Introduction to Computing Anticipatory Systems

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Abstract

This paper deals with an introduction to computing anticipatory systems starting with Robert Rosen's definition of anticipatory systems. Firstly, the internalist and externalist aspects of anticipation will be explained at an intuitive point of view. Secondly, the concepts of incursion and hyperincursion are proposed to model anticipatory systems. Thirdly, a simple example of a computing anticipatory system will be simulated on computer from an incursive harmonic oscillator. This oscillator includes an anticipatory model of itself in view of computing its successive states.

Keywords: Computing anticipatory systems, Externalist and internalist aspects of anticipation, Incursion, Hyperincursion, Incursive harmonic oscillator.

1. Introduction

In this introduction, I would like to define "computing anticipatory systems".
With computing power, systems are able to anticipate. Computation is not only related to "artificial computers" like a personal computer but also to natural systems which perform computations.

The verb "anticipate" comes from Latin word "anticipare" which means "to take before". "To anticipate" means to realise beforehand, to foresee, to look forward to, to act in advance to prevent, to forestall.

Robert Rosen (1985, p. 341), in the famous book Anticipatory Systems "tentatively defined the concept of an anticipatory system: a system containing a predictive model of itself and/or of its environment, which allows it to state at an instant in accord with the model's predictions pertaining to a later instant."

Robert Rosen considers that anticipatory systems are related to the final causation of Aristotle. A future cause could produce an effect at the present time. Then the causality principle seems reversed. Robert Rosen relates some anticipatory systems to feedforward loops.

So, for such anticipatory systems, it is perhaps better to speak of a finality principle and to see the process at a non-local or global point of view instead of seeing locally the causality process. In cybernetics and control theory, a goal and objective, defined at the present time by an engineer, drives the future states of a system by feedback loops.

It is interesting to point out that in physics, recursive causal systems can be formally expresses in a global and equivalent way from the principle of least action of Maupertuis.

An important class of anticipatory system is a system with multiple potential future states for which the actualisation of one of these potential futures is determined by the events at each current time. Such an anticipatory system is thus a system without an explicit future objective.

An anticipatory system could be also a system which contains a set of possible responses to any potential or, even, unpredictable external events. In this sense, the co-operative dynamics of the immune systems, for example, is a self-organising system which can be considered as an anticipatory system. Then all learning and evolutionary systems belong also to this class of anticipatory system.

In this paper, I will begin to define anticipatory systems in considering the externalist and internalist aspects of anticipation from two simple examples.

Then I introduce the concepts of incursion and hyperincursion. An incursion is an inclusive or implicit recursion for which the state of a system is a function of past, present by also future potential states. Hyperincursion is an incursion with multiple solutions.

Finally a simple example of a computing anticipatory system will be simulated on computer in considering an incursive harmonic oscillator.

2. Externalist and Internalist Aspects of Anticipatory Systems

Externalist anticipation refers to external events which can be anticipated.

For example: I take my umbrella because I anticipate a wet weather today. This is related to forecasting or prediction of the weather for the near future as a function of the configuration of the sky at the present moment. So, we have created in our mind a model of the evolution of the weather from an initial condition at the present time which permit us to extrapolate to the future. So, we can say that the prevision of the bad weather in the future is the cause of a present effect which consists in taking our umbrella. But, the prevision of the future is not a certitude: perhaps the weather will be sunny. So the future is potentially multiple and the realisation at each moment collapses all these possibilities to only one. The present and the past are actually unique meanwhile the future is always potentially multiple. A priori, the future states are potentially multiple and a posteriori, there is only one realised present state.

A paradox appears in the case of a sunny weather: indeed, the effect "to have the umbrella" will have no more its cause "wet weather" a posteriori. In fact, the actual cause is the act of prevision (a computation) which was performed before the effect "to take the umbrella". The result of the anticipatory computation is also memorised in our brain: so, three types of memory must be defined: a direct memory for present events, a long term memory for past events, and an anticipatory memory storing the future events. The future events can be potentially multiple but their memorisation is actually real. With the evolution of the current time, the multiple potential events in this memory of the future collapse to unique actual events which will enlarge the long term memory.

Internalist anticipation deals with this memory of the future. But also with the anticipation of events we create ourselves.

