Toward the Investigation of Vorticity Development and Fragmentation of Structures in 3D Media with the Memory Effects

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Abstract
The new model equations for hydrodynamics with memory effects are investigated. The possibilities of collapses, structures and fragmentation are discussed. The results of numerical evolution of the torus as the initial conditions are presented. The process of vorticity development and fragmentation at high speed of flow is displayed.
Keywords: Hydrodynamics, Vortices, Memory effects, Collapses

1. Introduction

The collapses origin problem in hydrodynamics has a long history since XVIII century. It is known that this problem is very complicated. Collapses origin is also closely connected to the considering the energy flows between different scales in turbulence. One of the tools for considering such problems is mathematical modelling, that is investigation of physical phenomena by models. The common models for hydrodynamics are Navier- Stokes, Burgers, non- viscid flows, Helmholtz equation for vortices and others (Frish, 1995; Danilenko et al, 1992; Ladyzhenskaya, 1996; Makarenko, 2000). But general problem still is open. One way for searching solution of general problem consists in considering more and more correct equations. Remark that in the case of smallest space scales the primary dynamical equations for particles is useful. But for intermediate scales the hydrodynamics equations with memory and non-locality accounting is intrinsic (see reviews of literature in (Joseph&Prezuosi, 1980, Mory, 1958; Danilenko et al, 1992; Makarenko, 2000). One of the most known are viscid-elastic equations. Just as say Navier- Stokes equations, complete equations with memory and non-locality effects are very complex object for mathematical and numerical investigations. This follows to the necessity of more simple models. The examples of such simple classical models are Burgers equations, Lorentz equations, Korteveg- de Vriz equations and many others. For equations with the memory effects earlier it had been proposed hyperbolic modification of Burgers equation and low-dimensional counterpart for Lorentz equations.

Some results with such equations had been described in (Makarenko, 1996; 2000). In one- dimensional case it was found the fragmentation of solutions and origin of collapses (blow- up regimes) in the case of high velocity of flow. So it is next natural step to investigate the collapses and structures origin in 2D and 3D case. In this paper

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we propose such modal equations for 2D and 3D cases and results of numerical modelling of collapses origin in model equations with memory effects.

Till now the system of Navier-Stokes equations is one of the basic tools for description the hydrodynamic systems. Up to this time still nobody succeeded in integrating it in the general case. Nevertheless, it is widely used for practical computations of the real flows of liquids.

However in force of limitations and suppositions, which were stopped up at its construction, this system takes into account many physical phenomena which are not practically noticeable at small speeds of flow, but which have a substantial influence at consideration of fast processes. So in the real gases and liquids the turbulent phenomena and research of processes, that take place on hyper-sound speeds, the effects of relaxation, heat conductivity, molecular dissociation and ionization begin to play important role. For adequate description a researcher must try to take into account many processes. Remark that in turbulence theory some difficult problems exist: vorticity development in the fast flows without external forces, energy transfer in flows from large scale to smaller, fragmentation and collapses and others. But because of the complexity of original hydrodynamic equations the exploiting of model equations is necessary. Thus the task of introducing and investigation of new classes of equations, which would give adequate description tc the above - mentioned processes get up.

In proposed paper we describe the results of numerical simulation of one such type of model equations for hydrodynamics with memory effects in three – dimensional case. Evolution of torus in flow had been the main objects of investigations.

At subsequent sections we briefly describe the model equations, posing the problem with the torus in the laminar flow as the initial condition and propose the results on numerical simulations of such problems.

2. Model Equations and Some Results of Previous Investigations

The basic equations for heat transfer and hydrodynamics are usually parabolic heat equation and the Navier-Stokes hydrodynamic equations. But it is known very well that these equations lose their applicability in extended media when characteristic scales of parameters change are less then correlation time and correlation length (relaxation or memory and non-locality effects). Especially many examples of non-applicability were found in turbulence. Thus more correct equations should be applied in such cases.

