The Mass Deficiency Correction to Classical and Quantum Mechanical Descriptions: Alike Metric Change and Quantization Nearby an Electric Charge, and a Celestial Body
Part I: A New General Equation of Motion for Gravitationally, or Electrically Bound Particles

Tolga Yarman,[*] Vladislav B. Rozanov[†]

Abstract

Via Newton's law of gravitation between two static masses exculsively, and the energy conservation law, in the broader sense of the concept of energy embodying the relativistic mass & energy equivalence, on the one side, and quantum mechanics, on the other, one is able to derive the end results aimed by the General Theory of Relativity. The energy conservation law, in the broader sense of the concept of "energy" embodying the relativistic mass & energy equivalence, is anyway a common practice, chiefly nuclear scientists make use of. Yet amazingly, besides it is not applied to gravitational binding, it also seems to be overlooked for atomic and molecular descriptions. Thus herein, next to the reestablishment of celestial mechanics, we propose to reformulate the relativistic quantum mechanics on the basis of Coulomb Force, but assumed to be valid only for "static electric charges"; when bound though, the rest mass of an electric charge, must be decreased as much as the "binding energy" it delineates.

Along the same line, one can remarkably derive the de Broglie relationship, for both electrically and gravitationally interacting objects. Our results, furthermore, seem capable to clarify the results of an experiment achieved long time ago, at the General Physics Institute of the Russian Academy of Sciences, but left unveiled up to now.

The frame we draw amazingly describes in an extreme simplicity, both the atomic scale, and the celestial scale, on the basis of respectively, Coulomb Force (written for static electric charges, exclusively), and Newton Force (written for static masses, exclusively), in exactly the same manner. Our approach yields precisely the same metric change and quantization, at both scales, in question.

For simplicity, the presentation is made based on just two particles, one very massive, the other one very light, at both scales, without though any loss of generality.

Our predictions, perfectly agree with all available experimental results.

Keywords: Mass Deficiency, Gravitation, Quantization, Electric Charge, Metric Change

[*] Department of Engineering, Okan University Istanbul, Turkey & Savronik, Eskisehir, Turkey (tyarman@gmail.com)

[†] Laser Plasma Physics Theory Department, Lebedev Institute, Moscow, Federation of Russia (rozanov@sci.lebedev.ru)

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1. Introduction

This article is prepared based on the presentation the authors made at the PIRT (Physical Interpretations of the Theory of Relativity), held in the Summer of 2005 in Moscow [1]. The main idea consists in the "decrease of the mass" of the bound particle, owing to the energy conservation law. A unique matter architecture cast yeld by both "quantum mechanics", and the "Special Theory of Relativity", independently from each other, as we will soon unveil, tells how the metric is accordingly altered. The fact that, "quantum mechanics" and the "Special Theory of Relativity" frame, independently from each other, the same matter architecture, thus the same metric change, based on the mass decrease we undertake in this article, tacitly delineates the organic interrelation between the two disciplines. Three other articles presented by the first author to the mentioned PIRT Conference, along with the first one, draw a complete picture, covering both the micro atomistic world, and the macro celestial world, in utterly similar terms [2,3,4]. The bound particle in consideration may be bound to any field it interacts with. This allows us to treat all fields in the same manner (and not restrict us to consider the gravitational field as a privileged field, a pitfall necessarily induced by the General Theory of Relativity).

Below we first tackle with the "gravitational field" (Section 2), then with the "electric field" (Section 3). This constitutes the content of Part I, condensed into a partial conclusion (Section 4). In both of the cases in question, "quantization" follows immediately and in exactly the same way; this constitutes the content of Part II (Sections 5 and 6). Next a general conclusion is drawn (Section 7).

2. Mass Decrease of a Gravitationally Bound Particle: The End Results of the General Theory of Relativity, via Just Newton's Law of Gravitation, Energy Conservation and Quantum Mechanics

In a previous work [5], a whole new approach to the derivation of the Newton's Equation of Motion was achieved; this, well led to the end results of the General Theory of Relativity, were the velocity of the object at hand, not considered negligible as compared to the velocity of light. Thus, one starts with the following postulate, in fact nothing else, but the law of conservation of energy, though in the broader relativistic sense of the concept of "energy".

