Constrained Thermal Plant Control Based on the Second Order Models

Martin Kamenský, Mikuláš Huba
Slovak University of Technology in Bratislava,
Fac. Electr. Eng. & Info. Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia
fax: +421 2 654 11 004 - e-mail: martin.kamensky, mikulas.huba@stuba.sk

Abstract
This paper deals with the control design for a thermal plant with constrained input signal based on the second order time delayed models. The design follows the requirement of the fastest possible transient processes without overshoot. It is limited to the case when the control signal can potentially attack both the upper and lower saturation limit. For the basic PD controller tuning it is usually enough to use approximation of the original plant by the I2Td model (double integrator + dead time). The piecewise constant (or slowly varying) disturbances are compensated by a windupless integral action added to the controller output. Here, the loop properties can be improved by using also more sophisticated 2^{nd} order plant approximation.
Keywords: thermal plant, constrained pole assignment controller, dead time, windupless I-action, disturbance reconstruction and compensation.

1 Introduction

The PID controllers are known as the basic and most frequently used instruments in the industrial automation and control. In the form we know them today they are already produced for more then 60 years. According to their functional simplicity one could expect that all basic problems relevant for their utilization have already been clarified many decades ago. However, in fact, contrary is the case. This situation in process control can be well characterized by the note given in Åström & Hägglund (1995) "...derivative action is frequently switched off for the simple reason that it is difficult to tune properly"... It is easy to show that using the linear controller design, the PD controller cannot be properly tuned in real (constrained) situations! Up to date, practically at each conference oriented to the control systems design, new papers devoted to an "optimal" design or tuning of PID controllers can be found. So, already this single moment is enough to indicate a till now hidden aspects necessary for a reliable controller design. The key factors for this „inflation" were shown by Huba (1999), Huba & Bisták (1999) as **the distribution of the dynamical terms within the control loop** in combination with the **control signal saturation**. This leads to the existence of two different dynamical classes of PI-controllers and three classes of PID controllers. By the dynamical class we denote the number of the control signal phases attacking possibly the given control signal constraints. According to this we speak then about PD_2, PID_0, PID_1 and PID_2 controllers. It is interesting to note that only the dynamical class of PID_0 and PI_0 controllers can be rigorously analyzed by the classical linear methods. The new ways to generally acceptable windupless PI_1-solutions are treated in the parallel paper Kamenský & Huba (2005). The fully new PID_2-controllers based on extending the PD_2 controllers by reconstruction and compensation of input disturbances were firstly treated by Huba and Bisták (1999).

In this paper, several practical steps of the new windupless controllers design are demonstrated for the dynamical class of PID_2 controllers by controlling thermal plant used in education.

2 Model of the Thermal Plant

Thermal plant consists of the heating light bulb, temperature sensor, filter, fan and control electronics.

The diagram of the thermal plant is shown in Fig.1. Table Tab.1 contains data of the input-output characteristic of the plant, the measured dependency is shown in Fig.2.

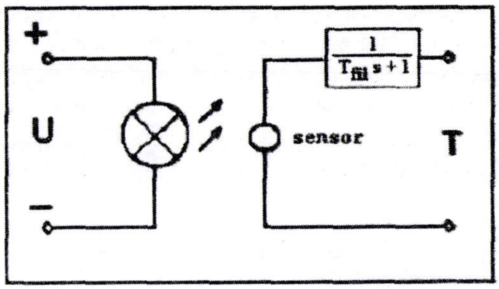

Figure 1: The thermal plant with filtered output (filter time constant $T_{fil} = 5s$)

Table 1: Measured input-output characteristic values of thermal plant

T [°C]	u [V]
23	-2.2
25	-2.28
27	-2.32
29	-2.34
30	-2.38
31	-2.42
33	-2.44
35	-2.46

The input-output steady state characteristic was approximated by a 2^{nd} degree polynomial computed by the least square method. The result is described by equation

$$T = 67.0031u^2 + 268.2099u + 288.7043 \quad (1)$$

Fig. 3 shows an upward step response of the thermal system. The step of the input voltage was made from -2.3V to -2.6V. The system output behaviour corresponds at first to 2nd order static system and then it turns to behaviour similar to the integral 2nd order

