Quantum Physics and Matter Self-Organization

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Abstract

This paper would like revaluing Albert Einstein's famous sentence

"God does not play dice with the Universe"

in quantum mechanics by unification of measure and probability concepts in a complete Boolean representation of quantum mechanics.

Keywords: Quantum physics, Probability, Sentence Logic, Hiley algebra.

1 Introduction

Malatesta¹ has proven a strict link between probability and polyadic connectives in standard sentence logic.

Grappone² has consequently proven the equivalence between probability and sentence sets by using a hyperincursive approach³.

Quantum Physics interprets subatomic particles as frequency distribution in the physical space.

Hiley algebra⁴ permits to eliminate the space-time in the description of quantum phenomena.

These premises allow us to give a contribution as to the hypothesis of matter selforganization.

If a subatomic particle can be seen as a frequency distribution in the physical space, then quantum phenomena can be described in terms of correspondences between physical space regions and probabilities.

But all the probabilities correspond to sentence sets.

Thus quantum phenomena can be described in terms of correspondences between physical space regions and sentence sets.

So timed quantum physics corresponds to a timed sentence logic.

Hence Hiley algebra puts indirectly a correspondence between timed quantum physics and a non-timed sentence logic.

Finally, if quantum physics corresponds to a sentence logic, we can conclude that matter self-organization is trivial in it.

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¹ See Malatesta (1989).

² See Grappone (2006).

³ See Dubois (1997). See Dubois and Resconi (1992) too.

⁴ See Hiley (2002).

2 Measures as Vectors

2.1 Definitions

2.1.1 Measure

Let **X** be a set, \mathbb{A} an algebra on **X**.

Definition 2.1.1.1: A function μ : $\mathbb{A} \rightarrow \mathbb{R}_+ \cup \{\infty\}$ is called "a measure" if

1. $\mu(A) > 0$ for any $A \in \mathbb{A}$ and $\mu(\emptyset) = 0$;

2. if $(A_i)_{i\geq 1}$ is a disjoint family of F sets in \mathbb{A} $(A_i \cap A_j = \emptyset$ for any $i\neq j$) such that $\bigcup A_i \in \mathbb{A}$,

then
$$\mu\left(\bigcup_{i=1}^{\infty}A_i\right) = \sum_{i=1}^{\infty}\mu(A_i).$$

Definition 2.1.1.2: A "signed measure" is a measure with sign $\pm \mu(A)$.

2.1.2 Measure Error

Let *m* be a signed measure. Suppose that the errors of its measurement procedure are completely random and they place the correct value of *m* into the interval $[m - \Delta m, m + \Delta m]$.

Thus we can reasonably say that |m| is a member of a population with normal distribution with mean $\mu_m = m$ and standard deviation $\sigma_m = \Delta m/4$ so that $[m - \Delta m, m + \Delta m]$ contains practically all this population (consider the normal distribution common tables).

Gauß' error propagation law⁵ allow us to carry out parallel calculations of measures and errors. Indeed, we have:

$$y = f(x_1, x_2, \dots, x_n) \text{ if and only if } \sigma_y = \sqrt{\left[\left(\frac{\partial y}{\partial x_1}\right)\sigma_{x_1}\right]_{\mu}^2 + \left[\left(\frac{\partial y}{\partial x_2}\right)\sigma_{x_2}\right]_{\mu}^2 + \dots + \left[\left(\frac{\partial y}{\partial x_n}\right)\sigma_{x_n}\right]_{\mu}^2}$$

where the index μ in the partial derivatives states that the numerical values of the partial derivatives are to be calculated as the mean values of the measured values $x_1, x_2, ..., x_n$.

Definition 2.1.2.1: Given a signed measure *m* whose correct value is at most in the interval $[m - \Delta m, m + \Delta m]$, define its error σ_m as $\Delta m/4$.

⁵ See Bronshtein, I. N., Semendyayev, K.A., (2005).

