# The Mass Deficiency Correction to Classical and Quantum Mechanical Descriptions: Alike Metric Change and Quantization Nearby an Electric Charge, and a Celestial Body
# Part II: Quantum Mechanical Deployment for Both Gravitationally, and Electrically Bound Particles

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**Abstract**

Herein a full quantum mechanical deployment is provided on the basis of the frame drawn in the previous Part I. Thus it is striking to find out that occurrences taking place at both atomic and celestial scales, can be described based on similar tools. Accordingly, the gravitational field, is quantized just like the electric field. The tools in question in return are, as we have shown, founded on solely the energy conservation law.

The relativistic quantum mechanical equation we land at for the hydrogen atom, is equivalent to the corresponding Dirac's relativistic quantum mechanical set up, but is obtained in an incomparably easier way. Following the same path, a gravitational atom can be formulated, in a space of Planck size, with particles bearing Planck masses.

For simplicity, we will enumerate the sections, as well as the equations, in continuity with the corresponding sections and equations drawn in the previous Part I.

**Keywords:** Mass Deficiency, Gravitation, Quantization, Electric Charge, Metric Change, Planck Size, Planck Mass, High Energy Cosmic Rays, Gravitational Waves

## 5. Full Quantum Mechanical Deployment of Our Approach

Here again we will consider for simplicity the hydrogen atom, where to begin, we assume the proton at rest, without however any loss of generality. We will also neglect the spin-orbit interaction, still without any loss of generality.

Let us then evaluate the difference $\mathcal{D}$ based on the usual relativistic definition of the momentum $p(r_0)$ of the electron on the orbit, i.e.

$$p(r_0) = m(r_0)v_0(r_0); \tag{14}$$

recall that here $m(r_0)$ is the overall mass, defined along our Eq.(13).

Thus

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International Journal of Computing Anticipatory Systems, Volume 17, 2006

Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-03-2

$$\mathcal{D} = m^2(r_0)c_0^4 - m_0^2(r_0)c_0^4 = \left( \frac{m_{0\infty}^2 c_0^4}{1 - \frac{v_0^2}{c_0^2}} - m_{0\infty}^2 c_0^4 \right) \left( 1 - \frac{Ze^2}{r_0 m_{0\infty} r_0 c_0^2} \right)^2 = p^2(r_0)c_0^2 . \quad (15)$$

Recall on the other hand that $m(r_0)c_0^2$ is nothing else, but (in the relativistic sense) the total energy $E_{Total}$.

Then

$$p^2(r_0)c_0^2 = E_{Total}^2 - m_{0\infty}^2 c_0^4 \left( 1 - \frac{Ze^2}{r_0 m_{0\infty} c_0^2} \right)^2 , \quad (16)$$

or

$$p^2(r_0)c_0^2 + m_{0\infty}^2 c_0^4 \left( 1 - \frac{Ze^2}{r_0 m_{0\infty} c_0^2} \right)^2 = E_{Total}^2 . \quad (17)$$

We can compose the correct relativistic quantum mechanical equation for a stationary case, to replace the classical Klein-Gordon Equation, via the usual quantum mechanical symbolisms of the momentum, and the energy, i.e.

$$p(r_0) = -i\hbar\nabla , \quad (18)$$

$$E_{Total}^2 = -\hbar^2 \frac{\partial^2}{\partial t_0^2} . \quad (19)$$

Thus

$$-\hbar^2\nabla^2\Phi(r_0,t_0)c_0^2 + m_{0\infty}^2 c_0^4 \left( 1 - \frac{Ze^2}{r_0 m_{0\infty} c_0^2} \right)^2 \Phi(r_0,t_0) = -\hbar^2 \frac{\partial^2\Phi(r_0,t_0)}{\partial t_0^2} , \quad (20)$$

*(correct relativistic equation written out of the "overall mass" expression, embodying the mass deficiency of the bound electron, instead of the classical Klein-Gordon Equation)*

where $\Phi(r_0,t_0)$ denotes the space and time dependent wave function; note that by "correct relativistic equation", we mean, "equation taking into account the mass deficiency of the bound electron, next to the mass dilation of it, due to its motion".

Eq.(17), for a stationary case should be written as

$$-\hbar^2\nabla^2\psi(r_0)c_0^2 + m_{0\infty}^2 c_0^4 \left( 1 - \frac{Ze^2}{r_0 m_{0\infty} c_0^2} \right)^2 \psi(r_0) = E_{Total}^2\psi(r_0) , \quad (21)$$

*(correct relativistic eigenvalue-eigenfunction equation)*

where $\psi(r_0)$, in short $\psi_0$, is the eigenfunction of the relativistic description of the stationary case in consideration; $E_{Total}$ then becomes the corresponding eigenvalue. We will see below that Eq.(21) well reduces to an equation involving $E_{Total}$ instead of

$E^2_{Total}$, so that at the level of this equation, one does not have to question the classical meaning of the wave function $\psi(r_0)$.