For example, the organisation of this conference CASYS'97, August 11-15, 1997, was planned one year ago. The cause "the conference" leaded to many effects in the past, that is to say during the year before its practical realisation. Each participant and author had managed to prepare their travel and also to write their papers. During one year, all the participants of this conference have had memorised in their memory of the future, the potential future event "the conference CASYS'97". There is a difference between the act to take an umbrella and to manage for attending a conference. In the first case, the anticipation is based on an external event "the weather", and in the second case, the anticipatory events are constructed by the actors themselves (the men). This is the internalist anticipation. In the first case, the externalist anticipation is dependent of the environment and in the second case, the internalist anticipation creates its own future events and manages to meet these anticipated events. In the internalist anticipation, there are one or several future potential objectives, the realisation of one of which being practically a certitude. I assume that each of the readers have an agenda and practically all the appointments written by anticipation in this agenda become actual realisations. The content of the agenda can be dynamically separated in three parts at each current time. The past events are a memory of the realised appointments. At the current time, the appointment are realising positively or not. The future appointments are potential memorised events.

In general, any human action at each current time takes into account the past events, the current situation in the environment, and the future anticipated events.
The anticipation in human actions deals with conscious and intentionality, a self-referential finality.

Anticipatory systems deal with the question "WHY ?" related to a finality: "Why did you take your umbrella, the weather is so sunny?", "Because I thought that the weather will be bad".
Anticipatory computation deals with potential final conditions. This is the basis of the intelligence and comprehension: "to comprehend" which means also "to include as a whole" .

Robert Rosen tried in his book anticipatory systems to build a logico-mathematical framework for taking into account explicitly the "why ?" in the reasoning task of intelligence. Until now, it seems that no formalism exists to explicit the why. This belongs only to the language.

Classical systems deal essentially with the question "HOW ?". Recursive computation is related to a current computation as a function of past computations. The explanation is to be found in the past memory of the system and for Newtonian Systems, in the actual initial conditions.

What means computing anticipatory systems for systems without a conscious and intentionality? The why does no more exists. The "how" is the subject of science in explaining natural systems by using mathematical models, for example. But the question "why" is the subject of philosophy. Science transforms the "why" into "how" by recursive arguments. Until now, only recursive functions are computable by artificial devices, the computers.

Robert Rosen in his book anticipatory systems says that what differentiates living systems and inorganic systems is anticipation. He considers the learning processes, for example in neuronal systems, as anticipatory systems. More, he proposes the same model for learning processes and evolutionary processes, based both on anticipation.

Many Biologists would prefer Lamarckian comprehension of evolution answering the "why" rather than Darwinian explanation answering the "how" question.

At my point of view, Darwinian and Lamarkian systems are two complementary models of evolutionary natural systems, similarly to the two descriptions of physical systems, on one hand, from a causal principle and, on the other hand, from a least action principle.

In the neo-Darwinism, random mutations create different species which are then selected by the environment. Thus a neo-Darwinian system is an anticipatory system with mutiple potential mutations which are selected by the environment.

In the Lamarckism, species possess organs with multiple potential functions which are then selected by the environment. Consequently, the function actualises particular functions for each organ. Thus a Lamarkian system is an anticipatory system with multiple potential functions and the selection by the environment actualises some of these.

3. The Concepts of Incursion and Hyperincursion.

Classically, dynamical discrete systems are defined in a recursive way of the type

$$x(t+1) = f[\ldots, x(t-1), x(t), p] \quad (1)$$

where x is the state variable of the system, t the time and p the control parameter. The future state of the variable x at time t+1 is a function f of this variable at past and/or present times.

An incursion is an inclusive or implicit recursion (Dubois, 1996ab):

$$x(t+1) = f[\ldots x(t-2), x(t-1), x(t), x(t+1), \ldots, p] \quad (2)$$

is an extension of recursion. An incursive system computes the future state of the variable x as a function f of this variable at past and/or present times but also at future times.

A simple example of incursion is the following inclusive (or implicit) recursion which is an extension of the recursion in the following way:

$$x(t+1) = f[x(t), x(t+1), p] \quad (3)$$

where the value of the variable at each instant t + 1 is a function of the value of this variable at the preceding time step t, but also at time t + 1. This defines a self-referential system which is an anticipatory system of itself. The function f describing the dynamics of a system contains a model of itself; indeed x(t + 1) in the function f can be replaced in the following way:

$$x(t+1) = f[x(t), f[x(t), x(t+1), p], p] \quad (4)$$

where the system explicitly contains a predictive model of itself.

There is a paradox which in fact is easy to understand. If an anticipatory system contains a model of itself, this means that the model of itself must include also the model of itself and so on until infinity. There are an infinity of embedded models in each other. Mathematically, there is a way to solve such a paradox. Such an anticipatory system has fixed points which represents its implicit finality or teleonomy. The goal or objective of this anticipatory system is not explicitly imposed from outside the system like in control theory but is determined by the system itself.

