# From Frequencies to the Forecasting Processes

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### Abstract

This communication aims to show and explain the frequency influences over the signal evolutions. We draw an operational analogy between the modulations and the detection of periodic and aperiodic behaviours. Consequently  $g(t)$  is related to  $G(s)$  its Laplace picture as a frequency generator .To win morc easiness in the structure analysis we will use the "Bond Graph" methodology and therefore we shortly notice its main specificities.

After the description of the reactivity which causes the kchnological memory we show the operational connection from this one to the (pole - zero) pairs and underline the delivered future characteristics. From inspecting the Laplace space we remark that any root locus =  ${R L}$  structure corresponds to a set of Newton's fields what supplies a divergence field in the  $(Lp_sp.) = Laplace space$ . This Laplace distribution helps for determining the  $g(t)$  evolution characteristics. From the  ${R L}$  it is possible to deduce the system decomposition into a fint-order one and a set of second-order ones, what brings a drastic analysis reduction of forecasting processes. The convolution of a system  $g(t)$  with an inflow signal  $x(t)$  will be transfered in a (Lp.sp.) what yields the angular convolution of both {R L} stars. In (Lp.sp.), are injected horizontal conjugated heliphasors for each

fixed frequency pair with  $exp(-\Delta\sigma)$  as damping variation of the amplitude, resulting from the imaginary graduation and vertical damped or amplified Fourier phasors supporting ( $\Delta\omega$ ) frequency variation for  $exp(-\sigma_k)$  as fixed amplitude. The supply of

synchronous curves following the time evolution along the  ${RL}$ , acts as an extension of the Fourier wavelets. Afterwards we deduce a shortage of the (Lp.sp.) into a sector of the 2"stability quadrant.This accessible window is defined by the uppemost possible frequency.

An extended (Lp.sp.) configuration describes the evolution and the management of a library or a documentation centre. For this last use, the informations will be located in the first quadrant where we can display the obtained gains and developments  $(=$  information increase) on the real axis and the use frequency of each knowledge on the imaginary one. This configuration shows the interest grade for each type of knowledge and will supply a valuable guidance to improve the future development.

Keywords : Forecasting, "Bond Graphs", Reactivity, Laplace Transform, Knowledge Management.

# 1 Introduction

This communication is conceived as an extension and a geometrical exploration of our previous one " Laplace Spaces and their Anticipatory Characteristics "(C.A.S.Y.S. 2003) From the observation of musician plays we can deduce that the emission of any melody

International Journal of Computing Anticipatory Systems, Volume l7,2O06 Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-03-2 is strongly related to the rhythm of players what points the fequencies as source or generator of any time evolution. This fact shows the explanatory function of frequency spaces about the time phenomena.Therefore the key idea is to develop bridges from G(s) geometry to g(t) future. By means of "Bond Graphs" we underline the functional analogies between the Frequency Modulations and the Fourier and Laplace Transformations. To understand the importance of the (pole - zero) pairs we have to explain the reactivity and its influence in our world. In  $(Lp(sp.)$  we will settle an analogy between the G(s) profiles and the electrical fields what will allow a rapid and accurate view of the frequency variations.When we are comparing the (Lp.sp.) with the Fourier phasors we deduce that the (Lp.sp.) may be considered as a support of both kind of modified Fourier phasors: the horizontal ones or heliphasors and the vertical ones: the radial damped or amplified Fourier phasors. Due to the synchronous curves it is possible to evaluate the velocities and accelerations of the time evolutions along each branch of the {R L}, what is helpful for the forecastings.The suiting of the (Lp.sp.) for the knowledge managing is given to prove that the Laplace topology can be used in various domains where the mastering is primordial.The main advantage of the (Lp.sp.) is delivered by the graphical transposition of any dynamical scheme.

