The New Concept of Deterministic Anticipation in Natural and Artificial Systems

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Abstract

This paper presents the new concept of Deterministic Anticipation which is related to the hyperincursive discrete harmonic oscillator, as an example. The hyperincursive discrete oscillator is, in this case, a pure recursive system which is a deterministic system that is separable into two incursive discrete harmonic oscillators which are characterized by a deterministic anticipation.

Keywords: hyperincursive harmonic oscillator, deterministic anticipation, incursive harmonic oscillator, discrete systems

1 Introduction

The definition of anticipation deals with the concept of program. Indeed, the word program, comes from "pro-gram" meaning "to write before" by anticipation, and means a plan for the programming of a mechanism, or a sequence of coded instructions that can be inserted into a mechanism, or a sequence of coded instructions, as genes or behavioural responses, that is part of an organism. Any anticipatory natural and artificial programs are thus related to rewriting systems (Dubois, 2010). The new concept of "deterministic anticipation" in natural or artificial systems is related to many natural and artificial systems which are characterized by an anticipatory program. For example, an agenda and a planning can be viewed as a pro-gram. Indeed, firstly, an agenda is a personal organizer where man writes appointments by anticipation. Secondly, a planning is the process of creating and maintaining a plan by anticipation, for an organization. In fact, the concept of "deterministic anticipation" characterizes "rewriting systems" which work with a "deterministic program". Thus, the genetic code of living systems is a program, which is a natural anticipator (Dubois, 2010) is also related to a "deterministic anticipation". The "deterministic anticipation" may be related to a "self-determinated anticipation". As such, any "deterministic anticipation" is a fundamental property of intelligent behaviour Then, the program of a Turing machine, a computer with deterministic algorithms which defines an artificial anticipator (Dubois, 2010), is also related to a "deterministic anticipation". It must be pointed out that the "deterministic anticipation" can be unpredictable, as the deterministic chaos.

This paper will show that a recursive algorithm of a program may contain incursive algorithms with "deterministic anticipation".

A very general system of a discrete harmonic oscillator will be taken as an example for demonstration of the concept of deterministic anticipation.

This paper is organized as follows. The section 2 deals with a survey on the incursive and hyperincursive discrete harmonic oscillators, based on research by Adel Antippa, Daniel M. Dubois and Eugenia Kalisz. The section 3 will present new properties of hyperincursive discrete harmonic oscillator separable into incursive oscillators with forward and backward Energies. Simulations of hyperincursive and incursive oscillators will show the conservation of energy of the discrete harmonic oscillator. The hyperincursive discrete harmonic oscillator is a deterministic recursive system that is separable into two incursive discrete harmonic oscillators with deterministic anticipation. The section 4 gives the figures of simulations with different parameters.

2 Survey on Incursive and Hyperincursive Discrete Harmonic Oscillators

Let us recall the example of the harmonic oscillator (Dubois and Kalisz, 2004), with *m* the oscillating mass and *k* the spring constant, represented by the ordinary differential equations:

$$dx(t)/dt = v(t)$$
$$dv(t)/dt = -\omega^2 x(t) \qquad (2.1a\text{-}b)$$

where x(t) is the position and v(t) the velocity as functions of the time t, and the pulsation ω is related to *k* and *m* by $\omega^2 = k/m$. The solution, with the initial conditions x(0) and v(0), is given by

$$x(t) = x(0)\cos(\omega t) + [v(0)/\omega]\sin(\omega t)$$
$$v(t) = -\omega\, x(0)\sin(\omega t) + v(0)\cos(\omega t) \qquad (2.2a\text{-}b)$$

In the phase space, given by [x(t), v(t)], the solutions are given by closed curves (orbital stability). The period of oscillations is given by $T = 2\pi/\omega$.