**5.1.Derivation of the Schrodinger Equation from Our Approach**

Schrodinger most likely did not think to take into account the mass decrease of the bound electron [33,34,35,36], i.e. the term between brackets, multiplying $m^2_{0\infty}c^4_0$, at the LHS of Eq.(21), nor seemingly did anyone after him.

We can easily derive the Schrodinger Equation, based on Eq.(21). Thus, note first that the eigenvalue $E_{Schr}$ yeld by the Schrodinger Equation is less than $E_{Total}$ as much as $m_{0\infty}c^2_0$, the energy equivalent of the rest mass of the electron, i.e.

$$E_{Schr} = E_{Total} - m_{0\infty}c^2_0 \ . \tag{22}$$

*(definition we make in conformity*
*with the eigenvalue of the Schrödinger Equation)*

Recall that $E_{Total}$ is, by definition [cf. Eq.(13)] a positive quantity. Therefore $E_{Schr}$ is a negative quantity, and as a first approach, should expected to be, as usual

$$E_{Schr} \cong m_{0\infty}c^2_0(1-\frac{1}{2n^2}Z^2\alpha^2) - m_{0\infty}c^2_0 = -m_{0\infty}c^2_0\frac{Z^2\alpha^2}{2n^2} \quad \text{(for small Z's)}, \tag{23-a}$$

n being the usual principal quantum number, and the fine structure constant $\alpha_{fs}$

$$\alpha_{fs} = \frac{2\pi e^2}{hc_0} \cong \frac{1}{137} \ . \tag{23-b}$$

Eq.(22) thus can be rewritten as

$$E_{Total} = m_{0\infty}c^2_0\left(1+\frac{E_{Schr}}{m_{0\infty}c^2_0}\right) \ . \tag{24}$$

Via Eq.(22), Eq.(21) will be written as

$$-\hbar^2\nabla^2\psi_0 c^2_0 + m^2_{0\infty}c^4_0\left(1-\frac{Ze^2}{r_0 m_{0\infty}c^2_0}\right)^2\psi_0 = m^2_{0\infty}c^4_0\left(1+\frac{E_{Schr}}{m_{0\infty}c^2_0}\right)^2\psi_0 \ . \tag{25}$$

*(correct relativistic quantum mechanical equation written based on his overall mass expression, to be set to a constant, on a given energy level)*

Note that this equation is rigorous; thus so, at this stage, is $E_{Schr}$.

Now, let us arrange the brackets at both sides of this equation, noting that both $Ze^2/(m_{0\infty}r_0c^2_0)$ and $E_{Schr}/(m_{0\infty}c^2_0)$ are generally very small as compared to unity.

Thus:

$$-\frac{1}{2m_{0\infty}}\hbar^2\nabla^2\psi_0 - \frac{Ze^2}{r_0}\psi_0 = E_{Schr}\psi_0 \ , \tag{26-a}$$

or the same

$$-\frac{1}{2m_{0\infty}}\hbar^2\nabla^2\psi_0 + V(r_0)\psi_0 = E_{Schr}\psi_0 \quad \text{(c.q.f.d.)}, \tag{26-b}$$

*[classical Schrodinger Equation derived from the correct relativistic equation, with the approximation that* $Ze^2/(m_{0\infty}r_0c_0^2)$ *and* $E_{Schr}/(m_{0\infty}c_0^2)$ *are very small as compared to unity]*

where $V(r_0)$, as usual denote the potential energy $-Ze^2/r_0$.

## 5.2. Correct, Simple, Relativistic Quantum Mechanical Equation

Via taking into account the terms we have neglected in Eq.(25), we arrive at an equation which can be considered well equivalent to Dirac's relativistic equation, incorporating though the mass decrease of the bound electron:

$$-\frac{\hbar^2}{2m_{0\infty}}\nabla^2\psi_0=\left(E_R+\frac{1}{2}\frac{E_R^2}{m_{0\infty}c_0^2}+\frac{Ze^2}{r_0}-\frac{1}{2}\frac{Z^2e^4}{r_0^2m_{0\infty}c_0^2}\right)\psi_0, \tag{27}$$

*(correct relativistic quantum mechanical equation, derived from the correct relativistic equation omitting the spin-orbit interaction)*

where $E_R$ is the "rigorous total energy" (diminished by the energy content of the rest mass), i.e.

$$E_R=E_{Total}-m_{0\infty}c_0^2\ ; \tag{28}$$

this is the same definition as the one we provided via Eq.(22), with the difference that $E_R$ of Eq.(27), now points to the "rigorous result" [whereas $E_{Schr}$ of Eq.(26-a) constituted a first approximation to it].

For the reason which will become clear soon, we will write Eq.(27) in a simpler form:

$$-\frac{\hbar^2}{2m_{0\infty}}\nabla^2\psi_0+U_S\psi_0=E_S\psi_0\quad , \tag{29}$$

where we define $E_S$ and $U_S$ as

$$E_S=E_R+\frac{E_R^2}{2m_{0\infty}c_0^2}\ , \tag{30-a}$$

$$U_S=-\frac{Ze^2}{r_0}+\frac{1}{2}\frac{Z^2e^4}{r_0^2m_{0\infty}c_0^2}\quad . \tag{30-b}$$

Note that once $E_S$ is known, interestingly there are two roots $E_R$, corresponding to it.