## 2 Structures of Transfers between Time and Frequency Spaces

### 2.1 Operational Advantages of the "Bond Graphs"



"Bond Graphs" or (B.G.) are graphical tools to describe synoptically the power flow structures because any power may be considered as the product of a qualitative or level variable with a quantitative or extensive one.This is easy to notice by "Bond Graphs" (Fig. 1). We remark that the level variable is connected to the referential and plays like a basis vector of this referential.

Consequently these "Bond Graphs" are also suited to notice the vector components  $(=$ contravariante variables) embedded in any particular referential with their correspondent basis vectors  $(= covariant variables)$  (Fig.2).





### Table I : Analysisof "Bond-Graphs" Characteristics

### 2.2 Frequency Distribution and Evolutive Processes

Frequency seems to be the key factor for forseeing the evolution of any time function. The best way to justify this point of view results from musical plays because the frequency beats on a drummer or on the piano keys are producing the melody what is an acoustical evolution.Therefore the (Lp. sp.) geometry helps to explain and to describe tbe future behaviours. Consequently the forecasting of any  $s(t)$  will be related to its generator  $G(\omega)$ .

### 2.3 Modulations and Referential changes (Figs.  $5 \& 6$ )

Any frequency modulation aims to transfer a signal  $g(\Delta\Omega t)$  into a carrier referenrial ( $\omega c$ ) to provide  $G(\omega c \pm \Delta \Omega)$  which is better matched for long distance emission. The used carrier is an exponential with an imaginary exponent because this has a uniry magnitude what avoids to alter the transferred signal:  $I \exp(j\omega ct) I = 1$ 

We note also that a carrier with an imaginary exponent can extract the periodic oscillations In modulation the functional carrier  $exp(j\omega ct)$  is sweeping the whole time basis of the signal for transfering information into the neighbourhoud of the frequency  $(\omega c)$  (Fig.4). Mathematical modeling of a frequency modulation:

### $\int g(\Omega t) \exp(i\omega ct) dt = G(\omega c \pm \Delta\Omega)$  (1)

It is to observe that the carrier is working simultaneously in the both spaces (t)  $\& \alpha$  ( $\alpha$ ) and

is performing a bridge from the source space (t) into the arrival one ( $\omega$ c). This observation shows the operational similarity between a modulation and a referential change.The relation (1) is analogous to each component of a Fourier Transformation.

Fourier Transformation aims to retrieve from any signal shape the weights of the source frequencies. Indeed Fourier is using a large lot of carriers and therefore is performing a set of frequency modulations. In Fourier Transformation the (t) sweeping is the more speedy incrementation because the whole time distribution has to contribute to each

frequency  $(\omega)$ . This is done by the time integral. All these transfers are working by means of a convolution of the signal  $g(t)$  with the specific carriers.

Mathematical modeling of a Fourier Transformation:

$$
\int g(t) \exp(-j\omega t) dt = G(\omega)
$$
\n
$$
\text{or in discrete form: } \Sigma_k s(k) \exp(-j k \cdot 2\pi/N) = G(1)
$$
\n
$$
(2a)
$$

or in discrete form:  $\Sigma_k$  s(k) exp[-j k l (2 $\pi$ /N)] = G(l)

where: k is time parameter, I frequency parameter and  $N = 1$ max kmax



### 2. 4 Operational Specificities of the Laplace Carrier (Fig. 7)

The (Lp.sp.) is embedded by a real axis which supports the frequency decay ( $\sigma$ ) or increase and by an imaginary axis which supports the frequency set  $(i\omega)$ 

The Laplace carrier is:  $exp(-st) = exp[-(\sigma + i\omega)t] = exp(-\sigma t) exp(-i\omega t)$  (3)

" where exp(-ot), real operator, is extracting the aperiodic trends from the signal to put them on the rcal axis by means of a real convolution.

Consequently the Real Laplace transformation:  $\int g(t) \exp(-\sigma t) dt = G(\sigma)$  (4)

 $\circ$  where exp(- jot), imaginary operator, is extracting the periodic behaviour from the signal to put them on the imaginary axis by means of an imaginary convolution. Indeed this is the Fourier carrier which is identical to the imaginary Laplace transformation.