Postulate: The rest mass of an object bound to a celestial body amounts to less than its rest mass measured in empty space, the difference being, as much as its binding energy vis-à-vis the gravitational field of concern.

A mass deficiency conversely, via quantum mechanics (whose basis, i.e. the wave equation, together with, in a way the de Broglie relationship, is already fully consistent with the Special Theory of Relativity), yields the "stretching of the size" of the object at hand, as well as the "weakening" of its internal energy, via quantum mechanical theorems proven elsewhere [6,7,8,9]. We summarize them herein.

Theorem 1: In a "real wave-like description" (thus, not embodying artificial potential energies), if the masses m_{i0}, i = 1, ..., I of different constituents involved by the object, are all over multiplied by the arbitrary number χ, then concurrently, a) the total energy E_0 associated with the given clock's internal motion of the object, is increased as much, or the same, the period T_0 of the motion associated with this energy, is decreased as much, and b) the characteristic length or the size $\mathcal{R}_0$ to be associated with the given clock's motion [10] of the object, contracts as much; in mathematical words,[‡]

$$[(m_{i0}, i = 1, ..., I) \rightarrow (\chi m_{i0}, i = 1, ..., I)] \Rightarrow \{ [E_0 \rightarrow \chi E_0], [T_0 \rightarrow \frac{T_0}{\chi}], [\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\chi}] \}$$

This, together with the above postulate, yields at once the next two theorems.

Theorem 2: A wave-like clock in a gravitational field, retards via quantum mechanics, due to the mass deficiency it develops in there, and this, as much as the binding energy it displays in the gravitational field; at the same time and for the same reason, the space size in which it is installed, stretches as much.

Theorem 3: A wave-like clock interacting with any field, electric, nuclear, gravitational, or else (without loosing its "identity"), retards as much as its binding energy, developed in this field; at the same time and for the same reason, the space size in which it is installed, stretches as much.

This can further be grasped rather easily, as follows. The mass deficiency the wave-like object displays in the gravitational field (or in fact, any field with which it interacts), weakens its internal dynamics as much, which makes it slow down. Thence, one arrives at the principal results, stated above. In order to calculate the binding energy of concern, we make use of the classical Newtonian gravitational attraction law, yet with the restriction that, it can only be considered for "static masses". Luckily we are able to derive the $1/r^2$ dependency of the "classical gravitational force" between "two static masses", here again, based on just the Special Theory of Relativity [5]. This can be achieved easily by noting that the quantity [force] x [mass] x $[\text{distance}]^3$ is Lorentz

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[‡] Note that as the "overall mass" of the object increases by the arbitrary factor χ, and this already at rest, its internal dynamics speeds up as much; or the same, its de Broglie wave-like frequency is increased as much [2]. One can show that, only if such a characteristic is drawn, the internal dynamics slows down as much, in the case where the object is brought to a uniform translational motion, χ then becoming the usual Lorentz dilation factor.

invariant.[§] (In fact, dimensionally speaking, it amounts to the square of the Planck Constant, which in return is Lorentz invariant.) On the other hand, it is known that the electric charges are Lorentz invariant. (If not, say in excited atoms, energetic electrons would exhibit electric charge intensities different than the electric charge intensity of the electrons at the ground level, which is not the case.)

Now suppose we have a "dipole" of a given mass at rest, bearing a given length r at rest. Coulomb Force reigns between the electric charges. Suppose we assume that Coulomb Force is, as usual, expressed as proportional to the electric charges coming into consideration, also to $1/r^n$, where though we do not know, a priori the exponent n. Suppose then we bring the dipole to a uniform translational motion, along the direction delineated by the line connecting the electric charges making it. Since then, [mass] x [length] remains invariant, it becomes evident that the Lorentz invariance of [force] x [mass] x $[\text{distance}]^3$ shall hold, only if Coulomb Force, dimensionally behaves as $[\text{charge}]^2 / r^n$, n being exclusively 2, given that charges are Lorentz invariant.

Note that the same holds, if "charges", in question, are gravitational charges; in this case however, the product of charges should be considered together with the universal gravitational constant.

Thus, the framework in consideration is fundamentally based on the Special Theory of Relativity.

The related metric (just like the one used by the General Theory of Relativity) is altered by the gravitational field (in fact, by any field the "measurement unit" in hand interacts with); though in the present approach, this occurs via quantum mechanics (anyway nailed to the Planck Constant, a universal Lorentz invariant constant).