Let us give the simple example of the well-known chaotic map given by the discrete Pearl-Verhulst equation

$$x(t+1) = a\, x(t)\, [\, 1 - x(t)\,] \tag{5}$$

where x is the population variable, t the time and a the growth rate of the population with a saturation effect given by $s(t) = [\, 1 - x(t)]$. For a varying from 0 to 4, this system shows fixed points, bifurcations and then chaos.

Let us construct an anticipatory Pearl-Verhulst system in considering an anticipatory saturation effect given by $s(t+1) = [\, 1 - x(t+1)\,]$. Eq 5 becomes then

$$x(t+1) = a\, x(t)\, [\, 1 - x(t+1)\,] \tag{6}$$

This incursive equation can be transformed to the following recursive equation

$$x(t+1) = a\, x(t) \,/\, [\, 1 + x(t)\,] \tag{7}$$

where chaos is no more present.

In some cases, there are multiple potential future states at each time steps: this is what I defined by hyperincursion. A simple example is given by the following hyperincursive equation

$$x(t) = a\, x(t+1)\, [\, 1 - x(t+1)\,] \tag{8}$$

which can be transformed to the hyper-recursive system

$$x(t+1) = 1 / 2\, [\, 1 \pm \sqrt{[\, 1 - 4\, x(t) / a\,]}\,] \tag{9}$$

where there are two solutions given by the sign plus/minus at each time steps. Such an anticipatory system shows successive bifurcations. Each bifurcation presents two potential future branches and if the system selects itself a branch of the bifurcation, a self-organising anticipatory system is defined. If the system possesses a selection rule at each current time, the set of potential solutions collapses to one realised solution. This is selected by environment for externalist evolutionary systems, and by the system itself for internalist self-organising systems. Without selection, this system will cumulate in itself all the potential solutions. This is similar to the immune system.

The selection process could be explicitly related to objectives to be reached by the state variable of this system. This is important to point out that the selections do not influence the dynamics of the system but only guide the system which creates itself the potential futures.

This hyperincursive anticipatory system was proposed as a model of a stack memory in neural networks (Dubois, 1996c).

4. Hyperincursive Discrete Harmonic Oscillator

Let us consider the classical differential equations of a harmonic oscillator

$$dx(t) / dt = v(t) \qquad (10a)$$
$$dv(t) / dt = - \omega^2.x(t) \qquad (10b)$$

in defining by x(t) the position and v(t) the velocity of a particle in a harmonic potential, where t is the time and ω the pulsation. The analytical solution of this system is

$$x(t) = x(0).\sin(\omega t + \phi) \qquad (11a)$$
$$v(t) = \omega.\ x(0).\cos(\omega t + \phi) \qquad (11b)$$

which is given by oscillations of period $T = 2\ \pi/\omega$, the amplitude of which being determined by the initial position and velocity x(0) and v(o) at time t = 0. This system shows an orbital stability.

Numerical simulations on computer of this system is only possible from a discrete model of these eqs. 10ab.

From the classical definition of the discrete time derivative

$$dx(t) / dt = [\ x(t+\Delta t) - x(t)\ ] / \Delta t \qquad (12a)$$
$$dv(t) / dt = [\ v(t+\Delta t) - v(t)\ ] / \Delta t \qquad (12b)$$

where Δt is a finite time interval, eqs. 10 ab can be written in the following finite difference equation system

$$x(t+\Delta t) = x(t) + \Delta t.v(t) \qquad (13a)$$
$$v(t+\Delta t) = v(t) - .\Delta t.\omega^2.x(t) \qquad (13b)$$

Unfortunately, with such a discretisation, the harmonic oscillator does no more show an orbital stability. This recursive system is unstable in presenting oscillations with growing amplitudes.

In view of keeping the same stability property of the original harmonic oscillator, I have proposed to compute the discrete equations in a sequential order from the following model

$$x(t+\Delta t) = x(t) + \Delta t.v(t) \qquad (14a)$$
$$v(t+\Delta t) = v(t) - .\Delta t.\omega^2.\mathbf{x(t+\Delta t)} \qquad (14b)$$

where the value of the position **x(t+Δt)** is propagated to the equation of the velocity. This is what I called an incursive system, because the value of the velocity at the future time step v(t+Δt) depends on the future value of the position x(t+Δt). This is an inclusive recursion, called incursion. Such an incursive harmonic oscillator shows an orbital stability.

