the absolute threshold of the neuron Q. Then the static function of neuron shifts to the right by the value of Q, or, alternatively, the X-axis moves to the left. In many cases it makes sense to consider the threshold to be zero, and introduce another inhibitory input from a neuron (pacemaker), which generates a stable maximal frequency X_M, and whose action at the synapse with a certain weight S will ultimately determine the threshold $Q=SX_M$.

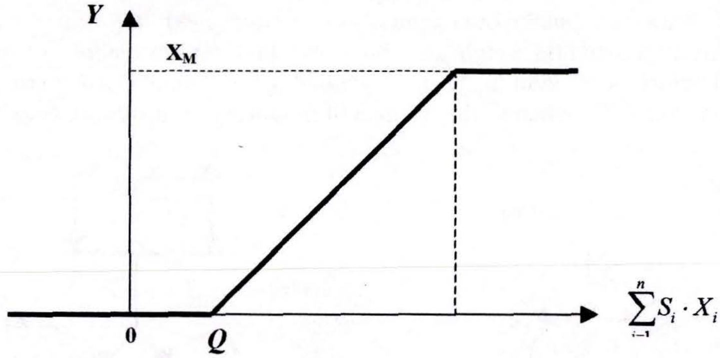


Figure 2: The static functional characteristics of analog quasineurones.

It is easy to find the condition under which the neuron is "unsaturable". This is the case when all the inhibitory inputs are silent, i.e., equal zero, and all the excitatory inputs j are carrying the $\mathbf{X_M}$ frequencies. In this state of maximal excitation the neuron cannot reach and only approaches the saturated $\mathbf{X_M}$ value. Then the following inequality must hold true:

$$\sum_{j}^{m} S_j \leq 1 \qquad (3)$$

This means that the larger the number of excitatory synapses on a neuron, the ssmaller the weight of every synapse. If all the weights of the excitatory synapses are equal and their number is **m**, then **S<1/m**. If we assume that neurons often react only to the difference between the inputs, i.e., they do not react when all the input frequencies are equal, we come to the conclusion that the sums of the excitatory Sj and inhibitory Si synaptic weights are equal, and their sum (taking into account the signs) is zero. That indicates that

$$\sum_{j} S_j \cong \sum_{i} S_i \qquad (4)$$

In some cases very strong inhibition is observed in pyramidal neurons. This effect has been ascribed to some somatic inputs Z_1 and Z_2 with big inhibitory synaptic weights (neuromorphologists relate it to the action of "basket" neurons). Such inhibitory synapses may realize logical prohibition operations, universal logical Pirs's arrows (Dagger functions) or Shaffer's functions. Such a pyramidal neuron sums up its input signals and produces an output, which in this case is a logical operation. The pyramidal neuron becomes an algebraic/logical functional device.

Considering dynamical properties of a neuron, it makes sense to characterize it as a first-order summate with a time constant T, describing the functioning of the neuron by a first-order differential equation

$$T \cdot \frac{dX}{dt} = \sum_{i=1}^{n} S_i \cdot U_i - X \tag{7}$$

In some cases in addition to the synaptic weight S_i, every synapse may also be characterized by its time constant T_i.

Therefore, from the functional point of view, every neuron may be characterized as an inertial algebraic summate with a time constant T, ***n*** non-negative inputs (frequencies) with their synaptic weights S_i, and the neuron's non-linear ("diode-like") characteristic N. Its output is also a spike frequency X, which may take on only non-negative continuous values that's do not exceed X_M. In some cases the neuron may also be a logical summate.

