Science Hyperincursive Integration

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Abstract

This paper describes the possibility of science hyperincursive integration by construction of a formal theory \mathbb{L} which becomes a universal language for the sciences and where it is possible to build their integration process. After this integration any scientific theory assumes all the data and ideas that can be useful for it from the other scientific theories. Dubois' incursive algorithm scheme permits a good two by two integration process of all the sciences but the continous making of new scientific ideas and data in the outside environment of the integrarion process implies the necessity to can change it during its execution by opporrune control parameters which represents the new scientific data and ideas which we can introduce. Thus we have really an hyperincursive process to integrate the sciences among them.

Keywords: epistemology, formal theory, hyperincursivity, incursiviry. science integration.

I Introduction

Is it possible to design a global scientific theory? Any science gives and takes ideas and data from other sciences. Apparently, a partial science hierarchy exists. Roughly: logics \rightarrow mathematics \rightarrow physics \rightarrow chemistry \rightarrow biology \rightarrow psychology \rightarrow ethologic sciences \rightarrow sociology and applied sciences \rightarrow history and culture history \rightarrow philosophy. Really, science history teaches us that any science has taken ideas and data from any science: e.g. mathematics and philosophy take ideas and data from logics but this last one takes also ideas and data by mathematics and philosophy. Thus sciences form a net that has to be described by anticipatory relations. This anticipatory net may be used as global theory. To manage it: before, we build a formal theory where any science can be representend, after, we build a logic on the truth relations between the sciences with a set of logic operators whose arguments are scientific theories, after, we define nets and nodes of scientific theories, finally, we give a science hyperincursive integration algorithm.

International Journal of Computing Anticipatory Systems, Volume 17,2006 Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-03-2

2 A Formal Theory L to Represent Sciences

2.1 Formal Theory Definition

2.1.1 General Definitions

A formal theory \Im is defined when the following conditions are satisfied:

- l*) There is a set of symbols which is at most countable and which is the set of the symbols of $\mathbb S$. A finite sequence of symbols of $\mathbb S$ is called *expression* of $\mathbb S$.
- 2^*) There is a subset of the expressions of S which is called set of well formed formulas² (abbreviate with wffs, singular wff). Usually, there is an automatic procedure to decide if any expression of $\mathbb S$ is wff.
- 3*) There is a subset of the wffs of S which is called set of the *axioms*. S is called *axiomatic* iff there is an automatic procedure to decide if a wff of \Im is axiom.,
- 4*) There is a finite set of relations $p_1, ..., p_n$, among wffs that are called *inference* rules. For every ρ_i there is a sole integer positive j such that for every set of j wffs and for every wff α one can decide if the j wffs are in relation ρ_i with α . In this case α is called *direct consequence* of the *j* wffs by ρ_i .

2.12 Proof Definition

A proof in S is a sequence $\alpha_1, \ldots, \alpha_n$, of wffs such that for every i, α_i is axiom or direct consequence of a subset of previous wffs.

A theorem α in S is the last wff of one or more proofs. Such proofs are called proofs of α .

 $\mathbb S$ is called *decidable* iff there is an automatic procedure to decide if any wff of $\mathbb S$ is a theorem.

A wff α of S is called *consequence* of a set Γ of wffs of S iff there is a sequence $\alpha_1, \ldots, \alpha_n$ of such that wffs for every i, α_i is axiom or direct consequence of a subset of previous wffs or $\alpha_i \in \Gamma$. Such a sequence $\alpha_1, \ldots, \alpha_n$ is called proof (or deduction) of α from Γ . The elements of Γ are called hypotheses or premises of α . Read ' $\Gamma \rightarrow \alpha$ ' "The wffs of the set Γ are premises of α'' , in other words, "The wffs of the set Γ deduce α'' . If

¹ We consider only standard logic with non-contradiction principle in this work. We think that the hyperincursion principles (see above) can be applied to any logic with any formalism but we do not prove this fact in this paper for space reasons. However, our convinction is based on the following achievements: Fuschino has proved the reducibility of fuzzy logics to standard one (Fuschino 1999); Rutz (1973) and Grappone (1995) have proved the reducibility of many-valued logics to standard one; Grappone (1988) has given a translation of Matte Blanco's bilogic in terms of standard logic; Malatesta (1982) proved that non-classical logics cannot take a step without a stock of tautologies belonging to classical one, which are laws of non-contradiction. According the author either the set of laws of a non-classical logic is a proper subset of the classical one or there is an intersection between the sets of laws

of classical logic and a non classical one, without which the last cannot work.
² Assume that a *well formed formula* is a symbol which means a given proposition in S.

 $\Gamma \equiv \emptyset$, then $\Gamma \rightarrow \alpha$ iff α is a theorem. So, we can denote " α is a theorem" with the expressions ' $\emptyset \mapsto \alpha$ ' and ' $\mapsto \alpha$ '.

The concept of consequence has the following properties:

1**) If $\Gamma \subset \Delta$ and $\Gamma \rightarrow \alpha$ then $\Delta \rightarrow \alpha$.

 2^{**}) $\Gamma \rightarrow \alpha$ iff there is such a finite set Δ that $\Delta \subset \Gamma$ and $\Delta \rightarrow \alpha$.

3**) If Γ deduces every wff of Δ and $\Delta \rightarrow \alpha$, then $\Gamma \rightarrow \alpha$.

2.1.3 Bi-logic Formal Theory Definition

A given formal theory is $bi\text{-}logic3$ iff all its meaning expressions, which are not operators, are fusions of other its meaning expression, which in turn are not operators. and such that its inference schemes include fusions and divisions of meaning expressions which are not operators (Matte Blanco's symmetry principle⁴ and symmetrical-asymmetrical relationship³).

2.2 Definition of of the bi-logic formal theory

2.2.1 Language of L

L01) σ denotes a void or inexisting expression;⁶

L02) a, b, c, \ldots denote generic terms;

L03) if α , β , γ , ... are terms or term series then α , $\alpha\beta$, $\alpha\beta\gamma$, ... are atomic terms;

L04) if α , β , γ , ... are terms then (α) , $(\alpha\beta)$, $(\alpha\beta\gamma)$, ... are terms;

L05) if α is a term then (α) is a term which is a constant;⁷

L06) if α , β , γ , ... are terms then $((\alpha)\beta\gamma...)$ is the term which is the achievement of the application of α on β , γ , ... where the constant (α) is used as functional letter,⁸

L07) if α is term then $\hat{\alpha}$ and $\hat{\alpha}$ are terms;

L08) If a row contains only ø then this expression is sentence;

L09) a, b, c, \ldots denote generic sentences;⁹

L10) if $\alpha, \beta, \gamma, \ldots$ are sentences or sentence series then $\alpha, \alpha\beta, \alpha\beta\gamma, \ldots$ are atomic sentences;

L11) if $\alpha, \beta, \gamma, \ldots$ are terms then $\{\alpha\}, \{\alpha\beta\}, \{\alpha\beta\gamma\}, \ldots$ are sentences;

L12) if α , β , γ , ... are sentences then $[\alpha]$, $[\alpha\beta]$, $[\alpha\beta\gamma]$, ... are sentences;

L13) if α , β , γ , ... are sentences then $\alpha\beta$, $\alpha\beta\gamma$, ... are sentences;

L14) if α , β , γ , ... are terms then $\{\langle \alpha \rangle \beta \gamma \dots \}$ is the sentence which is the achievement of the application of α on β , γ , ... where the constant (α) is used as predicate;¹⁰

⁵ See Matte Blanco (1975b).

⁹ Observe that a, b, c, \ldots can be \emptyset too (see L08).

³ See Matte Blanco (1988).

⁴ See Matte Blanco (1975a).

 $6\,\omega$ plaies in L the same function of zero in mathematics.

⁷ To define a constant see Mendelson (1964).

⁸ To define a functional letter see Mendelson (1964).

¹⁰ To define a predicate see Mendelson (1964).

