A Mathematical Theory of Dynamic Systems Built on Differential Calculus in Semi-Normed Spaces

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Abstract
The paper contains several results belonging to the theory of systems with infinite memory, using the differential calculus in locally convex spaces. These results are the following: the general constitutive functional can have, as a first approximation, an integral representation; the constitutive functional could be expressed by a double integral, so obtaining a better approximation; the speed of the present state modification is in a linear dependence on the history of the speed by which the inputs were changed, the whole time elapsed till the present moment; the state of the system in a next moment is obtained also by means of some formulae using the derivative of the constitutive functional; the problem of optimal control of the system evolution, formulated for this general functional representation, leads to the equations of the Calculus of Variations.

Key words: input history, Fréchet-Marinescu differential, process, optimal control.

1 Introduction

From the very beginning, the general theory of systems with fading memory achieved an integrating expression for the most diverse phenomena belonging to some domains of cognition that seemed, under all detectable aspects, disjunctive. The strong nature of mathematics with regard to the power of abstracting and of creating models has been proved once again in the theory of systems with infinite fading memory. Mathematics has not been enriched much with new elements since the apparition of the general systems theory, except more definitions for the concept of system; nevertheless the well-established mathematical notions, such as function, equation, matrix, sum, derivative, integral, vector space, and so on, have become common assets of other scientific disciplines.

Our research object was the dynamic system with infinite fading memory. For such a system, its initial state could be considered at the moment $(-\infty)$.

The results hereby presented, the most of them I already published (Manzatu, 1983; Otlacan, 2000, 2004) have two ideas as a starting point:

1.The causes of a phenomenon unfold in time, so they can be functions of one or more real variables. The effects numerically expressed at a certain moment are values of some real or vector functionals defined on sets of functions that describe the causes. In brief, the relationship cause – effect is one of functional type.

2.In the general study of phenomena we discuss about greater or smaller effects, close or remote causes, the speed of modification of effects or causes. From the mathematical point of view, we have to establish the contiguity, to fix the topology on the set of the functions considered as causes, and adopt an adequate definition for the functional derivative.

I have obtained a satisfactory answer to these questions and important results for the theory of systems with infinite fading memory, by imagining a locally convex linear topological space for the set of functions that describe in time the causes of a phenomenon that is presented numerically at a given moment. The topology in this space is defined by semi-norms linked to this given moment. The differential calculus for functionals defined on locally convex spaces that I used was created by Gheorghe Marinescu (1963), as a generalization of the Fréchet's differential on normed spaces, and uses the so called Fréchet-Marinescu's differential, a notion that will be presented below.

2 A pattern of the relationship between the input history and the state of a dynamic system with infinite fading memory

Let **S** be a dynamic system, $x(t)$ one of its state parameter $x_1(t)$, $x_2(t)$, ..., $x_n(t)$, at the present moment t, t_0 an initial moment, $t_0 < t$, and the real m-dimensional vector function $u_{[t_0,t]}(\tau)$ defined for $\tau \in [t_0,t]$, describing the inputs which acted upon the system **S** in the time interval $[t_0,t]$. The correspondence from the function $u_{[t_0,t]}(\tau)$ to $x(t)$ is given by a *state transition function* Φ, according to Kalman, Falb and Arbib (1969). In fact, this function is a functional depending on the function $u_{[t_0,t]}(\tau)$ and also on t_0, t, $x_i(t_0)$, $i=1,2,\ldots,n$, as $n+2$ real parameters:

$$x(t) = \Phi[t_0, t, x_1(t_0), x_2(t_0), \ldots, x_n(t_0); u_{[t_0,t]}(\tau)] \tag{2.1}$$