2.1.3 Measure Probability Density

Let *m* and σ_m be a signed measure and its error. Consider the normal distribution

$$y = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Definition 2.1.3.1: Define the "probability density δm " of a signed measure *m* the probability density that *m* has in a normal distribution with mean 0 and standard deviation σ_m i.e. $\delta m = \frac{1}{\sigma_m \sqrt{2\pi}} e^{-\frac{m^2}{2\sigma_m^2}}$.

2.2 Measure Vectors

2.2.1 (Signed Measure, Its Indetermination, Its Probability Density) Vector

As
$$\delta m = \frac{1}{\sigma_m \sqrt{2\pi}} e^{-\frac{m^2}{2\sigma_m^2}}$$
 (Definition 2.1.3.1), we have consequently:

F-2.2.1.1
$$m = \pm i\sigma_m \sqrt{2 \ln \delta m} + 2 \ln \sigma_m + \ln 2 + \ln \pi$$

F-2.2.1.2⁶
$$\sigma_m = \sqrt[4]{\frac{m^4}{(2\ln\delta m + 2\ln\sigma_m + \ln2 + \ln\pi)^2}}$$

Observe that F-2.2.1.2 is evidently an incursive equation that can be solved by Dubois' methods and by using 1 as starting value for σ_m .

Thus m, σ_m , δm are quantities such that two of them determine the third one. So we can affirm that $(m, \sigma_m, \delta m)$ is a vector. Let **m** be $(m, \sigma_m, \delta m)$.

Observe that we can consider σ_m the *m* indetermination too. Hence **m** is finally the vector (measure, its indetermination, its probability density).

⁶ Remember that σ_m can be only null or positive because it is an error measure. Only the positive solution of the root can be accepted.

2.2.2 (Signed Measure, Its Indetermination, Its Probability) Vector

Remember that the integral of the normal distribution $y = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is the

cumulative distribution $y = \int \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right)$ where Gauß' error

function $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^{2}} dt.$

Thus
$$\Pr(-\infty \le m) = \int_{-\infty}^{m} \delta x dx = \int_{-\infty}^{m} \frac{1}{\sigma_m \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_m^2}} dx = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{m}{\sigma_m \sqrt{2}}\right) \right).$$

This approach makes $Pr(-\infty \le m)$ a mathematical transformation of the probability density δm . Thus we can define the vector $\mathbf{Pm}=(m, \sigma_m, \Pr(-\infty \le m))$ from the vector $\mathbf{m}=(m, \sigma_m, \delta m)$.

2.2.3 (Signed Measure, Its Characteristic Sentence Set) Vector

Consider any sentence set $S = \{p_1, p_2, ..., p_n\}$. Standard tautology calculus identifies any sentence p_i with an oportune truth function φ_i of 2^m elements that are equal to 1 (true) and/or 0 (false). Thus we have: $\{p_1, p_2, ..., p_n\} \leftrightarrow \{\varphi_1, \varphi_2, ..., \varphi_n\}$. Let Φ_S be $\{\varphi_1, \varphi_2, ..., \varphi_n\}$.

For every truth function φ_i , we can consider the ratio ρ_i between the number of elements *1* and the number of all the elements of φ_i . So we have: $\{p_1, p_2, ..., p_n\} \leftrightarrow \{\varphi_1, \varphi_2, ..., \varphi_n\}$. Let **P**_S be $\{\rho_1, \rho_2, ..., \rho_n\}$.

Every $\rho_i \in \mathbf{P}_{\mathbf{S}}$ occurs in $\mathbf{P}_{\mathbf{S}}$ with a given statistical frequency $\delta \rho_i$. Thus $\mathbf{S} = \{p_1, p_2, ..., p_n\}$ defines finally a statistical frequency distribution through $\mathbf{P}_{\mathbf{S}} = \{\rho_1, \rho_2, ..., \rho_n\}$.

 $S = \{p_1, p_2, ..., p_n\}$ is a data set in nominal scale. So its adequate position index of its statistical frequency distribution is the mode $\ddot{\mathcal{R}}_s$ of $P_s = \{\rho_1, \rho_2, ..., \rho_n\}$.

