A Fuzzy Group-Decision Making Model Applied to the Choice of the Optimal Medicine in the Case of Symptoms not Disappearing after the Treatment

Elisabeth Rakus-Andersson

The University College of Karlskrona/Ronneby Department of Health and Natural Sciences Campus Annebo S-371 79 Karlskrona, Sweden Tel. +46455385408, Fax +46455385407 E-mail: Elisabeth.Andersson@ihn.hk-r.se

Abstract

Fuzzy Set Theory has used many auxiliary methods into the trials of solutions of some medical problems. One of the attempts was the finding of the optimal level of the drug action in the case when the clinical symptoms disappeared completely after the treatment (Gerstenkorn and Rakus, 1994; Rakus, 1991). However, there can occur such a morbid process in which the symptoms do not disappear after the treatment. The medication improves too high or too low level of the quantitative symptom but it still indicates the presence of the disease. It sometimes makes some problems to choose the medicine which acts best because it can happen that most of them influence the same symptoms while they do not improve the others.

A fuzzy decision making model tries to make easier to find such a drug which affects most of the symptoms in the highest degree. To solve this problem I propose the using of discrete values of the membership degrees in the model instead of the continuous ones which were tested in the paper of Rakus-Andersson and Gerstenkorn (1997). It should improve the thoroughness of the method and heighten the reliability of the accepted decision. It is also considered how to choose the best medicine in the circumstances when some decision-makers have different opinions about the priority of the tested drugs.

Keywords: fuzzy decision making, fuzzy sets as membership degrees, drug action, optimal drug, many decision-makers.

1 Introduction

The models which are going to be described also use non-fuzzy sets. It is remarkable to see how Fuzzy Set Theory joins the classical set to this one mentioned as fuzzy. Some readers may be fascinated with a possibility of translating "spoken" languages into numbers what can make easier to feel the richness of applications which this mathematical theory offers

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2.1 Presentation of the Problem

Let us introduce the notions of a space of states-results $X = \{x_1, x_2, ..., x_m\}$ and a decision space $A = \{a_1, a_2, ..., a_n\}$.

If a rational decision maker takes a decision $a_i \in A$, i=1, 2, ..., n, concerning states-results $x_j \in X$, j=1, 2, ..., m, then a problem is reduced to the consideration of the ordered triplet (X, A, U) where X is a set of states-results, A - a set of decisions and U - the utility matrix (Keeny, 1976; Jain, 1977; Kacprzyk, 1986)

$$U = \begin{bmatrix} U_{11} & U_{12} \cdots U_{1m} \\ U_{21} & U_{22} \cdots U_{2m} \\ \vdots \\ U_{n1} & U_{n2} \cdots U_{nm} \end{bmatrix}$$
(1)

in which each element U_{ij} , i=1, 2, ..., n, j=1, 2, ..., m, is a fuzzy set defining the fuzzy utility following from the decision a_i with the result x_j (Fishburn, 1970; Jain, 1977; Kacprzyk, 1986).

Assume now that the state-result is not known exactly, but as a fuzzy set $S \subset X$ given in the form

$$S = \frac{\mu_{S}(x_{1})}{x_{1}} + \frac{\mu_{S}(x_{2})}{x_{2}} + \dots + \frac{\mu_{S}(x_{m})}{x_{m}}.$$
 (2)

The theoretical model with the triplet (X, A, U) and the fuzzy set of states S, thus very shortly sketched, can find its practical application in the processes of choosing an optimal drug. If a given disease is connected with the symptoms accompanying it, by giving a medicine we try to liquidate these symptoms or at least to reduce their unfavourable influence upon the patient's health. Not all symptoms retreat after the cure has been carried out. One can only sometimes soothe their negative effects by, for example, the lowering of an excessive level of the indicator, the relief of pain, and the like, but cannot ascertain that the patient is fully free from them. The problem of choosing an optimal drug (decision) soothing the symptoms but having no power to liquidate them in full corresponds to the model described above. In order to show the algorithm for finding such a decision, let us consider a model with n drugs $a_1, a_2, \ldots, a_n \in A$, which, on the basis of the physician's decision, are prescribed to the patient (thus may be treated as decisions a_1, a_2, \ldots, a_n) with a view to have an effect on m symptoms $x_1, x_2, ..., x_m \in X$ representing certain states characteristic of the given morbid unit. To simplify the symbols, let us further assume that each symptom $x_j \in X$, where X is a space of symptoms (states), is understood as the result of the treatment of this symptom after the cure with the drugs $a_1, a_2, ..., a_n$ has been carried out. On the basis of earlier experiments, the physician knows how to define in words the effectiveness of the cure of these symptoms, which we shall describe in terms of the list named "the effectiveness of the cure of a symptom" = { R_1 =none, R_2 =almost none, R_3 =very little, R_4 =little, R_5 =medium, R_6 =large, R_7 =very large, R_8 =almost complete, R_9 =complete}. Each notion from this list of terms is the name of a fuzzy set. Assume that all these sets are defined on the space Z = [0,100] which is suitable to measure a number of patients corresponding to each name.