For a system with infinite fading memory the initial moment could be at $-\infty$ and so the inputs are described by a vector function u: $(-\infty,t] \to \mathbf{R}^m$, $u = u(\tau)$, and called *the global history of inputs* (*input history*). Introducing the constitutive functional F, the relationship will become:

$$x(t) = F\{u(\tau); \tau \in (-\infty, t]\} \tag{2.2}$$

More suggestive is the following writing:

$$x(t) = \underset{\tau=-\infty}{\overset{t}{F}} [u(\tau)] \tag{2.3}$$

Sometimes a variable s with an opposite sense from the variable time τ is more convenient, so we also introduce the following notations instead of $u(\tau)$, $\tau \in (-\infty,t]$:

$$u^t(s) = u(t-s),\ s = t-\tau,\ s\in[0,+\infty) \tag{2.4}$$

Thus the constitutive equation written for $x(t)$ will be:

$$x(t) = \mathop{F}_{s=0}^{\infty}[u^t(s)] \tag{2.5}$$

For simplicity, sometimes we will write $F[u]$ or $F[u^t]$.

The object of the mathematical modelling in this paper is given by the following definition:

DEFINITION 2.1. A *system with infinite fading memory* is a system for which the relationship between the global input history $u(\tau), \tau\in(-\infty,t]$, and any parameter $x(t)$ of its state at the present moment t is given by one of the equivalent formulae (2.3) or (2.5), where the values of $u(\tau)$ taken on $\tau < t$, but τ close to the moment t, bring a much more important contribution to the value $x(t)$ than those values of $u(\tau)$ taken for τ in intervals of time very remote of t, that is when $\tau\to-\infty$.

In other words, if $u(\tau)$ had modifications for $\tau < t-\mu_\varepsilon$, $\mu_\varepsilon > 0$ a large enough number, then these modifications are insignificant for the value $x(t)$, being less than an arbitrary little number $\varepsilon>0$.

To have concrete results for the mathematical modelling of systems with infinite fading memory, it is necessary to impose:

1) a topology of the set Ω^t of the functions $u(\tau)$ admissible as input histories of the system **S**;
2) properties of the constitutive functional F.

DEFINITION 2.2. The *space of admissible input histories till the present moment t* is a linear topological locally convex space Ω^t of vector functions $u(\tau) = (u_1(\tau), u_2(\tau), \ldots, u_m(\tau))$, for $\tau\in(-\infty,t]$, respectively $u^t(s)$ for $s\in[0,+\infty)$, $u^t(s) = (u_1^t(s), u_2^t(s), \ldots, u_m^t(s))$, functions with any order derivatives, the topology of Ω^t being given by a family of semi-norms linked by the moment t:

$$|u|_\lambda = \sup_{[t-\lambda,t]} |u(\tau)|,\ \text{where}\ |u(\tau)| = \left[\sum_{k=1}^{m} u_k^2(\tau)\right]^{1/2} \tag{2.6}$$

respectively

$$\left|u^t\right|_\lambda = \sup_{[0,\lambda]}\left|u^t(s)\right|,\ \text{where}\ \left|u^t(s)\right| = \left[\sum_{k=1}^{m} u_k^2(t-s)\right]^{1/2} \tag{2.7}$$

In consequence, a neighbourhood around the function $u(\tau)$ is formed by all the functions $v(\tau)$ belonging to Ω^t and meeting the condition $|v-u|_\lambda < \delta$ for certain numbers $\lambda>0$, $\delta>0$.

A first result for the modelling of systems with infinite fading memory is given with the hypotheses of the continuity of the constitutive functional:

THEOREM 2.1. If the constitutive functional F of a system **S** is *defined and continuous* on the locally convex space Ω^t to which the input history $u(\tau)$ belongs, then the correspondence given by the formula (2.3) will describe the present state of a system with infinite fading memory.

Indeed, the continuity of F means that if $v(\tau)$ is another function different of $u(\tau)$, the difference between $F[u(\tau)]$ and $F[v(\tau)]$ will be inferior to an arbitrary small number $\varepsilon>0$, if $|u(\tau)-v(\tau)|$, for all $\tau\in[t-\lambda_\varepsilon, t]$, is smaller than a number $\delta_\varepsilon>0$, λ_ε and δ_ε depending on ε. But when $\tau\in(-\infty, t-\lambda_\varepsilon)$ the difference $|u(\tau)-v(\tau)|$ could be no matter how large *without affecting* too much the value of the functional F, respectively the value $x(t)$.