In fact, there is a second incursive harmonic oscillator in propagating the value of the velocity to the equation of the position in the following way

$$v(t+\Delta t) = v(t) - .\Delta t.\omega^2.x(t) \qquad (15a)$$
$$x(t+\Delta t) = x(t) + \Delta t.\mathbf{v(t+\Delta t)} \qquad (15b)$$

This second incursive harmonic oscillator shows also an orbital stability. So we define such incursive systems, a hyperincursive system because there are two solutions.

There are some remarkable properties of these hyperincursive eqs. 14 ab and 15 ab.

Firstly, in replacing Δt by - Δt in eqs. 14ab, eqs. 15ab are obtained. The two incursive systems 14ab and 15 ab are time invertible of each other.

Secondly, the incursive system corresponds also to an implicit recursion in using forward and backward discrete derivatives.
Eqs. 12ab define forward discrete derivatives. Backward discrete derivatives are defined by

$$dx(t) / dt = [\, x(t) - x(t-\Delta t) \,] / \Delta t \quad (16a)$$
$$dv(t) / dt = [\, v(t) - v(t-\Delta t) \,] / \Delta t \quad (16b)$$

in replacing Δt by - Δt in eqs. 12ab, eqs. 16ab are obtained.
In making a time translation - Δt in eq. 14b, eqs. 14ab become

$$x(t+\Delta t) = x(t) + \Delta t.v(t) \quad (17a)$$
$$v(t) = v(t-\Delta t) - .\Delta t.\omega^2.\mathbf{x(t)} \quad (17b)$$

where we recognize the forward derivative in eq. 17a and the backward derivative in eq. 17b.
Similarly in making a time translation - Δt in eq. 15b, eqs. 15ab become

$$v(t+\Delta t) = v(t) - .\Delta t.\omega^2.x(t) \quad (18a)$$
$$x(t) = x(t-\Delta t) + \Delta t.\mathbf{v(t)} \quad (18b)$$

where we recognize the backward derivative in eq. 18a and the forward derivative in eq. 18b.
So these hyperincursive systems mix forward and backward discrete derivatives.

Thirdly, in replacing v(t) of eq. 17a in eq. 17b, we obtain

$$x(t+\Delta t) - 2.x(t) + x(t-\Delta t) = - \Delta t^2.\omega^2.x(t) \quad (19a)$$

which is the discrete model of the differential harmonic oscillator:

$$d^2x(t) / dt^2 = .- \omega^2.x(t) \quad (20)$$

In replacing v(t) of eq. 18b in eq. 17a, we obtain

$$x(t+\Delta t) - 2.x(t) + x(t-\Delta t) = - \Delta t^2.\omega^2.x(t) \quad (19b)$$

which is the same equation as eq. 19a. This eq. 19 is time invertible in replacing Δt by - Δt.
As a conclusion, when only one variable is taken into account, the position for example, the two different incursive eqs. 17ab and 18ab give the same eq. 19. If we like to know at the same time the position and the velocity of the particle, there is an uncertainty due to the two types of incursive equations. But when we like to know the evolution of only one variable, the position or the velocity, there is no more uncertainty.

Fourthly, the hyperincursive models are computing anticipatory systems. Indeed, in eqs. 14ab, the velocity is computed as a function of the position at a later time step. In eqs. 15ab, the position is computed as a function of velocity at a later time step.

Fifthly, a constant of movement can be defined in relation with the energy.
In adding eq. 14a multiplied by $\omega^2.x(t+\Delta t)/2$ and eq. 14b multiplied by $v(t)/2$, the following equation is obtained:

$$\omega^2.x^2(t+\Delta t)/2 + v(t+\Delta t).v(t)/2 = \omega^2.x^2(t)/2 + v(t).v(t-\Delta t)/2 \qquad (20a)$$

or

$$\omega^2.x^2(t+\Delta t)/2 + v^2(t+\Delta t)/2 + \Delta t.\omega^2.x(t+\Delta t).v(t+\Delta t)/2.=$$
$$\omega^2.x^2(t)/2 + v^2(t)/2 + \Delta t.\omega^2.x(t).v(t)/2 = C \qquad (20b)$$

where C is a constant of movement. At the limit of Δt tending to zero, this constant is the total energy E of the particle, that is the kinetic energy T plus the potential energy V:

$$\omega^2.x^2(t+\Delta t)/2 + v^2(t+\Delta t)/2.= \omega^2.x^2(t)/2 + v^2(t)/2 = T + V = E \qquad (21)$$

