Advancing Anticipatory Systems Analysis with Hyperincursive Processes, Parity Logic, and Fuzzy Logic

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Abstract

This paper outlines four major topics of tantamount importance to computing anticipatory systems. Section 1 introduces the reader to several historical facts regarding Daniel Dubois' hyperincursive modeling approach and its relationship with Gérard Langlet's work and the author's conception of parity logic systems. It provides the connection of Dubois' hyperincursivity theory and fractal machines with parity logic engines, a special class of binary integro-differential cellular automata. Section 2 on modeling anticipatory systems recalls first the essence of Dubois' anticipatory systems approach by comparing briefly recursivity, incursivity, and self-referentiality. Their impact on modeling cognitive anticipations is then discussed by rendering Piaget's recursive concept of anticipatory schemata into incursive schemata. Section 2 closes with an unresolved problem regarding anticipatory conflicts. Section 3 exhibits in a more formal way the difference between recursivity and incursivity by explicating Dubois' most important digital equations, how they apply to hyperincursive fractal machines, and how they are related to parity logic engines. This includes self-organized processing of parity intergrals and differentials, self-organized development of binary transforms, and several group theoretic implications of transforming parity matrices generated with fractal machines or parity logic engines. Finally, in section 4, further perspectives are outlined for the advancement of anticipatory systems by considering causal predictor systems in terms of fuzzy cognitive maps. This includes the law of concomitant variation, non-Aristotelian causality, the relationship between fuzzy causality, fuzzy subsethood and fuzzy causal cross-impact analysis.

Keywords: Hyperincursivity, Fractal machines, Parity logic engines, Anticipatory modeling, Cognitive anticipation, Integro-differential automatons, Non-Aristotelian causality, Law of concomitant variation, Fuzzy cross-impact analysis

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1 Introduction to Hyperincursive Computing and Hypercubical Calculus with Parity Logic

Hyperincursivity is a central property of self-referential models and processes which are "running on beyond past and present states by including future states" according to $x(t+1) = F[..., x(t-1), x(t), x(t+1), ...]$. It comprises a new mathematical theory due to Daniel Dubois that extends conventional recursive processes (Rosen 1985) to inclusive recursion, henceforth incursion, thereby allowing to develop digital, analogue and hybrid models of anticipatory systems which may include themselves in a self-referential manner (Dubois 1990, 1992, 1997, 1998; Dubois and Resconi 1992).

Incursive and hyperincursive processes shed new light on the arrow of time, since they generate simultaneously Boolean antiderivatives and derivatives, hence binary integrals and differentials. Computational procedures of this sort permit internal and external times scales for considering past, present, and future state vectors. The computational approach itself is based on Dubois' general non-linear and non-monotonous quadratic scalar function

$$x_{n+1} = \alpha + 4\mu x_n(1 - \beta x_n) \tag{1}$$

with $x_n = s_1x_1 + s_2x_2$ where $x, \mu, \alpha \in [0,1]$ and $s, \beta \in [-1,1]$. From this function, a large set of other special functions are derivable, in particular all Boolean truth functional rules, including XOR, the eXclusive-OR rule with $s_1 = s_2 = 1$, $\mu = \frac{1}{2}$, $\alpha = 0$, and $\beta = \frac{1}{2}$, thus

$$y = 2x(1 - x/2) \tag{2}$$

with $x = x_1 + x_2$ (Dubois 1990). Equation (2) is not only a fractal non-linear learning law that solves the XOR-problem through a single formal neuron as opposed to classical neural networks which require hidden units for solving it, but also a **universal fractal propagation law** for artificially or naturally excitable media.

A related hypercubical calculus approach is based on the generalized logical eXclusive-OR operator for parity integration. This operator was first introduced in the programming language APL (Iverson 1962, 1979). It is based on the logical function "unequal" ($x \neq y$), which denotes XOR, and on the reduction operator "scan" (\), thereby yielding the vectorial XOR-operator called Unequal-Scan ($\neq$\) in APL. Gérard A. Langlet called this operator the binary vector integral

$$\mathbf{x}_{n+1} \leftarrow \neq\backslash\, \mathbf{x}_n \in \mathcal{B}^l = \{0,1\}^l, \tag{3}$$

with $\mathbf{x}_n$ and $\mathbf{x}_{n+1}$ being binary vectors in the Boolean hypercube $\mathcal{B}^l$. This alternative approach was Langlet's choice in view of Dubois' foundational work and the work of Dubois and Resconi (Dubois 1990a, 1991, 1992; Dubois & Resconi 1992; Langlet 1991, 1992). In 1994, when I became involved with scientific modeling from scratch, I adopted Langlet's approach with great enthusiasm, since it proved to be rather promising to algorithmic compression. Unfortunately, Langlet never informed me about the works of Dubois and Resconi (Dubois 1992; Dubois & Resconi 1992). I found out about this by direct contact with D. Dubois regarding anticipatory aspects of fuzzy cognitive maps in spring 1999. In 1995, I concentrated my work on the mathematical foundations of crisp and fuzzy XOR, whereby it turned out that the Boolean XOR-operation and its generalized operators are special instances of the **parity function**, from which I derived the term **parity logic**. A **parity logic system** is straightforwardly defined by using the standard notation of set-theoretical predicates (Zaus 1999).

Definition 1.1: The quadruple $\langle \mathcal{B}^l, \oplus, \bigoplus_{i=1}^l x_i \in \mathbf{x}, \bigoplus_{i=1}^{l} x_i \in \mathbf{x} \rangle$ is called a finite Parity Logic System if, and only if, $\mathcal{B}^l$ is the Boolean hypercube $\{0,1\}^l$ such that

(i) elements $\mathbf{x,y,w,z} \in \mathcal{B}^l$ are binary vectors of length l, the vertices of $\mathcal{B}^l$,

(ii) $\mathbf{x} \oplus \mathbf{y}$ is the eXclusive-OR operation, defined on vectors $\mathbf{x,y}$ in $\mathcal{B}^l$, where $\oplus$ is

$$\text{symmetric: } \mathbf{x} \oplus \mathbf{y} = \mathbf{y} \oplus \mathbf{x},$$
$$\text{associative: } (\mathbf{x} \oplus \mathbf{y}) \oplus \mathbf{z} = \mathbf{x} \oplus (\mathbf{y} \oplus \mathbf{z}),$$
$$\text{bisymmetric: } (\mathbf{x} \oplus \mathbf{y}) \oplus (\mathbf{z} \oplus \mathbf{w}) = (\mathbf{x} \oplus \mathbf{z}) \oplus (\mathbf{y} \oplus \mathbf{w}).$$