Consequently the Imaginary Laplace transformation:  $\int g(t) \exp(-i\omega t) dt = G(\omega)$  (4a or 2) The Laplace Transformation is more powerful than the Fourier because it contains more informations along the  $\sigma$ ) axis and so can record the rates of time evolutions.

Laplace variable (s) is a complex frequency and consequently possesses an apparent magnitude:  $\text{IsI} = [(\sigma)^2 + (\omega)^2]^{1/2}$  and a direction in  $(\text{Lp}.\text{sp.})$ :  $\tan(\theta) = (\omega) / (\sigma)$  (5-5a)



## 2. 5 Fourier subspaces in  $(Lp_sp.)$   $(Fig. 8)$

On each fixed (ok) value is locating a vertical altered Fourier space ( $\Delta\omega$ ), where in each phasor the rotating magnitude is multiplied by the same  $exp(-\sigma_k)$ : danping in the 2<sup>o</sup> & 3<sup>o</sup> quadrants or amplification in the  $1^{\circ}$  & 4 $^{\circ}$ ones. It results from the real graduation.

On each fixed ( $\omega$ ) value is locating an horizontal heliphasor ( $\Delta \sigma$ ), where the rotating magnitude, of fixed velocity ( $\omega$ ), varies with the operator exp( $-\Delta\sigma$ ). Besides the heliphasors must be associated in conjugate pairs because of the symmetry with respect to the real axis caused by any realizable signal. It results from the imaginary graduation. Each Fourier subspace displays the frequency composition for each fixed damping. Here is underlined the common substance between laplace and Fourier spaces.

# 3 Location of Memories

### 3.1 Reactivity and Memories

The reactive elements such as generalized inductances and capacitances perform time derivatives or integrals acting over various time levels of potential and therefore reactivity plays as memory support.To construct forecasting schemes it is necessary to use past records for extrapolating future strategies and this underlines the high value of reactivity to bridge from the past to the future. We must remark that reactivity is inserted everywhere in the universe because any mass is inertial reactive. On an other side reactivity as memory support is essential for progress. Scripture and engraving were the first elementary reactive procedures because they have stabilized our memory. The reactivity advantages are poinæd out in the table 2

Time differential  $\Delta_t g = g(t) - g(t - \delta t)$  and Time integral  $g(t) + \Sigma_k g(t - k \delta t)$  use various time records.

### 3. 2 Pairs of Poles and Zeros (Figs. 9 10)

The poles and zeros are introduced from the mathematical description of the reactive behaviours. For instance an inductance L and its resistance R imply the differential relation:  $(LD_t + R)I = V$  where the tension V is valuating from the current I.

The corresponding transfer relation with I causality :  $(Ls + R) I = V$  (6) where I and V are the Laplace tranformations of I and V. When V disappears this relation

becomes (Ls + R) I = 0 what is reached for  $s = -(R/L)$ , which gives the zero of this relation, equivalent to the inverse of the time constant, related to the system size . It is also possible to invert the  $(6)$  relation for evaluating I from  $V$ .  $I = V/(Ls+R)$ (7)



Table 2: Reactivity Influences in our World

In this (7) rel.with V causality, the value  $s = -(R/L)$  is a pole and  $1/(Ls + R)$  is an

integrator. The causality exchange has transformed a zero into a pole.



Conclusion: a derivative operator produces a zero and an integrator gives a pole. Besides pole and zero are the same root of a reactive operator respectively considered as a derivative or an integrator by exchange of causality what is shown on the reactive (B G). These well known considerations underline the importance of (pole- zero) in (Lp. sp.).

#### 3.3 Meaning of the pairs (pole - zero)

Each reactive element introduces a zero or a pole and is also a memory container. Consequently each pole (p) or zero  $(z)$  points out a memory storage. The number of roots (= zero or pole) of a transfer function eqnals the number of reactive elements contained in this system. More roots of the transfer function, more dynamical stability of the system, more extrapolting power and more easiness to drive this one, because the root locations determine the time evolution.