3 Reciprocal neurons

An additional parallel may compensate for the non-linearity of a neuron

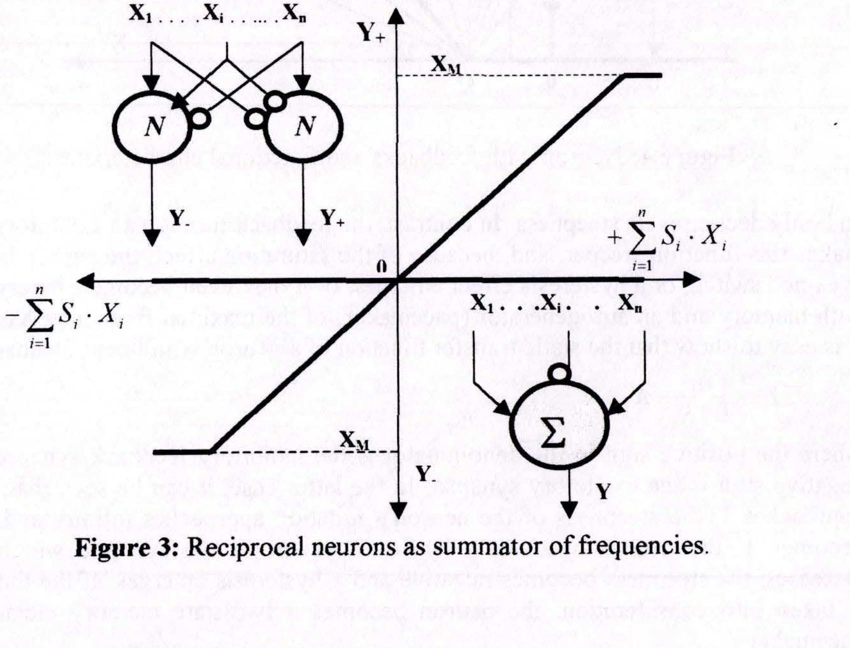


Figure 3: Reciprocal neurons as summator of frequencies.

which has exactly the same absolute values of its synaptic weights, but the signs of these weights are reversed. Such a pair of neurons satisfies the condition of "nonsaturability" and becomes a simple linear summator. Its diagram and functional properties are depicted in Fig. 3.

4 Neuron with Feedback

Generally, feedback may radically change the functional properties of a neuron. The feedback through an inhibitory synapse does not qualitatively change a neuron function

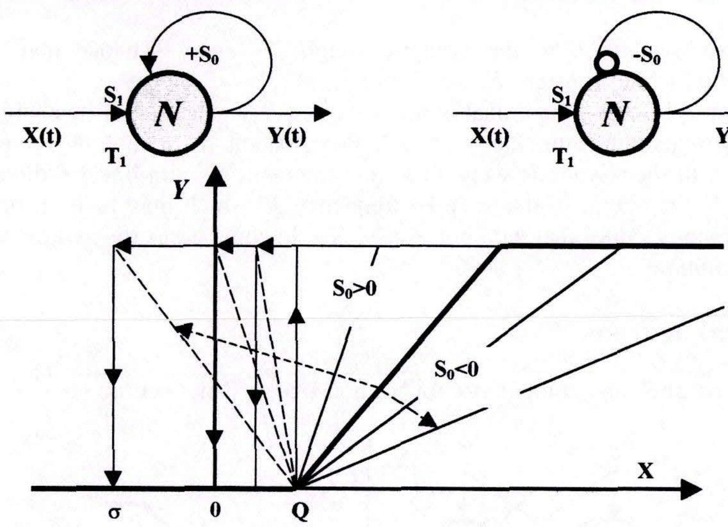


Figure 4: Neuron with feedbacks and functional characteristics.

and only decreases its steepness. In contrast, the feedback through an excitatory synapse makes this function steeper, and, because of the saturation effect, the neuron becomes a "yes-no" switch, or a hysteresis effect emerges, or it may even become a binary element with memory and an autogenerator (pacemaker) of the maximal frequency $\mathbf{X_M}$ (Fig. 7). It is easy to show that the static transfer function of a neuron with linear feedback is

$$Y = \frac{S_1}{1 \mp S_0} \cdot X , \tag{5}$$

where the positive sign in the denominator is the inhibitory feedback synapse and the negative sign is the excitatory synapse. In the latter case, it can be seen that, when S_0 approaches 1, the steepness of the neuron's function approaches infinity and, when it becomes 1, the neuron becomes a "yes-no" switch. If the synaptic weight further increases, the steepness becomes negative and a hysteresis emerges. If the threshold $\boldsymbol{Q}$ is taken into consideration, the neuron becomes a two-state memory element, or a pacemaker.

Such feedback also changes the time constant T:

$$T = \frac{T_1}{1 \mp S_0} \tag{6}$$

It can be seen that in the case of the excitatory feedback (when the synaptic weight approaches 1), the neuron becomes an integrator (T approaches infinity), whereas an increase in the inhibitory synaptic weight, in contrast, improves the neuron's dynamical function (T decreases).

5 Neuronal Analog Logic

Analog neuronal summators with "diode-like" non-linearity can realize all operations of continuous or analog logic (Kirvelis, 1967). Neuronal schemes of the simplest analog logic functions are presented in Fig. 5. Here the logic operations: constant .CONST., reiteration .RE. and negation .NOT., realized as inversion are represented.