- L15) if α is sentence then $\hat{\tau} \alpha$ and $\hat{\tau} \alpha$ are sentences;
- L16) \prod ab is sentence
- L17) $\sum ab$ is sentence¹¹
- 2.2.2 Metalanguage of L
- M01) Call $\prod a$ universal quantifier of **b** in $\prod_{i}ab$;
- M02) call $\sum a$ particular quantifier of **b**;
- M03) call a variable of $\prod a$ in $\prod ab$ and of $\sum a$ in $\sum ab$;
- M04) call **b** scope of \prod ain \prod and scope of \prod ain \prod ab;
- M05) call α free in **b** if neither it is the variable of any $\prod_{r} \alpha$ or any $\sum \alpha$ nor it is in their scopes in b and call \overline{a} linked if it is not free;
- M06) call any \overline{a} which is free in **b** linked by $\prod_{i} \overline{a}$ in $\prod_{i} \overline{a}$ and linked by $\sum_{i} \overline{a}$ in $\sum_{i} \overline{a}$ b;
- M07) call *a free for* \overline{b} *in c* if all its internal terms are free after its substitution to \overline{b} in *c*;
- M08) call sentence in prenex normal form a sentence a whose quantifiers are in its start;

 $M(09)$... are generic expression limits;

- M10) call closed sentence a sentence a if there are not free variables in it;
- M11) if α , β , γ , ... are expressions then $\alpha(\beta\gamma...)$ is an expression α which contains β , γ ,
- expressions which mean that α contains respectively at less 1, 2, 3, ... distinct groups of occurrences of β ;
- M13) if α , β and γ are expressions then $\alpha\langle...\beta/\gamma...\rangle$ is the achievement of the substitution of β with γ in α ;
- M14) if α and β are expressions then $\alpha\langle...\beta...\rangle$ and $\alpha\langle...\beta\langle...\rangle...\rangle$ are expressions which mean that α does respectively not contain β and $\beta \langle \ldots \rangle$;
- M15) $\alpha = \alpha \langle ... \rangle$ is the sentence: " α is a sentence which contains ...";

M16) Vab c means:

a is a sentence in prenex normal form, $^*)$

all the occurrences of \overline{b} are linked by a single particular quantifier, $***)$

***) all the occurrences of \overline{c} are linked by a single particular quantifier or free, M17) \cup ab c means:

> *a* is a sentence in prenex normal form, *)

$$
a=a\langle a|b|c\rangle\rangle;^{12}
$$

M18) \mathbb{Y} a \overline{b} \overline{c} means:

 $*)$ ab c

**) all the occurrences of \overline{b} in *a* are linked by a single universal quantifier,

***) all the occurrences of \overline{c} are linked by a single universal quantifier,

¹¹ To semplify exposition of the algorithm for deduction building in $\mathbb L$ we do not use the standard definiton of a particular quantifier $(\exists x)A$ in terms of universal quantifier, i. e. $-(\forall x)$ -A. Mendelson (1964).
¹² I. e.: the sentence **a** contains no atomic sentences which contain either the term b or or the term c.

****) if α is an expression which is obtained from a by the following sequential procedure:

> $01#$ erase all the quantifiers, 02# replace any atomic term which is neither $|b|$ nor $|c|$ with § 03# replace any $\S... \S$ with \S , 04# replace any $d\$ ge with de , 05# replace any (\S) with \S , 06# return to 3# till you obtain changes, 07# replace any $\{\S\}$ with \S , 08# replace any $\S... \S$ with \S , 09# replace any $f\$ g with fg , 10# replace any [§] with §, 11# return to 08# till you obtain changes,

and α is not \S , then α is a sentence which is true at most in two of these four cases:

1\$ true $\{b\}$ and true $\{c\}$, 2\$ true $\{b\}$ and false $\{c\}$, 3\$ false $\{b\}$ and true $\{c\}$, 4\$ false $\{b\}$ and false $\{c\}$;¹³

M19) $\int a$ means: a does not contain two quantifiers with the same variable and any quantifier links some variable in its scope;

M20) $\alpha \mapsto \beta$ means: α deduces β ;

M20) $\alpha \leftrightarrow \beta$ means: α deduces β and vice versa;

M22) $\alpha \mapsto ([] \leftrightarrow \beta)$ means: α deduces β by a term wrong fusion;

2.2.3 Axiom of L

 $AX01$) \emptyset

2.2.4 Inference schemes of L

 $\text{ISO1}\right)\left[\right]\left[\right]\left[\right]\left[\left[\left[\cdot\right]\right]\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\left[\cdot\right]\right]\left[\$ $IS03)$ $[[a(\uparrow abc...ba \downarrow abc... bcc...]][a(\uparrow abc...b \downarrow abc... bcc...]][a(\uparrow abc...bc \downarrow ab$ $c \dots |ab \dots |$...] $\leftrightarrow \quad a \langle \uparrow | abc \dots | \downarrow abc \dots \rangle;$ ISO4) $a\langle \text{Tab}\rangle \leftrightarrow a\langle \text{Tab}\rangle \land a\bigcup ab\langle \text{Lab}\rangle$ IS05) $\left[\left[a\left(\sqrt{abc\dots}\right)\right)\right]$ $a\left(\left(a\sqrt{bc\dots}\right)\right)$ $a\left(\left(ab\sqrt{c\dots}\right)\right)$ \rightarrow $a\left(\sqrt{abc\dots}\right)$; IS06) $\left[\left[a\langle(\hat{a}bc...\rangle)\right]\right]$ $a\langle(a\hat{b}c...\rangle)\right]$ $a\langle(ab\hat{c}...\rangle)\left[\ldots\right] \leftrightarrow a\langle\hat{a}bc...\rangle$;

¹³ This last property of α can easily be verified by standard truth function calculus. E. g., see Malatesta (1997a).

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IS07) [[{\{\downarrow}abc...}{\uparrow}][{\{a\downarrow bc...}\}][{\{ab\downarrow c...}\}]{\downarrow} \rightarrow \downarrow {\{abc...}\};
IS08) [\{\uparrow abc...\}\][\{a\uparrow bc...\}\][\{ab\uparrow c...\}\]\rightarrow \uparrow \{abc...\}IS10) a \langle \downarrow \overline{b} \rangle \langle \overline{b} \rangle \leftrightarrow a \langle \downarrow \overline{b} \rangleIS11) a \langle \uparrow \overline{b} \vert \overline{b} \rangle \leftrightarrow a \langle \uparrow \overline{b} \vert \overline{b} \rangleIS12) \lbrack \lbrack \uparrow abc... \rbrack \rbrack a \uparrow bc... \rbrack \rbrack ab \uparrow c... \rbrack ... \rbrack \leftrightarrow \downarrow \lbrack abc... \rbrackIS(13) \left[\downarrow abc\ldots\right]\left[a\downarrow bc\ldots\right]\left[ab\downarrow c\ldots\right]\ldots \leftrightarrow \uparrow [abc\ldots];IS14) \Pi b \uparrow c \leftrightarrow \uparrow \Pi b c;
IS15) \sqrt{b} \uparrow c \leftrightarrow \uparrow \sqrt{b}c;
IS16\uparrow a \mapsto \left( \left[ \right] \leftrightarrow a \right);[S17) [a[b]c][a[d]c] \leftrightarrow [a[bd]c];IS18[\] \leftrightarrow a[\]b;IS19) a[bc]def \leftrightarrow a[bec]def;
IS20) abc[de]f \leftrightarrow abc[dbe]f;
IS21) abcd \leftrightarrow abcbd;
[S22]a \leftrightarrow [[a]];
IS23) a(b) \leftrightarrow \prod b a(b);
IS24) \Box ba \mapsto \Sigma ba;
IS25) if \overline{\mathbb{U}}alb\overline{C} and \overline{\mathbb{U}}alb\overline{C} and \overline{\mathbb{U}}alb\overline{C} and \overline{\mathbb{U}}alb\overline{C} and \ldots in \sumba\langle \overline{C}\overline{C}\overline{C}\overline{C}\ldots then
                  \sum ba(\prod b \mid c \mid d \mid e \ldots);IS26) if \bigcup and \bigcup in \bigcup ba(b \bigcap alge \bigcup ) then
                  [[a\langle b\vert bc \vert c\vert bc \vert d]e \ldots] [[a\langle b\vert bc \vert c \vert d\vert b\vert e \ldots] [[a\langle b\vert bc \vert c \vert d]e\vert be \ldots]]\ldots] \mapsto\sumba\langleb dad...);
IS27) if \mathbb{Y}ab \bigcap and \mathbb{Y}ab \bigcap and \mathbb{Y}ab \bigcap and \mathbb{Y}ab \bigcap and \ldots in \bigcap \bigcap[[a\langle b\rangle b c|\langle c\rangle b c|\overline{d}e] \ldots][a\langle b\rangle b d|\langle d\rangle b d|e] \ldots][a\langle b\rangle b e|\langle d|e\rangle b e] \ldots] \mapsto\Piba\overline{\Phi}BBee ... );
\text{IS28)}\,\text{if}\,\text{Id}[\ldots b\langle a\rangle \ldots] \leftrightarrow \text{If}\ldots \sum ab\langle a\rangle \ldots];IS29) \textcircled{1} \textcircled{1} \ldots \textcircled{1} \cdots \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \ldots \textcircled{1} \ldots \textcircled{1}\overbrace{IS30} \overbrace{a \langle ... \overline{b} \overline{b} \overline{c} \langle d\overline{b} \rangle ...}^{\text{IG}} \times \overbrace{I \cup \overline{b} \langle d\overline{b} \rangle ...}^{\text{IG}} \rangle \leftrightarrow \overbrace{a \langle ... \overline{b} \overline{b} \rangle ...}^{\text{IG}} \overbrace{I \cup \overline{b}}^{\text{IG}} \overbrace{a \langle b \rangle ...}^{\text{IG}} \rangle\overline{1532}) a(b) \leftrightarrow \overline{110}a(b);
ISS3) a\langle b\rangle \leftrightarrow \sum ba\langle b\rangle.
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2.2.5 Abreviations of L^{14}