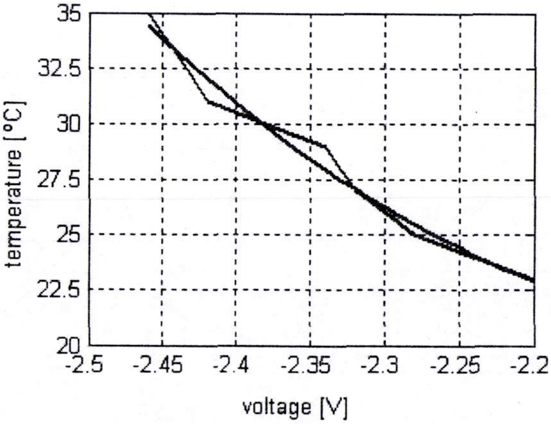

Figure 2: The input-output steady state characteristic of the thermal plant (broken line) and its approximation.

system with the 1st order integrator (system output does not tend to reach a steady state). Neither in case of a downward step the temperature value does stabilize simply. First it drops down and then it grows up slowly. This does not correspond to the 2nd order integral system and this characteristic of the system we could model as disturbance affecting always in one direction or to describe it by a time-variant parameter of the system. Because of such uncertain behavior of the system it seems reasonable to use simple I_2T_d approximation of the system.

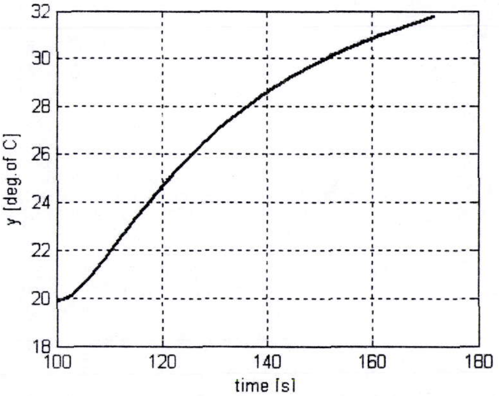

Figure 3: Step response of the thermal plant

The I_2T_d approximation of the response corresponding to an upward step is shown in Fig. 4. The acquired model concentrating on a good approximation of the initial phase of the transient response is:

$$F(s) = \frac{K_S}{s} e^{-T_d s}$$

$$K_S = -0.4; \; T_d = 0.3\, s \tag{2}$$

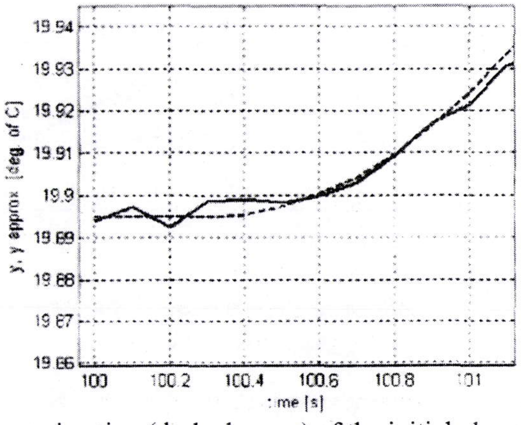

Figure.4: I_2T_d approximation (dashed curve) of the initial phase of an upward step response of the thermal plant, $K_s = -0.4$, $T_d = 0.3$ s

For designing the reconstruction-based compensation of disturbances (I-action) we will use also the approximation by a static 2nd order model denoted as PT_2T_d approximation as it is shown in Fig. 5. The acquired model using the same value of dead-time as the I_2T_d model is:

$$F(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-T_d s}; \tag{3}$$

$$K = -42; \; T_1 = 22\, s; \; T_2 = 11\, s; \; T_d = 0.3 s$$

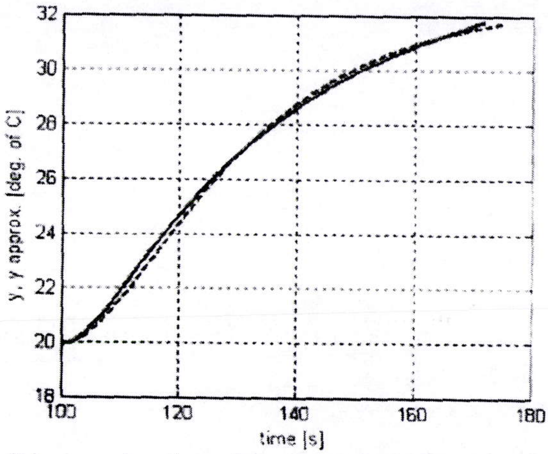

Figure 5: The PT_2 Td approximation of the upward step response of the thermal plant, K = -42, T1 = 22 s, T2 = 11s, Td = 0.3s

3 Constrained Control of the 2nd Order Systems

3.1 Controller Design

Using algorithms designed for the double integrator with transfer function

$$F(s) = \frac{K_s}{s^2} \tag{4}$$

it is possible to control a plenty of linear and nonlinear 2nd order systems. The control algorithm was derived following the idea of regular distance decrease of reference point from the next lower invariant set, see (Huba, 1999; Huba & Bisták, 1999; Huba et al, 1999; Huba, 2003).