Let us propose the membership functions for the fuzzy sets from the list which is called "the effectiveness of the cure of a symptom" by applying the simple functions which do not cause any trouble for a possible user (Adlassnig, 1980; Rakus, 1991, 1999)

$$L(z; \alpha, \beta) = \begin{cases} 0 & \text{for } z \le \alpha, \\ \frac{z - \alpha}{\beta - \alpha} & \text{for } \alpha < z \le \beta, \\ 1 & \text{for } z > \beta, \end{cases}$$
(3)

and

$$\pi(z;\alpha,\gamma,\beta) = \begin{cases} 0 & \text{for } z \le \alpha, \\ L(z;\alpha,\gamma) & \text{for } \alpha < z \le \gamma, \\ 1 - L(z;\gamma,\beta) & \text{for } \gamma < z \le \beta, \\ 0 & \text{for } z > \beta, \end{cases}$$
(4)

where z is an independent variable and α , β , γ are some parameters.

Let us define

$$\mu_{R_k}(z) = 1 - L(z; \alpha_k, \beta_k) \text{ for } k = 1, 2, 3, 4, \ \mu_{R_k}(z) = L(z; \alpha_k, \beta_k) \text{ for } k = 6, 7, 8, 9$$

and $\mu_{R_k}(z) = \pi(z; \alpha_5, \gamma, \beta_5)$ (5)

in which $z \in Z$, and α_k , β_k , γ are borders for the fuzzy supports also belonging to the set Z.

Let us further decide the values of the borders α_k , β_k , γ in order to introduce the constraints for the fuzzy sets from the mentioned list "the effectiveness of the cure of a symptom". In this way the following membership functions can be designed

$$\begin{split} \mu_{R_1}(z) &= \mu_{\text{none}}(z) = 1 - S(z;0,20), \\ \mu_{R_2}(z) &= \mu_{\text{almost none}}(z) = 1 - S(z;10,30), \\ \mu_{R_3}(z) &= \mu_{\text{very little}}(z) = 1 - S(z;20,40), \\ \mu_{R_4}(z) &= \mu_{\text{little}}(z) = 1 - S(z;30,50), \\ \mu_{R_5}(z) &= \mu_{\text{medium}}(z) = \pi(z;30,50,70), \\ \mu_{R_6}(z) &= \mu_{\text{large}}(z) = S(z;50,70), \\ \mu_{R_7}(z) &= \mu_{\text{very large}}(z) = S(z;60,80), \\ \mu_{R_8}(z) &= \mu_{\text{almost complete}}(z) = S(z;70,90), \\ \mu_{R_9}(z) &= \mu_{\text{complete}}(z) = S(z;80,100) \end{split}$$

for $z \in \mathbb{Z}$.

The parameters α_k , β_k and γ in eqs. 6 have been proposed in conformity with the physician's suggestion. In order to show these functions one is able to sketch them underneath.



Fig.1: The membership functions of the fuzzy sets R_1 - R_9

Each fuzzy set had its representative in the form of one value of the membership degree in the paper of Rakus-Andersson and Gerstenkorn (1997) but one can doubt if the thoroughness was accurate in such a case. Therefore I have decided to accept the finite fuzzy sets corresponding to R_1 - R_9 with the membership degrees greater than 0.5 (I have taken their essential part into consideration) modelled on the basis of eq. 6. The continuous membership functions have allowed to calculate the membership degrees for some chosen elements of the supports of the sets instead of guessing their values intuitively. In this way one could obtain the "discrete" fuzzy sets defined, for instance, below