(iii) $(\bigoplus_{i=1}^l x_i \in \mathbf{x}) = (x_1 \oplus x_2 \oplus \cdots \oplus x_l) = \begin{cases} 1 & \text{for an odd \# of 1s in } \mathbf{x} \\ 0 & \text{for an even \# of 1s in } \mathbf{x} \end{cases}$

(iv) $(\bigoplus_{i=1}^{l} \; x_i \in \mathbf{x}) = (x_1, (x_1 \oplus x_2), \cdots, (x_1 \oplus x_2 \oplus \cdots \oplus x_l)) = \mathbf{z} \in \mathcal{B}^l$, where the resulting binary vector $\mathbf{z} = (z_1, z_2, ..., z_l)$ is the **parity integral** of $\mathbf{x} \in \mathcal{B}^l$, whereas $\mathbf{x}$ is in turn the **parity differential** of $\mathbf{z}$ $\diamond$. Section 3 will make this more transparent.

Parity integration is thus an asymmetrical operator which preserves all structural properties of XOR, in particular the bisymmetry property which guarantees entropy preservation for binary vectors $\mathbf{x,y,z,w} \in \mathcal{B}^l$. Unlike Dubois' propagated fractal object of equation (2), which is a numerical scalar function, this one in expression (4) below is a non-numerical operator and operates exclusively at a computer's binary processing level. It serves not only as a power tool for incursive or hyperincursive parity integration

$$\mathbf{x}_{n+1} \hookleftarrow \bigoplus_{i=1}^{l} x_i \in \mathbf{x}_n \in \mathcal{B}^l, \tag{4}$$

but also as the main operator for $\mathcal{B}^l$ becoming a „*computing space*" in the spirit of Konrad Zuse (Dubois 1997, Zuse 1969). In what follows we will relax our formal notation a bit by using the symbol $\mathcal{P}$ for parity integration and $\mathcal{P}^{-1}$ for its inverse, i.e. parity differentiation. So, $\mathcal{P}(\mathbf{x}) = \mathbf{z}$ and $\mathcal{P}^{-1}(\mathbf{z}) = \mathbf{x}$ are two central concepts in parity logic, and the pair $PLS = \langle \mathcal{B}^l, \mathcal{P} \rangle$ turns out to be a nucleus for hypercubical calculus and hyperincursive computing. It implies a variety of parity logic engines, whose special nature depends on $\mathcal{P}$'s argument, i.e. whether it is a binary vector, an array or hyperarray, whose entities are processed in parallel through $\mathcal{P}$ (Dubois 1997, Zaus 1999).

Dubois' fundamental hyperincursivity theory, hyperincursive processes of Dubois and Resconi, and their binary integro-differential counterparts due to Langlet and myself share many computational procedures as based upon fractal machines and parity logic engines. Fusing and exploiting the power of both approaches is now our primary objective in order to advance anticipatory systems research.

2 Modeling Anticipatory Systems

Let us first recall Dubois' hyperincursive modeling approach to anticipatory systems, before we discuss its connection with cognitive anticipations. Dubois extended Rosen's approach to anticipatory computing decisively as follows (Rosen 1985, Dubois 1998a, 1998b). If the future state of a system S and the model M at time $t + \Delta t$ is a function F of this system S at time t and of the model M at a later time step $t + \Delta t$, then one obtains basically two fundamentally different relationships which, however, coincide in one functional term:

$$\Delta S/\Delta t = [S(t + \Delta t) - S(t)]/\Delta t = F[S(t), M(t + \Delta t)] \quad (5)$$

$$\Delta M/\Delta t = [M(t + \Delta t - M(t)]/\Delta t = F[S(t), M(t + \Delta t] \quad (6)$$

This looks strange at first glance, but if the model is the system itself, then $M = S$, and equations (6) and (7) reduce to

$$\Delta S/\Delta t = [S(t + \Delta t) - S(t)]/\Delta t = F[S(t), S(t + \Delta t)] \quad (7)$$

which is INclusively reCURSIVE, hence an incursive system. Incursivity holds when the future state of a system $S(t + \Delta t)$ depends not only on past and present states, but also on future state(s) of the system, hence on the sequence ..., $S(t - \Delta t), S(t), S(t + \Delta t), S(t + 2\Delta t), ...$, which constitutes one of Dubois' central equations.

$$S(t + \Delta t) = F[..., S(t - \Delta t, S(t), S(t + \Delta t), ...] \quad (8)$$

To compare the fundamental differences between the recursive, incursive, and self-referential approaches at a glance, where p refers to a command parameter, we obtain the following setup.

1. Recursivity $x(t + 1) = f[(x(t), p]$
2. Incursivity $x(t + 1) = f[x(t), x(t + 1), p]$
3. Self-reference $x(t + 1) = f[x(t), f[x(t), x(t + 1), p], p]$

The second and third expressions provide a first principles approach to a theoretically well-established framework of anticipatory and self-referential systems research. Dubois solved thus the old riddle of self-containment which – when viewed from a strictly recursive point of view – leads ultimately to an infinite regress.

Minsky's famous "mind, matter and models" analysis collapsed precisely for this reason, because infinite recursion seemed to be an inescapable problem (Minsky 1982). The same holds for Piaget's theory of cognitive equilibration as based upon recursive coordinative interaction schemes (Piaget 1975). The way Heinz von Foerster modeled them through recursive computations („cognition computes its own cognitions in order to reach eigenvalues viz. equilibria") was valuable in as much as it demonstrated the limitations of recursive computations due to their restriction to past and present states, and due to their omission of the issue of self-referentiality (v. Foerster 1976). Piaget's three distinguished concepts of cognitive, organic, and morphogenetic anticipation reveal basically similar recursive drawbacks (Piaget 1974). However, Piaget's conjecture that anticipations emerge from recurrent and extrapolative schemata extensions, where the latter may create a virtual future within internal representations, makes it a viable candidate for revising it from the hyperincursive point of view by *eliminating* the schema's atemporality.