The algorithm has grantedly 2 advantages: respecting of the control signal constraints and the possibility to choose the poles with respect to the identified dead time. From a geometric point of view, the chosen poles are quotients of a distance decrease in approaching the next lower invariant sets (the Reference Braking Curve – RBC and the origin of coordinate system) by the reference point. We denote the controller operating according to the proposed algorithm as Constrained Pole Assignment PD_2 Controller (CPAC).

The control algorithm design consists of four steps:

1. Transformation of the reference state to the origin (invariant set of the dimension 0). For a given reference signal w and the real output yr it can be done using equations

$$\begin{aligned} y(t) &= y_r(t) - w(t); \\ \dot{y}(t) &= \dot{y}_r(t) - \dot{w}(t); \\ \ddot{y}(t) &= \ddot{y}_r(t) - \ddot{w}(t) \end{aligned} \tag{5}$$

The system description is then

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}_0\mathbf{x} + \mathbf{b}_0 u \ ; \ \mathbf{x} = [y \quad \dot{y}]^t \tag{6}$$

Its time solution corresponding to the initial state x_0 and to $u=const$ is

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}_0 + \mathbf{b}(t)u \ ;$$

$$\mathbf{A}(t) = e^{\mathbf{A}_0 t} \ ; \ \mathbf{b}(t) = \int_0^t e^{\mathbf{A}_0 \tau}\mathbf{b}_0 d\tau \tag{7}$$

2. For given control signal constraints $u \in \langle U_1, U_2 \rangle$ and a chosen closed loop pole α_1 determination of the line segment of the Reference Braking Curve traced out by the eigenvector with vertices

$$X_0^{\ j} = (\alpha_1\mathbf{I} - \mathbf{A}_0)^{-1}\mathbf{b}_0 U_j \ ; \ j = 1, 2 \tag{8}$$

and determination of the continuation of the RBC corresponding to the limit control values U_j with the end at $X_0^{\ j}$ described by equation

$$X(\tau)^j = \mathbf{A}(-\tau)X_0^j + \mathbf{b}(-\tau)U_j \tag{9}$$

3. Definition of distance ρ of the representative point $\mathbf{x}$ from RBC, e.g. by choosing a direction for its measurement.

4. Deriving the control algorithms decreasing the distance ρ according to

$$\frac{d\rho}{dt} = \alpha_2 \rho \tag{10}$$

with α_2 being the 2nd closed loop pole.

For the double integrator and the distance measured along the y-axis one gets control algorithm, which is far from to be complicated (as usually considered about the constrained control design):

$$u = \mathbf{r}^t\mathbf{x} \ ; \ \mathbf{r}^t = \left[\frac{\alpha_1\alpha_2}{K_s} \quad \frac{\alpha_1+\alpha_2}{K_s}\right] \ ; \ \dot{y} \in \left(\frac{K_s U_2}{\alpha_1}, \frac{K_s U_1}{\alpha_1}\right) \tag{11}$$

$$u = \left[1 - \alpha_2 \frac{y - \frac{1}{2}\left(\frac{\dot{y}^2}{K_s U_j} + \frac{K_s U_j}{\alpha_1^2}\right)}{\dot{y}}\right] U_j \ ; \dot{y} \notin \left(\frac{K_s U_2}{\alpha_1}, \frac{K_s U_1}{\alpha_1}\right) \tag{12}$$

The controller operates in 2 modes: linear and nonlinear one, where the linear mode represents the control law of linear PD controller (11).

In case of the 2nd order system the control action consists of two dominant intervals – acceleration phase and braking phase that correspond to moving of the reference point to the RBC and along the RBC towards the origin.

3.2 Windupless I- action

Now we complete the controller design by a windupless I action. The possible control structures are based on reconstruction and compensation of an input disturbance via inverse model of the plant (Fig.6). For the case of the measured output derivative the I-action is introduced in the same way as in controlling the 1[st] order systems (Kamenský and Huba, 2005). The only difference is that the reconstructed disturbance influences also the effective limit values of the CPA PD_2 controller.