AB01) No is $[\alpha]$;¹⁵

¹⁴ With $n \ge 2$. We use a polyadic extension of Polish language for standard sentence logic. See Lukasiewicz (1958) and Malatesta (1997b). It is not complete because to complete it is useless for our purposes.

¹⁵ N α is the negation of α i. e., in standard language, $-\alpha$.

AB02)
$$
\mathring{V}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}
$$
 is α_{1}^{16}
\nAB03) $\mathring{A}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[(\alpha_{1}][\alpha_{2}][\alpha_{2}][\alpha_{3}]...[\alpha_{n-1}][\alpha_{n}]];^{17}$
\nAB04) $\mathring{B}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[(\alpha_{1}]\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}]^{18}$
\nAB05) $\mathring{B}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[(\alpha_{1}]\alpha_{2}[\alpha_{3}... \alpha_{n-1}\alpha_{n}]^{18}$
\nAB06) $\mathring{C}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{1}[\alpha_{2}][[\alpha_{2}]\alpha_{3}]...[[\alpha_{n-1}][\alpha_{n}]]^{19}$
\nAB07) $\mathring{C}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{1}[\alpha_{2}][[\alpha_{2}[\alpha_{3}]]...[\alpha_{n-1}][\alpha_{n}]]^{21}$
\nAB08) $\mathring{D}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{1}[\alpha_{2}]\alpha_{3}... \alpha_{n-1}\alpha_{n}]^{122}$
\nAB09) $\mathring{E}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}]^{122}$
\nAB10) $\mathring{F}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{1}];$
\nAB11) $\mathring{G}\alpha_{1}\alpha_{2}\alpha_{3}... \alpha_{n-1}\alpha_{n}$ is $[\alpha_{n}];$
\nAB12) $\mathring{H}\alpha_{1}\alpha_{2}\alpha_{$

¹⁶ $\overline{V}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is polyadic tautology.

¹⁷ ${}_{A}^{n}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is polyadic logicsum, i.e., instandard language, $\alpha_1 \vee \alpha_2 \vee \alpha_3 \vee ... \vee \alpha_{n-1} \vee \alpha_n$.

¹⁸ $\beta\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is Malatesta's converse sequence implication, i. e., in standard language, $((\dots((\alpha_1 \subset \alpha_2) \subset \alpha_3) \subset \dots) \subset \alpha_{n-1}) \subset \alpha_n$. See Malatesta (1989a).

¹⁹ ${}_{BC}^n \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is Malatesta's converse chain implication, i. e., in standard language, $(\alpha_1 \infty)$ $(\alpha_2 \infty, \alpha_3) \wedge ... \wedge (\alpha_{n-1} \infty)$. See Malatesta (1989b).

²⁰ $\alpha_0 \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is Malatesta's *sequence implication*, i. e., in standard language, $\alpha_1 \in (\alpha_2 \in (\alpha_3 \in (\alpha_3 \in \alpha_1) \in \alpha_n)$...). See Malatesta (1989c).

²¹ ${}_{G}^{n}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is Malatesta's *chain implication*, i. e., in standard language, $(\alpha_1 \in \alpha_2) \wedge (\alpha_2 \in \alpha_3) \wedge ... \wedge (\alpha_{n-1} \in \alpha_n)$. See Malatesta (1989d).

²² $\mathring{D}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is polyadic logic sum of sentence negations i. e., in standard language, $-\alpha_1 \vee -\alpha_2 \vee -\alpha_3 \vee \ldots \vee -\alpha_{n-1} \vee -\alpha_n$

²³ $\mu^2 \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is Malatesta's *polyadic equivalence*, i. e. is the equivalence of all its arguments. See Malatesta $(1989e).$

²⁴ $\int_a^b \alpha_1 \alpha_2 \alpha_3 \dots \alpha_{n-1} \alpha_n$ is Galens's *complete battle*, i. e. it is the truth of one and only one argument. See Galen, ed. Kalbfleisch (1896) and Malatesta (1992a).

AB16) $\mathring{K}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is $\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n^{20}$ AB17) $\mathring{L}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n$ is $\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\alpha_n$. AB18) $\hat{L}\alpha,\alpha,\alpha_1...\alpha_n,\alpha_n$ is $[[\alpha,[\alpha_1]][\alpha,[\alpha_1]]...[\alpha_n][\alpha_n]]]$;²⁸ AB19) $\mathring{M}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n$ is $[\alpha_1]\alpha_2\alpha_3...\alpha_n\alpha_n^{29}$ AB20) $\mathring{M}_{\alpha,\alpha,\alpha,\ldots,\alpha,\alpha}$ is $[[[\alpha,\alpha,\alpha,\alpha,\ldots][\alpha,\alpha,\ldots][\alpha,\alpha,\alpha,\alpha,\alpha,\alpha,\ldots][\alpha,\alpha,\alpha,\alpha,\ldots]]$ AB21) $\chi_{\alpha,\alpha,\alpha_1...\alpha_n}$ a is $[\alpha_1][\alpha_2][\alpha_3] \dots [\alpha_n][\alpha_n]^{3!}$ AB22) $\mathring{\mathcal{O}}\alpha$, $\alpha_1 \alpha$, ... α , ... is $[1]$ ³²

2.2.6 Calculus of the Simplest Sentence which Deduces a Given Sentence z of L

Let $\langle z \rangle$ be the simplest sentence which deduces a given z in L. To calculate it:³³ step $0!$: write z:

- step 02: let v be the top row (z in this case); if $v=v\langle O\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write AB22 at right of v and write $v\langle O\alpha_1\alpha_2\alpha_3... \alpha_n\alpha_n\langle 1\rangle$ on v; go to the start of this step till changes happen;
- step 03: let v be the top row; if $v = v \langle X \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n \alpha_n \rangle$ then write AB21 at right of v and write $v\langle \chi \alpha_1 \alpha_2 \alpha_2 \ldots \alpha_n \alpha_n \alpha_n | [\alpha_1][\alpha_2][\alpha_3] \ldots [\alpha_{n-1}][\alpha_n] \rangle$ on v; go to the start of this step till changes happen;

²⁶ Kα, α, α, ... α, a, ispolyadiclogic product, i.e., instandard language, α, α , α , α , ... α (α _{n-1} α),

²⁷ La, a, a, ...a, a, is negation of Malatesta's sequence implication, i. e., in standard language, $\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \wedge \ldots \wedge \alpha_n$. $\Lambda \sim \alpha_n$.