 $R_1 = \text{none} = \frac{1}{9} + \frac{0.92}{2} + \frac{0.84}{4} + \frac{0.76}{6} + \frac{0.68}{8},$ $R_2 = \text{almost none} = \frac{1}{10} + \frac{0.912}{12} + \frac{0.814}{14} + \frac{0.716}{16} + \frac{0.648}{16},$ (6)

$$R_{3} = \text{very little} = \frac{1}{20} + \frac{0}{22} + \frac{0}{24} + \frac{0}{26} + \frac{0}{28},$$

$$R_{4} = \text{little} = \frac{1}{30} + \frac{0}{32} + \frac{0}{34} + \frac{0}{26} + \frac{0}{9},$$

$$R_{5} = \text{medium} = \frac{0}{42} + \frac{0}{24} + \frac{0}{44} + \frac{0}{46} + \frac{0}{9},$$

$$R_{6} = \text{large} = \frac{0}{62} + \frac{0}{24} + \frac{0}{66} + \frac{0}{9},$$

$$R_{7} = \text{very large} = \frac{0}{72} + \frac{0}{74} + \frac{0}{8},$$

$$R_{8} = \text{almost complete} = \frac{0}{82} + \frac{0.7}{84} + \frac{0.8}{86} + \frac{0.9}{88} + \frac{1}{90},$$

$$R_{6} = \text{complete} = \frac{0.6}{82} + \frac{0.7}{84} + \frac{0.8}{86} + \frac{0.9}{88} + \frac{1}{90},$$

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The mentioned fuzzy sets U_{ij} from the utility matrix U can be replaced with the so-built fuzzy sets R_1 - R_9 . To describe a connection between a_i (medicine) and the effectiveness of the retreat of x_j (symptom) the physician uses the word from the list "the effectiveness of the cure of a symptom" and this word is "translated" into the fuzzy set R_k , k=1, 2, ..., 9.

Let us further accept that the physician possesses a general experience as to the "difficulties" in the retreating of the symptoms x_j , j=1, 2, ..., m. His medical knowledge, based on observations, can contribute in a classification of symptoms that are harder to treat and symptoms easier to retreat during the treatment process. Via the words from the list "the effectiveness of the cure of a symptom" one may assign to each symptom a general ability to retreat, fixed, for instance, by observing the cure of many patients with different drugs. Such a general classification of symptoms found its place in the fuzzy set S defined theoretically by eq. 2 in which the membership degrees $\mu_S(x_j)$, j=1, 2,..., m, correspond now to the fuzzy sets R_k , k=1, 2,..., 9. By the "cure" one can mean the level of the retreating of the symptom, the decrease of the heightened index, and the like.

The fuzzy utility (Jain, 1976, 1977; Kacprzyk, 1986; Rakus-Andersson and Gerstenkorn, 1997; Rakus-Andersson, 1999) for each decision-drug a_i , i=1, 2, ..., n, with the fuzzy state $S \subset X$ characterized by means of the membership degrees $\mu_S(x_i)$ is defined to be the set

$$U_{i} = \frac{\mu_{S}(x_{1})}{U_{i1}} + \frac{\mu_{S}(x_{2})}{U_{i2}} + \dots + \frac{\mu_{S}(x_{m})}{U_{im}}$$
(8)

for i=1, 2, ..., n. This set allows to observe the relationship between the general ability to soothe and this effect in soothing which the drug a_i causes for each symptom x_j . Both the membership degrees $\mu_S(x_j)$ and the elements U_{ij} in the support of the set U_i are the fuzzy sets of the type R_1 - R_9 . It is not possible to make further calculations on such sets which have the "double" support and we need an operation reducing this family of fuzzy sets to a fuzzy set which contains the single support with clearly prescribed membership degrees. One can propose the following operation

$$U_{i} = \sum_{j=1}^{m} \frac{\mu_{S}(x_{j})}{U_{ij}} = \sum_{x_{j} \in X} \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{t})}{\sum_{c=1}^{q} \mu_{R_{k_{\phi}}}(z_{c})} = \sum_{t=1}^{q} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{c})}{z_{c}}} = \sum_{t=1}^{r} \frac{\mu_$$

$$= \sum_{\substack{\{z_{t}, z_{c}\} \in \\ \{z_{1}, z_{2}, \dots, z_{r}\} \times \\ \{z_{1}, z_{2}, \dots, z_{q}\}}} \frac{\min(\mu_{R_{k_{\phi}}}(z_{t}), \mu_{R_{k_{\phi}}}(z_{c}))}{\left| \frac{z_{t} + z_{c}}{2} \right|} = \sum_{z \in \mathbb{Z}} \frac{\mu_{U_{i}}(z)}{z}$$

where $\mu_{S}(x_{j})$ is the set equal to $\sum_{t=1}^{r} \frac{\mu_{R_{k_{\phi}}}(z_{t})}{z_{t}}$ and U_{ij} is the fuzzy set given as