## 4. Operational Characteristics of a (Lp. sp.)

### 4.1 Compensation Effect between a pole and a zero

Because of the  $g(t)$  Laplace Transformation takes an infinite value in each pole neighbourhood and cancels at each zero, it is obvious that these  $(p-z)$  pairs define the profile of  $G(s)$  and consequently are the main points of the  $\tilde{G}(s)$  topography distribution. If a pole and a zero are very near of each other, they form a short pair  $(p-z)$ and perform a very strong mutual compensation or cancelling.Such short pairs are sometimes artificially constructed to improve or to suit the system behaviour.

The topography axes are the connections between each pole and its nearest zero (its dual one) to constitute compensated pairs, due to the frequency variations.

When the number of poles is different from this of zeros we have to link these isolated or free roots to their inverted ones, virtually located on the infinite border hence virtuals.Tbe compensation effects along these infinite connections are very weak.

### 4.2 Analogy between the  $G(s)$  Profile and Electrical Fields  $(Fig.11)$

If we consider each pole as a positive electrical charge and each zero as an earthing point, the (p-z) connections in this electrical analogy are assimilated to gradient lines. Between each pair (p-z) we find a divergential or Newton's field which is : Grad $[G(s)] = E(s)$  (8)

where  $G(s)$  is a scalar potential and E (s) the rate of (G) frequency variation what indicates also the intensity of compensation of its pair  $(p-z)$ .

Relation:  $Div[Grad(G)] = Div(E) = m G(p_k) = m q^+$  (9)

Consequently the G topography is analogous to the well known electrical potential and field and supports the same properties.This makes the G(s) explorations easier. It is to remark that the (m)phase (RL) star forms the operational Laplace subspace and consequently introduces space neguentropy due to their regular angular symmetry .

#### 4.3 System Regulation by Closed Loop (Fig. 11 - 12)

Here the polynomial forms of the tranfer functions are denoted as follows:

for  $G = P^n$ : polynomial of n-order; for  $B = P^m$ : polynomial of m-order When a system of forward response G is introduced in a closed loop with a feedback operator B, its regulation becomes easier but the closed-loop transferfunction wins more additional poles which are due to the (m) zeros of  $B(s)$ . This pole excess, denoted free poles, emanate from the closed-loop transfer function which is:  $G/[1+GB]$  (10) poles, emanate from the closed-loop transfer function which is:  $G/[1+GB]$ The poles are obtained for  $GB = -1$  what, in complex, is splitting in polar forms:

$$
1 = IGI IBI \qquad (11) \qquad \text{and:} \qquad \qquad \pi = \text{Angl}(G) + \text{Angl}(B) \qquad (11a)
$$

The poles of  $G/[1+GB]$  are the  $(m+n)$  zeros of  $1+GB = P(m+n)$ . Consequently the root number of  $(1 + GB)$  is  $(n+m)$ , higher than the  $(n)$  root quantity of  $(G)$  and this implies an excess of m free poles for the transfer function with a  $P<sup>m</sup>$  feedback operator.



**Fig. 11:**  $G(s)$  Profile along a (p-z) Connection as feed back operator



# 5 Simplification of the (Lp. sp.) Topography

## 5.1 Introduction of (m) Symmetrical Star in (Lp. sp.)

Each (p-z) pair at finite distance introduces a finite connection which belongs to the ${R}$ L). But the (m) free poles don't find finite compensations,therefore they have to emit each one a search antenna to the infinity border of the (Lp. sp.) like in the section 4.2. Root condensation: by comparison with the infinite distances, it seems reasonable to condense all finite elements of  ${RL}$  on a "polygonal point" located on the gravity centre of the roots obtained by weighting the poles with a positive unity and the zeros with a negative one; what implies this relation:  $\Sigma_k [R(p_k)] - \Sigma_l [R(z_l)] = (RC)$  (12) where R ( $p_k$ ) & R( $z_1$ ) are the ( $p_k \& z_1$ ) locations and (RC) the reactivity centre.