The energy e(t) of the harmonic oscillator is constant and is given by

$$e(t) = k\,x^2(t)/2 + m\,v^2(t)/2 = k\,x^2(0)/2 + m\,v^2(0)/2 = e(0) = e_0 \qquad (2.3)$$

The numerical simulation needs the discretization of these eqs. 2.1ab. With the current time, t, and the interval of time, $\Delta t = h$, the discrete time is defined as: $t_k = t_0 + kh$ with $k = 0,1,2,\ldots$ where t_0 is the initial value of the time and k is the counter of the number of interval of time h. With the discrete variables, $x_k = x(t_k)$ and $y_k = y(t_k)$, the discrete equations have the general form

$$x_{k+1} = A\,x_k + B\,v_k \text{ and } v_{k+1} = C\,v_k - D\,\omega^2 x_k \qquad (2.4ab)$$

where A, B, C, D are coefficients with values specific to the numerical algorithm. The conditions for obtaining an orbital stability of the numerical method are given by (Dubois and Kalisz, 2004)

$$(A + C)^2 < 4 \text{ and } A\,C + B\,D\,\omega^2 = 1 \qquad (2.5a\text{-}b)$$

In my paper (Dubois, 1995), I defined a generalized forward-backward discrete derivative

$$D(w) = w\,D_f + (1 - w)\,D_b \tag{2.6.1}$$

where w is a weight taking the values between 0 and 1, and where the discrete forward and backward derivatives on a function f are defined by

$$D_f(f) = \Delta^+ f / \Delta t = [\, f_{k+1} - f_k \,] / h \quad \text{and} \quad D_b(f) = \Delta^- f / \Delta t = [\, f_k - f_{k-1} \,] / h \qquad \text{(2.7a-b)}$$

The generalized incursive discrete harmonic oscillator is given by (Dubois, 1995) as:

$$(1-w)\,x_{k+1} + (2w-1)\,x_k - w\,x_{k-1} = hv_k \text{ and}$$
$$w\,v_{k+1} + (1-2w)\,v_k + (w-1)\,v_{k-1} = -\,h\omega^2 x_k \qquad \text{(2.8a-b)}$$

When $w = 0$, $D(0) = D_b$, this gives the first incursive equations:

$$x_{k+1} - x_k = h\,v_k \quad \text{and} \quad v_k - v_{k-1} = -\,h\,\omega^2\,x_k \qquad \text{(2.9a-b)}$$

When $w = 1$, $D(1) = D_f$, this gives the second incursive equations:

$$x_k - x_{k-1} = h\,v_k \quad \text{and} \quad v_{k+1} - v_k = -\,h\,\omega^2\,x_k \qquad \text{(2.10a-b)}$$

From the orbital stability conditions 2.5a-b we have, in both eqs. 2.9a-b and 2.10a-b,
$(A + C)^2 = (2 - h^2\,\omega^2)^2 < 4$, for $h\,\omega < 2$ and $A\,C + B\,D\,\omega^2 = 1 - h^2\,\omega^2 + h^2\,\omega^2 = 1$ which give an orbital stability for $h\,\omega < 2$, to the two discrete incursive harmonic oscillators.
When $w = 1/2$, $D(1/2) = [D_f + D_b]/2$, this gives the hyperincursive equations:

$$x_{k+1} - x_{k-1} = 2\,h\,v_k \quad \text{and} \quad v_{k+1} - v_{k-1} = -\,2\,h\,\omega^2\,x_k \qquad \text{(2.11a-b)}$$

which behave alternatively as the two incursive equations (Dubois, 1995, 2000).

The simulations of these incursive algorithms (2.9a-b, 2.10ab) are stable, and the simulation of this hyperincursive algorithm (2.11a-b) is stable with the conservation of energy. A complete mathematical development of incursive and hyperincursive systems was presented by Adel F. Antippa and Daniel M. Dubois (2007, 2008, 2010). Antippa and Dubois (2010) re-discovered the forward-backward discrete derivative, and this time-symmetric discretization of the harmonic oscillator (2.11a-b).