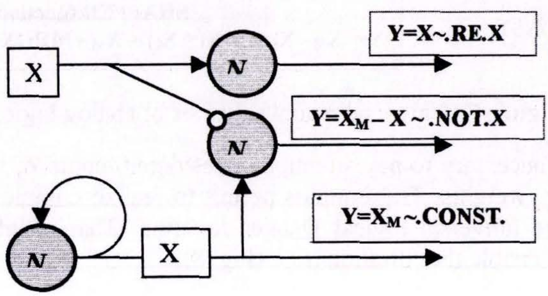


Figure 5: Neural schemes of analog logic operations .CONST., .RE., .NOT..

Other important logical functions are the analog MAX$\{X_1,X_2\}$ and MIN$\{X_1,X2\}$(Fig.6).

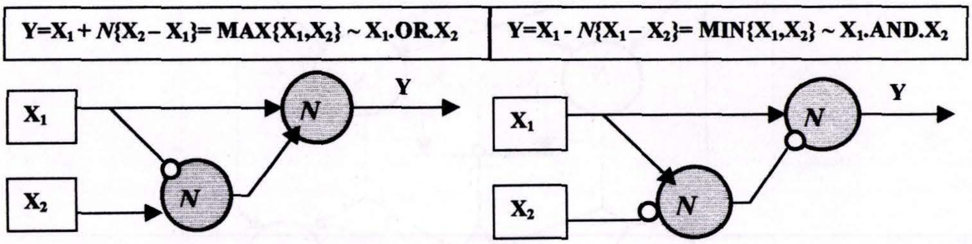


Figure 6: Neural scheme of analog logic .OR. and .AND. operations that correspond to MAX$\{X_1,X_2\}$ and MIN$\{X_1,X_2\}$.

The universal logical logic functions Pirs' arrow (or Dagger function) and Sheffer's link can be easily realized as well (Fig. 7).

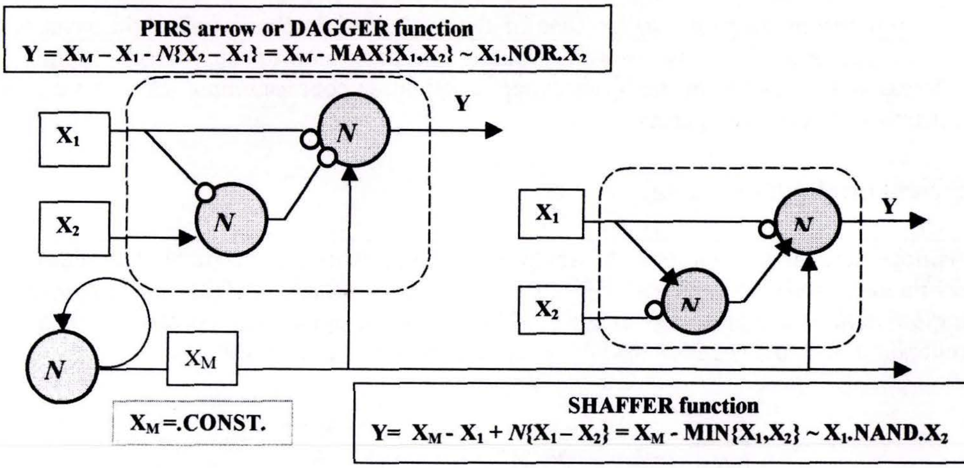


Figure 7: Universal neural schemes of analog logic.

It is especially necessary to pay attention to restricted inputs **Z**, which have very big negative inhibitory weights. These inputs permit to realize complex logical operations on the base of the universal logical Dagger function. That is informational control. Inputs of this kind enable the filtration *.EQ.* (Fig. 8).

$$.EQ.\{X_1, X_2\} = \begin{cases} \sum S_i \cdot X_i,,......IF\{Z_1.OR.Z_2\} = .FALSE. \\ 0,....................IF\{Z_1.OR.Z_2\} = .TRUE. \end{cases} \tag{7}$$