Henceforth, one does not require the "principle of equivalence" assumed by the General Theory of Relativity, as a precept, in order to predict the end results of this theory.

Let then $m_{0\infty}$ be the mass of the object in consideration, at infinity. When it is bound at rest, to a celestial body of mass $\mathcal{M}$, assumed for simplicity infinitely large as compared to $m_{0\infty}$; this latter will be diminished as much as the binding energy coming into play, to become m(r) [r being the distance of $m_{0\infty}$ to the center of $\mathcal{M}$], so that [5]

$$m(r) = m_{0\infty} e^{-\alpha(r)} , \tag{1-a}$$

where $\alpha(r)$ is

$$\alpha(r) = \frac{G\mathcal{M}}{rc_0^2} ; \tag{1-b}$$

G is the "universal gravitational constant"; r is the distance of m(r) to the center of $\mathcal{M}$, as assessed by the distant observer.

Note that m(r) becomes the "gravitational mass", if the object remains at rest. Otherwise, classically speaking, it is neither the "gravitational mass", nor the "inertial

[§] The dimension of "force", is as usual, [mass] x [length] x $[\text{period of time}]^{-2}$.

mass"; it is the" rest mass" of the gravitationally bound object (at rest). This will be clarified at the level of Theorem 4, stated below.

An explanation regarding the reason, for which energy should be retrieved from the mass of the tiny bound object, and not from the infinitely more massive celestial body hosting it, is provided in Appendix A.

We would like to recall that G is not Lorentz invariant, though classified as a universal constant. [One can immediately see this, as follows: Dimensionally speaking $GMm(r)$ is equivalent to (electric charge)^2; but the electric charge intensity is Lorentz invariant; thus so must be the quantity $GMm(r)$; mass is not a Lorentz invariant quantity; hence neither G can be, though the product $GMm(r)$ is].

Via differentiating Eq.(1-a), along with Eq.(1-b), it can be checked that we are indeed dealing with nothing else, but an "energy conservation equation", i.e.

$$dm(r)c_0^2 = G\frac{m(r)\mathcal{M}}{r^2}dr \quad ; \qquad (2)$$

in other words, the RHS of this equation is the "energy", one would have to furnish to m(r) at r, in order to carry it away from $\mathcal{M}$, as much as dr, and the LHS is the "energy, equivalent of the mass increase" dm(r), the mass m(r) delineates throughout, as imposed by the Special Theory of Relativity; "energy conservation" imposes that the two quantities of energy [the two sides of Eq.(2)], are equal to each other (c.q.f.d.).

Now suppose that the object of concern is in a given motion around $\mathcal{M}$; the motion in question, thus can be conceived as made of two steps:

i) Bring the object "quasistatically", from infinity to a given location r, on its orbit, but keep it still at rest.
ii) Deliver to the object at the given location, its motion on the given orbit.

The first step yields a decrease in the mass of $m_{0\infty}$ as delineated by Eq.(10). The second step yields the Lorentz dilation of the rest mass m(r) at r, so that the overall mass $m_\gamma(r)$, or the same the total relativistic energy of the object in orbit becomes

$$m_\gamma(r)c_0^2 = \frac{m(r)c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_{0\infty}c_0^2\frac{e^{-\alpha(r)}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \quad ; \qquad (3)$$

v_0 is the "local tangential velocity" of the object at r.

The total energy of the object in orbit [i.e. $m_\gamma(r)c_0^2$] must remain constant [11,12,13], so that for the motion of the object in a given orbit, one finally has**

** Amazingly the General Theory of Relativity predicts (as furnished by Reference 7)

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2\frac{\sqrt{1-2\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} \quad \textit{(relationship presented by Landau and Lifshitz)}, \qquad \text{(i)}$$

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2 \frac{e^{-\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} \ ; \tag{4}$$

(basis of the equation of gravitational motion)

v_0 (the relative velocity of m_0 in regards to $\mathcal{M}$), and c_0, in our approach, if written in terms of lengths and periods of time picked up along the trajectories in consideration, remain the same for both the local observer and the distant observer, similarly to what is framed by the Special Theory of Relativity.

