Polygons and Polynomials as Synoptic Tools for System Guidance

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Abstract
The purpose of this study is to construct a powerful topography for recording the evolution of any compound system composed of (n) subsystems. The essential idea is based on the fact that any regular (n) polygon (with n sides) acts as a vector operator which transforms the plane in a union of (n) concentric isosceles triangles wherein a complex phasor is inserted. Covariant and contravariant axes are introduced in each phasor to obtain equivalence with Bond-Graphs. In each triangle we can follow the behaviour of the correspondent subsystems by building their power balance.
The circular polynomials: $P^n(\theta) = \Pi_k [a_k(\theta - k2\pi/n)]$ play as an algebraic carrier for the transformation of the plane into a (n) star (with n branches). We represent any system working by a variable power triangle. The system coupling increases the stability, and accordingly we will choose the electrical machine as a particular anticipation modeling.
Keywords: Polygonal Configurations, Circular Polynomials, Bond-Graphs, Power Balances, Electrical Devices

1. Introduction

Here the basic idea is to develop a geometric record of the operational behaviour of the (n) different parts of a compound system by using (n)polygons. These synoptic configurations bring vector structures whose (n) components play as complex windows for scanning the time evolution of each elementary system. The introduction of covariant and contravariant variables allows to detect the working system characteristics by means of Bond-Graphs. It is important to develop power balances because any technical system is developed for converting power. For easiness and flexibility the study will explore electromagnetic devices. By constructing the complex vector diagrams to explain the periodic working of electrical devices we deduce that the power distributions always appear in a right angled triangle (= PTr). Besides each (PTr) is a condensation of a set of many Bond-Graphs around the system joint. These (PTr) can move in order to represent the power waves which carry the external influences and coupling effects.

The circular polynomials with angular variables are used to detect the branch directions of the stars and play as carriers of the space Fourier transformations. Their roots point out the orientations of phasor sectors in the (n)polygon.

The coupling development shows the stability growth against the external disturbances for any situation including also the future influence. Because the electrical machine contains a very intense net of magnetic and electrical connections it will be selected to constitute a particular stable anticipation modeling.

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2. Symbolic Geometry

Geometry contains spaces where it is possible to draw curves for representing state evolutions of any systems and various polygonal configurations for structuring the spaces into (n) sectors. These configurations are advantageous to win more easiness by exploring the whole system.

2.1. Polygonal Task

Creation of (n) complex windows containing each a phasor (a rotating complex plan). This is analogous to a space modulation because the two-dimensional plane is partitioned into (n) two-dimensional phasors. In each window it is possible to follow separately the evolution of each element or component of the whole. Indeed the radial expansion of the (n)polygon and phasors corresponds to the elapsed time. This allows to draw the state trajectory of each part because it is possible to draw the curves V(t) & I(t) and to deduce from these last ones the evolution: V[I(t)]
This star partition causes an entropy reduction because the splitting up (n) parts allows dividing by (n) the initial complexity.

2.2. Angular Distortion in the Phasors Inserted in a (n)Polygon

A usual phasor is a whole rotating complex plane, composed of 4 quadrants. However for describing the power distribution, the first quadrant is sufficient what requires an angular aperture of $\pi/2 = 90°$ But the (n)polygonal configuration gives for each phasor an angle of $(2\pi/n)$. The polygonal aperture $(2\pi/n)$ has to be multiplied by $(n/4)$ to find the true one, because $(2\pi/n)(n/4) = \pi/2$.

2.3. Selected Example

To illustrate our considerations we have represented a whole system composed of 6 elements what requires a hexagonal configuration which displays 6 observation windows or phasors. (Figs.1a &1b). (Table1) gives the relations between the covariant variables & between the contravariant ones.

Phasor Use: They are essentially created for presenting the sinusoidal behaviours in vector diagrams. (Figs 2, 3). This is an advantageous synoptic method.