The following summary, extracted from his monograph entitled „Biologie et connaissance", recalls Piaget's simplest model of how cognitive anticipations emerge[1] :Suppose a child aged 11-12 months pulls softly, but accidentally, a blanket (action A), thereby releasing a particular motion of some toy on that blanket (result B), thus $A \rightarrow B$. According to Piaget, result B gets immediately connected with action A through feedback such that A becomes associated with B. In this way, an accidental act has become an elementary schema AB with two possible extensions.

The first of which is called *extrapolation* by virtue of its forward direction into the future, and the second of which is called *recurrence* by virtue of its backward direction into the past. Extrapolation is based on a continuation of the resultant motion B to other positions B_2 or B_3 such that each new result is connected with the *initial* act A through feedback. The recurrent part consists in recognizing that action A was triggered by certain clues $-A_2$ and $-A_3$ prior to action A.

The total sequence $-A_3 \rightarrow -A_2 \rightarrow A \rightarrow B \rightarrow B_2 \rightarrow B_3$ emerges thus incrementally through multiple recurrent feedback, according to Piaget, and constitutes finally a cognitive anticipation schema in terms of a network, in which each instance of the now cyclic sequence can be joined with each other such that the external time order $A \rightarrow B$ of action A and consequence B gets lifted to the extent that A is reachable from B or vice versa. Thus, the initially non-anticipatory schema AB becomes an anticipatory schema through its forward and backward – hence bidirectional – extension (see figure 1 below). From now on, the child is ready to transfer and use that schema mentally in new situations before these actually occur over time. Piaget concluded that a particular extension in one of both directions suffices already for anticipation, because the extension resolves the schema into its constituent parts of recurrences and extrapolations [Piaget 1974, pp. 197 – 199].

Piaget's above characterization of anticipatory schemata is based on the assumption that conservation of information acquired earlier in time results at all higher cognitive stages in a considerably widespread anticipatory response behavior. One of the main functions of knowledge consists therefore in enabling human individuals to predict. Piaget claimed that at none of these cognitive stages, including the top level of thinking, is a **causa finalis** a prerequisite for anticipation, because anticipations derive exclusively from information acquired in the past through inferential thought processes, multiple feedback, perceptual transfer or sensori-motor-coordinations.

There are several aspects of Piaget's view which deserve a closer examination for epistemic and mathematical reasons. First, Piaget adopted an essentially non-finalistic,

[1] Piaget, J. 1967 Biologie et connaissance. Gallimard, Paris. We refer to its translation „Biologie und Erkenntnis", in particular §13, pp. 188-205 Fischer Verlag, Frankfurt/Main 1974.

if not anti-finalistic position. His rejection of a „causa finalis" as a prerequisite for anticipation is based partially on his rather doubtful complaints about the „ambiguous character of the concept *causa finalis*", and partially on traditional cybernetical reasons according to which any finalism has to be abandoned in favor of recursive feedback.

Second, although Piaget emphasized that *expectations* determine the whole sensorimotor organization, he disregarded any goal-directed implications of cognitive anticipations. Abandoning finalism is therefore too simple, since teleology, the study of final causes, needs to be reexplicated in terms of teleomaticity and teleonomy, where „causalized intentions" and „intentionalized causations" play a central role. In goal-oriented perception and action, cognitive anticipations rest on *anticipatory information*, i.e. on kinematical information that specifies kinetical or proprioceptive variables (Shaw & Kinsella-Shaw 1988; Runeson & Frykholm 1981; Zaus 1999). This is also evident from the intersensory size-weight illusion: If two stimuli viz. boxes A and B with identical weights $W(A) = W(B)$ but different sizes S with $S(A) \gg S(B)$ are first visually presented and then bimanually lifted, then stimulus A is judged lighter than B, i.e. $W(A) < W(B)$. Piaget referred to this illusion without recognizing that visually based kinematic information leads to an anticipatory bias which in turn „fools" the subject in comparing the weights of both stimuli, because less effort is anticipated for lifting B although it actually required more effort, thereby inducing the illusion that $W(A) < W(B)$ (Woodworth & Schlosberg 1954).

Third, Piaget argued that an anticipatory schema emerges recursively, and once established as an internal representation, it becomes *atemporal*, i.e. time has ceased to exist. The left diagram of figure 1 below illustrates Piaget's final anticipatory schema, in which each position is inferentially accessible from any other position. What remains is at most a spatial schema with interchangeable local contiguities.

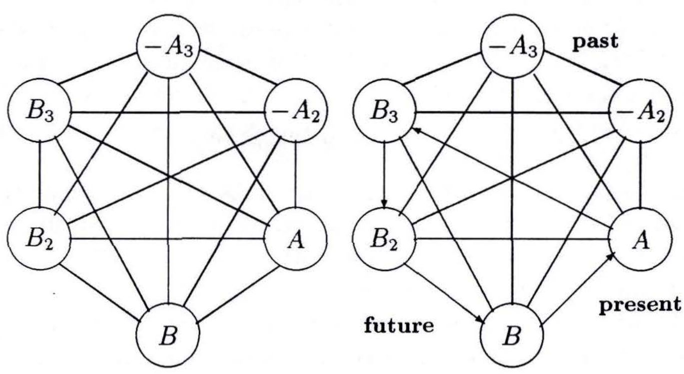


Fig. 1 Piaget's original and revised anticipatory schema AB

Fourth, Piaget's reasoning conflicts with the nature of temporal associative memories in general, and with goal-intended behavior in particular, since anticipatory information is defined kinematically as a temporal backflow of information from the target to the actor's current state. As shown in the right diagram of figure 1, Action A may then trigger the internal future instance B_3 with a temporal backflow of anticipatory information included in $A \to B_3 \to B_2 \to B \to A$, because that's the way it was internalized through multiple feedback in accordance with Piaget: $A \leftarrow B$, then $A \leftarrow B_2$, and $A \leftarrow B_3$, thereby admitting a 3rd order anticipation $A \to B_3$.

In Piaget's original view (left diagram of Fig. 1), the cyclic closure of schema AB consists in a network allowing forward-, backward- and cross-inferences *theoretically*. But atemporality would exclude internal time scales, temporal backflow of anticipatory information, and hyperincursiveness in terms of including past, present and future states for anticipating a future state (right diagram of Fig. 1). It would exclude also anticipatory scenarios which follow inverse paths by starting with the future and which work backwards to the present to discover what alternatives and actions are necessary to attain these futures, hence exploring the most efficient „causa finalis". In conclusion, Piaget's revised anticipatory schema admits not only mental associations from present to past or from past to present, but also mental associations from present to future or from future to present and past, whereby it becomes necessarily incursive or hyperincursive.