3 Fréchet-Marinescu differentiability hypothesis of the constitutive functional and its consequences

More than the continuity, we need the differentiability of the constitutive functional F. For that we resorted to the following definition:

DEFINITION 3.1. The real functional F, defined on the locally convex space Ω^t endowed with the family of semi-norms expressed by the formulae (2.6) or (2.7), is differentiable in Fréchet-Marinescu's sense in a point $u\in\Omega^t$ if there exist: a number $\lambda>0$ and a linear functional δF_u, depending on the point u, so as the following two equalities hold:

$$\underset{\tau=-\infty}{\overset{t}{F}}[u(\tau)+h(\tau)]-\underset{\tau=-\infty}{\overset{t}{F}}[u(\tau)]=\underset{\tau=-\infty}{\overset{t}{\delta F_u}}[h(\tau)]+\omega(u;h) \tag{3.1}$$

$$\lim_{h\to 0}\frac{|\omega(u;h)|}{|h|_\lambda}=0 \tag{3.2}$$

As consequences of the differentiability of the constitutive functional, we found more results that can be interpreted in connection with the modelling of dynamic systems.

A very important theorem asserts the possibility to have an integral expression of the differential δF_u of the constitutive functional.

THEOREM 3.1 (of integral representation): If the constitutive functional is differentiable in Fréchet-Marinescu's sense in the point $u\in\Omega^t$ and δF_u is its differential, that meets together with the number $\lambda>0$ the conditions (3.1), (3.2), then m real functions $a_1(\tau), a_2(\tau), \ldots, a_m(\tau)$, with integrable squares on $[t-\lambda,t]$ will exist, so that:

$$\underset{\tau=-\infty}{\overset{t}{\delta F_u}}[h(\tau)]=\int_{t-\lambda}^{t}\sum_{k=1}^{m}a_k(\tau)h_k^{'}(\tau)d\tau \tag{3.3}$$

Here $h(\tau)=(h_1(\tau),h_2(\tau), \ldots, h_m(\tau))$, $h_k^{'}(\tau)$, $k= 1,2,\ldots, m$, are the first order derivatives of the functions $h_k(\tau)$ and $h(\tau)=0$ for $\tau< t-\lambda$; the functions $a_k(\tau)$ depend on the input history $u(\tau), \tau< t-\lambda$ and do not depend on $h(\tau)$.

I have published a demonstration of this theorem (Otlacan, 2004).

An immediate consequence of this theorem is the expression of the difference between two values of the constitutive functional F:

$$\underset{\tau=-\infty}{\overset{t}{F}} [u(\tau)+h(\tau)] - \underset{\tau=-\infty}{\overset{t}{F}} [u(\tau)] = \int_{t-\lambda}^{t} \sum_{k=1}^{m} a_k(\tau)h_k^{'}(\tau)d\tau + \omega(u;h) \tag{3.4}$$

$\omega(u;h)$ has the property (3.2), namely its absolute value is smaller than the semi-norm $|h|_\lambda$ and this will tend to zero when h tends to the function zero, faster than $|h|_\lambda$.