### 5.2 Polygonal Distribution of the free Poles on the Reactivity Centre

The micro polygonal distribution of the (m) free poles seems the most symmetrical one. Consequently the (m) pole asymptotical antennas will propagate also the same polygonal

symmetry through the whole (Lp. sp.) to reach along their specific orientation the infinity border where virtual zeros are induced over a similar macro polygonal structure.

The macro polygon acts like a (m) faced mirror, each face reflecting a virtual zero to its free pole along the same direction and so induces a space neguentropy. This geometrical argumentation seems to follow the most logical way to justify the  $(m)$  phase regular $\{R \}$ star. "Finite geometry is reflecting over the infinity. These  $(m)$  asymptotes introduce  $(m)$ main axes running from the (RC) to the infinity border. These axes support also the  $Grad[G(s)]$  lines. Of this way the (Lp. sp.) may be considered as a symmetrical vector of

(m) phase asymptotes with  $(2\pi)/dn$  as angular opening: (Figs.13 - 14) where  $dn = number(p)$  - number(z) = m: the multiplicity of the pole excess.



compensation along each the pole conglomerate and running to the infinity border infinite branch Fig. 13: Root Locus: Regular star centered on

Here, we have shown that the transfer functions of any regulated systems present a "Hill" shape, with long stretched flanks. On the reactivity centre is the infinite peak or the "chimney"; over the infinity border is clamped the "O level" by the (m) virtual zeros. The regular star with (m) infinite branches can also be generated by a discrete Fourier distribution of  $(m)$  order around a J axis orthogonal to  $(Lp, sp.)$  situated on  $(RC)$ . Besides (Lp. sp.) is a mental concepton, therefore it is logical to introduce star symmetry as geometric neguentropy increase.This is reflecting the free pole multiplicity into a virtual division of the infinity border.

## 6 (Lp. sp.) Reductions

### 6.1 Consequence of the real Axis Symmetry

The transfer functions of any physically realizable system must have real coefficients and consequently all roots appear as real when singles or in conjugate pairs. Consequently the {R L} must also contain real roots with additional conjugate pairs. From this we identically apply the 3° & 4° quadrants respectively on the 2°  $\&$  1° ones. After this geometric operation we have only to keep the  $1^{\circ}$  &  $2^{\circ}$  quadrants; where each non real asymptote is in fact a double one and dispays the frequency evolution of a second-order subsystem. If there is a real asymptote this one is single and corresponds to a first-order subsystem.These double conjugate branches with the real ones form a regular half star. From these Laplace star configurations any system is splitted in a set of  $2^{\circ}$  order systems with an additional 1<sup>°</sup> order one.

6.2 Delinitation of the Radial Extcnsions in the both Residual Quadrants At present, it is also possible to limite the radial extension by considering the maximum value of the accessible frequencies.Consequently, Rmax, the accessible radius is determined by the max (s) value:  $R_{\text{max}} = [(\omega_{\text{max}})^2 + (\sigma_{\text{max}})^2]^{1/2} = s_{\text{max}}$  (13) what reduces the  $1^{\circ}$  &  $2^{\circ}$  quadrants to the "positive" fraction of a polygonal window. This polygonal line is a homologous reflection of the infinity border, with the same configuration. $($  = radial harmonic of the infinity polygon)

### 6.3 Elimination of the  $1^\circ$  quadrant

In the l°quadrant, supported by the angular interval  $[0 \rightarrow \pi/2]$ , are located the transient instable evolutions. Finally we only keep the 2° quadrant supported by the angular interval  $\lceil \pi/2 \rceil$   $\sim \pi$ , where are located the damped behaviours. The imaginary axis will play as a hermetic fence to avoid any crossing run of the susbsystem points.