An unresolved problem in recursive and incursive approaches are anticipatory conflicts, also related to Anochin's functional systems and anticipatory reflections of real world events in human and animals, as depicted in figure 2 below (Anochin 1962).

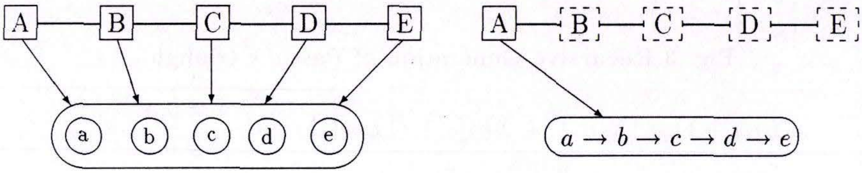


Fig. 2 Anochin's anticipatory reflection of future events

At left a serial processing of external events $(A, B, ..., E)$ at different time intervals induces gradually a corresponding internal disposition $(a, b, ..., e)$, i.e. a stored memory sequence as based on many replications. The right part of figure 2 shows how an organism might anticipate a future event E when the present cue event A happens, since A triggers a future oriented propagation chain $(a_t, b_{t+1}, c_{t+2}, d_{t+3}, e_{t+4})$ with a faster internal time scale than the real external time scale. In other words, the sequence $< A, B, ..., E >$ may take a large amount in (external) physical time, but only a minimal fraction of internal time for processing $< a, b, ..., e >$ to anticipate event E.

Although Anochin's conception looks incursive, it is in fact recursive for the following reason. Suppose an event sequence $< X, Y, Z >$ is usually followed by W, and another event sequence $< U, Y, Z >$ by event V. To anticipate the consequences of Z, the organism needs to know what occurred two steps before Z. Anticipation requires in this case a third-order transition, because to decide between W and V the organism needs to know three preceeding instances of the sequence. This problem generalizes to competition among k-fold anticipations and is currently investigated with multisets in neural network modeling (Kanerva 1988; Gluck & Meyers 1997; Zaus 2000).

3 Hyperincursive Fractal Machines and their Relationship with Parity Logic Engines

"There was a young lady named Bright, whose speed was faster than light; she went out one day, in a relative way, and returned the previous night" (A.H.R. Buller). Things which are impossible in reality aren't necessarily impossible computationally. It's like using a Lamarckian algorithm to solve an optimization problem successfully. Nobody would reject that algorithm if it saves lifes or advances robotics. Consider now the following one-dimensional cellular automatons for generating Pascal's triangle and its modulo 2 counterparts (Dubois 1997, 1998a). Notice especially equation (11) which represents Dubois' incursive digital equation as compared with equation (10).

$$X(n,t+1) = [X(n,t) + X(n-1,t)] \text{ with } \begin{matrix} t = 0,1,2,... \\ n = 0,1,2,... \end{matrix} \quad (9)$$

t\n	0	1	2	3	4
$t=0$	0	1	0	0	0
$t=1$	0	[1]	[1]	0	0
$t=2$	0	1	[2]	1	0
$t=3$	0	1	3	3	1
$t=4$	0	1	4	6	4

Example: $X(2,2) = X(2,1) + X(1,1) = 2$

Fig. 3 Recursive Generation of Pascal's triangle

$$X(n,t+1) = [X(n,t) + X(n-1,t) \bmod 2] \text{ with } \begin{matrix} t = 0,1,2,... \\ n = 0,1,2,... \end{matrix} \quad (10)$$

t\n	0	1	2	3	4
$t=0$	0	1	0	0	0
$t=1$	0	[1]	[1]	0	0
$t=2$	0	1	[0]	1	0
$t=3$	0	1	1	1	1
$t=4$	0	1	0	0	0

Example: $X(2,2) = X(2,1) \oplus X(1,1) = 0$

The future state $X(2,2)$ at $n=2$ and $t=2$ depends on present and past states (Recursivity)

Fig. 4 Recursivity of Pascal's triangle modulo 2

$$X(n,t+1) = [X(n,t) + X(n-1,t+1) \bmod 2] \text{ with } \begin{matrix} t = 0,1,2,... \\ n = 0,1,2,... \end{matrix} \quad (11)$$

t\n	0	1	2	3	4
$t=0$	0	1	0	0	0
$t=1$	0	1	[1]	1	1
$t=2$	0	[1]	[0]	1	0
$t=3$	0	1	1	0	0
$t=4$	0	1	0	0	0

Example: $X(2,2) = X(2,1) \oplus X(1,2) = 0$

The future state $X(2,2)$ at $n=2$ and $t=2$ depends on past, present and future states (Incursivity)

Fig. 5 Incursivity of Pascal's triangle modulo 2

Equation (11) regarding figure 5 is essentially the core of an **integro-differential cellular automaton,** a fractal machine whose existence was never mentioned before by Wolfram or other proponents of the theory of cellular automatons (Peitgen et al. 1992, Hameroff et al. 1993, Wolfram 1994). We expect a conceptual revision in that respect in Wolfram's forthcoming magnum opus entitled "The New Science of Complexity". While equation (11) centers upon temporal incursivity, it may be as well rewritten in terms of spatial incursivity by replacing time t by space s, which indeed resembles a Laplacien operator (Dubois 1998a):

$$X(n, s+1) = [X(n, s) + X(n-1, s+1) \bmod 2] \text{ with } \begin{array}{l} s = 0, 1, 2, ... \\ n = 0, 1, 2, ... \end{array} \quad (12)$$

We may add another local rule as shown in equation (13) which exhibits the nature of parity differentiation, the inverse of parity integration. Unlike equation (10), this one is going back to the past and reveals the higher ordered Boolean derivatives, where the result at $t = -1$ is the first parity differential of the leading argument at $t = 0$.