3 Hyperincursive Discrete Harmonic Oscillator Separable into Two Incursive Discrete Oscillators with Deterministic Anticipation

For the simulations and the demonstration of the general properties of the hyperincursive discrete harmonic oscillator, dimensionless variables X, V and H, are used for variables, x, v and h:

$$X(k) = \sqrt{[k/2]}\; x_k,$$
$$V(k) = \sqrt{[m/2]}\; v_k,$$
$$\tau = \omega t \text{ with } \omega = \sqrt{[k/m]},$$
$$\Delta\tau = \omega\Delta t = \omega h = H \qquad \text{(2.12a-b-c-d)}$$

With these dimensionless variables, the dimensionless energy is given by :

$$E(k) = X^2(k) + V^2(k) \quad (2.13)$$

With the dimensionless variables (2.12a-b-c-d), the eqs. (2.9a-b) and (2.10a-b) of the two incursive dimensionless harmonic oscillators are given by

$$X_1(k + 1) = X_1(k) + H\,V_1(k) \quad (2.14a)$$
$$V_1(k + 1) = V_1(k) - H\,X_1(k + 1) \quad (2.14b)$$

and

$$V_2(k + 1) = V_2(k) - H\,X_2(k) \quad (2.15a)$$
$$X_2(k + 1) = X_2(k) + H\,V_2(k + 1) \quad (2.15b)$$

These incursive discrete oscillators are non-recursive computing anticipatory systems. Indeed, in eq. 32.14b of the first incursive oscillator, the velocity $V_1(k + 1)$ at future next time step, k + 1, is computed from the velocity $V_1(k)$ at current time step, k, and the position $X_1(k + 1)$ at the future next time step, k + 1, which represents an anticipatory system represented by an anticipation of one time step, k.

Similarly, in eq. 2.15b of the second incursive oscillator, the position $X_2(k + 1)$ at future next time step, k + 1, is computed from the position $X_2(k)$ at current time step, k, and the velocity $V_2(k + 1)$ at the future next time step, k + 1, which represents an anticipatory system represented by an anticipation of one time step, k.

With the dimensionless variables (2.12a-b-c-d), the eqs. (2.11a-b) of the hyperincursive dimensionless harmonic oscillator are given by

$$X(k + 1) = X(k - 1) + 2\,H\,V(k) \quad (2.16a)$$
$$V(k + 1) = V(k - 1) - 2\,H\,X(k) \quad (2.16b)$$

This hyperincutsive discrete oscillator is a recursive computing system including a deterministic anticipation.

Indeed, let us demonstrate that the hyperincursive discrete harmonic oscillator, given by the eqs. (2.16a-b), is separable into two independent incursive harmonic oscillators, as shown in table 1A and table 1B:

The first Incursive Harmonic Oscillator with initial conditions, X(0), V(1), is given by

$$\mathbf{X(2k)} = X(2k - 2) + 2\,H\,V(2k - 1) \quad \text{and}$$
$$V(2k + 1) = V(2k - 1) - 2\,H\,\mathbf{X(2k)} \quad (3.1a\text{-}b)$$

and the second Incursive Harmonic Oscillator with initial conditions, V(0), X(1), is given by

$$\mathbf{V(2k)} = V(2k - 2) - 2\,H\,X(2k - 1) \quad \text{and}$$
$$X(2k + 1) = X(2k - 1) + 2\,H\,\mathbf{V(2k)} \quad (3.2a\text{-}b)$$

for k = 1, 2, 3, …

Let us remark that these two incursive discrete oscillators (3.1a-b) and (3.2a-b) are identical to the two incursive discrete oscillators (2.14a-b) and (2.15a-b), as we will explain.

These incursive oscillators are incursive, that means implicit non-recursive, because the order in which the iterations are made is important. Indeed, for the first incursive oscillator, (3.1a-b), the position X(k+1) is initially computed and then the velocity V(k+2) is sequentially computed, in taking into account the computed value of X(k+1). And for the second incursive oscillator, (3.2a-b), the velocity V(k+1) is initially computed and then the position X(k+2) is sequentially computed, in taking into account the computed value of V(k+1).
Thus the incursive systems are both characterized by a deterministic anticipation.

The Table 1A gives the iterations of the hyperincursive harmonic oscillator given by eqs. 2.16a-b.