The differentiation of the above equation leads to

$$-\frac{G\mathcal{M}}{r^2}\left(1-\frac{v_0^2}{c_0^2}\right)dr = v_0 dv_0 \ . \tag{5}$$

This equation can be put into the form

$$-\frac{G\mathcal{M}}{r_0^2}\left(\frac{e^{-\alpha_0}}{1+\alpha_0}\right)\left(1-\frac{v_0^2}{c_0^2}\right)\frac{\underline{r}_0}{r_0} = \frac{d\underline{r}_0^2(t_0)}{dt_0^2} \ , \tag{6-a}$$

written in terms of the "proper quantities", via the relationship

$$r = r_0 e^{\alpha} \equiv r_0 e^{\alpha_0} \ , \tag{6-b}$$

as induced by Theorem 2; $\underline{r}_0$ is the vector bearing the magnitude r_0, and directed outward.

Eq.(5) is the classical Newton's Equation of Motion, were v_0 negligible as compared to c_0.

Multiplying Eq.(6-a) by the "constant overall mass" $m_\gamma(r)$ at both sides, one for, $\alpha_0 << 1$, can state that (cf. Appendix B for the elucidation of a false contradiction between the present approach and the classical approach)

$$(\textit{Static Gravitational Force})(1-\alpha_0)e^{-\alpha_0}\sqrt{1-\frac{v_0^2}{c_0^2}} = (\textit{Overall Mass})\times(\textit{Acceleration}) . \tag{7}$$

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which coincides, up to the second order of the corresponding Taylor expansion, with Eq.(4); yet there does not seem any easy way to interpret the numerator, of Eq.(i), whereas, not only that it is possible to ascertain what the numerator of Eq.(4) is all about, but also, the set up of this latter equation is evident. Eq.(4), on the other hand, is fully consistent with what Yilmaz would have written, in the same way as that presented by Landau and Lifshitz, leading to Eq.(i), with the exponential correction (References 7, and 8) that Yilmaz proposed to Einstein's metric, i.e.

$$m_\gamma(r)c_0^2 = m_{0\infty}c_0^2 \frac{e^{-\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} \ ; \tag{ii}$$

(relationship that would have been written by Yilmaz, had he followed the same way as that presented by Landau and Lifshitz, together with the correction he proposed to Einstein's metric)

Thence, Newton's equation of gravitational motion, i.e. [Gravitational Force = Mass of the Planet x Acceleration], is broken, since an extra term, i.e. $e^{-\alpha}\sqrt{1-v_0^2/c_0^2}$ comes to multiply the "gravitational force", in its classical form.

Formally, this equation can be restored, if instead, one chooses to alter the "Newtonian gravitational force"; but then the "gravitational mass" and the "inertial mass", as classically defined, shall not be identical.

Thus one can establish the following theorem. (A rigorous proof of it is provided in Reference 5.)

Theorem 4: The gravitational mass $m_{\text{gravitational}}$, and the inertial mass m_{inertial}, as classically defined, are not the same; the theory summarized herein, to formally save *Newton's equation of gravitational motion,* predicts

$$m_{\text{gravitational}} = m_{0\infty}\exp(-\alpha)\sqrt{1-\frac{v_0^2}{c_0^2}} \,, \tag{8}$$

given that

$$m_{\text{inertial}} = \frac{m_{0\infty}\exp(-\alpha)}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \,; \tag{9}$$

though undetectable for most cases routinely considered, $m_{\text{gravitational}}$ and m_{inertial} differ.

Thence one arrives at

$$m_{\text{gravitational}} = m_{\text{inertial}}\frac{1}{\gamma^2} \,; \tag{10}$$

(relationship predicted by the author)

this result is amazingly the same as that predicted by Mie back in 1912, as a result of his "inverse problem set up" [14,15,16].

Nordström too predicted it, though through still a totally different way [17,18].

It is further interesting to recall the parallelism de Haas has drawn between de Broglie's clock-wise frequency and wave-like frequency, on the one hand, and Mie's gravitational mass and inertial mass, on the other [19].

Theorem 4 in short, tells us that the "Newton's second law of motion" along with "Newton's law of gravitation" can still be used, provided that the "gravitational mass" to be input to the "Newton's law of gravitation" is taken as the one given by Eq.(8), and the "inertial mass" to be input to the "Newton's second law of motion" is taken as the one given by Eq.(9).