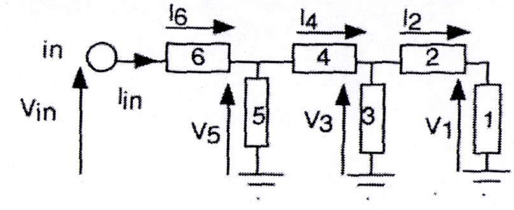

Fig.1a: Connection of 6 subsystems to constitute a Compound System.

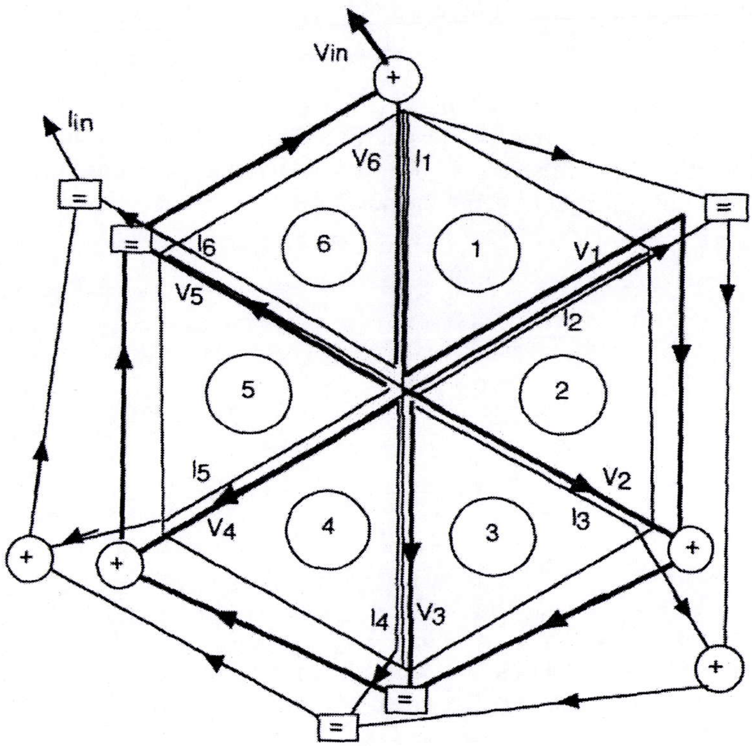

Fig.1b: Polygonal Display of the Windows corresponding to each Subsystem.

Table 1: Potential & Current Relations

Potential Relations (Covariant)	Current Relations (Contravariant)
V3 = V1 + V2	I1 = I2
V5 = V3 + V4	I4 = I1 + I3
Vin = V5 + V6	Iin = I6 = I4 + I5

3. Circular Polynomials

Definition: a circular polynomial of (n) degree is written:

$$P^n = {}_1\Pi_k{}^n\,(\theta - k2\pi/n) \qquad (1)$$

It acts as a rotating operator because each k^{th} factor is:

$$(\theta - k2\pi/n) = \mathbf{exp(j}k2\pi/n\mathbf{)} \qquad (2)$$

where **j** is the imaginary vector unit & $\mathbf{exp(j}2\pi/n\mathbf{)}$ is a rotation operator of angle $2\pi/n$. These circular polynomials are the algebraic operators homologous to the geometric (n)polygons.
They project on the plane regular stars of (n) branches. They play as carriers for these phase modulations and they may be considered as space Fourier distributors. Due to the previous properties they are algebraic forms similar to the polygons and they are well suited for computing operations about the polygons.

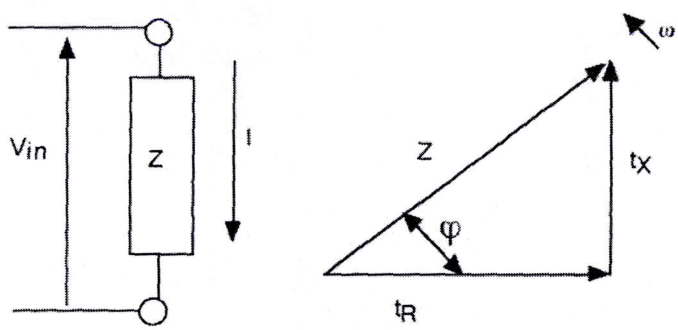

Fig.2: Electrical System and its internal Impedance in a right angled Triangle whose ortho sides are Resistances ^tR and Reactances ^tX.