## 5.3. Equation Equivalent to that of Dirac's Relativistic Equation

Above we preferred to define the quantities $E_S$ and $U_S$, for the following reason:

On the basis of the classical Schrodinger Equation [Eq.(26-a)], $E_R$ [from Eq.(30-a)] would be straight $E_S$; thus in this case $E_S$, $E_R$ and $E_{Schr}$ are all the same quantity. Through a better approach, but where we neglect the second term of the RHS of Eq.(30-b); $E_R$ of Eq.(30-a) becomes

$$E_R = E_{Schr} - \frac{E_R^2}{2m_{0\infty}c_0^2} \cong E_{Schr}\left(1 - \frac{E_{Sch}}{2m_{0\infty}c_0^2}\right) = E_{Schr}\left(1 + \frac{Z^2\alpha_{fs}^2}{4n^2}\right), \tag{31}$$

*(in the case where we neglect the potential energy alteration due to the mass decrease of the bound electron)*

where n is the principal quantum number.

$E_R$ can further be refined via calculating the two roots of Eq.(30-a):

$$E_R = m_{0\infty}c_0^2\left(-1 \pm \sqrt{1 - \frac{Z^2\alpha_{fs}^2}{n^2}}\right). \tag{32-a}$$

Thus the root $E_{R(+)}$ for the positive sign yields

$$E_{R(+)} = m_{0\infty}c_0^2\left(-1 + \sqrt{1 - \frac{Z^2\alpha_{fs}^2}{n^2}}\right), \tag{32-b}$$

which strikingly turns out to be the exact Dirac solution were (the second term at the RHS of Eq.(31) neglected, and) the spin-orbit interaction not taken into account [1].

We can right away estimate that, in this case the magnitude of $E_R$ is larger than that of the corresponding Schrodinger eigenvalue, as much as $E_R^2/2m_{0\infty}c_0^2$ (yielding a downward shift) [cf. Eq.(23-a)].

But, what we just have come to neglect essentially, is the effect of the mass decrease of the bound electron, altering the potential energy input to the classical Schrodinger Equation. In other words, the RHS of Eq.(31) or Eq.(32-b) represents the corrected Schrodinger eigenvalue based on only the relativistic effect due to the motion of the electron.

Recall yet that, along the line we pursue, Eq.(31) is incorrect, since Eq.(30-a) should be considered together with the RHS of Eq.(30-b) including not only the first term, but the second term, as well. This is what we will undertake below.

### 5.4. The Shift Discovered Along our Approach -Contrary to Dirac's Prediction- is Upward, and not Downward, and is Twice as Important as that of Dirac

In the case we consider Eq.(30-a), together with Eq.(30-b) as a whole, the eigenvalue $E^*_{Schr}$, in conformity with the classical Schrodinger Equation's eigenvalue still bears the form displayed by

$$E^*_{Schr} = E^*_R + \frac{E_R^{*2}}{2m_{0\infty}c_0^2} \tag{33}$$

but now on the basis of the perturbed potential, defined by the second term at the RHS of Eq.(30-b).

The quantity $E^*_{Schr}$ is to be compared with the eigenvalue $E_{Schr}^{Unpr}$ of the unperturbed classical Schrodinger Equation, i.e. [cf. Eq.(23-a)]

$$E_{Schr}^{Unpr} = -m_{0\infty} c_0^2 \frac{Z^2 \alpha_{fs}^2}{2n^2} \ . \qquad (34)$$

But, evidently

$$E_R^* \cong E_{Schr}^{Unpr} \ . \qquad (35)$$

Thus the perturbed Schrodinger eigenvalue $E_{Schr}^*$ of Eq.(21) can be expressed as

$$E_{Schr}^* \cong E_{Schr}^{Unpr} + \frac{(E_{Schr}^{Unpr})^2}{2m_{0\infty} c_0^2} = E_{Schr}^{Unpr} \left( 1 - \frac{Z^2 \alpha_{fs}^2}{4n^2} \right) \ . \qquad (36)$$

*(in the case where we take into account the potential energy alteration due to the mass decrease of the bound electron, along the relativistic dilation of the mass of the electron, in motion)*

Thus, it is question of – not a downward, but – an upward shift, with regards to $E_{Schr}^{Unpr}$, and this as much as $E_{Schr}^{Unpr} Z^2 \alpha_{fs}^2/(4n^2)$.

Recall that, as we discussed above, the upward shift we figure out, does not interfere with the successful quantum electrodynamical predictions, such as the prediction of Lamb shift (since here it is question of relative distances between energy levels); though, our finding as stated, should be expected to remedy the discrepancies between theory and experiments.

Anyhow, it is amazing that (supposing the proton is fixed), one can obtain the total energy of the electron (diminished by the energy content of its proper mass), embodying both its proper mass decrease due to binding, and the relativistic effect arising from its motion around the nucleus, from a simple equation just bearing the form of the quantum mechanical description written in the non-relativistic case, where merely the classical eigenvalue $E_{Schr}$ is altered by $-E_{Schr}^2/(2m_{0\infty} c_0^2)$ while the classical potential energy is altered by $Z^2 e^4/(r_0^2 2m_{0\infty} c_0^2)$, [cf. Eqs. (27)].