$$X(n, t-1) = [X(n, t+1) + X(n-1, t+1) \bmod 2] \text{ with } \begin{array}{l} t = 0, -1, -2, ... \\ n = 0, 1, 2, ... \end{array} \quad (13)$$

t\n	0	1	2	3	4
$t = 0$	0	1	0	0	0
$t = -1$	0	[1]	[1]	0	0
$t = -2$	0	1	[0]	1	0
$t = -3$	0	1	1	1	1
$t = -4$	0	1	0	0	0

Example: $X(2, -2) = X(2, -1) \oplus X(1, -1) = 0$

The past state $X(2, -2)$ at $n = 2$ and $t = -2$ depends here on present and future states, a typical form of inverse incursivity

Fig. 6 Iterative parity differentials modulo 2

A closer examination of equations (11) and (13) shows that we obtain their joint results by means of self-generating Boolean sequences through Dubois' fractal machine as based upon equation (11), or equivalently through iterated parity integration $\mathcal{P}_{it}(\mathbf{x})$ of the argument $\mathbf{x}(t-4) = 1\,0\,0\,0$ alone, subject to a parity logic engine (PLE):

$$\begin{array}{llll}
\mathbf{x}(t-4) = & (1 \;\; 0 \;\; 0 \;\; 0) & \downarrow \text{PLE-Stages} \\
\mathbf{x}(t-3) = & (1 \;\; 1 \;\; 1 \;\; \mathbf{1}) & = \mathcal{P}(\mathbf{x}(t-4)) \\
\mathbf{x}(t-2) = & (1 \;\; 0 \;\; \mathbf{1} \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t-3)) \\
\mathbf{x}(t-1) = & (1 \;\; \mathbf{1} \;\; 0 \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t-2)) \\
\mathbf{x}(t) = & (\mathbf{1} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t-1)) \\
\mathbf{x}(t+1) = & (1 \;\; 1 \;\; 1 \;\; \mathbf{1}) & = \mathcal{P}(\mathbf{x}(t)) \\
\mathbf{x}(t+2) = & (1 \;\; 0 \;\; \mathbf{1} \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t+1)) \\
\mathbf{x}(t+3) = & (1 \;\; \mathbf{1} \;\; 0 \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t+2)) \\
\mathbf{x}(t+4) = & (\mathbf{1} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0}) & = \mathcal{P}(\mathbf{x}(t+3))
\end{array}$$

Explanation: Notice that $\mathbf{x}(t)$ depends on past states $\mathbf{x}(t) = \mathcal{P}(\mathbf{x}(t-1))$ or on future states $\mathbf{x}(t) = \mathcal{P}(\mathbf{x}(t+3))$. Next, $\mathbf{x}(t)$ is the parity integral $\mathcal{P}(\mathbf{x}(t-1))$ and the parity differential $\mathcal{P}^{-1}(\mathbf{x}(t))$. The integration process generates implicitly three transforms indicated by the bold printed trigonal transforms (Dubois 1992; Langlet 1992; Zaus 1999).

Fig. 7 Compatibility of Fractal Machines with Parity Logic Engines

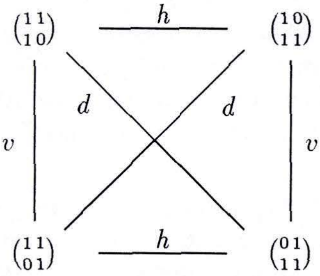
More specifically, $\mathbf{x}(t)$ is both a binary integral and differential, it is an integro-differential entity. The top-down-process involves iterated parity integration $\mathcal{P}(\mathbf{x})$ with a positive time arrow, whereas the bottom-up-process involves parity differentiation $\mathcal{P}^{-1}(\mathbf{x})$ with a negative time arrow, whereby parity logic engines or fractal machines become time reversible autonomous automatons. Thus, future states exist already in present and past states, as conjectured for anticipatory computing in fields as diverse as cellular, cognitive, organic or morphogenetic processes with synchronous and asynchronous viz. heterosynchronous development. For details in these respects the reader is referred to Dubois (1991, 1992, 1997, 1998a), Dubois & Resconi (1992), Langlet (1991, 1992, 1994, 1995), and Zaus (1994, 1996, 1999).

Of tantamount importance is the fact that iterated parity integration generates automatically transforms and transformation matrices **for any** binary vectors $\mathbf{x} \in \mathcal{B}^l$ whose length l is a power of 2. So far, a total of about 30 transforms have been identified, including Dubois transforms, Shegalkin transforms, and Langlet transforms.[2]

In what follows we restrict ourselves to some group theoretic implications. Let the elementary bit „1" be a unique precursor, then by XORing „1" with itself via $1 \oplus 1 = 0$ yields what „1" is not itself, hence „0". Through concatenation, we obtain the elementary sequence (1 0). Subjecting (1 0) to iterated parity integration $\mathcal{P}_{it}(1\,0)$ generates what Langlet christened the 2-geniton $\mathcal{G} = \binom{1\,1}{1\,0}$ (Langlet 1992). Next, a clockwise rotation of $\mathcal{G}$ around its barycenter by 90°, or equivalently, by introducing three involutive reflection operations vdh, generate

$$\mathcal{G} = \binom{1\,1}{1\,0} \rightarrow v(\mathcal{G}) = \binom{1\,1}{0\,1} \rightarrow d(\mathcal{G}) = \binom{0\,1}{1\,1} \rightarrow h(\mathcal{G}) = \binom{1\,0}{1\,1},$$

hence a four-group of transformation operators with $\mathcal{G}$, its vertical (v), diagonal (d), and horizontal (h) reflections. This four-group is related in binary algebra to the Kleinian 4-group, as depicted in figure 8.