Eqs. (8), (9), and (10) are interesting, for they suggest that a moving particle in a gravitational field would weigh less than the same particle at rest, in the same location in that field. V. Andreev effectively reported at the mentioned PIRT Conference that, a pendant load irradiated at the General Physics Institute of the Russian Academy of

Sciences, by high energy electrons, comes to weigh less than its untouched twin counterpart [20].

The first author of this article, right after Andreev's presentation, suggested that, the effect must be due to energizing the unpaired electrons of the atoms of the load in consideration (which happened to be duraliminium); these electrons, based on Eq.(8), become practically weightless. A quick calculation indeed proves this point of view, which shall be elaborated on, in a subsequent article.

Let us go back to Eq.(9). Taking into account the quantum mechanical stretching of lengths in the gravitational field [i.e. Eq.(6-b)], Eq.(4) can be transformed into an equation written in terms of the proper lengths [3,21], i.e.

$$\frac{\exp(-\alpha_0 e^{-\alpha})}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} , \tag{11}$$

where α_0, i.e.[††]

$$\alpha_0 = \frac{G\mathcal{M}}{r_0 c_0^2} , \tag{12-a}$$

is now expressed in terms of the proper distance r_0; note that the constant, appearing in the RHS of Eq.(11) is different than the constant appearing in the RHS of Eq.(1-b); the relationship between α_0 and α [via Eqs. (1-b) and (6-b)] is

$$\alpha = \alpha_0 e^{-\alpha} . \tag{12-b}$$

The use of Eq.(11) [instead of Eq.(4)], will lead to the replacement of $\exp(-\alpha)$, at the Right Hand Sides of Eqs. (8) and (9), by the exponential function $\exp(-\alpha_0 e^{-\alpha})$.

It should be recalled that, though consisting in a totally different set up, than that of the General Theory of Relativity, Eq.(11) amazingly yields results identical to those of this theory, within the frame of the second order of corresponding Taylor expansions.

3. Mass Decrease of an Electrically Bound Particle: Application of the Approach to the Hydrogen Atom

Based on the foregoing discussion, henceforth, we should take into account the proper mass decrease of the bound electron, as implied by the Special Theory of Relativity.

More specifically we can think that, the hydrogen atom is made in two steps:

1) We bring the electron from infinity to a given distance from the nucleus (supposing for simplicity, yet without any loss of generality, that the proton is

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†† The transformation between the "proper distance" r_0, of a given location P to the center of $\mathcal{M}$, as assessed by the observer situated at this location, and the same distance r, but as assessed by the distant observer, in effect, becomes (Reference 11)

$$r = r_0 e^{\alpha} \cong r_0 e^{\alpha_0} . \tag{i}$$

fixed); owing to the equivalence between mass and energy, this process reduces the electron's proper mass as much as much as the magnitude of its (in the classical sense) potential energy, at this location.

2) Next, we deliver to the electron its orbital kinetic energy (however, the orbit may be conceived); this process, in the familiar relativistic way, increases the already decreased proper mass, by the usual Lorentz factor.

Thus Dirac, just like Sommerfeld had considered the second process, but not the first one.

Recall that, Dirac's theory does not cover thoroughly the experimental results [22,23,24,25]; the measured doublets due to spin-orbit interaction remained narrower than predicted.

Our approach yields a shift of energy levels, upward (whereas the "relativistic quantum mechanics", just like "Sommerfeld's approach", predicts a shift of the Bohr energy levels, downward).

The upward shift in question depends on, just the principal quantum number, thus effects in exactly the same way, the shift of the $2S_{1/2}$ level, and shift of the $2P_{1/2}$ level, which makes that, it is not in any extent, responsible of the electrodynamical splitting of these levels (i.e. the Lamb shift). Yet it should account for the discrepancy between the theoretical prediction of the Lamb shift and the measured value of this.

We are going to base our approach on just hydrogen-like atoms. Further, for simplicity (though without any loss of generality), we shall neglect the mass deficiency undergone by the proton in the hydrogen atom, as compared to that displayed by the electron; along the same line, we can consider that, the reduced mass of the electron and the proton, is the mass of the electron, straight.