A dispersion index needs $S = \{p_1, p_2, ..., p_n\}$ now and common dispersion indexes are not adequate for data in nominal scale. Thus we introduce a new dispersion index.

Observe that: $1 \times 9=9$, $2 \times 8=16$, $3 \times 7=21$, $4 \times 6=24$, $5 \times 5=25$. In general, if the factor sum does not change, the difference between factors makes a reduction of their product. But a set of data in nominal scale is much more packed, the more that the other statistical frequencies are lower than the mode. Thus we can consider these dispersion indexes, we have:

F-2.2.3.1 $\sum_{i=1}^{n} (\ln \delta \rho_i)^2$ (*no*) **F-2.2.3.2** $\sum_{i=1}^{n} (\ln \delta \rho_i)^2 / n$ (*no*)

(nominal deviance)

(nominal variance)

F-2.2.3.3
$$\sqrt{\sum_{i=1}^{n} (\ln \delta \rho_i)^2} / n$$

(nominal standard deviation)

As all the $\delta \rho_i$ have Gauß' distribution it is evident that $\sum_{i=1}^{n} (\ln \delta \rho_i)^2$ has χ^2 distribution and additivity exactly as the parametric deviance. So nominal standard deviation has the same properties of parametric standard deviation and we can denote the nominal standard deviation of **S** with σ_s and the nominal variance with σ_s^2 .

Order every truth function in standard way (e.g.: $\varphi_{p_vq}=1110$, $\varphi_{p_hq}=1000$, $\varphi_{-p}=01$, $\varphi_{p\supset q}=1011$). A couple (\hat{X}_s, σ_s) corresponds in general to more sentence sets. Among these ones consider the sets that have less atomic sentences. Among these ones consider the sets that have less sentences. Among these ones consider the sets that have their truth functions with frequency \hat{X}_s with the most left elements *1*. Among these ones consider the sets that have their truth functions with more near frequency to \hat{X}_s with the most left elements *1*. If it is necessary, then we can choose once for all other criteria too, however, finally, we shall obtain that the couple (\hat{X}_s, σ_s) points one sentence set **B** (\hat{X}_s, σ_s) and vice versa.

Consider now a (signed Measure, its indetermination, iIts probability) vector $\mathbf{Pm}=(m, \sigma_m, \Pr(-\infty \le m))$. Remember that any its component is determined by the remaining two ones and, in particular, that $m=m(\sigma_m, \Pr(-\infty \le m))$. As the couple (\hat{X}_s, σ_s) points one sentence set $\mathbf{B}(\hat{X}_s, \sigma_s)$ and vice versa, we can put $\Pr(-\infty \le m) = \hat{X}_s$ and $\sigma_s = \sigma_m$ so that $(m, \mathbf{B}(\Pr(-\infty \le m), \sigma_m))$ is a vector because a component determines the other one. Call $\mathbf{B}(\Pr(-\infty \le m), \sigma_m)$ characteristic sentence set of the measure m. Thus we can define the vector $\mathbf{Bm}(m, \mathbf{B}(\Pr(-\infty \le m), \sigma_m))$ from the

vector $\mathbf{Pm} = (m, \sigma_m, \Pr(-\infty \le m))$. it is evident that:

F-2.2.3.4 $m \leftrightarrow \mathbf{B}(\Pr(-\infty \le m), \sigma_m)$ **F-2.2.3.5** $\mathbf{Pm} = (m, \sigma_m, \Pr(-\infty \le m)) \leftrightarrow \mathbf{Bm} = (m, \mathbf{B}(\Pr(-\infty \le m), \sigma_m))$