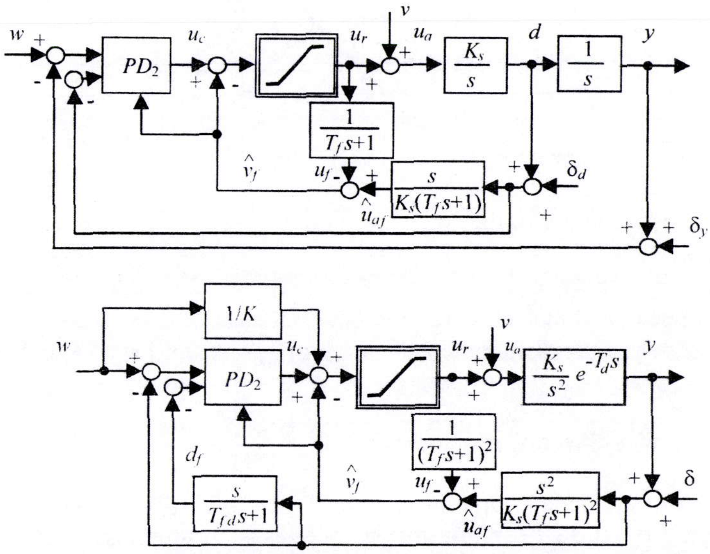

Figure 6: Constraint windupless control of the double integrator with measured output and its derivative (above) and just with measured output (below). The static feedforward control is to use only with the inverse model (16).

Implementation problems usually occur in the case of the 2[nd] order dominant systems with the non-measured output derivative. The transfer function of the filter on the controller output

$$F_{\mathrm{F}}(s) = \frac{1}{(T_f s + 1)^2} \tag{13}$$

does not cause any problems. However, these are related to the 2nd order derivation in the reconstruction block that is described by one of the following transfer functions

$$F_{\mathrm{R1}}(s) = \frac{s^2}{K_s (T_f s + 1)^2} \;; \qquad F_{\mathrm{R2}}(s) = \frac{s(T_1 s + 1)}{K_s (T_f s + 1)^2} \qquad (14), (15)$$

$$F_{\mathrm{R3}}(s) = \frac{(T_1 s + 1)(T_2 s + 1)}{K (T_f s + 1)^2} \tag{16}$$

If we use (14) or (15) in the reconstruction block, the feed-forward gain needs to be omitted, otherwise it has the value $1/K$ reciprocal to the steady state gain. Here we prefer use of the known filter time constants even in the case when the PD_2 controller is based on the double integrator approximations!

The signals added to the controller output (feed-forward gain and signal from I action), however, changes the effective control limits $U_{cj}; j = 1,2$ available to controller (Fig.7).

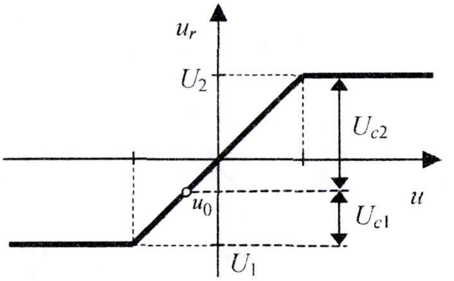

Figure 7: Transformation of the controller limit values $U_{cj}; j = 1,2$ in the case of a bias u_0.

The I-action designed in such a way integrates in right direction (against disturbance) also in the case when the disturbance occurs during transient process, independently of sign of the control error.

3.3 Step Response Based Controller Tuning

The couple of parameters required for the PD_2 controller tuning (K_s and poles $\alpha_{1,2}$) can be obtained from the I_2T_d approximation of the system step response. It is important and sufficient to get good approximation of the leading part of the measured step response. By using the integral based approximation, we can consider this approach as an extension of the well-known Ziegler-Nichols method to higher-order approximations.

The value of the appropriate closed loop poles is directly connected with the value of the identified dead time. For the I_2T_d system with the I-action according to Fig.6 above, the fastest possible transient processes are achieved if the PD controller

$$R(s) = r_0 + r_1 s = \alpha_1\alpha_2 - (\alpha_1 + \alpha_2)s \qquad (17)$$

corresponds to a quadruple real dominant pole that can be set to

$$s = -0.4158 \qquad (18)$$

by

$$r_0 = \frac{0.06587}{K_s T_d^2};\ r_1 = \frac{0.321}{K_s T_d};\ T_f = 5.45 T_d \qquad (19)$$