4. Power Balance of an Electrical Device (Fig. 2 &3)

4.1. Complex Analysis

By analyzing the working of an energy converter, and this is the essential function of any various device, we always detect both types of power: the reactive power noted (Qr) which stores potential energy in storing parts and the active power noted (Pa) which is converted by means of generalized resistances. (Pa) flows from the system to the surrounding. Among the (Pa) are also the losses (or anergy), which produce heat flows and reduce the functional efficiency.

Diversity of loss sources:
Firstly due to the power stream through the system (copper loss in electrical domain), all the friction effects
Secondly due to the " in & out movements" of (Qr). These dissipative effects of storage are indicated by:
the imaginary part of complex permeability $\mu = Re(\mu) - j\ Im(m)$
the imaginary part of complex permittivity $\varepsilon = Re(\varepsilon) - j\ Im(e)$
the imaginary part of complex heat $C = Re(c) - j\ Im(c)$
Proof: in the inductance we write:

$$j\,\mu\,\mathcal{G} = j\,\mathcal{G}\,[Re(\mu) - j\ Im(m)] = \mathcal{G}\,\{j\ Re(\mu) + Im(m)\} \quad (3)$$

where $\mathcal{G}$ is the geometric influence. The last term is real and corresponds to a resistance. For ε and **C** there is analogical explanations.
System study reveals that a sufficient level of potential energy, due to (Qr) is necessary for converting active power.

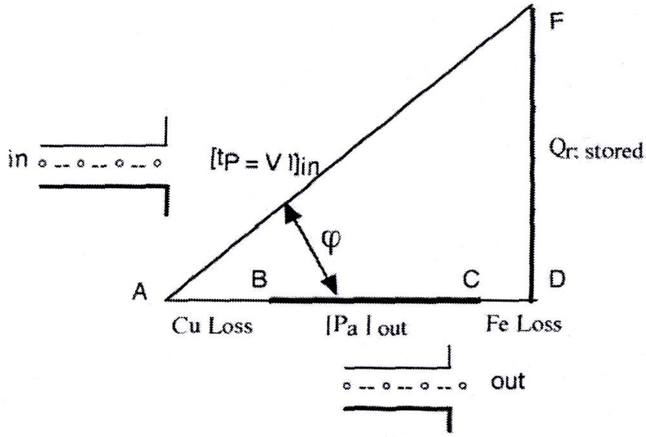

Fig.3: Power Triangle pointing out the Power Distribution during a Working Period.

Table2: Analysis of (PTr) of Fig.3

(AF): Hypotenuse	Total power
(AB): 1° part of the horizontal side	Streaming losses
(BC): 2° part of the horizontal side	Active power
(CD): 3° part of the horizontal side	Storage losses
(DF): vertical side	Stored reactive power

4.2. Result of Complex Analysis

The classic start is the voltage balance presented by the following relation

$V = RI + \mathbf{j}\, XI$ [V]: Ohm's law in complex form (4)

where V is the applied voltage, R: the resistance, X: the reactance, I: the current
By multiplying (4) by I, we obtain the power balance

$IV = RI^2 + \mathbf{j}\, XI^2$ [W] (5)

where RI^2 is the active power = (Pa) which contains losses and the useful power flow.
XI^2 is the reactive power = (Qr), stored in the reactive parts. It gives potential energy.
IV = S is the total or complex power which feeds the system.
The rels. (4&5) appear in the form of right angled triangles in their phasors.
(Table 2) indicates the meaning of the side of the power triangle noted by (PTr) in this report.