- ²⁸ $\int_{C}^{n} \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is negation of Malatesta's *chain implication*, i. e., in standard language, $(\alpha, \wedge \neg \alpha, \vee (\alpha, \wedge \neg \alpha, \vee \dots \vee (\alpha_{n-1} \wedge \neg \alpha_{n}).$
- ²⁹ $M\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is negation of Malatesta's converse sequence implication, i. e., in standard language, $\alpha, \wedge \neg \alpha, \wedge \neg \alpha, \wedge \dots \wedge \neg \alpha_{n-1} \wedge \neg \alpha_n$

³⁰ $M_c \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is negation of Malatesta's converse chain implication, i. e., in standard language, $(\alpha, \alpha) \wedge (\alpha, \alpha) \wedge \ldots \wedge (\alpha_{n-1} \alpha)$

³¹ ${}_{X}^{n}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is polyadic logic product of sentence negations i. e., in standard language, $-\alpha_1 \wedge -\alpha_2 \wedge -\alpha_3 \wedge \ldots \wedge -\alpha_{n-1} \wedge -\alpha_n$

³² $\mathcal{O}_{\alpha_1 \alpha_2 \alpha_3 \dots \alpha_{n-1} \alpha_n}$ is polyadic contradiction.

³³ Observe that *a* is theorem (contradiction) in L iff $\langle ax \rangle$ is \varnothing ([]); let contradictions be wrong term fusions (consider the historic development: set ingenuous concept \mapsto all class antinomy \mapsto division of the set ingenuous concept in set class and non-set class concepts), thus the steps 33, ..., 47 correct in $\langle a \rangle$ the contradictions of a .

²⁵ Pratically, $\hat{J}_i \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n$ is Gellius' *incomplete battle*, i. e. it is the truth of at most one argument. See Gellius, rec. Marshall (1968).