Involution	Commutativity
$v^2 = I$	$dhv = vdh$
$h^2 = I$	$vh = hv = d$
$d^2 = I$	$vd = dv = h$
$dhv = I$	$hd = dh = v$
$vdh = I$	

Fig. 8 The geniton $\mathcal{G}$ and its 4-group of transformations

Notice that $\mathcal{G}_v = \binom{1\,1}{0\,1}$ is self-inverse, i.e. $\binom{1\,1}{0\,1}^{-1} = \binom{1\,1}{0\,1}$, and that its square $\binom{1\,1}{0\,1}^2 = \binom{1\,0}{0\,1} = \mathcal{G}_U$, hence the unit matrix of $\mathcal{G}$. The structure $\langle \mathcal{G}_v, \mathcal{G}_U, \mathcal{P} \rangle$ constitute a **switching group**, since $\mathcal{P}[\binom{1\,1}{0\,1}] = \binom{1\,0}{0\,1}$ and $\mathcal{P}[\binom{1\,0}{0\,1}] = \binom{1\,1}{0\,1}$, respectively. This proves again that parity integration $\mathcal{P}$ is a fundamental operator for reversible computing in celluar automatons. In particular, the relationships demonstrated above hold for any order of upscaled genitons, i.e. for $n \times n$ parity matrices called paritons (Dubois 1991, 1992; Langlet 1992; Zaus 1999). The geniton $\mathcal{G}_v$, also called $S-matrix$ in binary computing (Iverson 1962), plays a central role in binary signal analysis (Shegalkin polynomials and transforms). Its impact on the binary counterpart of Fourier transforms is discussed in Zaus (1999), to which the reader is referred for more details.

[2] A comprehensive overview of these transforms is in preparation for the 4th International Conference on Computing Anticipatory Systems, CASYS '2000 (Zaus 1999, 2000a).

4 Fuzzy Logic and Non-Aristotelian Causality

A fuzzy set A is a set whose members $a_1, ..., a_l \in A$ belong to A with membership degrees $\alpha_i \in I^l = [0,1]^l$, where I^l is the l-dimensional unit hypercube. In hypercubical calculus, every set is therefore a fuzzy set, because crisp conventional sets are just degenerate fuzzy sets located as Boolean vectors at the vertices of $[0,1]^l$, whereas proper fuzzy set reside as fuzzy unit or fit-vectors $(\alpha_1, ..., \alpha_l) \in [0,1]^l$ inside the unit hypercube. Fractal machines and parity logic engines operate in hypercubes, and so do causal predictor systems in terms of fuzzy cognitive maps (FCMs).

This section outlines a non-Aristotelian concept of causality called fuzzy causality, derived from hypercubical calculus in neural network theory and fuzzy logic (Kosko 1986, 1988, 1992, 1997; Zaus 1999, 1999a). The mathematical backbone of fuzzy causality is the law of concomitant variation (Kosko 1988). In particular, let $\langle \mathcal{C}, e \rangle$ be a causal concept algebra for causal concepts C_i with $i = 1, 2, ..., n$, and a fuzzy causal edge function $e : \mathcal{C} \times \mathcal{C} \longrightarrow [-1,1]$ such that $e_{ij}(C_i, C_j) \in [-1,1]$, where $[-1,1]$ denotes the bipolar unit interval. The law of concomitant variation states how the causal edge weights e_{ij} between causal concepts C_i and C_j change over time.

$$\dot{e}_{ij} = -e_{ij} + \dot{C}_i(x_i)\dot{C}_j(x_j) = -e_{ij} + \frac{dC_i(x_i)}{dt}\frac{dC_j(x_j)}{dt}; \quad (14)$$

$$e_{ij}(t+1) = e_{ij}(t) + \dot{e}_{ij} \quad (15)$$

Thus, multiplying the derivatives of C_i and C_j, and upgrading this product with a causal decay factor $-e_{ij}$, we obtain the new causal edge weight $e_{ij}(t+1)$ at time $(t+1)$. The decay factor forces zero causality between unchanging concepts, because no changes imply no causal impact. To provide a simple but instructive example, we obtain the following FCM as displayed in figure 9 below.

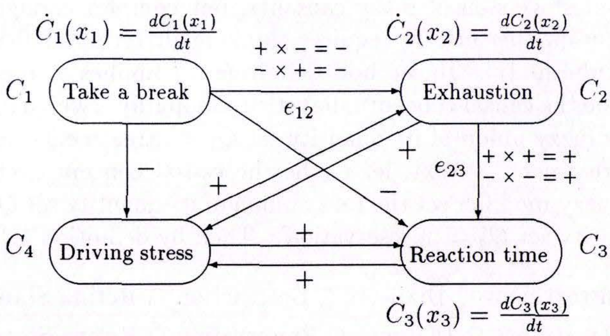


Fig. 9 Sign rule and Concomitant Variation in FCMs

A decisive property of the law of concomitant variation is that we can model concomitancy by products of differentials $\dot{C}_i\dot{C}_j$, and change over time through derivatives. Concomitant variation drives consequently the dynamics in the entire causal network. The sign rule in figure 9 indicates, that negative causality is coupled with negative differential products, and positive causality correspondingly with positive differential products. For example, an increasing break (+) causes a decrease of exhaustion (−), hence negative causality, while a decreased break (−) causes increased exhaustion (+), again negative causality. Contrary, an increase in exhaustion (+) causes an increase in reaction time (+), hence positive causality, while a decrease in exhaustion (−) causes a decrease in reaction time (−), again positive causality. Aristotelian causality concepts are subject to classical logic, i.e. bivalent logic (including probability theory, where the laws of bivalent logic and set theory are simply maintained).

Non-Aristotelian causality concepts, however, are subject to many-valued logic, i.e. fuzzy logic. Here, causal edge functions e_{ij} are many-valued in a partially ordered set P of linguistic variables, or infinitely valued in either the bipolar unit interval $[-1, 1]$ or in the unit interval $[0, 1]$. Using the causal edge function $e_{i,j} : \mathcal{C} \times \mathcal{C} \longrightarrow [0, 1]$ requires that we convert negative causality into positive causality by using dis-concepts like ,,instability" instead of ,,stability". Fuzzy subsethood $S(A, B)$, the degree$(A \subset B)$ to which fuzzy set A is contained in fuzzy set B, underscores non-Aristotelian causality, because the fuzzy causal edge function $e : \mathcal{C} \times \mathcal{C} \longrightarrow [0, 1]$ is representable by fuzzy subsethood:

$$e_{ij} = C_i \xrightarrow{e_{ij}} C_j = S(C_i, C_j) = \mu_{F(2^{C_j})}(C_i), \tag{16}$$

where $\mu_{F(2^{C_j})}(C_i)$ denotes the membership degree of fuzzy causal concept C_i in fuzzy causal concept C_j's fuzzy powerset, hence the degree of subsethood $S(C_i \subset C_j)$.