TABLE 1A: Hyperincursive harmonic oscillator, separable into two incursive harmonic oscillators (see table 1B)

	HYPERINCURSIVE HARMONIC OSCILLATOR	
	X(k + 1) = X(k – 1) + 2 H V(k)	**V(k + 1) = V(k – 1) – 2 H X(k)**
	Initial conditions: **X(0) = C_1, V(1) = C_2**, V(0) = C_3, X(1) = C_4	
k	**Iterations**	
1	X(2) = X(0) + 2 H V(1)	V(2) = V(0) – 2 H X(1)
2	X(3) = X(1) + 2 H V(2)	V(3) = V(1) – 2 H X(2)
3	X(4) = X(2) + 2 H V(3)	V(4) = V(2) – 2 H X(3)
4	X(5) = X(3) + 2 H V(4)	V(5) = V(3) – 2 H X(4)
5	X(6) = X(4) + 2 H V(5)	V(6) = V(4) – 2 H X(5)
6	X(7) = X(5) + 2 H V(6)	V(7) = V(5) – 2 H X(6)
ETC	---	---

TABLE 1B: Separation of the hyperincursive harmonic oscillator (see table 1A) into two independent incursive oscillators, with different initial conditions.

	FIRST INCURSIVE HARMONIC OSCILLATOR	**SECOND INCURSIVE HARMONIC OSCILLATOR**
	Initial conditions: X(0) = C_1, V(1) = C_2	Initial conditions: V(0) = C_3, X(1) = C_4
k	**Iterations**	**Iterations**
1	X(2) = X(0) + 2 H V(1)	V(2) = V(0) – 2 H X(1)
2	V(3) = V(1) – 2 H X(2)	X(3) = X(1) + 2 H V(2)
3	X(4) = X(2) + 2 H V(3)	V(4) = V(2) – 2 H X(3)
4	V(5) = V(3) – 2 H X(4)	X(5) = X(3) + 2 H V(4)
5	X(6) = X(4) + 2 H V(5)	V(6) = V(4) – 2 H X(5)
6	V(7) = V(5) – 2 H X(6)	X(7) = X(5) + 2 H V(6)
ETC	---	---

The difference between the two incursive oscillators, given by eqs. (2.14a-b, 2.15a-b) and (3.1a-b, 3.2a-b), holds in the labelling of the successive time steps. In the incursive oscillators, (2.14a-b, 2.15a-b), the position and velocity are computed at the same time step while in the incursive oscillators, (3.1a-b,3.2a-b), the position and the velocity are computed at successive time steps, but the numerical simulations of both give the same values. Each incursive oscillator is the time reverse of the other incursive oscillator, defined by time forward and time backward derivatives. So the two incursive oscillators are not reversible. Thus the superposition of the two incursive oscillators given by the hyperincursive oscillator is reversible.

For the simulations, the values of the initial conditions are given by: $X(0) = C_1 = 1$ and $V(0) = C_3 = 0$, so the energy 2.13 is given, $E(0) = X^2(0) + V^2(0) = E_0 = 1$.

Thus the values of the position and the velocity of the harmonic oscillator are given by the following analytical solution:

$$X_k = \cos(2k\pi/N) \quad \text{and} \quad V_k = -\sin(2k\pi/N) \qquad (3.4a\text{-}b)$$

where N is the number of iterates for a cycle of the oscillator.

The table 2 shows the numerical simulation of the hyperincursive oscillator.

The number of iterates is given by N = 12. So, the values for the two other initial conditions are given by, $X(1) = C4 = \cos(\pi/6) = 0.8660$ and $V(1) = -\sin(\pi/6) = -0.5$.

TABLE 2: The simulation of the hyperincursive harmonic oscillator (see eqs. 2.16a-b) gives exactly the theoretical values that represent alternatively the values of the two incursive harmonic oscillators, given at tables 3A and 3B. There is conservation of energy, E(k)=1.