*

The foregoing discussion allows us to consider Eq.(1) as a basis, instead of Eq.(13), in order to develop a straightforward relativistic quantum mechanical description with regards to gravitation, to be visualized whenever it may be necessary, and mostly for very strong fields.

This is what we undertake next.

## 6. Full Quantum Mechanical Deployment of Our Approach With Regards to Gravitation

Here again we will consider for simplicity just two objects, one very massive and the other one is very light, so that the former can be assumed throughout the motion at rest vis-à-vis the very light object.

Let us then evaluate the difference $\mathcal{D}$ based on the usual relativistic definition of the momentum $p(r_0)$ of the electron on the orbit, i.e.

$$p(r_0) = m(r_0) v_0(r_0) \ ; \qquad (37)$$

note that here $m(r_0)$ is the overall mass, defined along our Eq.(11) [along with Eqs. (12-a) and (12-b)].

Thus

$$\mathcal{D} = m^2(r_0)c_0^4 - m_0^2(r_0)c_0^4 = \left(\frac{m_{0\infty}^2 c_0^4}{1-\frac{v_0^2}{c_0^2}} - m_{0\infty}^2 c_0^4\right)\left(\exp - \alpha_0 e^{-\alpha}\right)^2 = p^2(r_0)c_0^2 \ . \tag{38}$$

Recall on the other hand that $m(r_0)c_0^2$ is nothing else, but (in the relativistic sense) the total energy $E_{Total}$. Then

$$p^2(r_0)c_0^2 = E_{Total}^2 - m_{0\infty}^2 c_0^4 \left(\exp - \alpha_0 e^{-\alpha}\right)^2 \ , \tag{39}$$

or

$$p^2(r_0)c_0^2 + m_{0\infty}^2 c_0^4 \left(\exp - \alpha_0 e^{-\alpha}\right)^2 = E_{Total}^2 \ . \tag{40}$$

We can compose the correct relativistic quantum mechanical equation for a stationary case, to replace the classical Klein-Gordon Equation, here again, via the usual quantum mechanical symbolisms of the momentum, and the energy, i.e.

$$p(r_0) = -i\hbar\nabla \ , \tag{41}$$

$$E_{Total}^2 = -\hbar^2 \frac{\partial^2}{\partial t_0^2} \ . \tag{42}$$

Thus

$$-\hbar^2\nabla^2\Phi(r_0,t_0)c_0^2 + m_{0\infty}^2 c_0^4\left(\exp - \alpha_0 e^{-\alpha}\right)^2\Phi(r_0,t_0) = -\hbar^2\frac{\partial^2\Phi(r_0,t_0)}{\partial t_0^2} \ , \tag{43}$$

*(correct relativistic equation written out of the "overall mass" expression, embodying the mass deficiency of the bound electron instead of the classical Klein-Gordon Equation)*

where $\Phi(r_0,t_0)$ denotes the space and time dependent wave function; note that by "correct relativistic equation", we mean, "equation taking into account the mass deficiency of the bound electron, next to the mass dilation of it, due to its motion".

Eq.(40), for a stationary case should be written as

$$-\hbar^2\nabla^2\psi(r_0)c_0^2 + m_{0\infty}^2 c_0^4\left(\exp - \alpha_0 e^{-\alpha}\right)^2\psi(r_0) = E_{Total}^2\psi(r_0) \ , \tag{44}$$

*(correct relativistic eigenvalue-eigenfunction equation written with regards to gravitation)*

where $\psi(r_0)$, in short $\psi_0$, is the eigenfunction of the relativistic description of the stationary case in consideration; $E_{Total}$ then becomes the corresponding eigenvalue. We will see below that Eq.(44) well reduces to an equation involving $E_{Total}$ instead of $E_{Total}^2$, so that at the level of this equation, one does not have to question the classical meaning of the wave function $\psi(r_0)$.

### 6.1.Derivation of a Schrodinger Equation for Gravitation Based on the Present Approach

We can easily derive a Schrodinger Equation, based on Eq.(44). Thus, note first that the eigenvalue $E_{Schr}$ yeld by the Schrodinger Equation is less than $E_{Total}$ as much as $m_{0\infty}c_0^2$, the energy equivalent of the rest mass of the electron, i.e.

$$E_{Schr} = E_{Total} - m_{0\infty}c_0^2 \ . \tag{45}$$

*(definition we make in conformity*
*with the eigenvalue of the Schrödinger Equation)*

Recall that $E_{Total}$ is, by definition [cf. Eq.(4)] a positive quantity. Therefore $E_{Schr}$ here again, is a negative quantity, and as a first approach, can be quickly predicted to be

$$E_{Schr} \cong m_{0\infty}c_0^2(1-\frac{1}{2n^2}Z^2\alpha^2) - m_{0\infty}c_0^2 = -m_{0\infty}c_0^2\frac{Z^2\alpha^2}{2n^2} \quad \text{(for small } Z\text{'s)}\,, \tag{46-a}$$

where we define the dimensionless quantity $Z$ as

$$Z = \frac{G\mathcal{M}m_{0\infty}}{e^2} \ , \tag{46-b}$$

in order to be able to keep the same formalism as that we used throughout the previous section. (Recall that the gravitational quantities were defined above.)