We thus make the following definitions.

r_0 : distance of the electron to the nucleus

$m_{0\infty}$: the electron's rest mass at infinity

$m_0(r_0)$: the electron's rest mass at a distance r_0 from the nucleus

$m(r_0)$: the electron's overall mass (which is its mass at infinity, decreased as much as its potential energy, and increased based on the Special Theory of Relativity, due to its "translational" motion), on a given orbit, at the location r_0

v_0 : the tangential velocity of the electron on the orbit (however the motion or the orbit may be conceived), at the location r_0

e : the charge intensity of the electron or that of the proton

Z : the number of protons of the nucleus of the hydrogen – like atom

Our idea is simple; on a given orbit the total energy of the electron, i.e. $m(r_0)c_0^2$, must remain constant. If the orbit is not circular, throughout the electron's journey on the orbit, however this may be, both r_0 and v_0 shall vary; but $m(r_0)c_0^2$, thus $m(r_0)$ must stay constant. Accordingly the overall mass, $m(r_0)$, or the total energy, $m(r_0)c_0^2$

of the electron, which we will call E_{Total}, at a distance r_0 from the nucleus bearing Z protons, shall be written as [26,27]

$$m(r_0)c_0^2 = m_0(r_0)c_0^2 \frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_{0\infty}c_0^2 \frac{1-\frac{Ze^2}{r_0 m_{0\infty} c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = E_{Total} = \textit{Consant on an Orbit}\ ; \qquad (13)$$

(overall relativistic energy of the electron, in a hydrogen-like atom, the setting of which to a constant, determines the given orbit's equation)

here, $1-Ze^2/(r_0 m_{0\infty} c_0^2)$ is the decrease factor of the proper mass $m_{0\infty}$ of the electron, when bound, at rest, to the nucleus of concern; thus Ze^2/r_0 as usual, is the magnitude of the potential energy, or the same, the binding energy at rest of the electron to this nucleus, at a distance r_0 from it, which makes that $Ze^2/(r_0 m_{0\infty} c_0^2)$ is the ratio of the potential energy to the original proper energy. Our approach interestingly generates, the improved Weber's Potential [28,29, 30,31,32]. ‡‡

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‡‡ The differentiation of Eq.(13) yields the following noteworthy, general orbit equation, for the motion of the electron around the nucleus:

$$-\frac{Ze^2}{m_{0\infty} r_0^2} \frac{1-\frac{v_0^2}{c_0^2}}{1-\frac{Ze^2}{r_0 m_{0\infty} c_0^2}} = v_0 \frac{dv_0}{dr_0} \quad . \qquad (i)$$

(differential expression of the energy conservation law, for the electron on the orbit)

One can transform this equation into a vector equation, with not much pain, and show that the RHS is accordingly transformed into the acceleration (vector) of the electron on the orbit.

Thus recalling that the LHS of Eq.(13), i.e. $m(r_0)c_0^2$, is constant, one can write

$$\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\frac{\underline{r}_0}{r_0} = m(r_0)\frac{d\underline{v}_0(t)}{dt}; \qquad (ii)$$

(equation of motion written by the author, via just the energy conservation law, extended to cover the mass – energy equivalence)

here, $\underline{r}_0$ is the radial vector of magnitude r_0, and $\underline{v}_0$ is the velocity vector of the electron.

The orbit would be as customary elliptical, for a small Z, thus a small v; otherwise it is open; in other words, the perihelion of it, shall precess throughout the motion.

This is anyway the same relationship as that proposed by Bohr, except that the electrostatic force intensity is now decreased by the factor $\sqrt{1-v_0^2/c_0^2}$.

According to our approach, it is in fact the decreased mass at rest, $m_0(r_0)$ at r_0, which is increased by the Lorentz factor $1/\sqrt{1-v_0^2/c_0^2}$, due to the electron's motion around the nucleus (and not the proper mass $m_{0\infty}$, measured in empty space, free of any field).

Dirac's theory, just like Sommerfeld's approach, misses the decrease factor $1-Ze^2/(r_0 m_{0\infty} c_0^2)$ we introduced in Eq.(1). This factor, as will soon become clear, is very small for small Z's, but may become quite important at big Z's; anyway (as we shall elaborate below) the inverse of it, is amazingly equal to the square of the Lorentz factor meaning that the overall mass (contrary to the actual wisdom and related mathematical formulation), is always smaller than $m_{0\infty}$.