The same controller setting can be achieved by the so-called equivalents poles (see Huba, 2003)

$$\alpha_{1,2} = (-0.16 \pm j0.23)/T_d \qquad (20)$$

substituted into (11). If we use the algorithm derived for real poles, the complex pole pair can be approximated by real part or the module of (20)

$$\alpha_{er} = -\frac{0.16}{T_d}, \text{ or } \alpha_{em} = -\frac{0.26}{T_d} \tag{21}$$

In such a case the dominant closed loop pole, however, moves to

$$s_r = (-0.12 \pm j0.037)/T_d \tag{22}$$

or

$$s_m = (-0.16 \pm j0.048)/T_d \tag{23}$$

It means that transients are reasonably slower than in the case of complex equivalent poles. In order to simplify the controller tuning, we can suppose the equivalent poles and the filter time constant as simple formulas

$$\alpha_e = -1/(cT_d)\,;\; T_f = cT_d \tag{24}$$

Fig.8 shows for some values of c all basic closed loop signals (controller output u, plant output y and the reconstructed - filtered disturbance signal v_f).

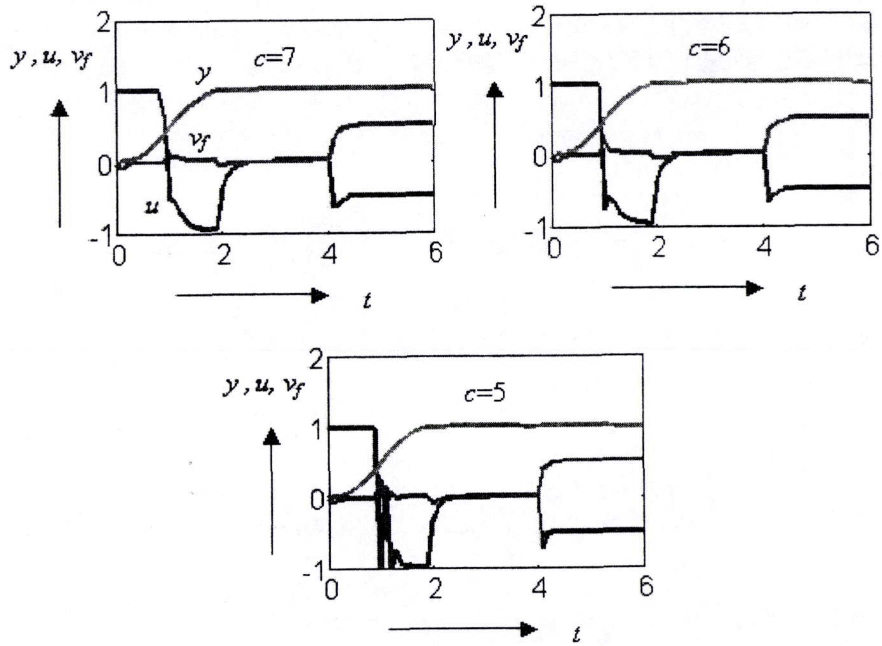

Figure 8: Transients of the loop with measured output derivative corresponding to the controller tuning $\alpha_e = -1/(cT_d)$; $T_f = cT_d$; Step of the input disturbance from 0 to 0.5 at $t = 4$; $U_1 = -1$; $U_2 = 1$; $K_s = 1$; $T_d = 0.02$

3.4 Experimental Results

The algorithm verification was performed on the thermal plant supplemented by the 1st order filter described by the transfer function

$$F_0(s) = \frac{1}{(5s+1)} \tag{25}$$

Because we know the filter time constant, we used the reconstruction block (15), where T_1 was set to 5s, the sampling period was set to 0,001s (the loop can be considered as a quasi-continuous one) and values K_s and T_d were taken from I_2T_d approximation of initial part of the step response. Experimentally the dynamics of transient processes was speeded up by decreasing the dead time value to 0,231s. This can be met also in controlling other stable plants. The presented experimental results correspond to the controller settings based on the parameters

$$K_s = \text{-}0{,}4; \qquad T_d = 0{,}231 \text{ s} \tag{26}$$

Fig. 9-12 show the signal of the reconstructed disturbance, the signal of the CPA PD_2 controller, the resultant control signal and the plant output converging to the changing reference value. The results show that the disturbance reconstruction represents the most noise sensitive part of the closed loop and that the use of the 2nd order models can be useful just with high quality signal measurement.