So, the constant of movement of the incursive system 14ab is

$$C = T + V + I = E + I = E + \Delta t.\omega^2.x(t).v(t)/2 \qquad (22)$$

and the constant of movement of the second incursive system 15ab is found in replacing Δt by - Δt in eq. 22

$$C = T + V - I = E - I = E - \Delta t.\omega^2.x(t).v(t)/2 \qquad (22a)$$

where I, an interaction energy, is related to the product of the position x(t) and the velocity v(t) of the discrete particle. This interaction energy measures the deviation between the energy of a continuous particle and the discrete hyperincursive particle. The average value of the constant of movement is equal to the energy of the continuous particle because the interaction energies are of opposite signs. The frequency of oscillation of this interaction energy is the double of the eigen frequency of the harmonic oscillator $2.\omega$. The temporal average of this interaction energy is zero.

Finally, let us point out a remarkable property of these hyperincursive systems. The successive iterates of the incursive system 14ab can be interpreted in two different ways. Indeed on one hand, one can plot in the phase space $v(t+\Delta t)$ as a function of $x(t+\Delta t)$, shown in Figure 1. In this simulation, $\omega^2 = 0.054$, $\Delta t = 1$ and the initial conditions are $x(0) = 25$, $v(0) = 0$.
On the other hand, one can plot v(t) as a function of $x(t+\Delta t)$, shown in Figure 2 with the same conditions as in Fig. 1. In fact, this represents the simulation of eqs. 15 ab if $v(t+\Delta t)$ is plot as a function of $x(t+\Delta t)$. This can be easily demonstrated: in recoding v(t) by $v(t+\Delta t)$ in eqs. 17ab, eqs. 15 ab are obtained, that is the second incursive system.
In Figs. 1 and 2, the curves are not symmetrical due to the interaction term I.
Symmetrical solutions can be obtained in plotting the mean value of the velocity $[v(t) + v(t+\Delta t)]/2$ as a function of $x(t+\Delta t)$, given in Figure 3, or in plotting $v(t+\Delta t)$ as a function of the mean position $[x(t) + x(t+\Delta t)]/2$, given in Figure 4. Figure 5 gives the superposition of the 4 Figures 1, 2, 3, 4.
In Figure 6, the same superposition as in Figure 5 in taking a smaller time interval $\Delta t = 0.05$.
All these solutions tend to the continuous solution of the harmonic oscillator for Δt tending to zero.