6.4 Comparison of the Durations of the Asymptotical Evolutions (Fig. 15) When a point is moving along an asymptote it is the angular orientation  $(\theta)$  that determines the rapport between the frequency variation and the frequency decay or damping progression:  $(\Delta \omega)/(\Delta \sigma) = \tan (\theta)$ . This rapport allows the determination of the transient evolutions, because the damping is the most important along the  ${R L}$  branches near of the negative real axis. The main  ${R L}$ branch is the nearest of the imaginary axis

because it corresponds to the longest duration. For a coarse analysis of a forecasting process it is possible to only record and explore the less damped or main second-order subsystem. This ultimate reduction of the (Lp.sp.) to a single branch allows to spare a lot

of record memory and analysis time because the determination of the slope  $\theta$  and the radial evolution are sufficient.

# 6.5 Polar Coordinates ( $\mathbf{R}_{\mathbf{k}}$ ,  $\theta_{\mathbf{k}}$ )

Due to the operational (Lp. sp.) decompositions into star spaces it is advantageous to use the polar coordinates to describe mathematically the evolutions of the system states.

# 7 Mapping of Pairs of Complex Systems

#### 7.t Graphical key for Mappings (Fig.16)

The laplace configurations of the mappings are deduced from the complex product rule in polar form. Considering in polar form, the well known mutiplication:

A exp(j $\alpha$ ) B exp(j $\beta$ ) = AB exp[j $(\alpha + \beta)$ ] = IABI with orientation ( $\alpha + \beta$ ) when the complex numbers may be considered as system transfer functions. Rel.(14) gives the operational key for mapping system interferences in (Lp.sp.). For sytems condensed into stars we have to perform the complex multiplication of their stars what gives a  $2 b<sub>y</sub>$  2 branch multiplication, because each star is a vector of branches.Each

branch (k) of the first one turns the second star by the angle  $(\alpha k)$ , its own orientation. Consequently this angular convolution produces  $max(k)$  second stars, each with an angular moving of  $(\alpha k)$  what gives an angular location of  $(\beta l + \alpha k)$  for each branch, where each  $(k)$  value is linked to each  $(l)$  value. The mapping operation gives:

 $IA(s)I \exp(i\alpha k) IB(s)I \exp(i\beta l) = IA(s) B(s)I \exp[i(\alpha k + \beta l)]$  $(15)$ 

(15) is providing a max( $\alpha k$   $\beta$ l) branch number, for the max. values of  $\alpha k$  and  $\beta$ l.

### 7.2 Power or Autoconvolution of a System

In this particuliar case is performing the angular autoconvolution between both identical (m)stars what provide a  $(m^2)$  branches with respective orientations (2 $\alpha$ k). This is a tensor product. For following the evolution of the active power transported by a signal we havè to perform the hermitian product what is realized by the angular convolution of a

conjugate pair of stars:  $IA(s)I \exp(j\alpha k) IA(s)I \exp(-j\alpha k) = IA(s)^2I$  (16) This result is in accordance with Parseval's theorem about the power invariance.

### 7.3 Mapping of a Convolution of Working Systems with crossing Flows

We have to perform a tensor multiplication of a pair of different stars and this is the operation already described by  $(15)$ , what can be produced by a tensor product of a pair of Fourier transformations.This used to forecast the interference between both systems.

# 8 Introduction of Isochronous Curves (Fig.15)

### 8.1 Record of the deformation times of Systems.

When we quote the time of fiequency occurrences along the star branches, we can record the past deformation velocities and accelerations of a system. This gives a time graduation on each branch.Time record is oft restricted on the main or dominant branch. Isochronous curves are the lines linking the same time on each star branch. This time quotation is the basis for any future extrapolation of reactive systems.This plays as an extension of the Fourier wavelets to the (Lp. sp.).