 $\sum_{c=1}^{q} \frac{\mu_{R_{k_{\varphi}}}(z_c)}{z_c} \text{ for } R_{k_{\varphi}} \text{ and } R_{k_{\varphi}} \text{ belonging to the class of the sets defined as } R_1 - R_9. \text{ Each } U_{ij} \text{ is a fuzzy set presenting the fuzzy utility following from the decision } a_i \text{ with the result } x_{j.}$

In the case of the same element of the support of the fuzzy set with the different membership degrees we apply the operation according to the formula

$$\mu_1(z) \oplus \mu_2(z) = \mu_1(z) + \mu_2(z) - \mu_1(z) \cdot \mu_2(z)$$

where \oplus denotes a symbolic addition of two different membership degrees for the same element of the support.

2.2 Solution of the Problem

The problem of choosing an optimal decision is solved according to the algorithm described by Jain (1976, 1977).

- We form a non-fuzzy set Y which is the union of the supports of the sets U_i, i=1, 2,..., n. This set contains the elements z∈Z which appear in all the sets U_i. In this way we have the knowledge about a range of the common utility.
- 2) We choose the maximal element of the set Y, so-called z_{max} .
- 3) We define the fuzzy sets U_i

$$U_{i}' = \sum_{z \in \mathbb{Z}} \frac{\mu_{U_{i}'}(z)}{z}$$
(10)

for the same supports which appear in the sets U_i . The membership degrees are computed by means of the formula

(9)

$$\mu_{U_i} = \frac{z_{U_i}}{z_{\max}} \tag{11}$$

where z_{U_i} stands for all the elements belonging to the support of the set U_i . These membership degrees evaluate the "distance" between the values belonging to the supports of U_i and the maximal z from the union of all U_i .

4) The next introduced fuzzy set has the form

$$U_{i0} = \sum_{z \in \mathbb{Z}} \frac{\mu_{U_{i0}}(z)}{z}$$
(12)

where the membership degree $\mu_{U_{i0}}(z)$ is calculated according to the rule

$$\mu_{U_{i0}}(z) = \min(\mu_{U_i}(z), \mu_{U_i}(z)).$$
(13)

The fuzzy utility U_{i0} , so-constructed for each medicine a_i , gathers all the possible factors which can influence the appreciation of the soothing power of a_i . The minimum-operation which is applied to the membership degrees prevents from taking into consideration too high values which can be the results of the mentioned operation \oplus . The support in eqs. 9-13 has the elements z belonging to Z for i=1, 2, ..., n.

5) Let us create a new fuzzy set A^* with the elements $a_1, a_2, ..., a_n$ of the support $(a_i \in A, i=1, 2, ..., n)$ as

$$A^* = \sum_{i=1}^n \frac{\mu_{A^*}(a_i)}{a_i}$$
(14)

where the membership degree for each a_i looks like

$$\mu_{A^{\bullet}}(a_i) = \text{mean value}(\mu_{U_{i0}}(z))$$
(15)

what practically means the computing of the arithmetic mean for the membership degrees in every set U_{i0} . This value expresses the decisive character of every a_i according to the rule: the higher value the better influence of the medicine on patient's health.

6) The optimal decision a^* is found as this element a_i whose membership degree satisfies the equation

$$\mu_{A^*}(a^*) = \max_{1 \le i \le n} (\mu_{A^*}(a_i)).$$
(16)

After choosing a^* we ascertain that the application of the drug a^* should yield the best effects in the process of the retreating of the symptoms.

2.3 Example

This model has been controlled on the clinical data coming from the investigation carried out among women who have suffered from the illness with three typical clinical symptoms. Three different drugs have been taken into consideration. The physician has decided that the set S and the matrix U should have following elements

$$S = \frac{\text{little}}{x_1} + \frac{\text{large}}{x_2} + \frac{\text{large}}{x_3}$$

and

	x_1	x_2	x_3
a_1	medium	large	medium
$U = a_2$	medium	very large	large
$\cdot a_3$	little	very large	almost complete

After the whole computing process one has obtained the final result, namely, the sets U_{i0} , i=1, 2, 3

and

The corresponding set A^* has been decided as

 $A^* = \frac{0.6195}{a_1} + \frac{0.6503}{a_2} + \frac{0.6934}{a_3}$

what means that the accepted optimal decision-drug should be a_3 .

It has been interesting to test such a case because a_3 has a little influence on the first symptom x_1 , but it affects both x_2 and x_3 in the substantial grade. It has been difficult to predict the solution looking at the set S and the matrix U only, but the received results are confirmed by the physician. He also prefers using a_3 to recommending the other drugs what can make sure that the proposed method functions even in the complicated cases.