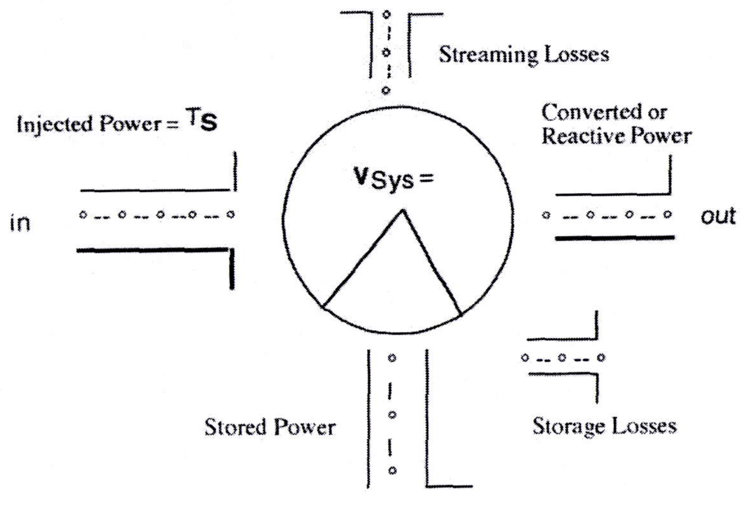

Fig.4: Power Distribution in a Dynamic Convector.

5. Bond Graphs Noted (BG)

Definition: (BG) are power channels composed of a pair of strokes; the thick one supports the covariant factor and the thin one carries the contravariant one.
The central axis displays the power flow which is the product of the stroke factors because any power is the product of an intensive factor with a quantitative one.

In (Fig.4) we can observe (BG)s with a pair of lips, they drive(Pa) flows and also (BG) with a single lip, to produce (Qr).
For more information over this subject see [References 1, 2 , 3 ,4].
(Fig.3) shows that the power balance contained in a (PTr) requires a set of 5 (BG). This underlines the large possibilities supplied by each (PTr).Each (BG) is a complex tensor.

6. Universality of (PTr)

6.1. Adaptation of (PTr) in any Technical Domain

The power conversion indicated by a (PTr) can be used in other domains different of the electrical one. These use extensions can be deduced by means of the analogical rules because losses, stored and converted powers are located in every system type. (Fig.5) shows the energy balance of a thermodynamic converter by (BG) methodology which can be directly transferred in a (PTr) (Fig.3).

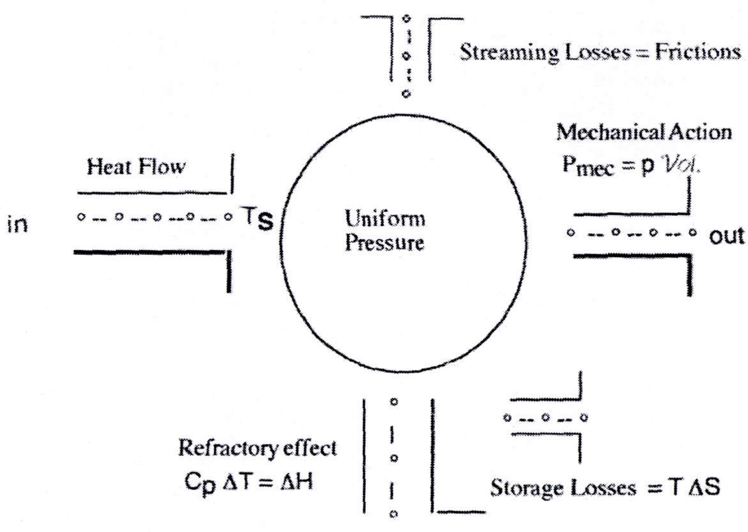

Fig.5: Description of a Thermodynamic Converter by means of Bond Graphs.

6.2. Fourier Decomposition of non Sinusoidal Workings

It is possible to transform any periodic behaviour as a sum of harmonics = sinusoidal functions of different frequencies. For the power we let remark that we only use the even frequencies because the product of two signals with the same frequency gives a result of a double frequency:

$$S = V \exp(jk\omega)\, I \exp(jk\omega) = V\, I \exp(j2k\omega) \quad (6)$$

(Fig. 6) indicates the vertical axis with a frequency graduation. It is logic to construct a harmonic (PTr) on each harmonic phasor what proves the large impact of the (Ptr)s.