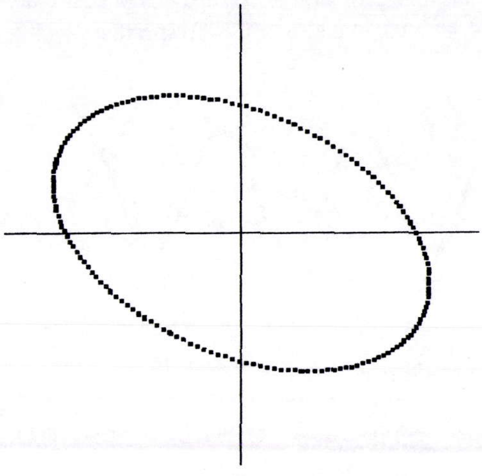

Figure 1

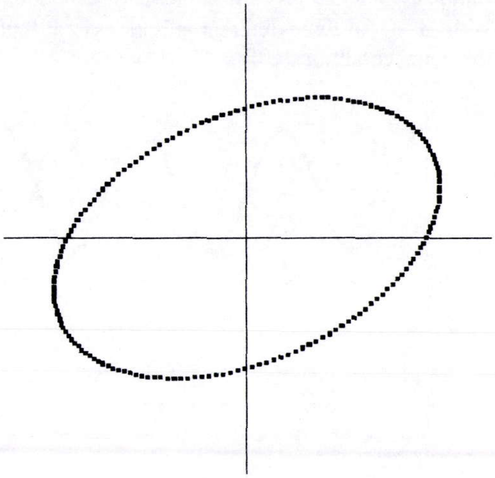

Figure 2

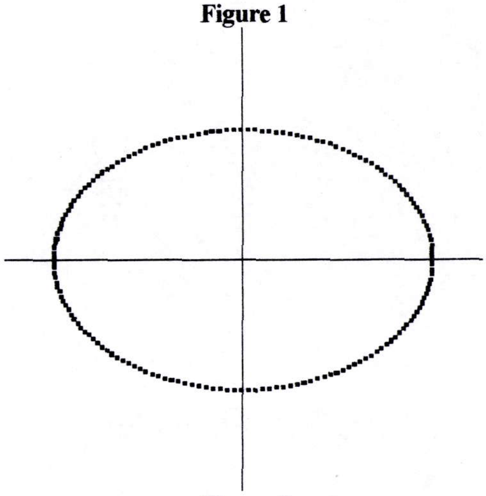

Figure 3

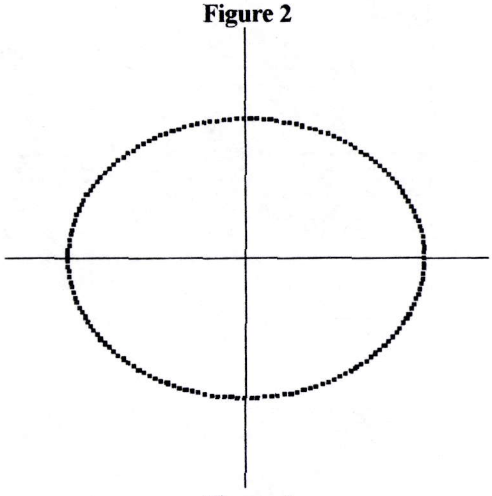

Figure 4

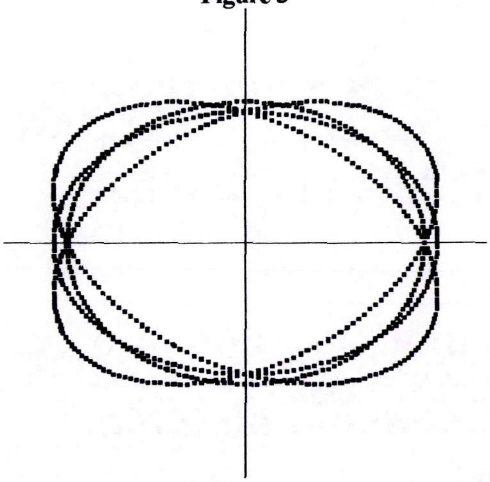

Figure 5

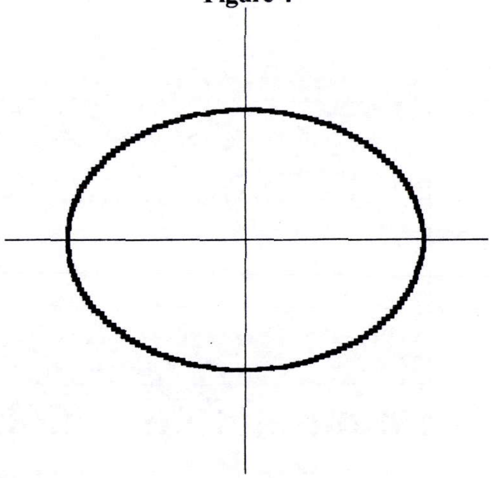

Figure 6

Figures 7, 8 and 9 give the numerical simulations of the time evolution of x(t) and v(t) of eqs. 14ab, with $\Delta t = 1$, at three different pulsations $\omega^2 = 0.0054$, $\omega^2 = 0.054$ and $\omega^2 = 0.54$, respectively with the initial conditions $x(0) = 25$, $v(0) = 0$.