2.3 Standard Operators and Quantum Operators among Measure Vectors

2.3.1 Standard Operators among Measure Vectors

Let $\mathbf{F}^{n}(\mathbf{Pm}_{1}',\mathbf{Pm}'_{2}',...\mathbf{Pm}_{n}')$ be a standard operator on $\mathbf{Pm}_{1}',\mathbf{Pm}_{2}',...\mathbf{Pm}_{n}'$ if it is equivalent to $\begin{pmatrix} f_{1}^{n}(m_{1},m_{2},...,m_{n}) \\ f_{2}^{n}(\sigma_{m_{1}},\sigma_{m_{2}},...,) \\ f_{3}^{n}(\mathbf{Pr}(-\infty \le m_{1}),\mathbf{Pr}(-\infty \le m_{2}),...,\mathbf{Pr}(-\infty \le m_{n})) \end{pmatrix}$, i.e. measure calculus,

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indetermination (error) calculus and probability calculus are parallel without interaction among them.

In fact, classical physics usually mantains distinguished parameter calculus, from indetermination (error) and probability calculus.

2.3.2 Quantistic Operators among Measure Vectors

Let
$$\mathbf{F}^{n}(\mathbf{Pm}_{1}^{\prime},\mathbf{Pm}_{2}^{\prime},...\mathbf{Pm}_{n}^{\prime})$$
 be a quantum operator on $\mathbf{Pm}_{1}^{\prime},\mathbf{Pm}_{2}^{\prime},...\mathbf{Pm}_{n}^{\prime}$ if it is
equivalent to
$$\begin{cases} f_{1}^{n}(m_{1},\sigma_{m_{1}},\operatorname{Pr}(-\infty \leq m_{1}),m_{2},\sigma_{m_{2}},\operatorname{Pr}(-\infty \leq m_{2}),...,m_{n},\sigma_{m_{n}},\operatorname{Pr}(-\infty \leq m_{n}))\\ f_{2}^{n}(m_{1},\sigma_{m_{1}},\operatorname{Pr}(-\infty \leq m_{1}),m_{2},\sigma_{m_{2}},\operatorname{Pr}(-\infty \leq m_{2}),...,m_{n},\sigma_{m_{n}},\operatorname{Pr}(-\infty \leq m_{n}))\\ f_{3}^{n}(m_{1},\sigma_{m_{1}},\operatorname{Pr}(-\infty \leq m_{1}),m_{2},\sigma_{m_{2}},\operatorname{Pr}(-\infty \leq m_{2}),...,m_{n},\sigma_{m_{n}},\operatorname{Pr}(-\infty \leq m_{n})) \end{cases}$$

i.e. measure calculus, indetermination (error) calculus and probability calculus are merged.

In fact, quantum physics merges parameter calculus, from indetermination (error) and probability calculus. We think that this characteristic of quantum physics may be more important than its usual study object, i.e. subatomic physics. Thus we suppose that we may have a quantum approach when we merge parameter calculus with indetermation (error) and probability calculi on any problem.⁷

Observe that every standard operator is a particular case of quantistic operator. Consider the numeric function of projection $x_i = U_i^n(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n)$. It transforms a set of arguments in one of them. Thus any standard operator can be transformed in a quantum operator by intoduction of opportune projection functions.

2.3.3 Quantum Nature of Operators on Characteristic Sentence Sets

Consider the measure vector $\mathbf{Bm}=(m, \mathbf{B}(\Pr(-\infty \le m), \sigma_m))$. It corresponds biunivocally to $\mathbf{Pm}=(m, \sigma_m, \Pr(-\infty \le m))$ (F-2.2.3.5). Every valid calculus on it has to be isomorphous with a valid calculus on $\mathbf{Pm}=(m, \sigma_m, \Pr(-\infty \le m))$.

Consider now that $\mathbf{B}(\Pr(-\infty \le m), \sigma_m)$ contains all the information of the parameter m by definition, thus every valid calculus on $\mathbf{B}(\Pr(-\infty \le m), \sigma_m)$ has to be isomorphyc to to a valid calculus on $\mathbf{Bm}=(m, \mathbf{B}(\Pr(-\infty \le m), \sigma_m))$ and, by transitive property, on $\mathbf{Pm}=(m, \sigma_m, \Pr(-\infty \le m))$.