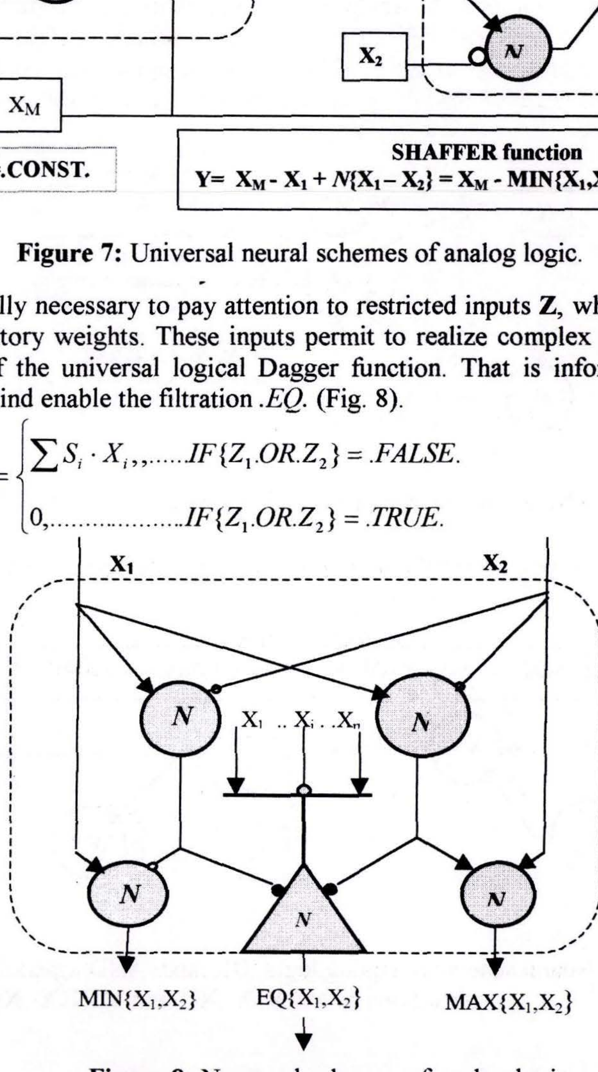


Figure 8: Neuronal schemes of analog logic.

In general

$$.EQ.\{...X_i...X_j...\} = \begin{cases} \sum_{i=1}^{n} S_i \cdot X_i, ...IF\{LOGICFUNCTION\} = .TRUE. \\ 0, ...IF\{LOGICFUNCTION\} = .FALSE. \end{cases} \quad (8)$$

6 Neuronal Regulator

Here we point out to the properties of an inertial (T large) pyramidal neuron with feedback through two complementary non-inertial ($T < T_0$) neurons which together carry out the difference $Z = X_0 - X = N\{X_0 - X\} + N\{X - X_0\}$. Here X_0 is a constant, X(t) is the neuron's reaction, and *N* is a non-linearity indicating the polarity of the difference. In the case of negative feedback, the first neuron acts through an excitatory synapse, and the second one through an inhibitory synapse (Fig. 10). This scheme models a classical neuronal regulator which stabilizes X, *i.e.*, it tries to maintain $X(t)=X_0$ constant. The solution of the function Z=F(X), intersecting the X-axis at a negative angle at point X_0, shows the pyramidal neuron's stable state in a "potential pit". By reversing the signs of the synaptic connections, we could get a dynamical system with the opposite effect, *i.e.*, a non-stable "potential hill" state. In the latter case, Z(X) would intersect the X-axis at a positive angle. That would correspond to positive feedback.