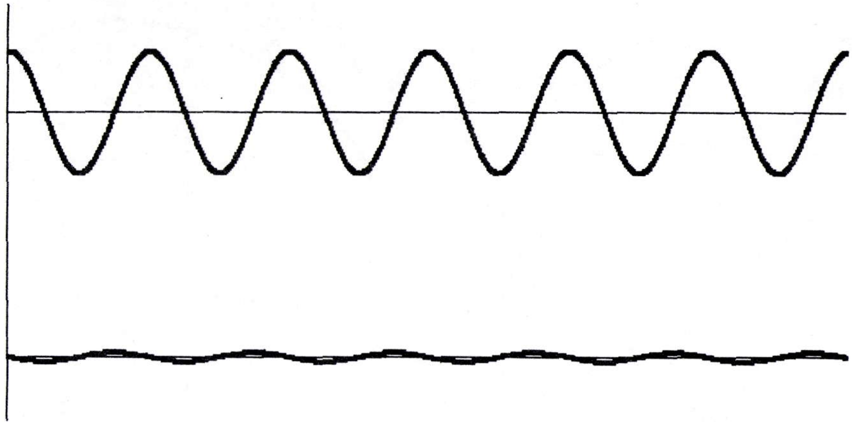

Figure 7

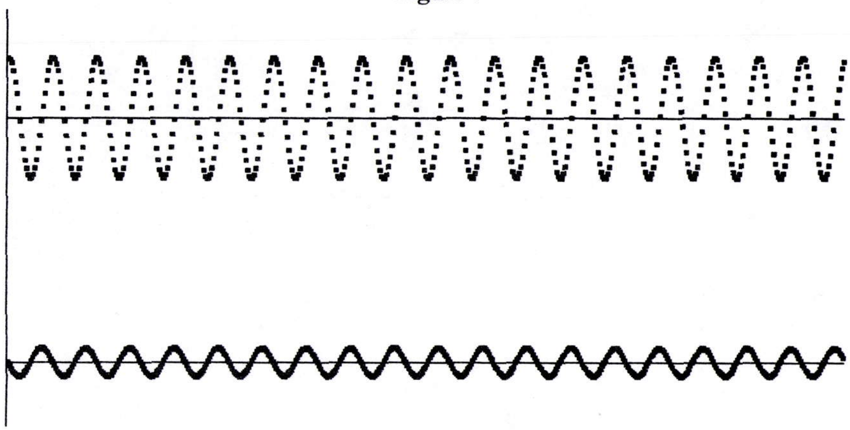

Figure 8

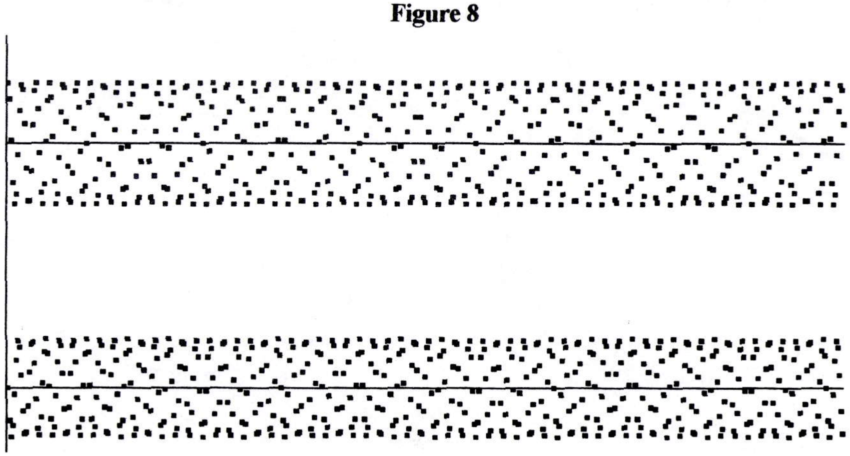

Figure 9

In these Figs. 7, 8 and 9, time steps are the horizontal axis and x(t), v(t) are the vertical axis. At low frequency, the curves are continuous and at high frequency, the pattern is complex.
Figures 10 and 11 give the numerical simulation of eqs. 14ab for successive values of the frequency: $\omega^2 = 0.54 + n/10^5$, n = 1 to 340 with $\Delta t = 1$. In Figure 10, the horizontal axis gives the iterates of the position x(t) as a function of the frequency given by the vertical axis, from top to bottom. Figure 11 gives the corresponding iterates of the velocity v(t).
It can be seen a well organized pattern with self-similar properties. An enlargement of a part of the pattern in Figure 10 is given in Figure 12 which shows a similar pattern.