HYPERINCURSIVE OSCILLATOR						**Analytical solution**	
N	**H**	**k**	**X(k)**	**V(k)**	**E(k)**	**X_k= cos(2kπ/12)**	**V_k= -sin(2kπ/12)**
12	0.5	0	**1.0000**	0.0000	1	1.0000	0.0000
		1	0.8660	**-0.5000**	1	0.8660	-0.5000
		2	**0.5000**	-0.8660	1	0.5000	-0.8660
		3	0.0000	**-1.0000**	1	0.0000	-1.0000
		4	**-0.5000**	-0.8660	1	-0.5000	-0.8660
		5	-0.8660	**-0.5000**	1	-0.8660	-0.5000
		6	**-1.0000**	0.0000	1	-1.0000	0.0000
		7	-0.8660	**0.5000**	1	-0.8660	0.5000
		8	**-0.5000**	0.8660	1	-0.5000	0.8660
		9	0.0000	**1.0000**	1	0.0000	1.0000
		10	**0.5000**	0.8660	1	0.5000	0.8660
		11	0.8660	**0.5000**	1	0.8660	0.5000
		12	**1.0000**	0.0000	1	1.0000	0.0000
		13	0.8660	**-0.5000**	1	0.8660	-0.5000

TABLE 3A : Simulation of the first incursive oscillator (see eqs. 2.14a-b). There is no conservation of energy, $E_1(k)$, but averaged energy on half a cycle is constant, $[E_1(k-1)+E_1(k)+E_1(k+1)]/(N/2) = E_0 = 1.0$. There is a conservation of FORWARD ENERGY, $E_F(k) = E_{F0} = 0.75$ (see eq. 3.4a-b-c)

FIRST INCURSIVE HARMONIC OSCILLATOR							
N	H	k	$X_1(k)$	$V_1(k)$	$E_1(k)$	$FE_1(k)$	$E_F(k)$
6	1.0	0	1.0000	-0.5000	1.25	-0.50	0.75
		1	0.5000	-1.0000	1.25	-0.50	0.75
		2	-0.5000	-0.5000	0.50	0.25	0.75
		3	-1.0000	0.5000	1.25	-0.50	0.75
		4	-0.5000	1.0000	1.25	-0.50	0.75
		5	0.5000	0.5000	0.50	0.25	0.75
		6	1.0000	-0.5000	1.25	-0.50	0.75
		7	0.5000	-1.0000	1.25	-0.50	0.75

TABLE 3B : Simulation of the second incursive oscillators (eqs. 2.15a-b). There is no conservation of energy, $E_2(k)$, but averaged energy on half a cycle is constant, $[E_2(k-1)+E_2(k)+E_2(k+1)]/(N/2) = E_0 = 1.0$. Moreover, there is a conservation of BACKWARD ENERGY, $EB(k) = E_{B0} = 0.75$ (see eq. 3.5a-b-c)

SECOND INCURSIVE HARMONIC OSCILLATOR							
N	H	k	$X_2(k)$	$V_2(k)$	$E_2(k)$	$BE_2(k)$	$E_B(k)$
6	1.0	0	0.8660	0.0000	0.75	0.00	0.75
		1	0.0000	-0.8660	0.75	0.00	0.75
		2	-0.8660	-0.8660	1.50	-0.75	0.75
		3	-0.8660	0.0000	0.75	0.00	0.75
		4	0.0000	0.8660	0.75	0.00	0.75
		5	0.8660	0.8660	1.50	-0.75	0.75
		6	0.8660	0.0000	0.75	0.00	0.75
		7	0.8660	0.0000	0.75	0.00	0.75

For the first incursive oscillator (2.14a-b), I introduce the concept of FORWARD ENERGY, $E_F(k)$ for $k = 0, 1, 2, 3, \ldots$, given by the sum of the energy, $E_1(k)$, and the forward H-dependent energy, $FE_1(k)$, as

$$E_F(k) = E_1(k) + FE_1(k) = E_{F0},$$
$$E_1(k) = X_1^2(k) + V_1^2(k), \text{ and } FE_1(k) = +H\,X_1(k)V_1(k) \qquad (3.4a\text{-}b\text{-}c)$$

and for the second incursive oscillator (2.15a-b), the concept of BACKWARD ENERGY, $E_B(k)$ for $k = 0, 1, 2, 3, \ldots$, given by the sum of the energy, $E_2(k)$, and the backward H-dependent energy, $BE_2(k)$, as

$$E_B(k) = E_2(k) + BE_2(k) = E_{B0},$$
$$E_2(k) = X_2^2(k) + V_2^2(k), \text{ and } BE_2(k) = -H\,X_2(k)V_2(k) \qquad (3.5a\text{-}b\text{-}c)$$