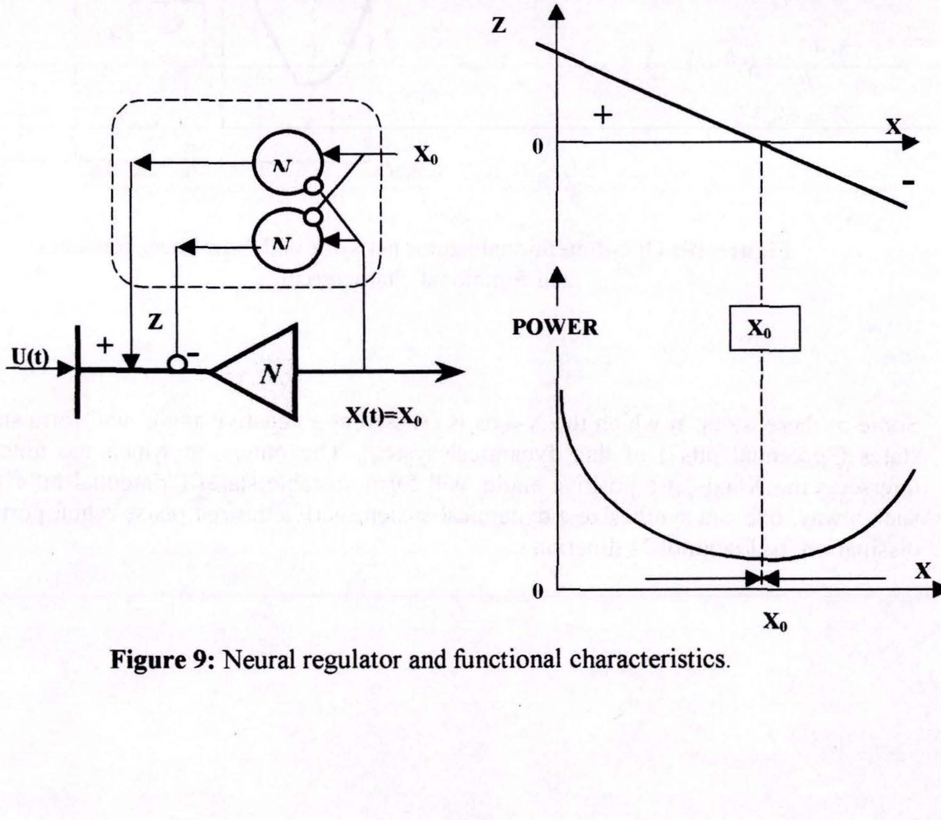
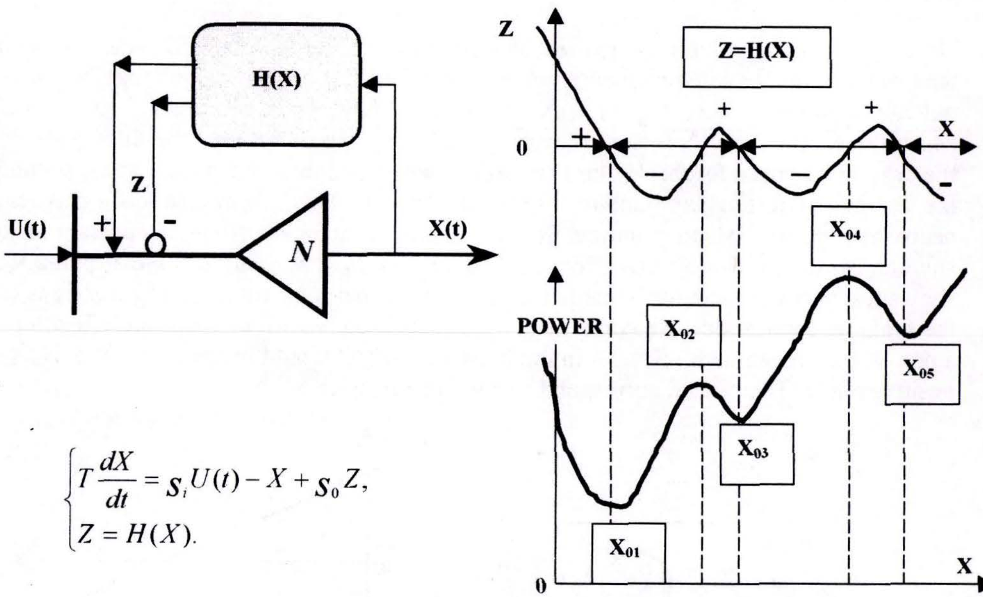


Figure 9: Neural regulator and functional characteristics.

The described neuronal structure gives us insight into a one-dimensional neuronal network with more complicated dynamical characteristics, where the feedback non-linear function Z=H(X) has many real solutions (Fig. 10).



$$\begin{cases} T\dfrac{dX}{dt} = s_i U(t) - X + s_0 Z, \\ Z = H(X). \end{cases}$$

Figure 10: One-dimentional neural network with non-linear feedback and functional characteristics.

Some of these roots, at which the X-axis is crossed at a negative angle, will form stable states ("potential pits") of this dynamical system. The others, at which the function intersects the X-axis at a positive angle, will form unstable states ("potential hills"). In such a way, one can synthesize a dynamical system with a desired phase relief, portrait, dissipation, or Liapunoff's function.