Eq.(45) thus can be rewritten as

$$E_{Total} = m_{0\infty}c_0^2\left(1+\frac{E_{Schr}}{m_{0\infty}c_0^2}\right) . \tag{47}$$

Via Eq.(45), Eq.(44) will be written as

$$-\hbar^2\nabla^2\psi_0 c_0^2 + m_{0\infty}^2 c_0^4 \left(\exp-\alpha_0 e^{-\alpha}\right)^2\psi_0 = m_{0\infty}^2c_0^4\left(1+\frac{E_{Schr}}{m_{0\infty}c_0^2}\right)^2\psi_0 \ . \tag{48}$$

*(correct relativistic quantum mechanical equation written with*
*regards to gravitation based on the overall mass expression)*

Note that this equation is rigorous; thus so, at this stage, is $E_{Schr}$.

Now, let us arrange the brackets at both sides of this equation, noting that both $\alpha_0$ and $E_{Schr}/(m_{0\infty}c_0^2)$ are generally very small as compared to unity.

Thus:

$$-\frac{1}{2m_{0\infty}}\hbar^2\nabla^2\psi_0 - G\frac{\mathcal{M}m_{0\infty}}{r_0}\psi_0 = E_{Schr}\psi_0 \quad , \tag{49-a}$$

or the same

$$-\frac{1}{2m_{0\infty}}\hbar^2\nabla^2\psi_0 + V(r_0)\psi_0 = E_{Schr}\psi_0 \quad \text{(c.q.f.d.)}\,, \tag{49-b}$$

*[classical Schrodinger Equation derived from the correct relativistic equation, with the approximation that* $-G\mathcal{M}m_{0\infty}/(r_0 m_{0\infty}c_0^2)$ *and* $E_{Schr}/(m_{0\infty}c_0^2)$ *are very small as compared to unity]*

where $V(r_0)$ now denotes the classical gravitational potential energy $-G\mathcal{M}m_{0\infty}/r_0$.

### 6.2. Correct, Simple Relativistic Quantum Mechanical Equation with Regards to Gravitation

Via taking into account the terms we have neglected in Eq.(25), but neglecting (without any loss of generality), terms higher than the second order term in the Taylor expansion of the exponential term, we arrive at an equation which can be considered well equivalent to Dirac's relativistic equation, written for gravitation:[†††]

$$-\frac{\hbar^2}{2m_{0\infty}}\nabla^2\psi_0=\left(E_R+\frac{1}{2}\frac{E_R^2}{m_{0\infty}c_0^2}+\frac{G\mathcal{M}m_{0\infty}}{r_0}-\frac{G^2\mathcal{M}^2m_{0\infty}^2}{r_0^2m_{0\infty}c_0^2}\right)\psi_0, \tag{50}$$

*(correct relativistic quantum mechanical equation for gravitation derived from the correct relativistic equation)*

where $E_R$ is the rigorous total energy (diminished by the energy content of the rest mass):

$$E_R=E_{Total}-m_{0\infty}c_0^2\ ; \tag{51}$$

this is the same definition as the one we provided via Eq.(45), with the difference that $E_R$ of Eq.(50), now points to the rigorous result [whereas $E_{Schr}$ of Eq.(49) constituted a first approximation to it].

We can write Eq.(50) in a simpler form:

$$-\frac{\hbar^2}{2m_{0\infty}}\nabla^2\psi_0+U_S\psi_0=E_S\psi_0\quad , \tag{52}$$

where we define $E_S$ and $U_S$ as

$$E_S=E_R+\frac{E_R^2}{2m_{0\infty}c_0^2}\ , \tag{53-a}$$

$$U_S=-\frac{G\mathcal{M}m_{0\infty}}{r_0}+\frac{G^2\mathcal{M}^2m_{0\infty}^2}{r_0^2m_{0\infty}c_0^2}\ . \tag{53-b}$$

Once again, our approach, allowed us to deploy, in a straightforward way, the quantum mechanical version of it. It is still amazing that (supposing $\mathcal{M}$ is fixed), one can obtain the total energy of the light mass (diminished by the energy content of its proper mass at infinity), embodying both its proper mass decrease due to binding, and the relativistic effect arising from its motion around $\mathcal{M}$, from a simple equation just bearing the form of the quantum mechanical description written in the non-relativistic case, where merely the classical eigenvalue $E_R$ is altered by $-E_R^2/(2m_{0\infty}c_0^2)$ while the classical potential energy is altered by $G^2\mathcal{M}^2m_{0\infty}^2/(r_0^2m_{0\infty}c_0^2)$.