There are some well-established facts in theoretical physics on the description of transport processes. The first famous idea is the existence of hierarchy of description levels. The choice of the level of description depends on the degree of deviation from equilibrium.

The second background idea is the existence of many interrelating relaxation processes and many time and space scales with different relaxation times and lengths. The memory and non-locality effects are common for all levels. The turbulence is a bright example of such complex phenomena. It should be stressed that each level of description has its own model equations with typical behavior of solution.

There are many phenomenological, experimental and mathematical models for hydrodynamic processes. Theoretical physics can give adequate understanding of transport phenomena in different media under different conditions. So we very briefly describe some main concepts from statistical physics relevant to modeling equations choice. Statistical physics considers the ensemble of system by introducing distribution functions for particles distribution probability at time t.

These stages with distribution functions named kinetic. Further averaging with one-particle distribution function leads to hydrodynamic equations for macro-parameters T (temperature), U (speed), and P (pressure). Usual procedures lead to well-known equations of hydrodynamic type: parabolic heat equation, Navier-Stokes equations and so on. But more correct description leads to more adequate equations with memory effects accounting.

The reason of memory effects origin under reduction processes is very well described in theoretical physics since the works of Mory, Zwanzig, Picirelly, Zubarev and many others, see review in (Mory, 1958; Picirelli, 1968; Danilenko et al, 1992; Makarenko, 1996, Makarenko et al, 1997). In such approach the hydrodynamic equations for macro-parameters with some constitution equations relating macro variables are received.

In general such constitution equations have the form of integral-differential equations. The kernels in such equations correspond to the accounting the memory and non-locality effects. Only in the case when the kernel is δ-function we receive usual heat conduction equation. The more realistic kernel for fast processes is of exponential type $K=C\exp(-t/\tau)$ where τ is the relaxation time. Then we receive the so-called hyperbolic heat conduction equation well known since Maxwell, Cattaneo, Vernotte, Lykov. Also there exists hyperbolic counterpart of the Navier- Stokes equations. Remark that integral-differential equations with such kernels may be interesting for considering anticipatory phenomena.

It should be stressed that in general it is known from the theoretical physics the precise abstract equations for different hierarchical levels of description. Usually theoretical physicists explore these very complex abstract equations (frequently qualitatively). But on hydrodynamic level it is especially interesting to search visible macro-effects. In such a case especially useful is the consideration of model equations. Thus as the Navier-Stokes equations as their hyperbolic counterpart are the model equations. In the next sections of the paper we describe the results on investigations of some new model equations with memory accounting proposed by author.

Accounting the memory effects leads to the new equations and new possible solutions. The simplest new equation is the one-dimensional hyperbolic modification of Burgers equation. The two and three-dimensional hyperbolic systems improving the Navier-Stokes equations was introduced by one of the authors. When the relaxation time is small remarked equations is the singular perturbation of the parabolic counterpart. It is important that new equations have more solution types then usual. One distinctive feature is that in a case of the large flow velocities the equations with memory accounting allow collapses (blow-up) in solutions (Frish, 1995; Samarsky et al, 1995). Remember that an existing of blow–up solutions in the Navier- Stokes systems is still under the question now.

2.1. Hyperbolic Hydrodynamics with Memory Effects.

As was mentioned more correct then the Navier-Stokes equations should be applied in such a case. Cattaneo, Vernotte, Joseph and many others had investigated more correct hyperbolic equations for heat conduction and for the thermoelasticity (see also (Danilenko et al, 1992; Makarenko, 1996; Makarenko et al, 1997)). The accounting of the memory effects also follows to the origin of new hydrodynamic models. The simplest modification of the Navier-Stokes equations is the Maxwell media equations, which take into account relaxation of tensions (memory effects), see for example equations form (Oskolkov, 1983):

$$\frac{\partial \bar{V}}{\partial t}+V_k\frac{\partial \bar{V}}{\partial x_k}+\tau\left(\frac{\partial^2 \bar{V}}{\partial t^2}+\frac{\partial V_k}{\partial t}\frac{\partial \bar{V}}{\partial x_k}+V_k\frac{\partial^2 \bar{V}}{\partial t \partial x}\right)-\nu\Delta\bar{V}=-(1+\tau\frac{\partial}{\partial t})gradP+F;$$

$$div\bar{V}=0; \qquad (1)$$

$$k=1,2,3;$$

where $\bar{V}$ – vector field of speed, P,F - pressure and force, τ – relaxation time, ν - viscosity.