But $\mathbf{B}(\Pr(-\infty \le m), \sigma_m)$ is finally only a sentence set and does not contains explicitly neither parameter nor indetermination measure nor probabilities: their information is clearly merged. Consequently, any operator that has it as argument is necessarily a quantistic operator.

Finally, as every physical measure with its indetermination (error) $m \pm \Delta m$ and every probability Pr(p) can be expressed in terms of sentence sets, we can make a physical theory and, particularly a quantum physical theory by working exclusively with sentence sets.

⁷ See Dubois (2008).

3 (Quantum) Physics in Standard Sentence Logic

3.1 Sentence Sets as Sentences

3.1.1 Making General Procedure

Given any sentence set $SS = \{p_1, p_2, ..., p_n\}$:

- 3.1.1.1 Consider its sentence p_i with more distinct atomic sentences and add it all the remaining distinct atomic sentences by conserving its truth function at less of duplications.
- 3.1.1.2 Individuate the element number of the truth function of $p_i 2^m$.
- 3.1.1.3 Assign truth functions of 2^m elements to every sentence among $p_1, p_2, ..., p_n$ by opportune dubling.
- 3.1.1.4 As the truth functions BSS of 2^m elements are exactly 2^{2^m} SS={ $p_1, p_2, ..., p_n$ } can be represented by a truth function of 2^{2^m} elements that has 1 in the rows that correspond to truth functions of $p_1, p_2, ..., p_n$ and 0 in the remaining ones.

Define BSS truth function of the sentence set SS. Observe that if SS has a truth function then it can be representend by one sentence.

3.1.2 A Practical Example

Given any sentence set $SS = \{p \lor q, \neg r\}$:

3.1.2.1
$$SS = \{p \lor q \lor (r \land \neg r), \neg r\}$$

	р	۷	q	v	(<i>r</i>	۸	~r)		
	1		1		1		0	1	
	1		1		0		1	1	
	1		0		1		0	1	
3.1.2.2	1		0		0		1	1	$8 = 2^3$ elements
	0		1		1		0	1	
	0		1		0		1	1	
	0		0		1		0	0	
	0		0		0		1	0)

3.1.1.1

3.1.1.2

3.1.3 Considerations

We have started in this paper from any physical measure at will $m \pm \Delta m$ and we have defined from it the indetermination as the error Δm and the probability as $Pr(-\infty \le m)$ in a Gauß' distribution. We have defined quantum calculus as any physical calculus that merges measures, probabilities and indeterminations.

Now, this paragraph 3 shows that measures, probabilities and indeterminations can be represented together by sentences through sentences set. Physics, and quantum physics particularly, become simply a sentence word and, evidently, physics relations and theories are sentential structures as we are in order to show.

3.2 Sentence Relations as Sentences

3.2.1 Generic Sentence Relation Definition

"When two objects, qualities, classes, or attributes, viewed together by the mind, are seen under some connexion, that connexion is called a relation." (A. De Morgan).⁸

⁸ See De Morgan (1858).

"A relation L over the sets $X_1, ..., X_k$ is a subset of their Cartesian product, written $L \subseteq X_1 \times ... \times X_k$." (Standard Set Theory).

How can we connect two sentences in the most possibile general way? We must previde the non-commutativity, while.

1

1

0

Consider the two sentences $p \wedge q$ and p. Their truth function are respectively $\begin{array}{c} 0 \\ 0 \end{array}$ and $\begin{array}{c} 1 \\ 0 \end{array}$

Let the ordered couple $\langle p \land q, p \rangle$ be 1,0-represented by (1000)(1100)=10001100, i.e the

ordered succession of the transposition of $\frac{0}{0}$ and $\frac{1}{0}$. We can generalize this definition

0

0

easily to the ordered sentence *n*-ple $\langle p_1, \ldots, p_n \rangle$.