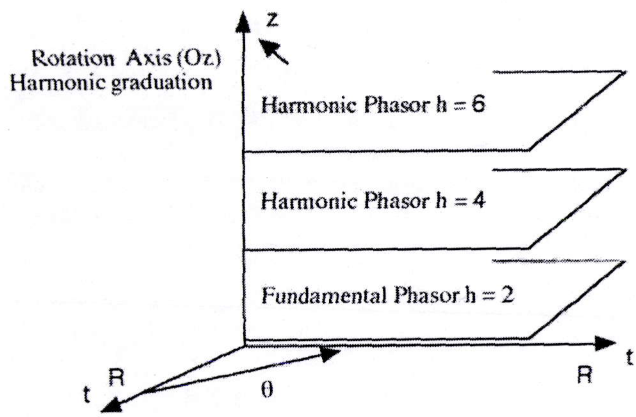

Fig.6: Harmonic Graduation along the Rotation Axis (Oz).

7. Mathematical Operations over the (PTr)s

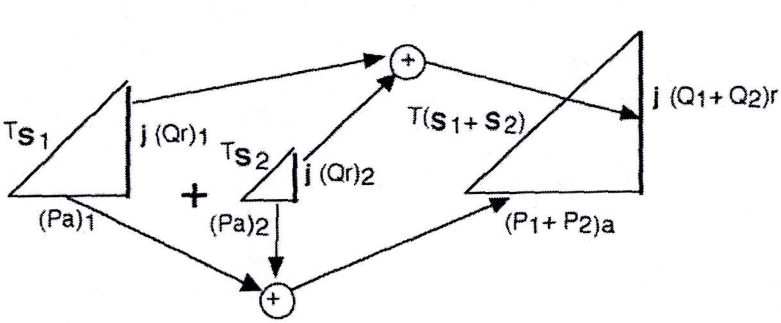

Fig.7: Sum or Integration of a Power Triangle: (PTr)1 + (PTr)2 = Σ_1^2(PTr).

7.1. Key Operational Principle or Dual Rule

To modify the power supported by a (PTr).it is necessary to act along the (Pa) axis and along the (Qr) axis separately This is the dual rule. It is logic because the (PTr)s. are complex numbers. Here the (PTr) is time varying from state (1) to state (2).

7.2. Integration or Sum

This operation has to follow the dual rule. Consequently the resulting (PTr) is always a right angled triangle whose ortho- sides are the sums of the homologous sides of the (PTr)1&2. See (Table 3 & Fig.7)

7.3. Derivative or Difference

This operation has to follow the dual rule. Consequently the resulting (PTr) is always a right angled triangle whose ortho- sides are the differences of the homologous sides of the (PTr)1&2.
See (table 3 & Fig.8).

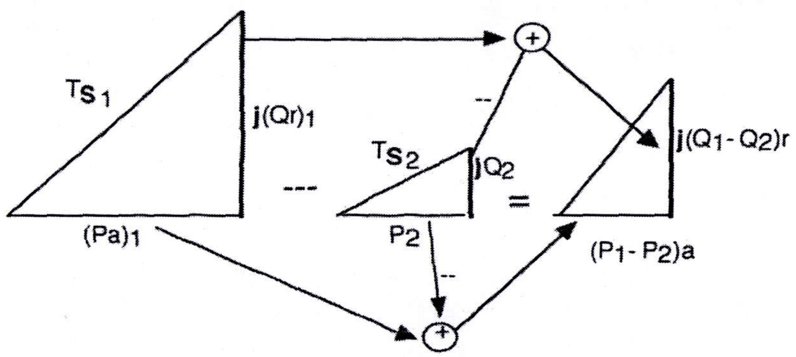

Fig.8: Difference or Derivative of a (PTr): (PTr)1 - (PTr)2 = Δ_1^2(PTr).