Take, for instance, $H(X)=S_0(X_01-X)(X-X_02)(X_03-X)(X-X_04)(X_05-X)$. This fifth order polynomial forms three "potential pits" and two "potential hills" in the feedback (Fig.12). A neuronal structure, realizing a third-order polynomial feedback and having two "potential pits" and "a hill" in between at desired values of *X*, can be made of three neuronal pairs, calculating differences. If their outputs are fed into the appropriate

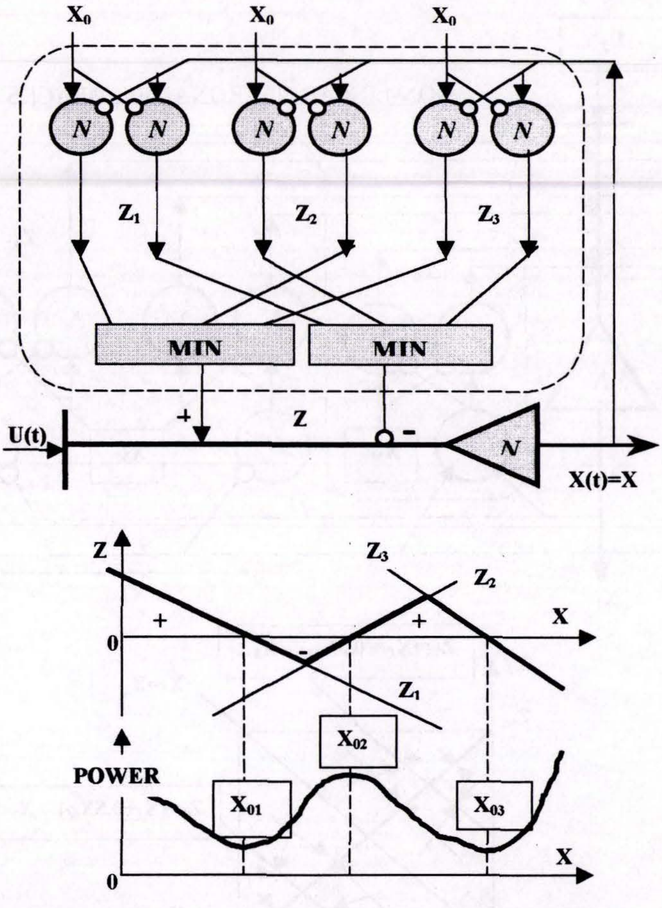


Figure 11: One-dimentional neuronal structure realizing non-linear system with two "potentials pits" and a hill".

neuronal structures passing on the minimal value, and one of which gives excitatory and the other inhibitory feedback effects, one gets a function made of broken lines, which approximates a third-order polynomial (Fig. 11).

7 Multidimensional Neural Net Structure with Non-Linear Feedback

The main feature of neuronal structures is parallel information processing of signal