Setting aside this term $\omega(u;h)$, the difference between the two values remains as an integral formula. We can interpret this result in relation with the state of the system. Let $v(\tau)=u(\tau)+h(\tau)$, $\tau\leq t$, the input history till the moment t. As $h(\tau)=0$ for $\tau\leq t-\lambda$, we have $v(\tau)=u(\tau)$ for $\tau\leq t-\lambda$, and so the value of $F[u(\tau)]$ gives the state parameter at the moment $t-\lambda$, $F[u(\tau)+h(\tau)]$ giving this state parameter at the moment t. We have the formula of the respective state parameter *evolution*:

$$x(t) - x(t-\lambda) = \int_{t-\lambda}^{t} \sum_{k=1}^{m} a_k(\tau)h_k^{'}(\tau)d\tau + \omega(u;h) \tag{3.5}$$

Only renouncing to the term $\omega(u;h)$, that depends both on the whole input history $u(\tau), \tau\in(-\infty,t)$, and on the input $h(\tau)$ that acted on the time interval $[t-\lambda,t]$, we have a simple integral. The weight functions $a_k(\tau)$ also depend on the whole input history $u(\tau), \tau\in(-\infty,t-\lambda)$, and they can be established only experimentally. Theoretically, we notice that the infinite memory of the system is implicitly expressed by these functions. Taking into account the mentioned dependence, we can write the functions $a_k(\tau)$ as values of certain functionals:

$$a_k(\tau) = \underset{\theta=-\infty}{\overset{t-\lambda}{A}} [\tau, x(t-\lambda), u(\theta)] \tag{3.6}$$

Differentiability properties of functionals with the use of integral theorem 3.1 lead us to an integral formula expressing the evolution of the state parameter x. The equality which follows is an approximation only, because we will renounce to the complementary term of the differential definition, that is $\omega(u;h)$ from (3.1), (3.5) and a similar term of the differential formula of the functional A from (3.6). In a new formula we used an interval of time $[t-\mu, t-\lambda]$ prior to $t-\lambda$. The formula emphasizes the synergy of the system (Otlacan, 2004), the multiplication of inputs that are coming from different paths and on different periods of time. The new formula is of the following type:

$$x(t)-x(t-\lambda)\cong \sum_{k=1}^{m} a_k(t)h_k(t)+\sum_{k=1}^{m}\int_{t-\lambda}^{t} q_k(\tau)h_k(\tau)d\tau+ \\ +\sum_{j=1}^{m}\sum_{k=1}^{m}\int_{t-\lambda}^{t} d\tau\int_{t-\mu}^{t-\lambda} p_{jk}(\tau,\theta)e_j(\theta)h_k(\tau)d\theta \tag{3.7}$$

Here $\mu>\lambda$, $h_k(\tau)$ are the input functions that acted on the interval $(t-\lambda,t)$, and $e_j(\theta)$ represent the input functions on the interval of time $(t-\mu,\ t-\lambda)$, for $k,j=1,2,\dots,m$.

We have to notice that the functions a_k, q_k, p_{jk} are not determined mathematically, they result from a theorem of existence.

4 The derivative of the state vector; formulae for the future state

The formulae (3.5) and (3.7) could be used to predict the value of the state parameter x at a moment $t+\alpha$. An approximate formula deduced from (3.7) replacing t by $t+\alpha$, $0<\alpha=\lambda<\mu$, is the following:

$$x(t+\alpha)-x(t)\cong \sum_{k=1}^{m} a_k(t+\alpha)h_k(t+\alpha)+\sum_{k=1}^{m}\int_{t}^{t+\alpha} q_k(\tau)h_k(\tau)d\tau+ \\ +\sum_{j=1}^{m}\sum_{k=1}^{m}\int_{t}^{t+\alpha} d\tau\int_{t-(\mu-\alpha)}^{t} p_{jk}(\tau,\theta)e_j(\theta)h_k(\tau)d\theta \tag{4.1}$$

To use this formula we must impose the inputs $h_k(\tau)$ on the time interval $[t,\ t+\alpha]$ and know the past input history $e_k(\theta)=h_k(\theta)$ on an interval $(t-\mu+\alpha,\ t)$ prior to the present moment. The weight functions q_k, p_{jk} are established by a past experience. It is necessary that the inputs $h_k(\tau)$ and $h_k(\theta)$ should not have too great values so that the error may not be too big.

A part of these inconveniences are removed using a formula based on the theorem of the derivative of the state-function of the system with infinite fading memory.