# 9 Laplace Topography used for the Knowledge Management

#### 9. 1 Development Structure of Documentation Centres

Here the management of the dynamical development of a documentation centre or  $(DC)$ has to be described with accuracy and the storage level increase estimated with the best probability. Therefore the Laplace Transpositions will be used as a graphical presentation. At first, the documentation purposes must accurately be defined what will point out the different related domains necessary to be collected for supporting the efficiency and the attractivity of the centre. Each domain has to be considered as component of the field vector which forms the basis for any iteration of the  $(DC)$  management. Therefore we use a particular (Lp.sp.) for indicating the progression of each covered knowledge domain presented by the state point  $B(\lambda t, \theta k)$ . Of this way we are performing a vector

introspection of the  $(DC)$  dynamical driving because the field vector  $( =$  domain set) is related to a (Lp.sp.) vector.



**Fig.17:** (Lp. sp.) for the Evolution Analysis Fig. 18: (B.G.) display of a (DC) of a (k) Domain in a (DC) Management with (k) Domains



#### 9.2 Structure of this Laplace Development

The common startpoint in each (Lp.sp.) is their origin: the common zero.Their used quadrant is the l" one because we must draw the development of the storage levels of each noticed domain. Here is no instability risk. The progression of each domain will be transposed on a specific radius: B(s) trajectory. Here, the slope displays the major infomations because its real projection indicates the storage increase and its imaginary ones the use frequencies. The common startpoint in each (Lp.sp.) is their origin: the common zero. Their used quadrant is the  $1^{\circ}$  one because we must draw the development of the storage levels of each noticed domain.The progression of each domain will be transposed on a specific radius: B(s) trajectory. Here, the slope displays the major infomations because its real projection indicates the storage increase and its imaginary projection, the exploration frequency. In these Laplace presentations we can compare rhe evolutions of each topic with easiness and accuracy from the reckoning of each numerical slope indicator which is for the (k) domain :

 $\left[ (\Delta \omega) / (\Delta \sigma) \right]$ k = tan( $\theta$ k) (17)

Time planification can be inserted in each  $(Lp(sp), k)$  by quoting reached dates on the planned time scale what allows evolution control. For the mathematical description of these managements the polar coordinates are used to locate the (k)domain states ( $R_k$ ,  $\theta_k$ ).



#### 9.3 Specific Supply of the Method

This Laplace management introduces the "conviviality" and the limpidity of the geometric configurations coupled to a vector decomposition for a more simple analysis. It is easy to record the effects of the various disturbances when they occur in each (k) domain as indicated on (Figs: 19 - 2O).

Fig.l9 presents a broken development due to an interest decrease for this (k) domain. Fig. 20 presents various evolutions when the document supplying is perturbed. Along  $(D1)$  for a document destruction or loss; along  $(D2)$  for impossibility to acquire additional documents; along (D3) when document depletion occurs.

# 10 Conclusion

Here was underlined the reactivity power for structuring the (Lp.sp.) because the (p-z) pairs are issued from the operational description of the reactive subsystems.The analogy between the Laplace distribution of the  $(p-z)$  and the electrical fields allows to understand rapidly the profile of the Laplace transfer functions which seems essential to explore the forecasting processes. Besides we have identified the (Lp.sp.) with a vector of Fourier Spaces with altered radii. This is a major support for discovering the common specificities between these both transformations. To extend further this essential idea we have remarked that the  ${R}$  L stars could also result from discret Fourier distibutions whose **J** axis would be orthogonal to their (Lp.sp.). The star products were also used to explain the Laplace transposition of the convolutions between system pairs.

Finally the extension of the Laplace method for the development of a  $(DC)$  has shown the (Lp.sp.) power for the analysis of various dynamical domains.

We notice that the key methodology of this communication is founded on geometrical deductions what gives a deep and speedy exploration of (Lp.sp.) and delivers important didactic wins. We let remark the advantages and easiness of the polar coordinates to follow the time evolutions of the subsystems in  $(L_p, sp.)$ .

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