- step 04: let v be the top row; if $v = v \langle M \alpha_0 \alpha_2 \alpha_2 \dots \alpha_n \alpha_n \rangle$ then write AB20 at right of v and write $v\langle \mathring{M}\alpha_1\alpha,\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/[[[\alpha_1]\alpha_2][[\alpha_2]\alpha_3]...[[\alpha_{n-1}]\alpha_n]]\rangle$ on v; go to the start of this step till changes happen;
- step 05: let v be the top row; if $v=v(\overline{M}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n)$ then write AB19 at right of v and write $v\langle M\alpha_1\alpha_2\alpha_2... \alpha_n\alpha_n/\alpha_1/\alpha_2\alpha_2...\alpha_n/\alpha_n\rangle$ on v; go to the start of this step till changes happen;
- step 06: let v be the top row; if $v = v \langle L \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n \alpha_n \rangle$ then write AB18 at right of v and write $v\langle L\alpha_1\alpha_2\alpha_2...\alpha_{n-1}\alpha_n/[[\alpha_1[\alpha_2]][\alpha_2[\alpha_3]]...[\alpha_{n-1}[\alpha_n]]]\rangle$ on v; go to the start of this step till changes happen;
- step 07: let v be the top row; if $v=v\langle \mathring{L}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write ABl 7 at right of v and write $v\langle \overline{L}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/\alpha_1\alpha_2\alpha_3...\alpha_{n-1}[\alpha_n]\rangle$ on v; go to the start of this step till changes happen;
- step 08: let v be the top row; if $v = v(K\alpha, \alpha, \alpha, \alpha, \alpha)$ then write AB16 at right of v and write $v\langle K\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n\rangle$ on v; go to the start of this step till changes happen;
- step 09: let v be the top row; if $v=v(\mathring{A}\alpha_1\alpha_2\alpha_3... \alpha_{n-1}\alpha_n)$ then write AB15 at right of v and write $v\langle \mathring{A}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n/[\lceil[\alpha_1][\alpha_2][\alpha_3]\ldots[\alpha_n][\alpha_n][\alpha_n][\alpha_1]\ldots[\alpha_{n-1}][\alpha_n] \rangle$ α_n^{\prime}]][[α_1^{\prime}] α_2^{\prime} [α_3^{\prime}]...[α_{n+1}][α_n^{\prime}]][[α_1^{\prime}] α_2^{\prime}] α_3^{\prime}] α_3^{\prime}][α_1^{\prime}][α_2^{\prime}][α_3^{\prime}] α_3^{\prime}]... α_{n+1}^{\prime} [$\alpha_n || [[\alpha_1][\alpha_2][\alpha_3] \dots [\alpha_{n-1}]\alpha_n]]$ on v; go to the start of this step till changes happen;
- step 10: let v be the top row; if $v=v\langle \overline{J}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write AB14 at right of v and write $v\langle \hat{J}\alpha_1\alpha_2\alpha_3...\alpha_n,\alpha_n\rangle \left[\left[\alpha_1[\alpha_2][\alpha_3]...\left[\alpha_n\right]]\left[\alpha_1[\alpha_1]\alpha_2[\alpha_3]...\left[\alpha_n\right]\right]\right]\right]$ $\lim_{n\to\infty} \left[\left[\alpha_n \right] \right] [\alpha_n] [\alpha_1] [\alpha_2] [\alpha_3] \dots [\alpha_n] [\alpha_n] [\alpha_2] [\alpha_3] \dots [\alpha_n] [\alpha_n] [\alpha_2] [\alpha_3] \dots [\alpha_{n-1}]$ α .]) on v; go to the start of this step till changes happen;
- step 11: let v be the top row; if $v=v\sqrt{\tau}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ then write AB13 at right of v and write $v \langle \overline{I} \alpha_1 \alpha_2 \alpha_3 \dots \alpha_{n-1} \alpha_n / \alpha_1 \rangle$ on ν ; go to the start of this step till changes happen;
- step 12: let v be the top row; if $v = v \langle H\alpha_1\alpha_2\alpha_3... \alpha_n \rangle$ then write ABl2 at right of v and write $v\langle H\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ on v ; go to the start of this step till changes happen;
- step 13: let v be the top row; if $v=v\langle \overrightarrow{G}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n \rangle$ then write AB11 at right of v and write $v(\sigma_{\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha}(\alpha)\cap v;$ go to the start of this step till changes happen;
- step 14: let v be the top row; if $v = v \langle F\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n \rangle$ then write AB10 at right of v and write $v\langle F\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n/[\alpha_1]\rangle$ on v; go to the start of this step till changes happen;
- step 15: let v be the top row; if $v=v(\mathring{E}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n)$ then write AB09 at right of v and write $v \langle E \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n / [\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n-1} \alpha_n] [\alpha_1] [\alpha_2] [\alpha_3] \ldots [\alpha_{n-1}] [\alpha_n]] \rangle$ on v; go to the start of this step till changes happen;
- step 16: let v be the top row; if $v=v\langle \overrightarrow{D}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write AB08 at right of v and write $v(\mathring{D}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/[\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n])$ on v; go to the start of this step till changes happen;
- step 17: let v be the top row; if $v=v\langle C_0\alpha,\alpha,\ldots,\alpha_{n-1}\alpha\rangle$ then write AB07 at right of v and write $v(\bar{C}_{\alpha}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/[\alpha_1[\alpha_2]][\alpha_2[\alpha_3]]...[\alpha_{n-1}[\alpha_n]])$ on v; go to the start of this step till changes happen;
- step 18: let v be the top row; if $v = v \langle C\alpha_1\alpha_2\alpha_3\ldots\alpha_n|\alpha_n\rangle$ then write AB06 at right of v and write $v\langle \tilde{C}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n/[\alpha_1\alpha_2\alpha_3...\alpha_{n-1}[\alpha_n]]\rangle$ on v; go to the start of this step till changes happen;
- step 20: let v be the top row; if $v=v\langle \mathring{R}_c\alpha,\alpha,\alpha,\ldots,\alpha,\alpha\rangle$ then write AB05 at right of v and write $v(\mathbf{R}^T\alpha_1\alpha_2\alpha_3...\alpha_n)\alpha_n/[(\alpha_1]\alpha_2][[\alpha_2]\alpha_3]...[[\alpha_{n-1}]\alpha_n])$ on v; go to the start of this step till changes happen;
- step 21: let v be the top row; if $v=v(\beta\alpha_1\alpha_2\alpha_3...\alpha_n,\alpha_n)$ then write AB04 at right of v and write ${\bf v}(\stackrel{\scriptscriptstyle\prime}{B}\!\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n/[[\alpha_1]\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n])$ on ${\bf v}$; go to the start of this step till changes happen;
- step 22: let v be the top row; if $v=v\langle A\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write AB03 at right of v and write $v\langle A\alpha_1\alpha_2\alpha_3... \alpha_n|\alpha_n|[\alpha_n][\alpha_n][\alpha_n][\alpha_n]...[\alpha_n][\alpha_n] \rangle$ on v; go to the start of this step till changes happen;
- step 23: let v be the top row; if $v=v\langle V\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n\rangle$ then write AB02 at right of v and write $\mathbf{v}'_x \mathbf{\hat{V}} \alpha_1 \alpha_2 \alpha_3 \dots \alpha_{n-1} \alpha_n / \mathcal{O}$ on v; go to the start of this step till changes happen;
- step 24: let v be the top row; if $v=v(N\alpha)$ then write AB01 at right of v and write $\nu\langle N\alpha/[\alpha]\rangle$ on ν ; go to the start of this step till changes happen;
- step 25: let v be the top row; if $v=v(\Sigma b \alpha \langle b \rangle)$ then write IS33 at right of v and write $\nu \langle \nabla \phi \phi \rangle / \phi \langle \phi \rangle$ on ν ; go to the start of this step till changes happen;
- step 26: let v be the top row; if $v=v\langle \prod \vec{b}a(\vec{b})\rangle$ then write IS32 at right of v and write $\nu(\Pi \overline{ba} \langle \overline{b} \rangle/a \langle \overline{b} \rangle)$ on ν ; go to the start of this step till changes happen;
- step 27: let v be the top row; if $v=v\langle a\langle ...b... \rangle d\langle b|c\langle d/b... \rangle)$ then write IS31 at right of v and write $v\langle a\langle ...b... \rangle a \langle b\rangle \langle b\rangle \langle b\rangle ... \rangle / a \langle ...b... \rangle a \langle b\rangle \langle b\rangle \langle b\rangle$ on v; go to the start of this step till changes happen;
- step 28: let v be the top row; if $v=v\langle a\langle \dots \overline{b}\rangle \dots \sqrt{a\langle b\vert}c\langle d/b\rangle \dots\rangle$ the write IS30 at right of v and write $v\langle a(\ldots b\ldots b\vert\ldots\vert d\vert b c\langle d\vert b\rangle\ldots\rangle/a\langle\ldots\vert b\ldots\vert d\vert c\langle d\vert b\rangle\ldots\rangle)$ on v; go to the start of this step till changes happen;
- step 29: let v be the top row; if $v=v(\hat{U}[\dots \Pi a]b\langle a \rangle \dots])$ the write IS29 at right of v and write $v(\hat{\mathfrak{gl}} \dots \Pi \hat{d}[\mathbf{b} \langle \hat{d} \rangle \dots] / \hat{\mathfrak{gl}} \Pi \dots \hat{d} \langle \hat{d} \rangle \dots]$ on v; go to the start of this step till changes happen;
- step 30: let v be the top row; if $v=v(\hat{\mathbb{I}}[...][\hat{\mathbb{I}}[b\langle a \rangle...])$ then write IS28 at right of v and write $v(\hat{\mathbb{I}}[...]\Sigma[a[b\langle a\rangle...]/\hat{\mathbb{I}}[\Gamma[a[...b\langle a\rangle...])$ on v; go to the start of this step till changes happen;
- step 31: if the steps 29 or 30 have added rows in their last execution return to step 29;
- step 32: if \vec{a} is free in $\vec{b}(\vec{a})$ then write IS23 at right of v and write $\vec{v}(\vec{b}(\vec{a})/\sqrt{|\vec{a}|\vec{b}(\vec{a})})$ on v; go to the start of this step till changes happen, finally store the top row as w ;
- step 33: let y be the top row; if $v = v \sqrt{|\beta q|} \sqrt{dq} \sqrt{dq}$...) and Yallocand Yallo \mathbb{Z} able and ... in \prod_{k} ble (ble and write IS27 at right of v and write v<Mba
blodde...>>>>>> [a
bloddeded="15" [a
blodded="2" [a
blodded="2"]... \vert on ν ; go to the start of this step till changes happen;
- step 34: let v be the top row; if $v=v\langle \sum b\alpha|bc|de... \rangle$ and $\cup ab|c|$ and $\cup ab|c|$ and $\cup ab|c|$ and able and ... in $\sum b a \langle b \rangle$ and write IS26 at right of v and write x Da Gdde... ([[a b|boc|bode...)][a b|bdc|drbde...)][a b|becddelbe...)]...] \rangle on ν ; go to the start of this step till changes happen;
- Uable and ... in $\Sigma b a \langle b|c|d|c... \rangle$ then write IS25 at right of v and write $v(\sum \vec{b} | \vec{a} \langle \vec{b} | \vec{c} | \vec{a} | \vec{e} \ldots \rangle | [\vec{a} \langle \vec{b} | \vec{b} \vec{c} | \vec{c} | \vec{c} | \vec{c} \ldots \rangle] [\vec{a} \langle \vec{b} | \vec{b} \vec{d} | \vec{c} | \vec{c} | \ldots \rangle] [\vec{a} \langle \vec{b} | \vec{b} \vec{c} | \vec{c} | \vec{d} | \vec{c} | \ldots \rangle] \ldots]$ step 35: let v be the top row; if $v=v\langle \sum \beta a \langle b \rangle c | a e \rangle$...)) and $\langle b \rangle a | b \rangle c$ and $\langle b \rangle a$ and $\langle b \rangle a$ and) on ν ; go to the start of this step till changes happen;
- step 36: let v be the top row; if $v=v(\sum b\alpha)$ then write IS24 at right of v and write $\nu\langle \sum \vec{b} \vec{a}/\sqrt{b} \vec{a} \rangle$ on ν ; go to the start of this step till changes happen;
- step 37: let v be the top row; if $v = v\langle \prod b \alpha \langle b \rangle \rangle$ then write IS23 at right of v and write $\nu\langle \prod b a \langle b \rangle / a \langle b \rangle \rangle$ on ν ; go to the start of this step till changes happen;
- step 38: let v be the top row; if $v=v(\vert[a]\vert)$ then write IS22 at right of v and write ν ([[a]]/a) on v; go to the start of this step till changes happen;
- step 39: let v be the top row; if $v = v$ (abcbd) then write IS21 at right of v and write $v(abcd/abcd)$ on v ; go to the start of this step till changes happen;
- step 40: let v be the top row; if $v=v(abc[dbc]f)$ then write IS20 at right of v and write $v(\mathbf{abc}[\mathbf{dbe}]\mathbf{f}/\mathbf{abc}[\mathbf{de}]\mathbf{f})$ on v ; go to the start of this step till changes happen;
- step 41: let v be the top row; if $v=v(a[\text{bec}]\text{def})$ then write IS19 at right of v and write $v(a\beta\,ec\,def/a\beta\,bc\,def)$ on v; go to the start of this step till changes happen;
- step 42: if the steps 39 or 40 have added rows in their last execution retum to step 39;
- step 43: let v be the top row; if $v=v\langle a|b\rangle$ then write IS18 at right of v and write $\nu(a[|b|)]$ on ν ; go to the start of this step till changes happen;
- step 44: if the steps 38 or 39 or 40 or 4l or 42 or 43 have added rows in their last execution return to step 38;
- step 45: let v be the top row; if $v=[a[bd]c]$; then write IS17 at right of v and write $[a[b]c][a[d]c]$ on v; go to the start of this step till changes happen;
- step 46: if the step 45 has added rows in its last execution return to step 38;
- step 47: let v be the top row; if v is not [] then end the algorithm otherwise write IS16 at right of v and write $\hat{\mathcal{L}}$ w (see step 31) on v;
- step 48: let v be the top row; if $v=v\sqrt{\sum a_i}$; then write IS15 at right of v and write $v \langle \uparrow \rangle$ α α β α ; go to the start of this step till changes happen;
- step 49: let v be the top row; if $v=v(\hat{T}(\vec{a}\cdot\vec{a}))$; then write IS14 at right of v and write ν (\int III) α α / $\prod \alpha$ \hat{i} α); go to the start of this step till changes happen;
- step 50: if the steps 48 or 49 have added rows in their last execution return to step 48;
- step 51: let v be the top row; if $v=v(\hat{\theta}[\boldsymbol{abc}...])$; then write IS13 at right of v and write $v\langle \hat{\Pi}[\text{abc}...]/[\hat{\Pi}[\text{abc}...][\text{abc}...][\text{ab}[\text{bc}...][...]]\rangle$; go to the start of this step till changes happen;
- step 52: let v be the top row; if $v=v\sqrt{[abc...]}$; then write IS12 at right of v and write $v \triangle \left[abc... \right] / \left[\int_a^b \phi(c...)[a \phi(c...][ab] \right]$; go to the start of this step till changes happen;
- step 53: if the steps 50 or 51 have added rows in their last execution return to step 50;
- step 54: let v be the top row; if $v=v\langle \hat{b}|\hat{b}|\hat{b}\rangle$; then write IS11 at right of v and write $\nu = \langle \uparrow b \rangle$; go to the start of this step till changes happen;
- step 55: let v be the top row; if $v=v\langle\sqrt{b^2+4}g\rangle$; then write IS10 at right of v and write $\nu = \langle \downarrow b \rangle$; go to the start of this step till changes happen;
- step 56: let v be the top row; if $v=v(\sqrt{[k(\{... \})}] \sqrt{[k(\{... \})}]$ then write IS09 at right of v and write $v(\hat{b}(\{...)\}, \mathbb{Z}) \leq \frac{1}{2}$ (c) \hat{d} \ $\hat{b}(\{... \} \times \{c\})$ { (c) \hat{d} }); go to the start of this step till changes happen;
- step 57: let v be the top row; if $v=v(\hat{\theta}{}_{a}bc...)$; then write IS08 at right of v and write $\nu\langle \hat{\Pi}^{\{\text{abc}\}}\rangle / \left[\{\hat{\Pi}abc...\}\right] \left[\{a\hat{\Pi}bc...\}\right] \left[\{ab\hat{\Pi}c...\}\right]$; go to the start of this step till changes happen;
- step 58: let v be the top row; if $v=v(\frac{1}{abc...})$; then write IS07 at right of v and write $v\langle \downarrow \{abc\} / [[\{\downarrow abc...\}][\{a\downarrow bc...\}][\{ab\downarrow c...\}]]$; go to the start of this step till changes happen;
- step 59: let v be the top row; if $v=v\langle a(\hat{a}(\hat{a}bc...))\rangle$; then write IS06 at right of v and write $v\langle a(\hat{\mathcal{T}}(abc...) \rangle / \langle [a(\hat{\mathcal{T}}abc...) \rangle][a(\hat{a}(\hat{\mathcal{T}}bc...))]$ $[a(\hat{a}(\hat{\mathcal{T}}bc...)]$...]); go to the start of this step till changes happen;
- step 60: let v be the top row; if $v=v\langle a\langle \downarrow (abc...) \rangle$; then write IS05 at right of v and write $v\langle a\langle\downarrow(abc...)\rangle/[a\langle(\downarrow abc...)\rangle][a\langle(a\downarrow bc...)\rangle][a\langle(ab\downarrow c...)\rangle]...]$; go to the start of this step till changes happen;
- step 61: let v be the top row; if $v=v(a \langle \hat{a} \rangle \langle \hat{a} \rangle \langle \hat{a} \rangle)$; then write IS04 at right of v and write $v\langle a\rangle \hat{\mid}$ ab| $\langle \downarrow a\phi \rangle / a \langle \hat{\parallel} a\phi \rangle$; go to the start of this step till changes happen;
- step 62: let v be the top row; if $v=v(a(\hat{a}bc...\hat{b}bcc))$; then write IS03 at right of v and write