Let us remark that in the expression of the H-dependent energy, the interval of time, H, is positive for the forward oscillator and negative for the backward oscillator.
Let us notice that the averaged energies on the two incursive oscillators give the constant energy,

$$[E_1(k) + E_2(k)]/2 = \text{constant} = E0 = 1.0 \quad (3.5)$$

There is the conservation of this remarkable uncertainty relation, depending on discrete time, H,

$$BFE(k) = [- BE_2(k) - FE_1(k)]/2 = H\,[\,X_2(k)V_2(k) - X_1(k)V_1(k)]/2 = \text{constant} = 0.25 \quad (3.6)$$

Moreover, there is a conservation of FORWARD and BACKWARD ENERGIES,

$$E_F(k) = E_{F0} = 0.75 \text{ and } E_B(k) = E_{B0} = 0.75 \quad (3.7a\text{-}b)$$

These functions are constants of motion for the two incursive discrete harmonic oscillators.

4 Simulations of the Hyperincursive and the Two Incursive Discrete Harmonic Oscillators with Different Parameters

This last section gives Figures 1 to 6 of the simulation of the hyperincursive discrete harmonic oscillator from eqs. 2.16a-b, with N = 3, 4, 6, 12, 24 and 48 time steps.

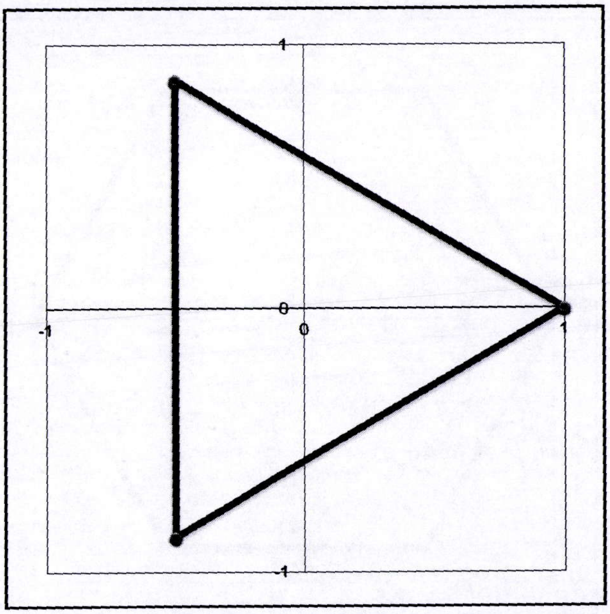

Figure 1: Figure of the simulation of the hyperincursive discrete harmonic oscillator with eqs. 2.16a-b, with N = 3 time steps. The horizontal axis represents the position X(k), and the vertical axis represents the velocity V(k) of the oscillator.

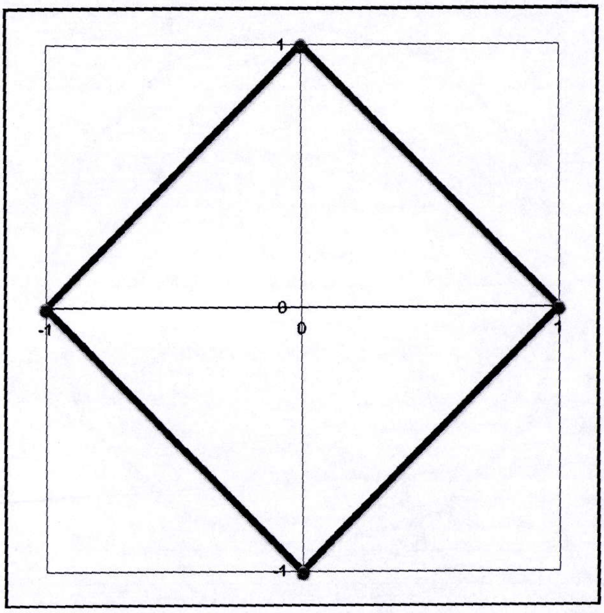

Figure 2: continuation of Figure 1 with N = 4 time steps.