3 The Choice of the Drug in the Case of Many Decision-Makers

3.1 Description of the Mathematical Model Solving the Problem

Each physician treated as a decision-maker would like to create the matrix U and the set S according to his own experience and judgement what leads to the different priorities in the set A^* . How to choose the best medicine in the case when the sets A^* differ a lot from each other? The question is answered by means of the algorithm created by Arrow and Blin (1977) which is adapted to so-sketched medical task.

Let us accept a set of physicians $P = \{P_1, P_2, ..., P_i\}$ who appreciate the drugs belonging to the set $A = \{a_1, a_2, ..., a_n\}$. We use the notation $a_i \succ a_j$ if a_i acts better than a_j . A fuzzy relation $\Re \subset A \times A$ with the membership function $\mu_{\Re}: A \times A \rightarrow [0,1]$, called the group order, has membership degrees $\mu_{\Re}(a_i, a_j)$ which describe in what grade the decision a_i is preferable in comparison with a_j . If we define

$$\delta_{ij} = \left\{ P_k : a_i \succ a_j \right\},\tag{17}$$

k=1, 2, ..., t, i, j=1, 2, ..., n, we will be able to compute the membership degrees in the relation \Re as

$$\mu_{\mathfrak{R}}(a_i, a_j) = \frac{\left|\delta_{ij}\right|}{t}.$$
(18)

Further, we apply the definition of the α -intersection

$$R_{\alpha} = \left\{ \left(a_i, a_j\right) : \mu_{\Re}\left(a_i, a_j\right) \ge \alpha \right\}$$
(19)

for each $\mu_{\mathfrak{R}}(a_i, a_j) = \alpha$. We look for such a set R_{α} with the total order which, for the

greatest α , contains all a_i in the pairs coming from the relation \Re . The directed graph with the vertices a_i and the single arrows in the appropriate directions, sketched according to R_a , illustrates the group decision.

3.2 Example

In order to test the model I have asked six physicians for evaluating the drugs a_1 , a_2 , a_3 used in the previously described disease. Their priorities are shown in the following schedule:

 $P_{1} = P_{3} = P_{4} = (a_{3} \succ a_{2} \succ a_{1}),$ $P_{2} = (a_{3} \succ a_{1} \succ a_{2}),$ $P_{5} = (a_{2} \succ a_{3} \succ a_{1}),$ $P_{6} = (a_{2} \succ a_{1} \succ a_{3}).$

The using of eq. 18 has allowed to decide the contents in the matrix R

$$\mathfrak{R} = a_{2} \begin{bmatrix} a_{1} & a_{2} & a_{3} \\ 0 & 0.17 & 0.17 \\ 0.83 & 0 & 0.33 \\ a_{3} \end{bmatrix} \mathfrak{l}_{0.83} \mathfrak{l}_{0.67} \mathfrak{l}_{0} \mathfrak{$$

All the α -intersections are determined as the sets of pairs on the basis of the eq. 19. One can list them as

$$R_{0,83} = \{(a_3, a_1), (a_3, a_2)\}, R_{0,67} = \{(a_2, a_1), (a_3, a_1), (a_3, a_2)\}, R_{0,33} = \{(a_2, a_1), (a_3, a_1), (a_3, a_2), (a_2, a_3)\}, R_{0,17} = \{(a_2, a_1), (a_3, a_1), (a_3, a_2), (a_2, a_3), (a_1, a_2), (a_1, a_3)\}.$$

The totally ordered α -intersection for the greatest value of α is determined as $R_{0.67} = \{(a_2, a_1), (a_3, a_1), (a_3, a_2)\}$ what can be illustrated by the graph stated below.



Fig. 2: The graph representing the set $R_{0.67}$

Looking at the directions of the arrows it can be noticed that the most effective medicine is a_3 and the worst one is a_1 .

4 Conclusions

The proposed model of testing the drug action is new and should be simple for this user who does not have special knowledge in mathematics and still would like to apply it. The method tests the data very thoroughly because of introducing the finite fuzzy sets instead of one representing value what was applied to the same problem in the paper of Rakus-Andersson and Gerstenkorn (1997). The obtained decision seems to be very reliable in spite of different interactions between the drugs and their influence on the symptoms. We can also observe the hierarchy of the drugs in the case of many decision-makers who present various points of view.

The potential user who has problems with the choice of the most efficacious drug in the similar case can check the possible effectiveness of all the therapeutic means on the basis of the above theory.

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