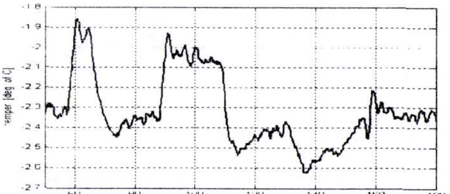

Figure 9: Filtered disturbance reconstruction. K_S = -0.4, T_d = 0.231, $\alpha_{1,2}$ = -1, T_f = 1.0018

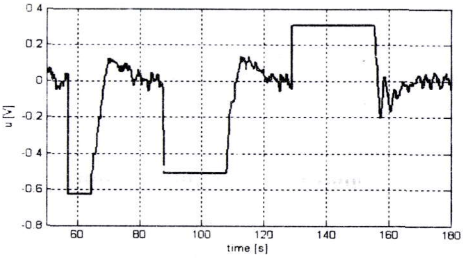

Figure 10: Control signal from the CPA PD_2 controller. K_S = -0.4, T_d = 0.231, $\alpha_{1,2}$ = -1, T_f = 1.0018

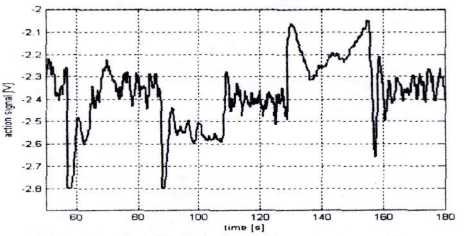

Figure11: Resulting control signal (CPA PD_2 + disturbance compensation). K_S = -0.4, $T_d = 0.231$, $\alpha_{1,2} = -1$, $T_f = 1.0018$

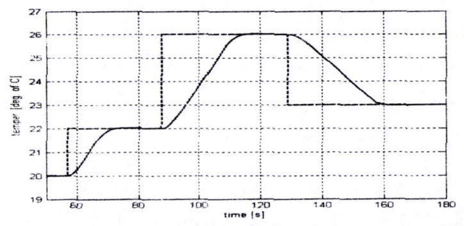

Figure 12: The reference and the plant output signal. K_s = -0.4, $T_d = 0.231$, $\alpha_{1,2} = -1$, $T_f = 1.0018$

4 Conclusion

In this paper we presented effective approach to the control design of the thermal plant with constrained input based on the 2nd order models. The proposed structure respects the control signal constraints and moreover, it enables to achieve monotonous transient processes with short settling time. The designed windupless I-action responds fast and is easy to implement. The structure is robust and can be applied to a broad range of systems with the dominant 2nd order dynamics (and not only on systems with clearly linear behavior). Results obtained from experiments on real plant are better than those obtained by traditional PID controllers supplemented with standard anti-wind-up structures. The computation of control is simple as contrasted to the most of alternative constrained control design approaches (e.g. the constrained predictive control).

Of course, by modifying the linear PID-controllers with different anti-windup structures one can get many solutions and it is far beyond the possibilities of a single paper to test all of them. But, we believe that the optimal solutions to constrained systems cannot be simply achieved by modifying the linear solutions on a heuristic basis (or by the trial-error procedures). So, we prefer solutions motivated by clear physical interpretation. Not those motivated by the mathematical properties (linearity), or simply by the traditional habits of practice.

Acknowledgements

This work has been partially supported by the Slovak Scientific Grant Agency, Grant No. 1/3089/06

References

Åström, K.J., Hägglund, T. (1995). PID controllers: Theory, design and tuning – 2. ed. Instrument society of America, Research Triangle Park, NC.

Huba, M. (2003) Constrained system design (Syntéza systémov s obmedzeniami). Vol. 1 Basic Controllers, Vol. 2 Basic Structures. STU Bratislava, 2003 (in Slovak).

Kothare, M.V., Campo, P.J., Morari, M., Nett, C.V. (1994) A Unified Framework for the Study of Anti-Windup Designs. Automatica, Vol. 30, 1869-1883.

Huba, M. (1999). Dynamical Classes in the Minimum Time Pole Assignment Control. In: Computing Anticipatory Systems - CASYS '98. Woodbury: American Institute of Physics.

Huba, M. - Bisták, P.(1999). Dynamic Classes in the PID Control. In: Proceedings of the 1999 American Control Conference. San Diego: AACC.

Huba, M., Sovišová, D. and I. Oravec (1999). Invariant Sets Based Concept of the Pole Assignment Control. In: European Control Conference ECC'99. VDI/VDE Düsseldorf.

Kamenský, M., Huba, M. (2005). Constrained Thermal Plant Control Based on the First Order Models. CASYS 2005, Liege

Huba, M. (2006). Constrained pole assignment control, In: Current Trends in Nonlinear Systems and Control, Boston: Birkhäuser, 163-183.