Obviously, that as numerical as theoretical analysis of such system are very bulky and not trivial. Therefore there is the natural desire to simplify the system, saving the characteristic features of its conduct. In this case the so-called model equation, which actually represent the features of the initial system begins become in the case.

Just as for the classic Navier – Stokes system the Burgers equation is a model, for the system (1) we will have its modification taking into account relaxation processes in an environment:

$$\frac{\partial \bar{V}}{\partial t}+V_k\frac{\partial \bar{V}}{\partial x_k}+\tau\left(\frac{\partial^2 \bar{V}}{\partial t^2}\right)-\nu\Delta\bar{V}=0; \qquad (2)$$

where τ – relaxation time, ν - viscosity, $\bar{V}$ – vector field of velocity in R^n, $k=1..n$, $n=3$ – dimension of space in problem.

Details of the investigations of this equation in 1D and 2D space cases were conducted in the papers by A.Makarenko (1996, 2000). In this cases some results had been found:

1. possibility of existence of solutions that can grow to infinity for the finite time for the 1D and 2D cases.

2. possibility of growing of characteristic scales number and diminishing characteristic length
3. for the conduct of the system a parameter is determining

$$M = \frac{\max\{|U_0(x)|\}}{C}$$

where

$$C = \sqrt{\nu / \tau}$$

In the case of M < 1 initial disturbances of the flow disappear in time, at M > 1 there is growth of the vorticity of field of speed and increase of quantity of fragments of small scale (fragmentation).

3. Initial-Value Problem Statement and Numerical Investigation

3.1 The Problem Description

Generalization and consideration of three-dimensional modification of equation became interesting and logical continuation of research of equation (2). Thus the task of investigation of three-dimensional generalization of the modified Burgers equation was put on. For rising of mathematical task it was needed to set initial and boundary conditions and choose spatial region for investigation. As initial conditions for a task was offered to choose torus disturbance in the stationary laminar stream. As to the space region a parallelepiped had been considered, with such size, that during the computation the indignations did not reach boundaries of region.

Formal problem has the following form.

Equations:

$$\frac{\partial \overline{V}}{\partial t} + V_k \frac{\partial \overline{V}}{\partial x_k} + \tau\left(\frac{\partial^2 \overline{V}}{\partial t^2}\right) - \nu \Delta \overline{V} = 0;$$

Initial conditions:

$$\overline{V}(0,\overline{x}) = \begin{cases} \varphi(\overline{x}) \cdot \left(\dfrac{-\alpha xz}{\sqrt{x^2+y^2}};\ \dfrac{-\alpha yz}{\sqrt{x^2+y^2}};\ \alpha(\sqrt{x^2+y^2} - R) \right) + \overline{V}_c, \\ \text{if } (\sqrt{x^2+y^2} - R)^2 + z^2 \le r^2 \\ \overline{V}_c, \text{ if } (\sqrt{x^2+y^2} - R)^2 + z^2 > r^2 \end{cases} \tag{3}$$

$$\frac{\partial \overline{V}}{\partial t}(\overline{x},0) = 0;$$

Boundary condition:

$\overline{V}(\overline{x},t)=\overline{V}_C$ when $x\in\Omega(D)$; $D=[a_1,b_1]\times[a_2,b_2]\times[a_3,b_3]$; $\Omega(D)-boundary$ of D

Such formulas follow to the initial disturbances of flow of torus type:

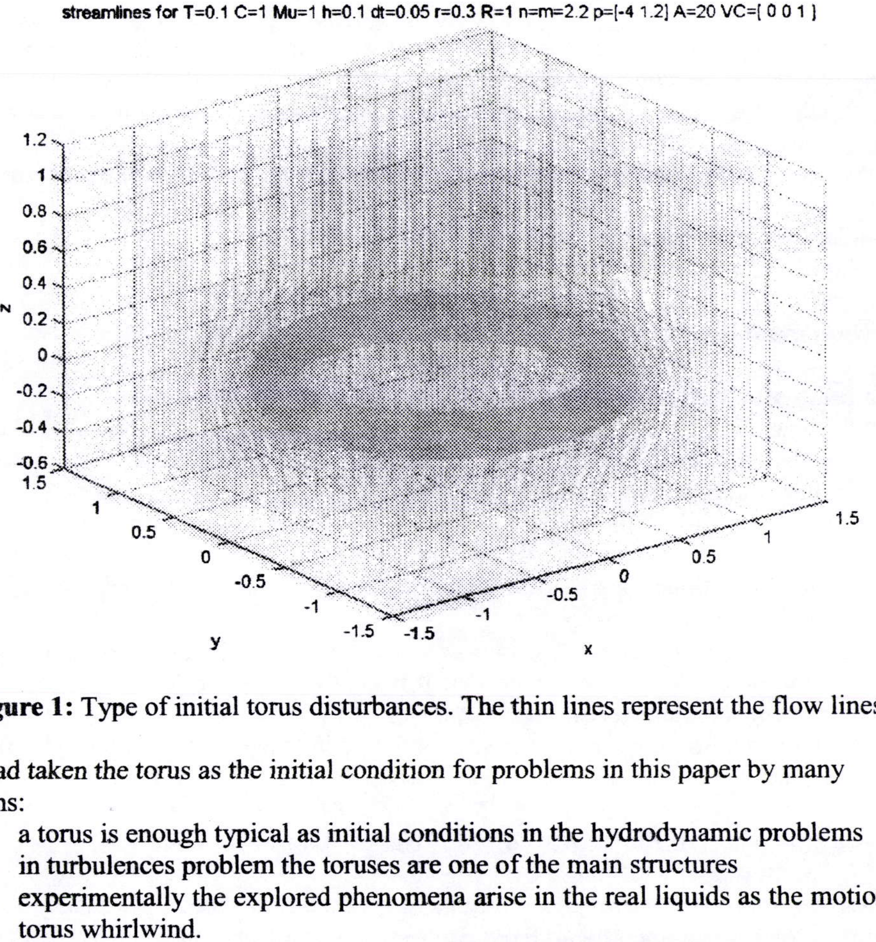

Figure 1: Type of initial torus disturbances. The thin lines represent the flow lines.

We had taken the torus as the initial condition for problems in this paper by many reasons:

1. a torus is enough typical as initial conditions in the hydrodynamic problems
2. in turbulences problem the toruses are one of the main structures
3. experimentally the explored phenomena arise in the real liquids as the motion of torus whirlwind.
4. similar turbulences structures are considered in physics of plasma

Thus due to a "typicalness" and well known results for the classic equations, the choice of torus as of initial conditions allows during investigations to compare the developed results to the known and natural processes, speaking about adequacy or inadequacy of description, and also to forecast new types of the phenomena.

Investigations had been made by rising of numerical experiment. As software environments the system MathLab was selected, as it has the wide arsenal of facilities for the visualization of the three-dimensional vector fields, which numerically had been calculated on the base of equations (3). For the numeral calculations it was selected the well-known simple numerical scheme "cross", at the choice of which deciding part was acted by the following factors:

- simplicity of realization, as it is an explicit scheme
- an explicit scheme is less whimsical comparatively with implicit schemes to the resources of processor time and memory. Taking into account three-dimensional nature of problems, this factor becomes crucial.
- the properties of scheme already had been explored for different types of the hyperbolical equations (including wave equation) by many researches, that allows to separate the features of scheme from the features of solutions of the concrete equation.