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[†††] The rigorous equation is

$$-\frac{\hbar^2}{2m_{0\infty}}\nabla^2\psi_0=\left\{E_R+\frac{1}{2}\frac{E_R^2}{m_{0\infty}c_0^2}+\frac{1}{2}\left[1-\exp-(2\alpha_0e^{-\alpha})\right]\right\}\psi_0, \tag{i}$$

together with [cf. Eq.(12-b)]

$$\alpha=\alpha_0e^{-\alpha}. \tag{ii}$$

### 6.3.A Quick Estimation of the Outcome

We can develop a feeling about the outcome of Eq.(50). For this, let us consider Eqs. (3), (5) and (6-a), for a circular orbit

$$\frac{G\mathcal{M}}{r^2} m_{0\infty} e^{-\alpha} \sqrt{1-\frac{v_0^2}{c_0^2}} = \frac{m_{0\infty} e^{-\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \frac{|d\underline{v}_0|}{dt} = m(r)\frac{v_0^2}{r} \tag{54}$$

Next, we should write the Bohr's postulate in an appropriate way. For this, it would be useful to recall de Broglie's doctorate thesis [2].

Along this approach, Bohr's postulate reduces to the expression of de Broglie wave (associated with the electron's motion), confined (thus, like any classical wave, bound to be quantized), on the orbit. But the momentum of the electron entering the de Broglie's relationship, must be the local relativistic momentum, where then, the "mass" should be taken as the overall mass, we defined at the stage of Eq.(1).

In fact, the first author was recently able to derive the de Broglie relationship, based on the main idea presented herein, i.e. the mass deficiency delineated by the bound particle, regarding either an electric field or a gravitational field, though inevitably inducing an interaction, at tachyonic speeds [37,38].[+++]

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[+++] Based on just the energy conservation law, we have come to figure out that, the gravitational motion depicts a "rest mass variation", throughout [cf. Eqs. (4) and (5)]. Consider for instance, the case of a planet in an elliptic motion around the sun; according to our approach, an "infinitesimal portion of the rest mass" of the planet, is transformed into "extra kinetic energy", as the planet approaches the sun, and an "infinitesimal portion of the kinetic energy" of it, is transformed into "extra rest mass", as it slows down away from the sun, while the total relativistic energy remains constant throughout. The same applies to a motion driven by electrical charges.

One way to conceive the mass exchange phenomenon we disclosed, is to consider a "jet effect". Accordingly, an object on a given orbit, through its journey, must eject mass to accelerate, or must pile up mass, to decelerate.

The speed U of the jet, strikingly points to the de Broglie wavelength $\lambda_B$, unavoidably coupled with the period of time $T_0$, inverse of the frequency $v_0$, delineated by the electromagnetic energy content $hv_0$, of the object; $hv_0$ is originally set by de Broglie, equal to the total mass $m_0$ of the object (were the speed of light taken to be unity). This makes that, the "jet speed" becomes $U = \lambda_B / T_0 = c_0^2 \sqrt{1-v_0^2/c_0^2} / v_0$.

This result seems to be important in many ways. Amongst other things, it may mean that, either gravitationally interacting macroscopic bodies, or electrically interacting microscopic objects, sense each other, with a speed much greater than that of light, and this, in exactly the same way. Furthermore, it induces immediately the quantization of the "gravitational field", in exactly the same manner, the "electric field" is quantized.

Thus we are to propose

$$2\pi m(r)v_0 r = nh\,,\ \ n=1, 2, 3, \dots . \tag{55}$$

*(de Broglie's relationship rewritten by the author, instead of Bohr's postulate, taking into account the overall mass decrease of the bound electron, confined on the given orbit)*

The two unknowns $v_{0n}$ and $r_{0n}$ (to be associated with the nth quantum level), for circular orbits can then be found to be [27]

$$v_{0n} = \frac{1}{\sqrt{1+\dfrac{n^2h^2c_0^2}{4\pi^2\left(G\mathcal{M}\,m_{0\infty}\,e^{-\alpha}\right)^2}}}\,c_0\,, \tag{56}$$

and

$$r_n = \frac{G\mathcal{M}e^{-\alpha}}{c_0^2}\left[1+\frac{n^2h^2c_0^2}{4\pi^2\left(G\mathcal{M}\,m_{0\infty}\,e^{-\alpha}\right)^2}\right]. \tag{57}$$

Here, the term in $n^2$, next to unity, in between brackets, for common celestial bodies, is infinitely small. This makes that the velocity of the rotating object for n=1, as well as for enormously high quantum numbers, easily attains the speed of light. Its orbit radius r, accordingly, becomes

$$r \cong \frac{G\mathcal{M}}{c_0^2}\ ; \tag{58}$$

r becomes sensitive to n, only around values of n, satisfying the relationship

$$\frac{n^2h^2c_0^2}{4\pi^2\left(G\mathcal{M}\,m_{0\infty}\,e^{-\alpha}\right)^2} \approx \text{unity}\ . \tag{59}$$

For a binary system, each of the stars bearing about the mass of our sun, one has

$$n \approx \frac{2\pi G\mathcal{M}\,m_{0\infty}}{hc_0} \cong \frac{6.28x6.67x10^{-11}x(2x30^{30})^2}{6.62x10^{-34}x3x10^8} \approx 8x10^{75}\,, \tag{60}$$

for which r becomes only twice as that furnished by Eq.(58), and the rotational speed of the bound object is about $\sqrt{2}c_0/2$.