Suppose now that $p_1, ..., p_n$ can be represented by truth function of 2^m elements. We have 2^{n2^m} possible *n*-ples as $\langle p_1, ..., p_n \rangle$. But $2^{(n2^m)}$ is exactly the length of a truth function. If we give the standard order of the tautology calculus to all the *1,0*-representations of the possible *n*-ples as $\langle p_1, ..., p_n \rangle$ then every their subset **Re** can be represented with a truth function that has *1* if the corresponding $\langle p_1, ..., p_n \rangle$ belongs to **Re** and 0 oerwise.

Observe that **Re** is a relation according to the standard set theory definition and, as it is identified by a truth function, a sentence too. Thus sentence relations can be transformed into sentences.

3.2.2 A Practical Example

 $p \wedge q \rightarrow -p \vee -q$ Consider the sentence relation R1: $p \vee q \rightarrow -p \vee -q$. We have the couples: $p \rightarrow -p$ $\langle p \wedge q, \neg p \vee -q \rangle$ $\langle p \wedge q, \neg p \vee -q \rangle$ $\langle p \wedge q, \neg p \wedge -q \rangle$ The 1,0-representation is: (1110)(0001) = 11100001 and the $\langle p, \neg p \rangle$ (1000)(0111) 11000011

corresponding sentence is:

1	1	1	1	1	1	1	1		0	
÷	:	÷		:	:	:	:		:	
1	1	1	0	0	0	1	0		0	
1	1	1	0	0	0	0	1		1	
1	1	1	0	0	0	0	0		0	
÷	÷	:	÷	÷	:	÷	÷		:	
1	1	0	0	0	1	0	0		$\theta_{(3,7)}$ $(2,6,2)$ $(4,7)$	$\langle \rangle$
1	1	0	0	0	0	1	1	=	$1 \approx \left \bigwedge_{i=1}^{n} p_i \wedge \bigwedge_{i=1}^{n} p_i \wedge p_s \right \left \bigwedge_{i=1}^{n} p_i \wedge \bigwedge_{i=1}^{n}$	p_i
1	1	0	0	0	0	1	0		$0 \qquad \qquad$	·)
:	÷	÷	:	÷	÷	:	÷		1	
1	0	0	0	1	0	0	0		0	
1	0	0	0	0	1	1	1		1	
1	0	0	0	0	1	1	0		0	
÷	:	÷	÷	:	:	:	÷		i.	
0	0	0	0	0	0	0	0		0	

3.3 Physical Theories as Sentences

3.3.1 Physical Theories as relations sets, i.e. sentences

Consider any physical theory. Finally, it is a set of relations between physical measures, i.e., given some measures, it allows us to predict the values of other measures. But we have proven in $\S2.3.3$ that every physical measure (its indetermination and probability included) is reducible to a sentence set, i.e. to a sentence (see $\S3.1$). Thus a physical measure relation becomes sentence relations and these ones are sentences too (see $\S3.2$). So a physical theory, i.e. a measure relation set, is a sentence set, i.e. a sentence (see $\S3.1$).

If such a sentence is obtained by the methods of this paragraph 3 then we can evidently obtain from it all the measure relations of the theory algorithmically, i.e our sentence is a codification of the whole physical theory in standard sentence logic.

Thus the final achievement is a complete representation of (quantum) physics in sentence logic.

3.3.2 Towards a New Information Theory

For coding purposes, Shannon's standard information theory uses sequences of bits (or q-bit in its quantum variation). This procedure has given very good results but it has not solved all the problems, especially in the dynamic systems.

We have introduced a structured coding of data and their relations by standard sentence logic and tautology calculus that look like managing the whole scientific theories. Also, information calculus becomes standard sentence logic with tautology calculus and with some numerical indexes in common with Shannon's standard theory. A development of this new system of data codification may obtain some advantages.

5 Conclusions

The whole representation of (quantum) physical theories with measure, indetermination and probability, in standard sentence logic implies the intrinsic self-organization of the matter, i.e.:

"God does not play dice" (Albert Einstein)

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