Definition 4.1: Let C_i and C_j be fuzzy concepts, and let Q_i, Q_j, Q_i^c, Q_j^c be their associated quantity and dis-quantity sets, where Q^c denotes the complement of Q. Then

$$C_i \text{ causes } C_j \text{ iff } Q_i \subset Q_j \text{ and } Q_i^c \subset Q_j^c$$
$$C_i \text{ causally decreases } C_j \text{ iff } Q_i \subset Q_j^c \text{ and } Q_i^c \subset Q_j \diamond$$

This is the simplest version of fuzzy causality, but complex enough to challenge the reader's imagination, because it requires thinking in terms of fuzzy subsethood in the unit hypercube $[0, 1]^n$. To see how definition 5.1 applies to medical cross-impact analysis, let C_1 be the causal concept ,,diabetic retinopathy", which intersects as a fuzzy modifier set the fuzzy union of its quantity set Q_1 = ,,treatment" and its dis-quantity set Q_1^c = ,,untreatment". Next, let C_2 be the causal concept ,,retina state", which intersects as a fuzzy modifier set the fuzzy union of its quantity set Q_2 = ,,destruction" and its dis-quantity set Q_2^c = ,,preservation". Then by definition 5.1 we get

$$\text{Untreatment} \cap \text{Diabetes} \subset \text{Destruction} \cap \text{Retina State} \tag{17}$$
$$\text{Treatment} \cap \text{Diabetes} \subset \text{Preservation} \cap \text{Retina State} \tag{18}$$

So untreated diabetic retinopathy causes retina destruction (D), hence blindness (B), and treated diabetic retinopathy causes retina preservation. The in- or decreased causal relationship is representable by the corresponding inclusion of intersections, as shown in figure 9 below. Notice also that the causal relationship reflects positive concomitant variation such that causal strengths are determined by the product of differentials, provided both fuzzy causal concepts C_1 and C_2 are measureable with respect to appropriate (dis-)quantity sets. The central message of this example is at least threefold: (1) Fuzzy causality $e_{1,2} : C_1 \to C_2$ is representable by fuzzy subsethood $S(C_1, C_2)$, (2) from fuzzy subsethood we may derive Bayes' theorem $P(C_2 \mid C_1) = S(C_1, C_2)$, thereby reducing probability to subsethood, and (3) by virtue of clause (2) fuzzy subsethood, and thus non-Aristotelian causality, qualifies as a predictor in modern cross-impact analysis (Kosko 1992, Zaus 1999, Zaus 2000b).

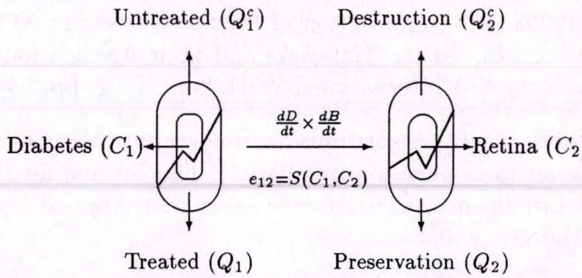


Fig. 10 Non-Aristotelian causality as a fuzzy predictor

Fuzzy causality in terms of $e_{ij}(C_i, C_j)$ takes spatio-temporal, time associative, and anticipatory behavior into account. Why anticipatory behavior? Because an FCM is a forward chaining inference engine, and a forward chain is a prediction of effects given the respective causes in the FCM's causal web network. Even simple FCMs with crisp causal concept nodes and fuzzy causal edges are predictor systems, since they predict limit cycles, hence fixed points in terms of single events, or oscillatory states with alternating events, or periodical states with reoccurring event sequences, or even chaotic states with aperiodical event sequences (Kosko 1997; Zaus 1999).

Forward chaining inference engines like FCMs answer ,,what happens, if ...?" questions, and these questions are always directed at the future. Now, if a causal edge $e_{ij}(C_i, C_j)$ is representable by fuzzy subsethood $S(C_i, C_j)$, and in fact it is, then the present cause C_i is contained in the future, i.e. in C_j to some degree. This partial containment is impossible in Aristotelian causality concepts, because the law of excluded middle states that every set either is or is not a subset of every other set. Moreover, the classical probability term $P(B \mid D)$, the probability of blindness (B) given diabetic retinopathy (D), represents only the degree of correlation between diabetes and blindness, whereas fuzzy subsethood represents $P(B \mid D) = S(D, B)$ as well as the causal relationsship $S(D, B) = e : D \to B$. Finally, diabetes retinopathy and blindness are matters of degree, and so is their causal relationship, but these matters of degree are infinitely-valued, thus neither bivalent nor probabilistic, hence non-Aristotelian, respectively. The idea of non-Aristotelian systems goes back to Korzybski (1994).

In conclusion, the law of concomitant variation and fuzzy causality extend the four Aristotelian causality concepts fundamentally, because they provide a causation-based hypercubical calculus **inside** the unit hypercube $I^l = [0, 1]^l$, where causal concepts and relations are turned into a computational algebra that expands the laws of Aristotelian classical bivalent logic and modern probability theory (Kosko 1997; Zaus 1999).

References

[1] Anochin, P.K. 1962 Vorgreifende Widerspiegelung der Wirklichkeit. In: Anochin, P.K. Beiträge zur allgemeinen Theorie des funktionellen Systems. Fischer, Jena

[2] Dubois, D. 1990 Le Labyrinthe de L'Intelligence: De l'intelligence naturelle à l'intelligence fractale. InterEditions/ACADEMIA, Paris/Louvain-la-Neuve

[3] Dubois, D. 1990a Self-Organisation of Fractal Objects in XOR Rule-Based Multilayer Networks. In: Neural Networks and their Applications, Neuro-Nimes '90, Proceedings of the 3rd International Workshop,, EC2, pp. 555-557

[4] Dubois, D. 1991 Fractal Algorithms for Holographic Memory of Inversible Neural Networks. Issues in Connectionism: Part II, The Journal for the Integrated Study of Artificial Intelligence, Cognitive Science, and Applied Epistemology, Vol. 8, No. 2, pp.137-189

[5] Dubois, D. 1992 The Fractal Machine. Presses Universitaires de Liège, Liège

[6] Dubois, D. & Resconi, G. 1992 Hyperincursivity: A New Mathematical Theory. Presses Universitaires de Liège, Liège

[7] Dubois, D. 1997 Generation of fractals from incursive automata, digital diffusion and wave equation systems. BioSystems 43, 97-114, Elsevier