Based on Eq.(59), we can further have an estimate about the mass m of relatively light objects, gravitationally bound to each other, on a quantized state. Thus let us consider a binary system, at n=1. Thus

$$m = \sqrt{\frac{hc_0}{2\pi G}} = \frac{6.62x10^{-34}x3x10^8}{6.28x6.67x10^{-11}} \approx 2.x10^{-8}\,\text{kg}\ , \tag{61}$$

which interestingly, turns to be the Planck mass.

The size of such a system based on Eq.(57), becomes

$$r \approx 2\frac{G\mathcal{M}}{c_0^2} = 2\sqrt{\frac{Gh}{2\pi c_0^3}} \approx 3.2x10^{-35}\,\text{m}\ , \tag{62}$$

which is twice the Planck length.

The total energy of such a massive atomic system, can further be estimated quickly, noting the following points:

i) It seems to be question of atoms made of Planck masses, bound to each other gravitationally. We would like to call such a system a gravitational atom. Thus, suppose, it is question of two Planck masses, bound to each other gravitationally.
ii) The total energy of such a system, as well as energies related to the transitions between states (just like in the hydrogen atom), is proportional to the reduced mass of the system, which evidently points to the order of magnitude of Planck mass.
iii) Consequently, the total energy of the gravitational atom, as well as energies related to the transitions between its states must be greater than those of hydrogen atom, as much as [the mass of the Planck mass] / [the mass of the electron], i.e. about $2x10^{-8}$ / $10^{-30}$, or $2x10^{22}$.

The energies in question in hydrogen atom, are few electron volts; this frame yields, energies for a gravitational atom, in the order of $10^{22}$ ev, which may clarify, the origin of so very high energies encountered amongst cosmic rays.

Note that our finding about the gravitational quantum states, necessarily induces the tendency of an object in motion at a high energy level, to move to lower energy levels. Here may be a clue for the creation of gravitational waves, in fact, according to the present approach, nothing but electromagnetic waves we expect to be emitted via transitions in question.

## 7. General Conclusion

The energy conservation law, in the broader sense of the concept of energy embodying the relativistic mass & energy equivalence, has been a common practice, chiefly nuclear scientists make use of.

Yet amazingly, besides it is not applied to gravitational binding, it also seems to be overlooked for atomic and molecular descriptions.

Thus, via Newton's law of gravitation between two static masses, and the energy conservation law, in the broader sense of the concept of energy embodying the relativistic mass & energy equivalence, on the one side, and quantum mechanics, on the other side, we have shown that one is able to derive the end results aimed by the General Theory of Relativity.

Likewise, we proposed to reformulate the relativistic quantum mechanics on the basis of Coulomb Force, but assumed to be valid only for static electric charges.

When bound though, the total mass, or the same, the overall energy of the electric charges at infinity, must be decreased as much as the binding energy coming into play.

The frame we draw amazingly describes in an extreme simplicity, both the atomic scale, and the celestial scale, on the basis of respectively, Coulomb Force (written for static electric charges), and Newton Force (written for static masses).

The decrease of the mass of the bound particle, via Theorem 1, this time applied to the internal dynamics of the bound particle, changes both the period of time and the size of space to be associated with the internal dynamics in question, in exactly the same manner, at either an atomistic scale or a celestial scale.

Thus, the frame we draw, yields exactly the same metric change and quantization, at both scales.

One important conclusion is that the metric change nearby a nucleus in regards to a charge is exactly the same as the metric change nearby a celestial body with respect to a mass.

For simplicity, we made our presentation on the basis of just two particles, one very heavy, the other one very light, at both scales, without though any loss of generality.

All of our predictions, perfectly agree with the experimental results.


## Acknowledgement

The authors would like to thank to Professor A. Kholmetskii, Professor S. Kocak, Professor S. Ozdemir, Professor V. Andreev, and Professor D. Yu. Tsipenyuk, for many hours of discussion. The authors would further like to extend their gratitude to Professor P. de Haas; he is the one who brought Mie's and Nordström's original articles (written in German), to the authors' attention, together with valuable effort in regards to their evaluation. The authors are further indebted to the Referee who very carefully reviewed the article, and proposed many useful comments.


## References


[1] T. Yarman, V. B. Rozanov, Physical Interpretations of the Theory of Relativity Conference, Bauman Moscow State Technical University, 4 – 7 July 2005.

[2] T. Yarman, V. B. Rozanov, N. Zaim, F. Ozaydin, The Architecture of Triatomic and Polyatomic Molecules, Physical Interpretation of The Relativity Theory, Moscow, 4 – 7 July 2005.

[3] T. Yarman, The End Results of the General Theory of Relativity, via Just Energy Conservation and Quantum Mechanics, Physical Interpretation of The Relativity Theory, Moscow, 3 – 7 July 2005.