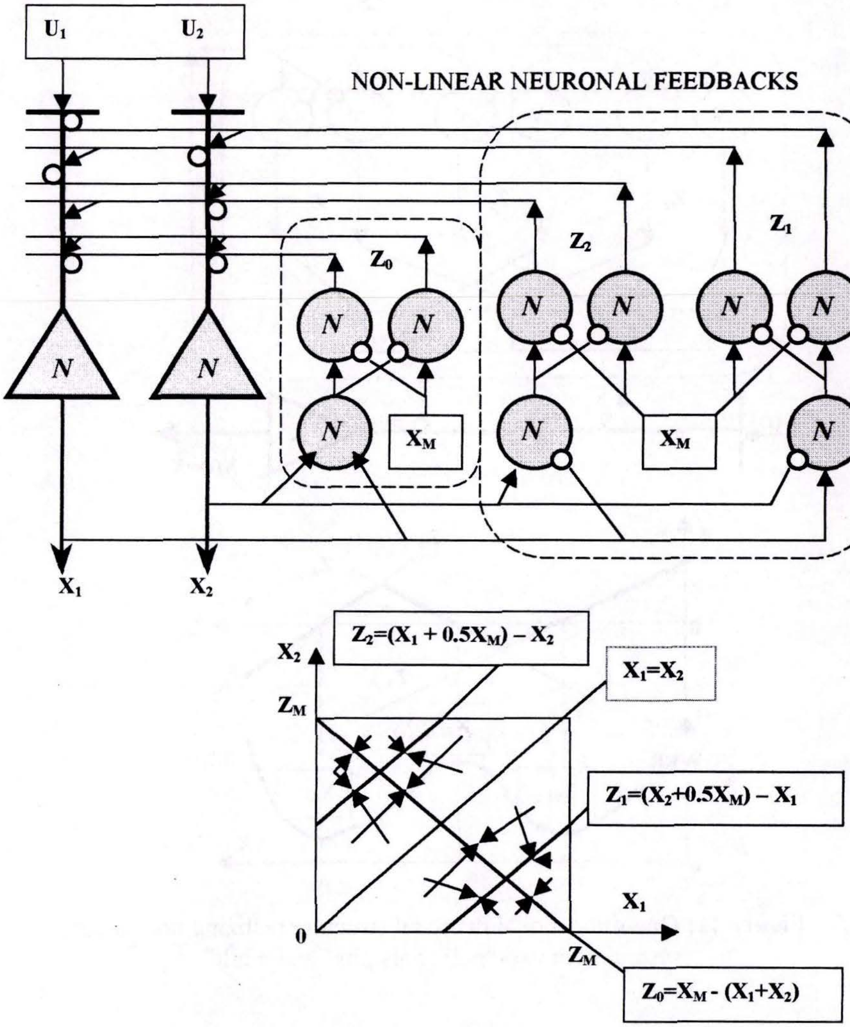


Figure 12: Two-dimentional neural swich with two "potential pits".

vectors. Therefore, it is important to understand the possibility of synthesizing a neuronal structure with desired properties and required multidimensional phase portrait. This can be realized by using a few or many simultaneously functioning pyramidal neurons with appropriate nonlinear negative feedback connections. The negative feedback keeps in check the basic structure elements, pyramidal neurons in this case, not allowing them to reach the saturation limit and, when they get to a certain point of excitation, pushes the system to a level of excitation which is less than X_M. When the level of excitation is low, positive feedback may come into action, too.

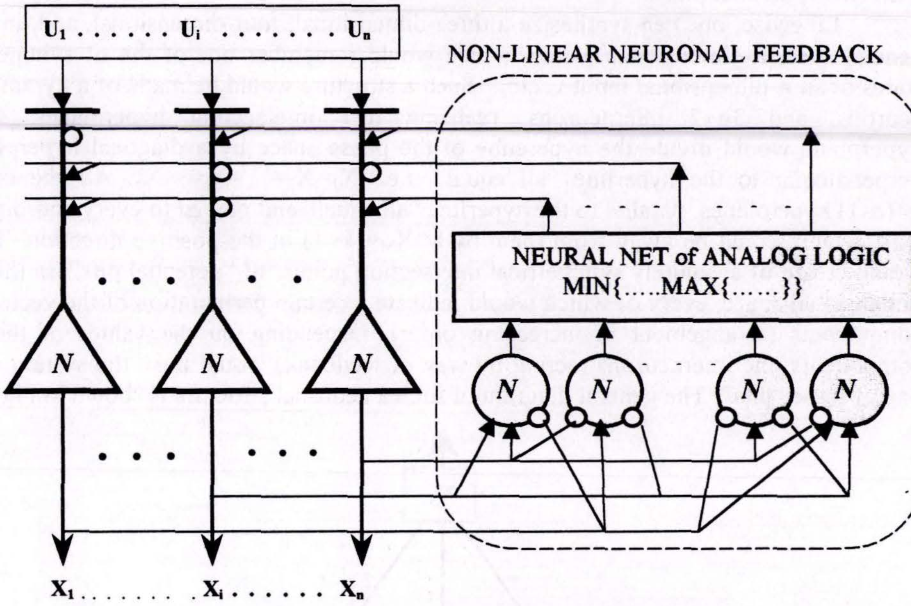


Figure 13: Multidimensional neural swiching net.

Considering the simplest case, suppose we have a two-dimensional structure with two inputs (U_1 and U_2) to two pyramidal neurons *N*, which in turn have two outputs (X_1 and X_2) with such interneuronal feedback that it creates a desired phase relief (portrait). For instance, this system may have two "potential pits" positioned symmetrically with respect to the line $X_1=X_2$, in the sectors $X_1>X_2$ and $X_1>X_2$, and on neither of the coordinate axes (Fig. 12). It would be an "on-off" switch, which could remember the state of the vector **U**, by which component of the vector was bigger. This property emerges in the interneuronal network composed of two parts functioning in parallel; the first part realizes the nonlinear algebraic equation $Z+=+S_0*N[X_M-(X_1+X_2)]$,

$Z = S_1*N[(X_1+X_2)-X_M]$, and the second one realizes the disjunctive (connected by the analog logical operation .OR.or V) expression $Z=S_0*\{N[N(X_1-X_2)-1/2*X_M]$ .OR. $N[1/2X_M-N(X_1-X_2)]$ .OR. $N[1/2X_M - N(X_2-X_1)]$ .OR.$N[N(X_2-X_1)-1/2X_M]\}$.