On can, based on Eq.(13), frame in a straightforward way, a corresponding quantum mechanical formulation. This is what we undertake next.

4. Conclusion of Part I

Herein we have shown (based on just the energy conservation law) that, both the atomistic and the celestial motions can be described along with similar concepts, provided that the mass deficiency of the bound particles is taken into account. This amazingly leads us, to all of the observed occurrences, though in a much simpler way than that classically followed.

Thereby the Coulomb Force, or the Newton Force, holds only for, respectively, static charges, and static masses.

Thus, we have shown that these force laws, are not any more valid if the test charge moves around the source charge, or similarly, the test mass moves around a given source mass.

In the next part, we are going to develop a full quantum mechanical deployment based on the findings, we have presented herein.

The cited references, are presented, altogether, at the end of Part II.

Appendix A

Why Mass Should be Retrieved From the Tiny Object Bound to an Infinitely More Massive Celestial Body?

Suppose indeed we set the very tiny object of mass m_∞ and the very massive object (MO) (originally assumed at rest), simultaneously free, in the reference system of the distant observer. Because of the attraction force, they will get accelerated toward each other, and at a given distance from each other, they will come to acquire the velocities v and v_{MO} (supposed far below that of the speed of light), thus the kinetic energies $\mathcal{M} v_{MO}^2/2$ and $m_\infty v^2/2$. Because of the conservation of the linear momentum, we have

$$\mathcal{M}\,v_{MO} = m_\infty v\,. \tag{A-1}$$

This makes that the fraction of the kinetic energy of m_∞ out of the total kinetic energy coming into play, turns out to be

$$\frac{\dfrac{m_\infty v^2}{2}}{\dfrac{\mathcal{M}\,v_{MO}^2}{2}+\dfrac{m_\infty v^2}{2}} = \frac{\mathcal{M}}{\mathcal{M}+m_\infty} \cong 1 \qquad ; \tag{A-2}$$

likewise the fraction of the kinetic energy of $\mathcal{M}$ out of the total kinetic energy in question, becomes

$$\frac{m_\infty}{\mathcal{M}+m_\infty} \approx 0 \qquad \text{(c.q.f.d.).} \tag{A-3}$$

The same philosophy well applies if there are more than two particles getting bound, since in this case, it appears sufficient to handle the problem in the frame of the center of mass of the system (at rest throughout). At a first glance, in effect it may seem that our result depends on the history of the recombining particles; this is not correct, in the frame of the reference of the center of mass.

Otherwise, through say the recombination of a proton and an electron (of initially random kinetic energies), yielding a hydrogen atom, the extra energy the system would acquire, in the laboratory frame of reference, exhibits itself, as the translational energy of the center of mass, or a rotational energy around the center of mass, or else. It is the mass deficit, different elements of the system displays, after the system as a whole comes to a rest, that we must account for, and the resulting picture is well free of the history of the recombining particles.

The essential idea is anyway that the overall mass of the bound particles, is less than the total mass of these particles when weighed at infinity, and this is less than the former, as much as the total binding energy coming into play.

One other issue has to be elaborated on, though. Suppose, around the gravitational attractor location of a galactic cloud, hydrogen atoms start to get closer and closer to make a star. In order to model this occurrence through a linearized approach, we should start up with just two atoms. Then, exactly half of the gravitational binding energy that will come into play, is to be subtracted (as the mass deficit), from the mass of each atom. Suppose we continue to manufacture the gedanken star, by bringing one by one hydrogen atoms, from infinity to the immediate neighborhood of the star's original seed. Based on the foregoing discussion; when the star is close to be built entirely, then, an hydrogen atom that we bring from infinity, to merge with it, i) will experience a much too greater binding energy as compared to that displayed by the very first uniting atoms (since the atom in question is now getting bound to practically the whole star, and not just a couple of hydrogen atoms making the very beginning of it), ii) the atom we visualize will further experience a mass deficit, practically the same as the entire binding energy, since (according to the foregoing discussion), in this latter case we ought to retrieve the equivalent of the binding energy coming into play, from just the single atom, and not the huge star.