 $v(a(\text{7}abc... \text{7}abc...)(a(\text{7}abc... \text{7}abc... \text{7}bc...)(a(\text{7}abc... \text{7}abc... \text{7}ac...)(b$

- $a(\sqrt[n]{abc...bc...bc...ab...})...$; go to the start of this step till changes happen; step 63: let v be the top row; if $v=v(\Sigma abc...|a|abc...)\$; then write IS02 at right of v and write $v(\Sigma abc...|a\langle ab|c.../\Sigma a\Sigma b\Sigma c...a\langle ab|c...)\rangle$; go to the start of this step till changes happen;
- step 64: let v be the top row; if $v=v\langle \prod_{a}abc...a \rangle a|b|c... \rangle$; then write IS01 at right of v and write $v(\prod_{a}abc...a(a)b|c...)\prod_{a}\prod_{b}\prod_{c}...a(a|b|c...)\$; go to the start of this step till changes happen;

step 65: return to step 33.

$\overline{\mathbf{3}}$ Scientific Theories and Their Logic in L

Any scientific theory can be represented in L by the logical product of the representations in L of its single affirmations or equations.³⁴ Thus any scientific theory is finally a sentence in L.

Any scientific theory has a potency which is either its event prevision power or its tecnical problem solving power. To define the logical relations among potencies of sentences of $\mathbb L$ when they represent scientific theories observe that the consideration of their simplest premises increases the efficiency and the elegance of the represented theories. Furthermore it conserves their potencies. Thus, for every sentence a , we use always $\langle a \rangle$ in its place when a represents a scientific theory; also, as a tautology gives no information,³⁵ it has no potency, thus «[««a» («a»]»»]» which represents a tautology in L, has no potency and so we can put that «[«a»]» is a theory which has exactly the potency that α has not and, in general, that «« a » (a » (b » (c »)... » is a theory which has exactly the potencies of all the theories a, b, c, \ldots .³⁶ Thus we can define the following connectives in $\mathbb L$ which represents the logical relations among the potencies of their arguments when they represent scientific theories:

IN01) $N\alpha$ is «[« α »]»;

IN02) $\hat{V}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n$ is \varnothing ; IN03) $\mathring{A}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n$ is $\langle [N\alpha_1N\alpha_2N\alpha_2N\alpha_3\ldots N\alpha_{n-1}N\alpha_n] \rangle$; IN04) $\vec{B}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n$ is $\vec{A}\alpha_1N\alpha_2N\alpha_3...N\alpha_n\alpha_n$ IN05) $\vec{B}_c \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_{n} \alpha_n$ is $\langle \vec{B} \alpha_1 \alpha_2 \vec{B} \alpha_2 \alpha_3 \ldots \vec{B} \alpha_{n} \alpha_n \rangle$; IN06) $\ddot{C}\alpha_1\alpha_2\alpha_3...\alpha_n\alpha_n$ is $\ddot{A}N\alpha_1N\alpha_2N\alpha_3...N\alpha_n\alpha_n$ IN07) $\ddot{\mathcal{C}}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is $\langle \dot{\mathcal{C}}\alpha_1\alpha_2\dot{\mathcal{C}}\alpha_2\alpha_3...\dot{\mathcal{C}}\alpha_{n-1}\alpha_n\rangle$;

 34 The equations are represented in L by representation of opportune mathematical formal theories.

³⁵ See Wittgenstein (1921).

³⁶ The contradiction ««a» («a») » has the potencies of all the theories, of the opposite theories too. Therefore it is completely useless because it has contradictory prevision of events and contradictory solutions of technical problems.