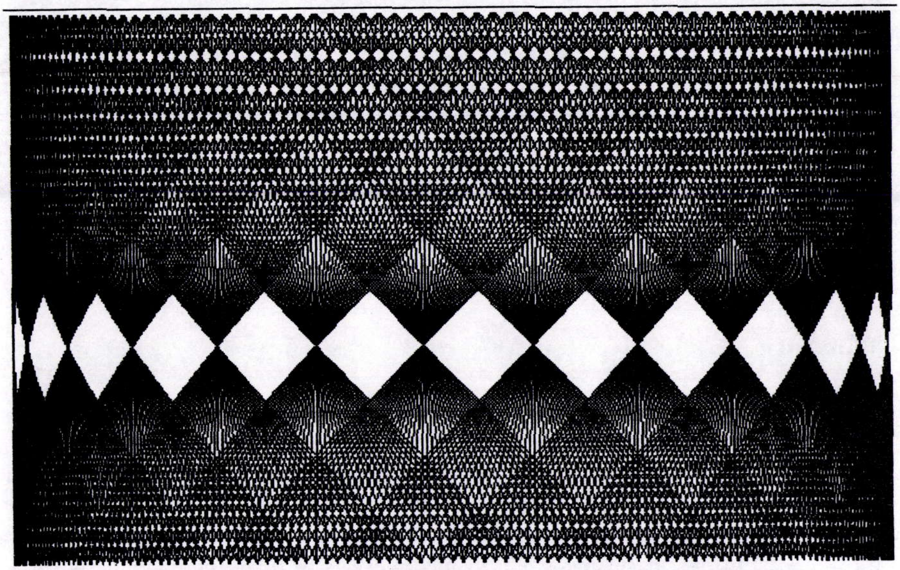

Figure 10

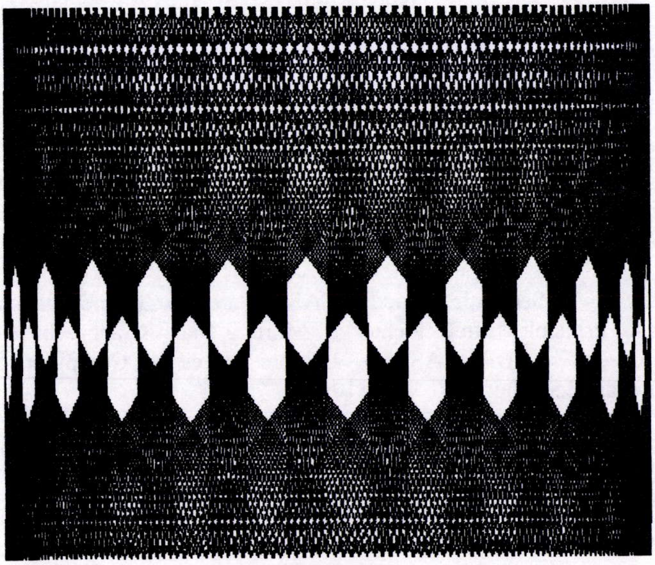

Figure 11

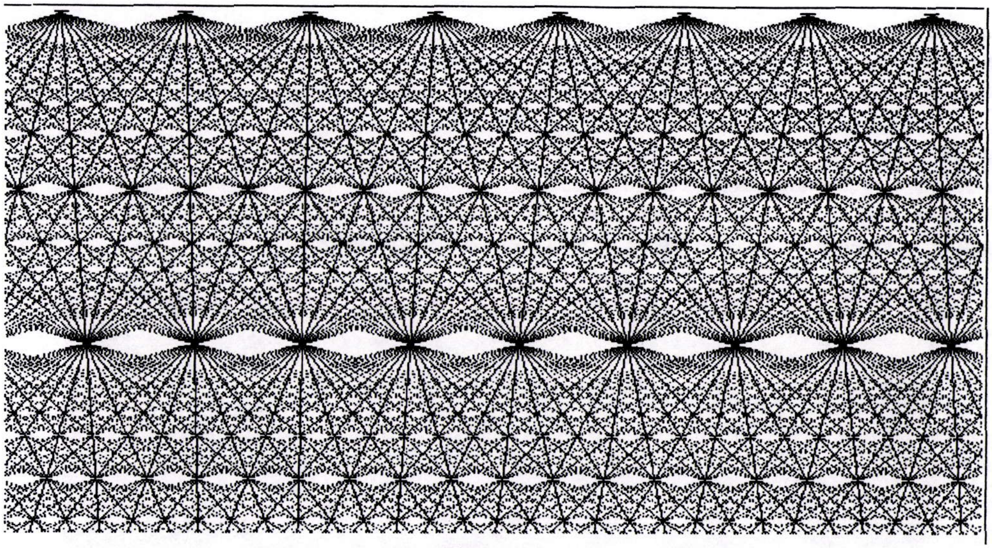

Figure 12

5. Conclusion

This paper shows some remarkable properties of a simple anticipatory system given by hyperincursive discrete harmonic oscillators. The position or the velocity of a particle in a harmonic potential is a function of the velocity or the position of this particle at a later future time step. The same hyperincursive system gives the dynamics of the discrete harmonic oscillator in plotting $v(t+\Delta t)$ as a function of $x(t+\Delta t)$ and its time inverse in plotting $v(t)$ as a function of $x(t+\Delta t)$ in the phase space. With small intervals of time, the continuous harmonic oscillator is obtained. For the interval of time tending to zero, the discrete hyperincursive harmonic oscillators tend to the classical differential equation, where the anticipatory hyperincursive property disappears.

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