THEOREM 4.1. Let the function $x(\theta)$ be the value at the moment $\theta\le t$ of a state parameter:

$$x(\theta)=\underset{\tau=-\infty}{\overset{\theta}{F}}[u(\tau)] \tag{4.2}$$

If the constitutive functional F: $\Omega^t\rightarrow\mathbf{R}$ is a differentiable functional in Fréchet-Marinescu's sense on Ω^t and its derivative δF_u: $\Omega^t\rightarrow L(\Omega^t,\mathbf{R})$ is also a continuous functional, then the function $x(\theta)$ defined on each $\theta\le t$ by the formula 4.2 admits a left derivative in the point t and the value of this derivative will be given by a value the derivative δF_u:

$$x'(t) = -\underset{\tau=-\infty}{\overset{t}{\delta F_u}}\left[\frac{du}{d\tau}\right] \quad (4.3)$$

I have given and published a demonstration of this theorem (Manzatu, 1983).

With the theorem 3.1 of integral representation for the functional δF_u and adapting the formula (3.3), we will have, with $\lambda > 0$:

$$x'(t) = -\int_{t-\lambda}^{t} \sum_{k=1}^{m} b_k(\tau) h_k''(\tau) d\tau \quad (4.4)$$

Here $h''(\tau)$ is the second derivative of $h(\tau)$.

Based on this formula and on Taylor's formula, with $\alpha > 0$ and $t+\alpha$ a moment in the future, we obtain the following, also an approximate formula, because the rest of Taylor's formula was put aside:

$$x(t+\alpha) \cong x(t) - \alpha \int_{t-\lambda}^{t} \sum_{k=1}^{m} b_k(\tau) h_k''(\tau) d\tau \quad (4.5)$$

The vector function $h''(\tau)$ may be considered the acceleration of the input $h(\tau)$.

We could obtain other formulae in conditions of derivability of the weight functions $b_k(\tau)$. Let us remember that $h(\tau)=0$ for $\tau \le t-\lambda$ and we will have the formulae:

$$x(t+\alpha) \cong x(t) - \alpha \sum_{k=1}^{m} b_k(t) h'(t) + \alpha \int_{t-\lambda}^{t} \sum_{k=1}^{m} b_k'(\tau) h_k'(\tau) d\tau \quad (4.6)$$

$$x(t+\alpha) \cong x(t) - \alpha \sum_{k=1}^{m} [b_k(t) h'(t) - b_k'(t) h(t)] - \alpha \int_{t-\lambda}^{t} \sum_{k=1}^{m} b_k''(\tau) h_k(\tau) d\tau \quad (4.7)$$

The advantage of the formulae (4.5) – (4.7), as compared to the formula (4.1), is the fact that in these formulae the knowledge of input functions $h_k(\tau)$ refers to the past time, prior to the present moment. Besides, the step $\alpha > 0$ does not depend on λ, this number imposed by the differentiability condition 3.1.

5 The problem of optimal control; relation with Calculus of variations

Let us consider again the defining correspondence (2.5).

More convenient is now to refer to the vector function $h(\tau) = (h_1(\tau), h_2(\tau), \ldots, h_m(\tau))$ belonging to Ω^t, defined on $(-\infty, t]$ and null before the moment $t-\lambda$, as a *process* π that took place in the system between the moments $t-\lambda$ and t. This function-process is added to the input history $u(\tau), \tau \leq t-\lambda = t_1$, transforming this history into the input history till the moment $t_2 = t$, $u(\tau), \tau \leq t$; I wrote (Otlacan 2005) $\pi(s)$ instead of $h(\tau)$ and defined this function on an interval $[t_1,t_2]$ as a difference:

$$\pi^{[t_1,t_2]}(s) = u^{t_2}(s) - u^{t_1}(s) \tag{5.1}$$

so that:

$$u^t(s) = u^{t-\lambda}(s) + \pi^{[t-\lambda,t]}(s) \tag{5.2}$$

In a problem of optimum leadership the introduction of commands in a system on an interval of time $[t-\lambda,t]$ presupposes an efficiency criterion to be satisfied, many times this signifying to obtain a minimum (or a maximum) of a real functional that could represent the costs (or benefits) of the system work. This functional depends on the reference (initial) state $x(t-\lambda)$, by this understanding all the n state parameters, and on the process π on the interval $[t-\lambda,t]$. To fix the ideas, let us take the cost of the system as the value of a functional L:

$$e(t, x(t-\lambda)) = \underset{\tau=t-\lambda}{\overset{t}{L}} [x(t-\lambda), \pi^{[t-\lambda,t]}(\tau)] \tag{5.3}$$

For the dynamical system with infinite fading memory we introduced a new functional, called the *total cost functional*, that has a theoretical interest.

DEFINITION 5.1. The *total cost* of a system with infinite fading memory **S** till the moment t is the value $E(t)$ of a real continuous functional $L : \Omega^t \to \mathbf{R}$, named the *total cost functional*, that will permit to express the system costs on different finite intervals of time:

$$E(t) = \underset{s=0}{\overset{\infty}{L}} [u^t(s)] \tag{5.4}$$

The argument $u^t(s)$ is the history of inputs which realized the state $x(t)$ at the moment t.

DEFINITION 5.2. The *cost* $E(t_1,t_2)$ of the system S on an interval of time $[t_1,t_2]$, $t_1 < t_2 \leq t$, is the difference between the two total costs:

$$E(t_1, t_2) = E(t_2) - E(t_1) \tag{5.5}$$

The general formula for this cost is the following difference:

$$E(t_1,t_2) = \mathop{L}_{s=0}^{\infty}[u^{t_2}(s)] - \mathop{L}_{s=0}^{\infty}[u^{t_1}(s)] \tag{5.6}$$

The relationship between the two functions $u^{t_2}(s)$ and $u^{t_1}(s)$ is expressed by means of the process π:

$$u^{t_2}(s) - u^{t_1}(s) = \pi^{[t_1,t_2]}(s), \text{ and } \pi^{[t_1,t_2]}(s) = 0 \text{ when } s \geq t_2 - t_1 \tag{5.7}$$

Within the theory of optimal control, usually, the study of the optimum evolution of a continuous system is given by a criterion of integral type:

$$J[u] = \int_{t_1}^{t_2} G(t; x(t), u(t))dt, \tag{5.8}$$

while the relationship from the input $u(t)$ to the system state is described by the differential systems:

$$\frac{dx}{dt} = g(x(t); u(t)) \tag{5.9}$$

Classically the problem of optimal control has the following formulation: to determine a sequence of command $u(t)$, with $t \in [t_1,t_2]$ and a trajectory $x(t)$ corresponding to the system (5.9) so that $J[u]$ takes the minimum (maximum) value and the values $x(t_1)$, $x(t_2)$ be those we impose.

For the beginning, *our formulation of the problem of optimal control applied to a system with infinite fading memory* described by a constitutive functional F from the input history $u^t(s)$, $s \in [0,\infty)$, to the state $x(t)$, with a total cost functional L having the value $E(t)$ in the same input history (relation 5.4), is the following:

With the given moments t_0 and t, $t_0 < t$, to determine the process $\pi^{[t_0,t]}(s)$, that is the function $\pi^{[t_0,t]}(s) = u^t(s) - u^{t_0}(s)$, equal to zero for $s \geq t - t_0$, so that the cost $e(t; x(t_0))$ should be minimum.

We proved that this formulation, available for general systems with fading memory, could lead us to the classical formulation of optimal control, namely the formulae (5.8), (5.9). Two statements were to be demonstrated:

I. The system S with infinite fading memory may be described by a differential system comparable with (5.9).

II. The functional of the costs of the system may have, in a first approximation, an integral expression, similar to (5.8).