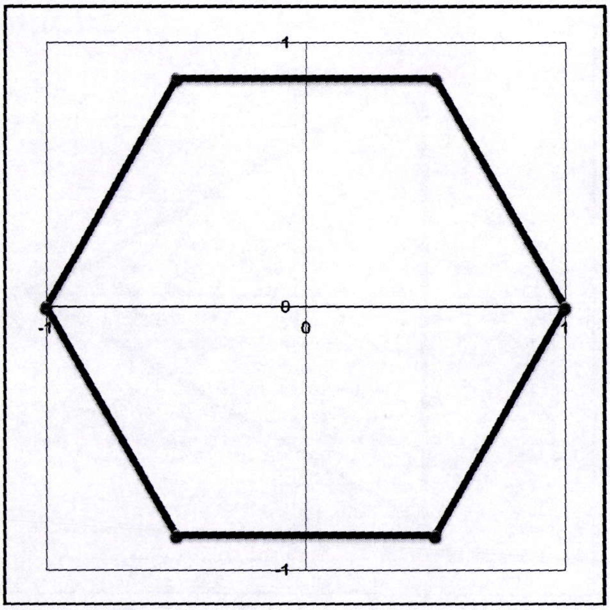

Figure 3: continuation of Figure 2 with N = 6 time steps.

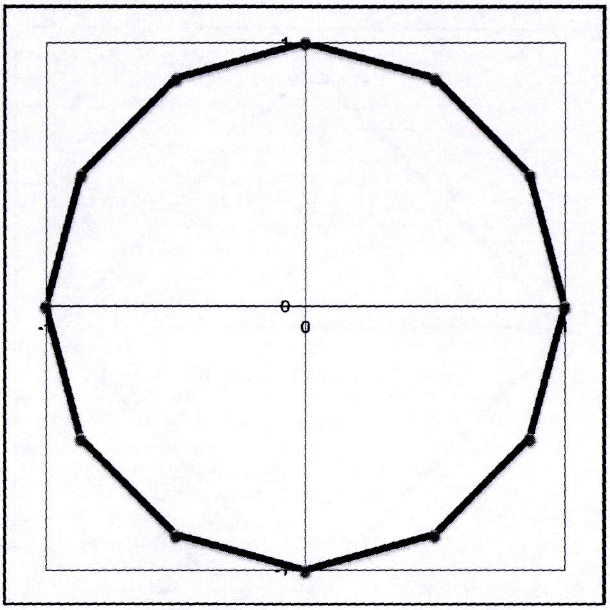

Figure 4: continuation of Figure 3 with N = 12 time steps, and this case corresponds to the numerical values given in Table 2.

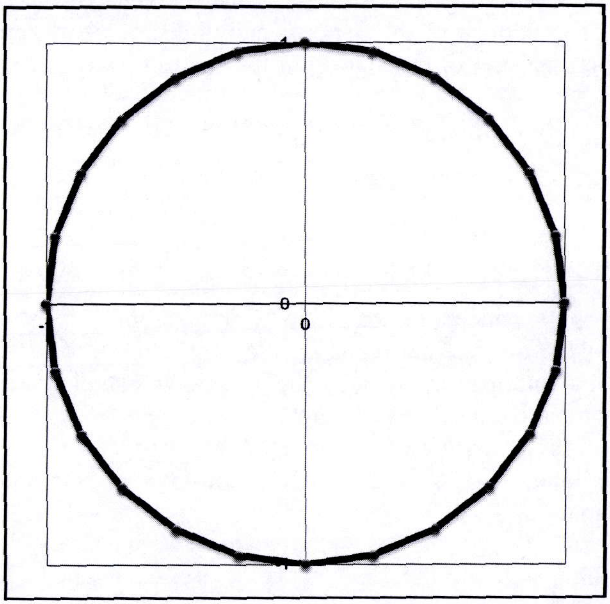

Figure 5: continuation of Figure 4 with N = 24 time steps.