Moreover, that to ascertain and evidently feel the properties of scheme operating the row of experiments on research of firmness of scheme had been made. This allowed to formulate recommendations on the choice of spatial and time steps and numerical calculation of blow- up solution decisions were formulated.

3.2 Numerical Results

As a result of numerical solutions the next was exposed

- depending on the parameters of the system a process develops after the three possible scenarios, two from which can be interpreted as the mode with sharpening (collapses, blow-up)
- for the origin of the mode with sharpening correlation is determining. However if for the 1D case the threshold value for M was 1, for given type of equation in the three-dimensional case the threshold value of M should be more (approximately an order grows).
- simultaneously with the origin of collapses there is the origin of a plenty of whirlwinds of small radiuses, that it is possible to interpret how the process of growing of characteristic scales number take place.
- the similarity of numeral solutions at the definite parameters of the system to the experimentally visible pictures of flows of liquids had been found

3.3 Interpretation of Blow-up Solutions (Modes With Sharpening)

Origin or absence collapses evolutions of maximum of speed were judged from the graphical representation. So at fixed values of ν and gradual increase of velocity amplitude of initial disturbances in numerical experiments the indignations are observed:

1. dissipation of initial disturbances (remember classical hydrodynamics)
2. after exceeding the critical value of velocity ($6 < M_{кр} < 10$), the first scenario of collapse shows up. Collapse time exceeds in a few times the relaxation time τ.

3. with the subsequent increase of initial amplitude, an initial torus whirlwind has not time to disintegrate and arises up the second scenario of blow - up. The time of collapse origin may be less then time of relaxation.

Thus we can speak about the origin the mode with sharpening, if there is sharp growth of the module of speed and exceeding by him threshold value after which the validity of the numerical calculation became problematic.

3.4 Growing of Scales Shallow

Here we for the illustration pose some examples of numerical calculation of the evolution torus in the fast laminar flow. At first we pose the example with growing of scales shallow. Here it is possible to see the lines of flows.