INOS)
$$
\vec{D}\alpha_1\alpha_2\alpha_3...\alpha_n_1\alpha_n
$$
 is $\vec{A}N\alpha_1N\alpha_2N\alpha_3...\overline{N}\alpha_{n-1}N\alpha_n$;
\nINOS) $\vec{E}\alpha_1\alpha_2\alpha_3...\alpha_n_1\alpha_n$ is $\vec{B}\vec{D}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$;
\nIN10) $\vec{F}\alpha_1\alpha_2\alpha_3...\alpha_n_1\alpha_n$ is $N\alpha_1$;
\nIN11) $\vec{G}\alpha_1\alpha_2\alpha_3...\alpha_n_1\alpha_n$ is $N\alpha_n$;
\nIN12) $\vec{H}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is $\alpha\alpha_n$;
\nIN13) $\vec{I}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is $\alpha\alpha_n$;
\nIN14) $\vec{J}\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n$ is $\vec{B}\vec{A}N\alpha_1\alpha_2\alpha_3...\alpha_{n-1}\alpha_n\vec{A}\alpha_1N\alpha_2\alpha_3...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2N\alpha_3...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_{n-1}\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_n\vec{A}\alpha_1\alpha_2...\alpha_n\vec{A}\alpha_1\alpha_2...\vec{A}\alpha_n\vec{A}\alpha_1\alpha_2...\vec{A}\alpha_n\vec{A}\alpha_1\alpha_2...\vec{A}\alpha_n\vec{A}\alpha_1\alpha_2...\vec{A}\alpha_n\vec{A}\alpha_1\alpha_2...\vec{A}\alpha_n\vec{A}\alpha_1\alpha_2$

Scientific Theory Nets and Nodes in L 4

Let $\ddot{\Phi} \alpha_1 \dots \alpha_n$, $\ddot{\Psi} \alpha_1 \dots \alpha_n$ be sentences at will which contain only atomic sentences and some of the following connectives: N, \ddot{V} , \ddot{A} , \ddot{B} , \ddot{B} , \ddot{C} , \ddot{C} , \ddot{C} , \ddot{D} , \ddot{E} , \ddot{F} , \ddot{G} , \ddot{H} , \ddot{I} , \ddot{J} , \ddot{J} , \ddot{J} , \ddot{K} , \ddot{L} \mathring{L} , \mathring{M} , \mathring{M} , \mathring{X} , \mathring{O} .

Let $\ddot{\Phi}_i \alpha_1 \dots \alpha_n \dots \alpha_n$ be a sentence in L which is obtained from $\ddot{\Phi} \alpha_1 \dots \alpha_n \dots \alpha_n$ by the following algorithm:

step 01: let $\mathbf{\Psi}\alpha_1 \dots \alpha_n \dots \alpha_n$ be $\mathbf{\Phi}\alpha_1 \dots \alpha_n \dots \alpha_n$;

step 02: for every $abc...$ which contains either terms, which α_i contains, or terms, which α_i does not contain, erase these later ones in the occurrences of $abc...$ in $\mathring{\Psi}\alpha_1 \dots \alpha_n \dots \alpha_n$ if such occurrences exist;

step 03: for every $|a|$ abc...) where a, b, c, ... are not contained in α , replace all its occurrences in $\Psi \alpha_1 \dots \alpha_n \dots \alpha_n$, if they exist, with a symbol among $\phi(\{...\})$, $\mathbf{d}(\{...\})$, $\mathbf{d}(\{...\})$, ... which has not been used before; step 04: Given the four sentences:

 Ψ_i^a : $\Psi \alpha_i \dots \alpha_i \dots \alpha_n$ is true and $\alpha_i \rightarrow \cdots \rightarrow \alpha_i$ is true,

 Ψ^a_2 : $\Psi^a_1 \ldots \alpha_i \ldots \alpha_n$ is true and $\alpha \leftrightarrow \infty$ is false,

 Ψ_1^{α} : $\Psi \alpha_1 \dots \alpha_n \dots \alpha_n$ is false and $\alpha \left\langle \frac{\alpha_1}{\cdots} \right\rangle$ is true,

 Ψ_a^a : $\Psi \alpha_1 \dots \alpha_r \dots \alpha_n$ is false and $\alpha \rightarrow \rightarrow \infty$ is false,

where $\mathbf{a} \langle \{..., \} \rangle$ occurs in $\mathbf{\Psi} \alpha_1 \dots \alpha_r \dots \alpha_n$, if $\mathbf{\hat{J}}_i \mathbf{\Psi}_1^a \mathbf{\Psi}_2^a \mathbf{\Psi}_3^a \mathbf{\Psi}_4^a$ or $\mathbf{\hat{K}} \mathbf{\hat{J}}_i \mathbf{\Psi}_1^a \mathbf{\Psi}_2^a \mathbf{\Psi}_3^a \mathbf{\Psi}_4^a$ then replace $\alpha\langle \{..., \rangle\}$ in $\Psi \alpha_1...\alpha_n...\alpha_n$ with [[]], otherwise if $\hat{J}_i N \Psi_i^a N \Psi_i^a N \Psi_i^a N \Psi_i^a$ or $\hat{\mathcal{K}}^3$ N**Y**^{*a*}₁N**Y**^{*a*}₄N**Y**^{*a*}₄ (then replace \mathbf{a} (\leftrightarrow) in \mathbf{a} \mathbf{a} ₁... α ₁... α _n with [],

step 05: let $\ddot{\Phi}_i \alpha_1 \dots \alpha_n \dots \alpha_n$ be $\ddot{\Psi} \alpha_1 \dots \alpha_n \dots \alpha_n$.

Observe that α , and $\ddot{\Phi}_1 \alpha_1 \dots \alpha_n$ have at less the same atomic terms, i. e. $\ddot{\Phi}_i \alpha_1 \dots \alpha_n \dots \alpha_n$ is the transformation of α_i by its interaction with $\alpha_1, \dots, \alpha_n, \dots, \alpha_n$ by $\ddot{\Phi}_i$.

Given a sentence $\ddot{\Phi} \alpha_1 \dots \alpha_n \dots \alpha_n$, let its *node* # $\ddot{\Phi}$ # be such an operator that if it receives the *n* inputs $\beta_1, ..., \beta_n, ..., \beta_n$, then it restores the *n* outputs $\hat{\Phi}_1 \beta_1 ... \beta_i ... \beta_n$, ... $\ddot{\Phi}_{\beta_1} \dot{\beta}_2 \dot{\beta}_3 \dot{\beta}_2 \dot{\beta}_3 \dot{\beta}_2 \dot{\beta}_3 \dot{\beta}_2 \dot{\beta}_3 \dot{\beta}_3 \dot{\beta}_3 \dot{\beta}_3 \dot{\beta}_3$

Given a dyadic node # $\ddot{\Phi}$ # with inputs α_1 , α_2 , whose outputs are obviously $\ddot{\Phi}_1 \alpha_1 \alpha_2$,

 $\oint_{\mathcal{D}} \alpha_1 \alpha_2$: denote this one by $\frac{\alpha_1}{\alpha_1} + \frac{\alpha_2}{\alpha_2} + \cdots + \frac{\alpha_n}{\alpha_1} \alpha_2}$, call α_1 *left input* and α_2 *upper*

call $\ddot{\Phi}_1 \alpha_1 \alpha$, right output and $\ddot{\Phi}_2 \alpha_1 \alpha$, lower output. Given input. also. $\frac{\alpha_2}{\pi \vec{\Phi} + \vec{\Phi} + \vec{\Phi}_1 \alpha_1 \alpha_2}$ and $\frac{\beta_2}{\beta_1 + \vec{\Psi} + \vec{\Phi}_1 \beta_1 \beta_2}$, if the right output of the former is $\frac{\vec{\Phi}_1 \alpha_1 \alpha_2}{\vec{\Phi}_1 \alpha_1 \alpha_2}$ α_{1}

the left input of the later, i. e. if $\ddot{\Phi}_1 \alpha_1 \alpha_2$ is β_1 , then we write $\frac{2}{\mu} \frac{\partial \psi}{\partial y} + \frac{\partial^2 \psi}{\partial z}$ and, if the lower output of the former is the upper # $\overline{\mathbf{\Phi}^2}$