The conclusion we land at is that, items gravitationally bound to each other, should exhibit different mass deficits. But at the same time, it seems legitimate to expect some sort of an energy exchange to take place, between these items (given that they can lend or gain the amount of energies that would have come into play, while getting bound or getting dissociated). Such an energy exchange process should be expected to insure a thermodynamic equilibrium, which should lead to a full (Maxwellian type of) spectrum of a gravitational red shift, and not just one classical red shift, to be associated with the star. And this is strikingly what is observed (References 3 and 4).

Appendix B

Elucidation of a False Contradiction Arising Between the Present Approach and the Classical Approach

Classically (cf. Reference 7); defining as usual, the momentum $\underline{p}$ and the force $\underline{F}$ (as vector quantities), for an object of mass $m_{0\infty}$, moving with the instantaneous velocity $\underline{v}$, under the influence of the force of strength F, as

$$\underline{p}=\frac{m_{0\infty}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\underline{v} \quad , \tag{B-1}$$

$$\underline{F}=\frac{d\underline{p}}{dt} \quad ; \tag{B-2}$$

one can write

$$\frac{d\underline{p}}{dt}=\underline{F}=\frac{m_{0\infty}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\frac{d\underline{v}}{dt}+\frac{m_{0\infty}}{\left(1-\frac{v_0^2}{c_0^2}\right)^{3/2}}\frac{v}{c_0^2}\frac{dv}{dt}\underline{v} \quad . \tag{B-3}$$

For a circular motion, v is constant; thus

$$\frac{d\underline{p}}{dt}=\underline{F}=\frac{m_{0\infty}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\frac{d\underline{v}}{dt} \, . \tag{B-4}$$

For a free fall, $\underline{v}$ and $d\underline{v}/dt$, are in the same direction; thus

$$\frac{d\underline{p}}{dt}=\underline{F}=\frac{m_{0\infty}}{\left(1-\frac{v_0^2}{c_0^2}\right)^{3/2}}\frac{d\underline{v}}{dt} \, . \tag{B-5}$$

At the first strike, this outcome seems to be contradictory in comparison with Eq.(7), if the gravitational force is considered to be

$$F = G\frac{\mathcal{M}\, m_{0\infty}}{r^2} , \tag{B-6}$$

where $\mathcal{M}$ is the mass of the celestial body, acting on the bound particle of original mass $m_{0\infty}$.

However the foregoing derivation ignores the fact that the *rest mass* is altered due to binding; this means that $m_{0\infty}$ cannot be kept constant throughout the differentiation operations we have achieved. The momentum should thus be written as

$$\underline{p} = \frac{m_{0\infty}\exp(-\alpha)}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\underline{v} , \tag{B-7}$$

instead of Eq.(B-1); this makes that, based on Eq.(3) of the text, the term multiplying $\underline{v}$ is constant, and we have straight

$$\frac{d\underline{p}}{dt} = \frac{m_{0\infty}\exp(-\alpha)}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\frac{d\underline{v}}{dt} . \tag{B-8}$$

Henceforth, contrary to what is formulated in Eq.(B-3) we have no terms in $\underline{v}$. Our force strength on the other hand, clearly, is not given by this classical expression; Eq.(B-6), based on our approach, is only valid for static masses, further assuming that they are not perturbed due to binding with each other. The static gravitational force F_{pa} acting on $m_{0\infty}$, as framed by the present approach [cf. Eq.(7) of the text], is

$$F_{pa} = G\frac{\mathcal{M}\, m_{0\infty}\exp(-\alpha)}{r^2} . \tag{B-9}$$

(static gravitational force as framed by the present approach)

In other terms Eq.(7) of the text, can explicitly be written as

$$\frac{d\underline{p}}{dt} = \frac{m_{0\infty}\exp(-\alpha)}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\frac{d\underline{v}}{dt} = G\frac{\mathcal{M}\, m_{0\infty}\exp(-\alpha)\sqrt{1-\frac{v_0^2}{c_0^2}}}{r^2}\frac{\underline{r}}{r} = \underline{F}_{pa}\sqrt{1-\frac{v_0^2}{c_0^2}} ; \tag{B-10}$$

here $\underline{r}$ is the inward looking vector, of length r.

Once again Eq.(B-3), as well as Eqs. (B-5) and (B-6) are invalid; the correct expression for the "change of the momentum with respect time" (i.e. "force"), is Eq.(B-10).