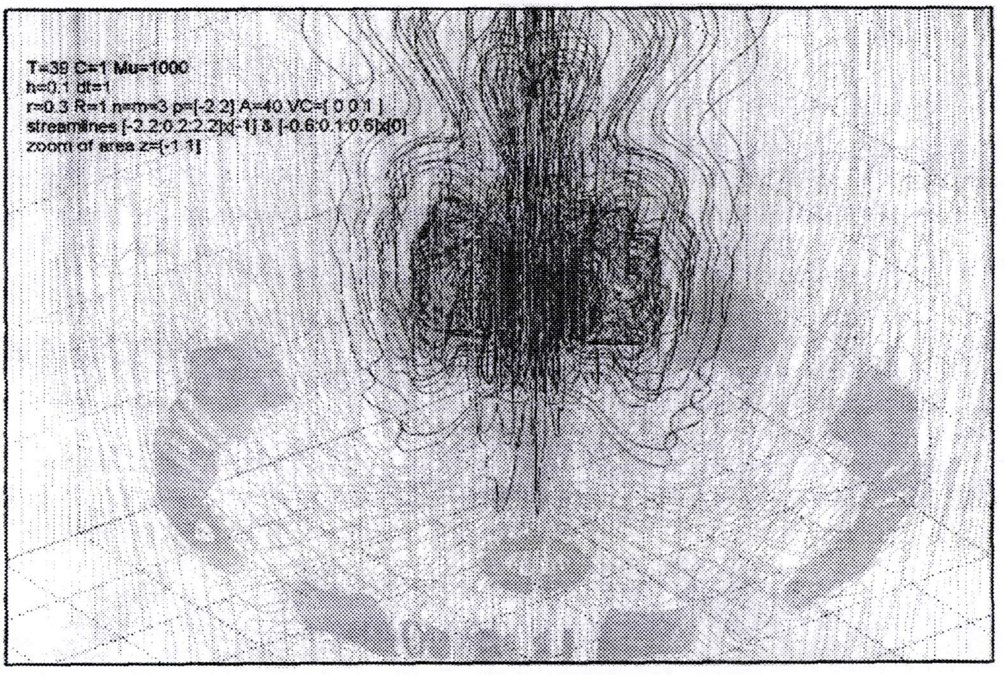

Figure 2: The example of fragmentation and developing of vorticity in the flow. Small dark regions (fragments) at the picture correspond to the regions with grooving rotations of fluid particles.

The second example illustrates the development of three - dimensional flows with many segment, fragments with growing rotations. In principle such distribution of the fragments may correspond to the flows in developed turbulence with many vortex lines (Frish, 1995; Prigogine, 1980).

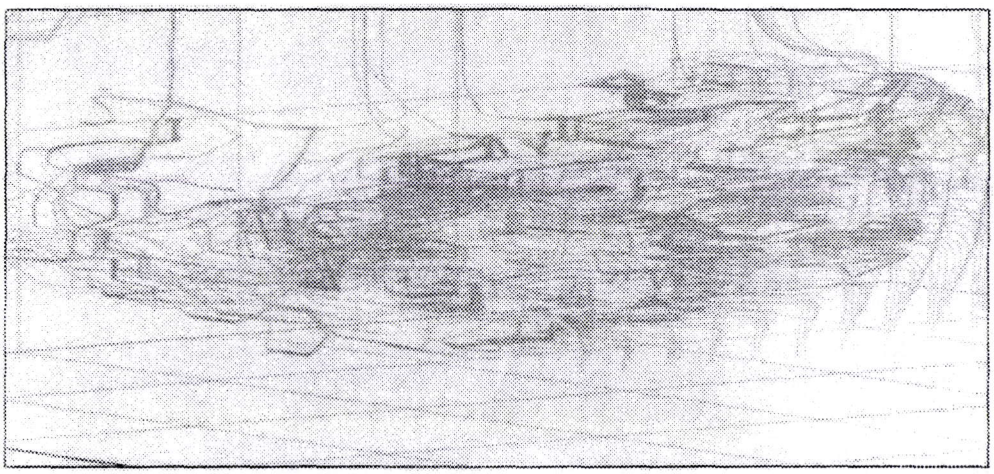

Figure 3: Development of complex flow from simple initial condition

Of course we have represented here only a little number of numerical calculations. But just these examples illustrate the possibilities of origin of absolutely new types of behavior in the media with memory effects in case of large speed of flows. Remark, that such effects are absent in the slow flows or without memory accounting.

4. Conclusions

Thus at given paper we had proposed the results of numerical investigation of model equations with the memory accounting in 3D case. It is shown, that consideration of memory effects (which correspond to relaxation of tensions) leads to new solution types appearance. The explored conduct of new types of solutions can be used in subsequent investigations for comparison with the real model experiments and other models. The approximate correlation of parameters for which it is possible to expect appearance of so-called blow – up (collapses) had been defined.

The main novelty in the received results is the illustration of strong difference of behavior of the systems with or without accounting memory effects. Proposed investigations may open the way for understanding many problems of self-organization (Prigogine, 1980; Loskutov&Michailov, 1992), include turbulence. For example the evident grows of vorticity in fast flows may receive simple modeling background. Also it may follow to the new problems posing in the systems with anticipation when integral-differential equations are forwarded from given moment of time to the future.

Also the numerical results represented at the paper may lead to interesting analytical investigations, for examples on estimation of time of collapse and searching analytical description of solutions near the collapse moment. Physical interpretation of collapses, fragmentation and so on in model equations is as we hope useful for understanding the process in many physical (and moreover biological, social etc. distributed systems with memory, non-locality and may be anticipatory property).

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