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input of the later, i. e. if $\oint_2 \alpha_1 \alpha_2$ is β_2 , we can write $\frac{\alpha_1}{\beta_1} + \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \frac{\alpha_1 \alpha_2}{\beta_2}$. Both last

abbreviations permit us to define a net to manage theory. In fact if $v_1^1, ..., v_n^1, v_1^2, ..., v_n^2$..., V_1^n , ..., V_n^n are dyadic nodes in L and $\alpha_1 \dots \alpha_n$, $\beta_1 \dots \beta_m$, $\gamma_1 \dots \gamma_n$ and $\delta_1 \dots \delta_m$ are theories in

 $\frac{\alpha_1}{\alpha_1} \frac{v_1^1 \cdots v_n^1}{v_1^1 \cdots v_n^1}$ is a *net* with left inputs $\alpha_1 \ldots \alpha_n$, upper inputs $\beta_1 \ldots \beta_m$,
 $\frac{\alpha_n}{\beta_1} \frac{v_n^1 \cdots v_m^n}{v_n^1 \cdots v_m^1}$ is a *net* with left inputs $\alpha_1 \ldots \alpha_n$, upper inputs $\beta_1 \ldots \beta_m$

left outputs $\gamma_1 \dots \gamma_n$ and lower outputs $\delta_1 \dots \delta_m$.

Observe that this net is not an n -adic node because whereas any node output is built from all the node inputs, in our net: γ_i is only built by $\alpha_i, \beta_1, ..., \beta_m$ but not by α_2 , ..., α_n ; γ_2 is only built by α_2 , β_1 , ..., β_m but not by α_1 , α_2 , ..., α_n ; and so on. To make our net a node we must use Dubois' incursive algorithm scheme,³⁷ i. e., we must repeat the calculus of $\gamma_1, \ldots, \gamma_n, \delta_1, \ldots, \delta_m$ and after replace every time respectively any α_i and β_i with their corresponding calculated γ_i and δ_i until $\gamma_1, \ldots, \gamma_n, \delta_1, \ldots, \delta_m$ do not change more. Let α be the application of Dubois' incursive algorithm scheme to net α . Thus α is always a node. To simplify the calculus, if α is a sentence (sentence column) then let α be an *identical node* where α (any sentence of α) is together input and output.

As all the connectives can be built by Sheffer's connective (i. e. the negation of a logic product),³⁸ so all the nodes can be built by a single dyadic node: let $\mathbb{N}[\alpha]$ be the node whose outputs are the

negations of the outputs of α , let $\mathbb{K}[\alpha]$ be the node

where $\alpha_1, \ldots, \alpha_n$

are the outputs of α and $\beta_1, ..., \beta_m$ are the outputs of β , let $\mathbb{D}[\alpha]\beta$ be NIK $\alpha'\beta$: $\mathbb{D}[\alpha]\beta$ is exactly the dyadic single node which can define any node because it corresponds to Sheffer's connective.

Thus, we can build a node algebra in L which is isomorphic to sentence logic:

³⁷ See Dubois and Resconi (1992).

³⁸ See Sheffer (1913).

Science Hyperincursive Integration in L 5

Let δ_1 , ..., δ_n be the available scientific data which are translated as formalised sentence columns and let τ_1 , ..., τ_n be the current scientific theories which are also translated as formalised sentence columns. Their incursive integration in L is obviously \mathbb{K} δ on the identical nodes $|\delta_{\rm u}|$ by the previous $|\tau_{\scriptscriptstyle 1}|$ lτ. hτ.

considerations. After this integration we should have the best possible interpretation of $\delta_1, \ldots, \delta_n$ and the best possible formulation of τ_1, \ldots, τ_n .

But the continuous production of new scientific data and new scientific ideas implies a continuous modification of our science integration process from the outside environment. If we interpret the new data and ideas as parameter modification then it is clear that to integrate sciences an incursive process is not sufficient but an hypercursive process is necessary.³⁹ Let $\Rightarrow \beta[\alpha] \dots \alpha$... be the updating of α in $\ldots \alpha$... by β . $\ldots \alpha$... has the following execution:

step 01: calculate completely the incursive process $\ldots \alpha \ldots$; step 02: if there is an updating β of α then calculate completely β β α ; step 03: replace any component of α in ... α ... with the corresponding output of $\mathbb{Z}[\beta]$ α ; step 04: if changes have happened in last loop return to step 01 otherwise end.

We can easily generalise $\Rightarrow \beta \alpha \dots \alpha \dots$ in $\Rightarrow \beta_1 \alpha_1 \dots \beta_n \alpha_2 \dots \alpha_n \dots$ by consecutive repetition of step 02 for every couple β , α , and, after, by consecutive repetition of step 03 for every couple β , α . The remaining algorithm is equal.

Finally, if δ_1 , ..., δ_n are the formalised available scientific data, $\nu \delta_1$, ..., $\nu \delta_n$ are respective updating of δ_1 , ..., δ_n , also, τ_1 , ..., τ_n are the formalised current scientific theories and $v\tau_1, ..., v\tau_n$ are respective updating of $\tau_1, ..., \tau_n$, then their best integration in L is:

$$
\stackrel{2a}{\Longrightarrow}\negthinspace\negthinspace\bullet\negthinspace\delta_1\stackrel{.}{\otimes_1}\negthinspace\cdot\ldots\negthinspace\bullet\negthinspace\bullet\negthinspace\delta_a\stackrel{.}{\otimes_2}\negthinspace\negthinspace\bullet\negthinspace\tau,\negthinspace\negthinspace\tau_1\dots\negthinspace\bullet\negthinspace\tau_{a}\stackrel{.}{\tau_{a}}\negthinspace\cdot\ldots\negthinspace\bullet\negthinspace\bullet\negthinspace\cdot\negthinspace\delta_{a}\negthinspace\cdot\ldots\negthinspace\bullet\negthinspace\tau_{a}.
$$

³⁹ To define incursive and hyperincursive processes Dubois write: 'The recursion consists of the computation of the future value of the variable vector $X(t+1)$ at time $t+1$ from the values of these variables at present and/or past times, t, t-1,t-2, ... by a recursive function: $X(t+1) = f(X(t), X(t-1),..., p)$ where p is a command parameter vector. So, the past always determines the future, the present being the separation line between the past and the future. ... Starting from cellular automata the concept of fractal machines was proposed in which composition rules where propagated along paths in the machine frame. The computation is based on what I called 'INclusive reCURSION', i. e. INCURSION ... An incursive relation is defined by: $X(t+1) = f(..., X(t+1), X(t), X(t-1), ..., p)$ which consists in the computation of the values of the vector $X(t+1)$ at time $t+1$ from the values $X(t-i)$ at time $t-i$, $i = 1,2,...$ as a function of a command vector p. This incursive relation is not trivial because future values of the variable vector at time steps $t+1$, $t+2,...$ must be known to compute them at the time step $t+1...$ In a similar way to that in which we define hyper recursion when each recursive step generates multiple solutions, I define HYPERINCURSION. ... I have decided to do this for three reasons. First, in relativity theory space and time are considered as a four-vector where time plays a role similar to space. If time t is replaced by space s in the above definition of incursion, we obtain $X(s+1) = f(..., X(s+1), X(s), X(s-1), ..., p)$ and nobody is astonished – a laplacean operator looks like this. Second, in control theory, the engineers control engineering systems by defining goals in the future to compute their present state, similarly to our human anticipative behaviour. ... Third, I wanted to try to do a generalisation of the recursive and sequential Turing machine in looking at space-time cellular automata where the order in which the computations are made is taken into account with an inclusive recursion'. See Dubois (1997).

5 Conclusions

The presented paper is only a first attempt to obtain an automatic exchange of ideas and data among sciences which often have no contact among them.

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