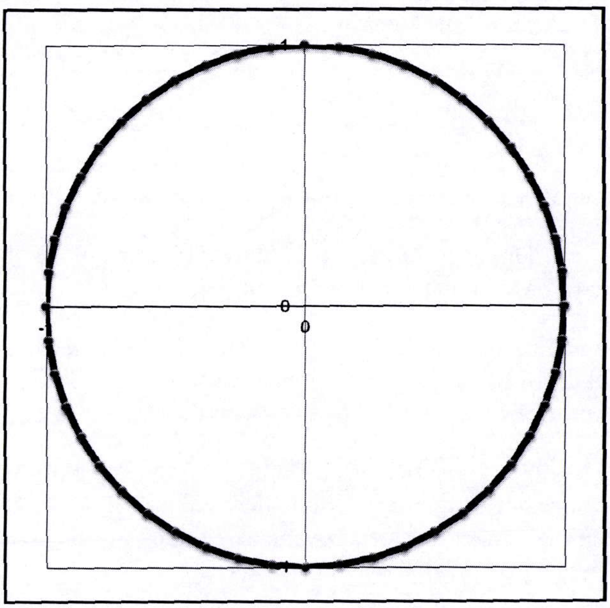

Figure 6: continuation of Figure 5 with N = 48 time steps.

The figures of the simulations of the hyperincursive discrete harmonic oscillator sow the stability and the precision of the algorithm for values of time steps N = 3, 4, 6, 12, 24 and 48.
The representation of the harmonic oscillator tends to a circle when the number of time steps increases

5 Conclusion

This paper deals with the concept of deterministic anticipation.
The general case of the discrete harmonic oscillator is taken as a typical example of a discrete deterministic anticipation given by the hyperincursive discrete oscillator that is separable into two incursive discrete oscillators.
The hyperincursive oscillator shows a conservation of energy.
The incursive oscillators do not show such a conservation of energy but show a deterministic anticipation.
It is proposed to add, to the energy equation, a forward energy depending on the positive discrete time, +H, for the first incursive oscillator, and a backward energy depending on the negative discrete time, – H.
The figures of the simulations of the hyperincursive discrete harmonic oscillation show the stability of the oscillator and the high precision of the numerical computed values, even for very small values of time steps.

References

Antippa, Adel F., and Dubois, D. M. (2007). Incursive discretization, system bifurcation, and energy conservation. Journal of Mathematical Physics, 48, 012701

Antippa, Adel F., and Dubois, Daniel M. (2008). Hyperincursive discrete harmonic oscillator. Journal of Mathematical Physics, 49, 032701

Antippa, Adel F., and Dubois, Daniel M. (2010). Time-Symmetric Discretization of The Harmonic Oscillator, in Computing Anticipatory Systems, edited by D. M. Dubois, AIP CP 1303, American Institute of Physics, Melville, New York, pp. 121–125

Dubois Daniel M. (1995). Total Incursive Control of Linear, Non-linear and Chaotic Systems, in Advances in Computer Cybernetics, volume II, G. E. Lasker (ed.), publ. by The International Institute for Advanced Studies in Systems Research and Cybernetics, Canada, pp. 167-171

Dubois Daniel M. (2000). Review of Incursive, Hyperincursive and Anticipatory Systems - Foundation of Anticipation in Electromagnetism. Computing Anticipatory Systems, edited by D. M. Dubois, AIP CP 517, American Institute of Physics, Melville, New York, pp. 3-30

Dubois, Daniel M. (2010). Introduction to the Natural Anticipator and the Artificial Anticipator, in Computing Anticipatory Systems, edited by D. M. Dubois, AIP CP 1303, American Institute of Physics, Melville, New York, pp. 17–24

Dubois Daniel M., Kalisz Eugenia (2004). Precision and Stability Analysis of Euler, Runge-Kutta and Incursive Algorithms for the Harmonic Oscillator. International Journal of Computing Anticipatory Systems, Volume 14, pp. 21-36, ISSN 1373-5411 ISBN 2-930396-00-8