[8] Dubois, D. 1998 Computing Anticipatory Systems with Incursion and Hyperincursion. AIP Conference Proceedings 437, Computing Anticipatory Systems: CASYS 1st International Conference, Liège, Belgium, Published by The American Institute of Physics, pp. 3-29

[9] Dubois, D. 1998a Hyperincursive Methods for Generating Fractals in Automata Related to Diffusion and Wave Equations. International Journal of General Systems, Vol. 27 (1-3), pp. 141-180

[10] Gluck, M.A. & Myers, C.E. 1997 Psychobiological Models of Hippocampal Function in Learning and Memory. Annual Review of Psychology, Vol. 48, 481-514

[11] Hameroff, S., Dayhoff, J., Lahoz-Beltra, R., Rasmussen, S., Insinna, E.M., & Koruga, D. 1993 Nanoneurology and the Cytoskeleton: Quantum Signaling and Protein Conformational Dynamics as Cognitive Substrate. In: Pribram, K.H. (Ed.) Rethinking Neural Networks: Quantum Fields and Biological Data. Lawrence Erlbaum Ass., Hillsdale

[12] Iverson, K.E. 1962 A Programming Language. John Wiley & Sons, New York

[13] Iverson, K.E. 1979 Notation as a Tool of Thought. Turing Award Lecture of the ACM. In McDonnell, E. 1981 A Source Book in APL, Palo Alto Press, Palo Alto, CA

[14] Kanerva, P. 1988 Sparse Distributed Memory. MIT Press, Cambridge

[15] Korzybski, A.H.S. 1994 Science and Sanity: An Introduction to Non-Aristotelian Systems and General Semantics (5th edition). Institute of General Semantics, Chicago

[16] Kosko, B. 1986 Fuzzy Cognitive Maps. International Journal of Man-Machine Studies, 24, 65-75

[17] Kosko, B. 1988 Hidden Patterns in Combined and Adaptive Knowledge Networks. International Journal of Approximate Reasoning, 2, 377-393

[18] Kosko, B. 1992 Neural Networks and Fuzzy Systems. A Dynamical Systems Approach to Machine Intelligence. Prentice Hall, Englewood Cliffs

[19] Kosko, B. 1997 Fuzzy Engineering. Prentice Hall, Upper Saddle River

[20] Langlet, G. 1991 Theorie des Images Fractales. In: Dubois, D. La Technologie de L'Image de la C.A.O. au Chaos Fractal. InterEditions/ACADEMIA, Paris/Louvain-la-Neuve

[21] Langlet, G. 1992 Towards the Ultimate APL-T.O.E., APL-Quote Quad, Vol. 23, No. 1, pp. 118-132

[22] Langlet, G. 1994 The Power of Boolean Computing in APL. La Hulpe (Belgium), SHARE Europe APL Spring Conference

[23] Langlet, G. 1995 Genetica Acta. Les Nouvelles d'APL, No. 15, 112-125, ISSN 1664-4699, Paris

[24] Minsky, M. 1982 Matter, Mind, and Models. In: Minsky, M. (Ed.) Semantic Information Processing, 425-432, MIT Press, Cambridge

[25] Peitgen, H.O., Jürgens, H. & Saupe, D. 1992 Chaos and Fractals: New Frontiers of Science. Springer, Berlin

[26] Piaget, J. 1975 The Development of Thought: Equilibration of Cognitive Structures. Basil Blackwell, Oxford

[27] Piaget, J. 1974 Biologie und Erkenntnis: Über die Beziehungen zwischen organischen Regulationen und kognitiven Prozessen. Fischer, Frankfurt

[28] Rosen, R. 1985 Anticipatory Systems. Pergamon Press, New York

[29] Runeson, S. & Frykholm, G. 1981 Visual Perception of Lifted Weight. J. of Experimental Psychology: Human Perception and Performance, Vol. 7, 733-740

[30] Shaw, R. & Kinsella-Shaw, J. 1988 Ecological Mechanics: A Physical Geometry for Intentional Constraints. Human Movement Science, 7, 155-200

[31] Von Foerster, H. 1976 Objects: Tokens for Eigen-Behaviors. Cybernetic Forum, Vol. VIII, Nos. 3 & 4, 91-96. In French: Inhelder, B. & Garcia, J. (Eds.) 1977 Hommage à Jean Piaget: Epistémologie génétique et équilibration. Voneche, Neuchatel

[32] Wolfram, S. 1994 Cellular Automata and Complexity: Collected Papers. Addison-Wesley, Reading

[33] Wolfram, S. 2000 The New Science of Complexity. Addison-Wesley, Reading, in press

[34] Woodworth, R.S. & Schlosberg, H. 1954 Experimantal Psychology, 3rd. Edition, Methuen & Co., London

[35] Zaus, M. 1994 La Logique de la Parité, Théorique et Appliquée. Les Nouvelles d'APL, No. 12-13, pp. 42-66, ISSN 1664-4699, Paris

[36] Zaus, M. 1996 Fondements Mathématiques de la Logique de la Parité. Les Nouvelles d'APL, No. 18, pp. 27-48, ISSN 1664-4699, Paris

[37] Zaus, M. 1999 Crisp and Soft Computing with Hypercubical Calculus. New Approaches to Modeling in Cognitive Science and Technology with Parity Logic, Fuzzy Logic, and Evolutionary Computing. 423pp, Springer, Heidelberg - New York

[38] Zaus, M. 1999a Fuzzy Cognitive Maps: Kausale Prädiktorsysteme in den Wirtschafts-, Sozial-, Politik-, Gesundheits- und Verhaltenswissenschaften. AFN-Berichte 1999, AG Fuzzy Logik und Soft Computing Norddeutschland, ISBN 3-932243-04-8

[39] Zaus, M. 2000 Cognitive Anticipations and Anticipatory Information Processing. Institute for Cognitive Science, University of Oldenburg, in preparation

[40] Zaus, M. 2000a Hypercubical Calculus with Hyperincursive Transforms. 4th International Conference on Computing Anticipatory Systems, August 7-12, CASYS '2000, in preparation

[41] Zaus, M. 2000b Causal Cross-Impact Analysis with Fuzzy Logic. Institute for Cognitive Science, University of Oldenburg, in preparation

[42] Zuse, K. 1969 Rechnender Raum. Vieweg, Braunschweig