Based on the results presented before, the following theorem is true:

THEOREM 5.1. If the total cost functional L accomplishes the same conditions of differentiability as the constitutive functional F, and if the magnitude of the process $\pi^{[t-\lambda,t]}(s)$ is sufficiently small on the interval $[0, \lambda]$, i.e. the command $u(t)$, $\tau \in [t-\lambda,t]$, has a small magnitude (formula 2.6 for $|u(\tau)|$) then the cost of the system in this interval of time will have approximately the integral expression:

$$E(t-\lambda,t) \cong \int_0^{\lambda} \sum_{j=1}^{m} g_j(s)\frac{du_j(s)}{ds}ds, \tag{5.10}$$

the functions $g_j(s) = G[u^{t-\lambda}(s);s]$ being certain functionals on the history of inputs till the moment $t-\lambda$.

The theorem is an approximate representation of the cost on the interval $[t-\lambda, t]$, but it explains the successful use of the integral (5.8) in the problem of optimal control.

We give here the following formulation of the optimal control:

To determine the process $\pi = (\pi_1, \pi_2, \ldots, \pi_m)$ on the interval of time $[t-\lambda, t]$ so that it has to:

**give the minimum value of the integral of cost:*

$$E(t-\lambda,t) = \int_0^{\lambda} \sum_{j=1}^{m} g_j(s)\frac{d\pi_j(s)}{ds}ds \tag{5.11}$$

*** accomplish the conditions $\pi(\lambda)=0$, $\pi(0)=u(t)-u(t-\lambda)$, with $u(t-\lambda)$ known and $u(t)$ imposed (therefore also known),*

**** realize imposed values to the derivative $\dot{x}(t)$ of the parameter state, expressed by their dependence on π_j in the following formula:*

$$\left.\frac{dx}{d\theta}\right|_{t=t} = \int_0^{\lambda} \sum_{j=1}^{m} f_j(s)\frac{d^2\pi_j(s)}{ds^2}ds \tag{5.12}$$

We observe that this is an isoperimetric problem with n integral links.

This way, from the general model of the correspondence between the input history and the present state of a dynamic system, we arrived to the Calculus of variations and could use Euler-Lagrange theory.

6 Conclusions

The differential calculus in semi-normed spaces is more adequate to deduce mathematical models of the behavior of dynamic systems, taking into account the infinite memory of these systems.

Obviously, the results here presented have a theoretical importance, but they constitute a mathematical accreditation of a lot of formulae which are used in many and diverse domains, from thermodynamics to biology. So, in the mechanics of the deformable medium every material, except the perfectly elastic ones, has memory, that means that the tensor of tensions $x(t)$ depends not only on the deformation gradient at the moment t and on the current temperature, but also on the whole previous history of the deformations and temperature. The functions $u_k(\tau)$, $k=1,2,\ldots,6$, represent the components of the function-gradient of the deformations, $u_7(\tau)$ being the history of the temperature variation. If a formula of (3.5) type (without ω) is frequently met in works about visco-elasticity and thermo-visco-elasticity, the formula (4.4) could be interpreted as the derivative of the tensor of tensions and could be used to estimate the speed of tension change in a point of a material.

In biology, the functions $u_k(\tau)$, $\tau\in(-\infty,t]$, are the history of stimuli which acted till the moment t, $x(t)$ is the answer in the special nervous fibre of a living organism at the moment t. In this field, the interpretation of the formula (4.4) of the derivative is rather surprising: the modification of the speed of every possible answer $x(t)$ to the stimulus-function $u(\tau)$, $\tau\in(-\infty,t]$, is linearly dependent on the derivative $\dot{u}(\tau)$, namely on the speed of the input modifications, but the same derivative $\dot{x}(t)$ is in a non-linear dependence on the function $u(\tau)$.

Besides the interpretations in thermodynamics and biology, the mathematical theory hereby exposed provided coherent explanations in the theory of observation operators (Otlacan, 2000).

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