[4] T. Yarman, A Novel Approach to the Bound Muon Decay Rate retardation: Metric Change Nearby the Nucleus, Physical Interpretation of The Relativity Theory, Moscow, 3 – 7 July 2005.

[5] T. Yarman, The General Equation of Motion Via the Special Theory of Relativity and Quantum Mechanics, AFBL, Volume 29, Numéro 3, 2004.

[6] T. Yarman, Invariances Based on Mass and Charge Variation, Manufactured by Wave Mechanics, Making up The Rules of Universal Matter Architecture, Chimica Acta Turcica, Vol.27, 1999.

[7] T. Yarman, An Essential Approach to the Architecture of Diatomic Molecules. Part I. Basic Theory, Optics & Spectroscopy, Volume 97 (5), 2004 (683).
[8] T. Yarman, An Essential Approach To The Architecture of Diatomic Molecules, Part II: How Size, Vibrational Period of Time And Mass Are Interrelated?, Optics & Spectroscopy, Volume 97 (5), 2004 (691).
[9] T. Yarman, Elucidation of the Complete Set of $H_2$ Electronic States' Ground Vibrational Data, International Journal of Hydrogen Energy, 29, 2004 (1521).
[10] L. de Broglie, Recherches Sur La Théorie Des Quanta, Annales de Physique, Annales de Physique, 10e Série, Tome III, 1925.
[11] L. D. Landau & E. M. Lifshitz, The Classical Theory of Fields, Chapter 10, Paragraph 88, Butterworth & Heinemann.
[12] H. Yilmaz, Einstein, the Exponential Metric, and a Proposed Gravitational Michelson-Morley Experiment, Hadronic Journal, 2: 997, 1979.
[13] H. Yilmaz, Towards a Field Theory of Gravity, Nuovo Cimento, 107B: 941, 1992.
[14] G. Mie, Ann. Phys. 37, 511-534, 1912.
[15] G. Mie, Ann. Phys. 39, 1 - 40, 1912.
[16] G. Mie, Ann. Phys. 37, 1 - 66, 1913.
[17] G. Nordström, Phys. Zeitschr., 8, 1126-1129, 1912.
[18] G. Nordström, Ann. Phys., 40, 856-868, 1913.
[19] E. P. H. de Haas, The Combination of de Broglie's Harmony of the Phases and Mie's Theory of Gravity in a Principle of Equivalence for Quantum Gravity, AFLB, Volume 29, Numéro 4, 2004.
[20] D. Yu. Tsipenyuk, V. A. Andreev, Physical Interpretations of the Theory of Relativity Conference, Bauman Moscow State Technical University, 4 – 7 July 2005.
[21] T. Yarman, The End Results of General Theory of Relativity, via Just Energy Conservation And Quantum Mechanics (Extended Version of References 3 and 5): Communication to Professor A. A. Logunov, Director of Particle Physics Laboratory, Protvino, Russia, 2005. Article Under Review in Foundation of Physics.
[22] W. E. Lamb, Jr., R. C. Retherford, Fine Structure of the Hydrogen Atom by a Microwave Method, Physical Review, 72, 3, 1947.
[23] W. V. Houston, Physical Review, 51, 446, 1937.
[24] R. C. Williams, Physical Review, 54, 558, 1938.
[25] S. Pasternack, Physical Review, 54,1113, 1938.
[26] T. Yarman, APS Meeting, University of California, Riverside, 26-31 August, 2004.
[27] T. Yarman, Mass Deficiency Correction to The Relativistic Quantum Mechanical Approach: Metric Change Nearby The Nucleus, Article Under Review in Foundation of Physics.
[28] W. Weber, Annalen der Physik 73, 1848.
[29] W. Weber, Wilhelm Weber's Werke, Springer, Berlin, 1892-1894.

[30] H. V. Helmholtz, Philos. Mag. 44, 1872.
[31] J. C. Maxwell, A Treatise of Electricity and Magnetism (Chapter 23, Volume 2), Dover, New York, 1954.
[32] V. F. Mikhailov, The Action of an Electrostatic Potential on the Electron Mass, Annales de la Fondation Louis de Broglie, Volume 24, 1999.
[33] E. Schrödinger, Der Steige Übergang von der Mikro- zur Makromechanik, Die Naturwissenschaften 14, 664, 1926.
[34] E. Schrödinger, Quantisierung als Eigenwertproblem, No.6, Band 79, Vierte Folge, pages 489 - 527, Annalen der Physik, 1926.
[35] E. Schrödinger, Quantisierung als Eigenwertproblem, No.18, Band 81, Vierte Folge, pages 109 - 139, Annalen der Physik, 1926.
[36] E. Schrödinger, An Undulatory Theory of the Mechanics of Atoms and Molecules, Second Series, Volume 28, No. 6, Physical Review, December 1926.
[37] T. Yarman, Tachyonic Interaction or the Same de Broglie Relationship, Part I: Electrically Bound Particles, Article Under Review in Foundation of Physics.
[38] T. Yarman, Tachyonic Interaction or the Same de Broglie Relationship, Part II: Gravitationally Bound Particles, Article Under Review in Foundation of Physics.