These in non-linear equations, embedded in neural networks, not unlikely as in the case of the regulator, "push" the state of pyramidal neuron excitation towards one of the points of intersection between the lines $X_1+X_2=X_M$, $X_2=X_1+1/2*X_M$, $X_1=X_2+1/2*X_M$, *i.e.*, towards one of the two possible states: either $X_1>X_2$, or $X_2>X_1$. Such a nine-neuron dynamical system with non-linear feedback has a phase portrait with two "potential pits".

Likewise, one can synthesize a three-dimensional, four-dimensional, and, in the general case, **n**-dimensional switch, which would remember one of the **n!** symmetric states of an **n**-dimensional input vector. Such a structure would be made of n pyramidal neurons, and **3n+2** interneurons, realizing **n+1** intersecting hyperplanes. One hyperplane would divide the hypercube of the phase space by a diagonal hyperplane perpendicular to the hyperline "all equal", i.e., $X_1=X_2=\ldots=X_i=\ldots X_n$. All the other **n*(n-1)** hyperplanes, parallel to the hyperline "all equal" and moved to every coordinate axis, which would be away from them by $\mathbf{k}*X_M$, (k<1) in the positive direction. That would create **n!** absolutely symmetrical intersection points, **n!** "potential pits", in the **n**-dimensional space, every of which would indicate a certain permutation of the vector **U** components (arrangement in increasing order). Depending on the values of the **U** components, the interneurons (acting by way of feedback) would push the system into one of these "pits". The general diagram of such a neuronal structure is shown in Fig.13.

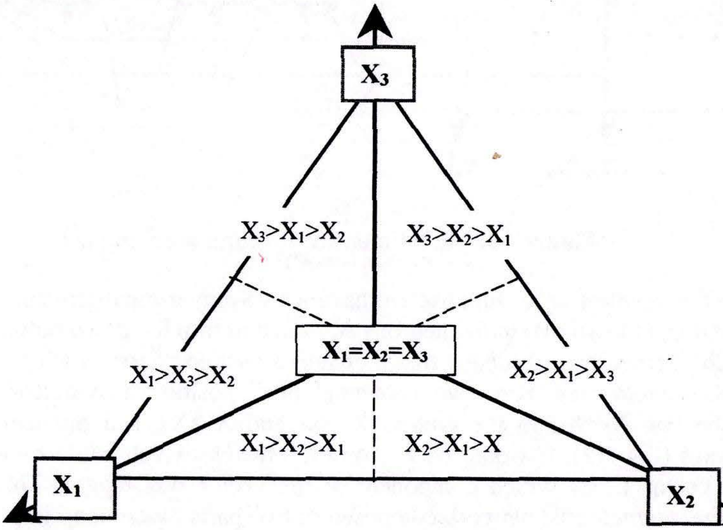


Figure 14: Three-dimensional cube representing factorial states.

The transition to the vector form allows us to understand opportunities of synthesis of multidimensional neuronal structures. Such structures form potential holes not at the tops of the multidimensional cube, but in any place inside the cube. The analog logic allows us to form complex physical surfaces with many potential holes and barriers in the desirable places inside the cube. Thus, it is possible to synthesize various neuronal triggers, qualifiers, filters and any structures processing information. For example, the 3-dimensional cube is presented in Fig. 14.

It is easy to see that similar methods may be used to synthesize n-dimensional dynamical structures with a rather complex phase portraits. We can call them *factorial switches*. If the binary logic is used to analyze the states of a neural net, then n neurons can have 2^n states, whereas the factorial logic of analog neurons can see as many as $\mathbf{M=2^n * n!}$ states. Every hyperquandrant of the phasic space can have **n!** stable states. It can be attained by virtue of the feedback non-linearity of analog interneurons.

Conclusions

- Neuronal investigations need interpretation of neuron as analogue "diode-like" summator of continuous quantities (spike frequencies), different from the present binary formal neuron explanations on the basis of threshold logic;
- The concept of an analogue (continuos) logic presented in this paper can be useful in joining together binary and analogue or fuzzy neural logic nets in a purposively functioning neural schemes (in a point of view of informational transformations and control);
- Dynamical systems with a non-linear neuronal feedback based on analogue logic structures may generate the needed phase portrait and find a way to synthesize the appropriate functions;
- Neuron nets with analogue structures of a non-linear neuronal feedback can synthesize an **n**-dimensional factorial switch, which would remember one of the **n!** symmetric states of n-dimensional input vectors.

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