8. Traveling Waves

8.1. Definition

Each wave is a power flow traveling from a sender to a receiver (Fig.9). Consequently a wave may be represented by a moving (BG) or a moving (PTr). Therefore the start and arrival points and the velocity are mentioned on both representing tools.

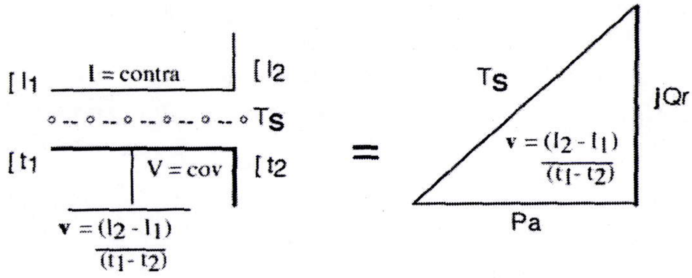

Fig.9: Traveling Wave equivalent to a (PTr), with its Velocity (**v**) & Addresses [l & [t.

8.2. Wave Capture by a (PTr)

By the wave capture an additional power enters the (PTr). This modification of the (PTr) has to follow the dual rule alike the previous operations in (7.2 & 7.3).
Complex power carried by a wave:

$$Sw = (Pa)w + \mathbf{j}(Qr)w \quad (7)$$

See (Fig.10)

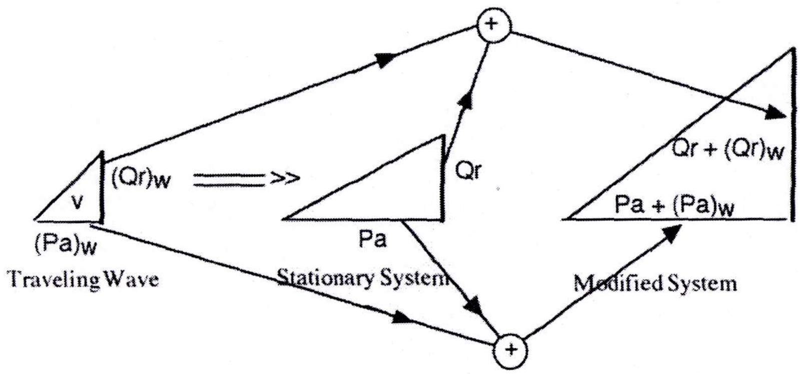

Fig.10: Capture of a Wave by a System. Energizing Effects.

Table 3: Procedures of Integration & Derivative of (PTr)

Power types	Pa	Qr
Integration	(Pc)1 + (Pa)2	(Qr)1 + (Qr)2
Derivative	(Pa)2 – (Pa)1	(Qr)2 – (Qr)1
Wave Capture	(Pa)1 + (Pa)w	(Qr)1 + (Qr)w

All these operations prove the strong stability and flexibility of (Ptr) configuration.

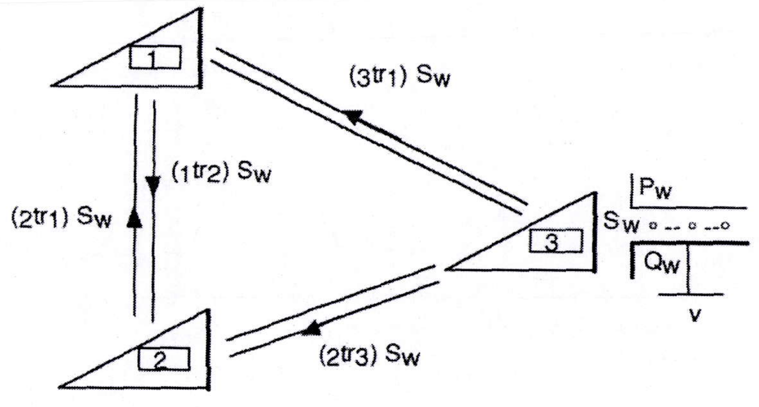

Fig.11: Damping of Wave Disturbance in the subsystem (1) through the Net of coupled systems with Scatter Compensation.

9. System Coupling or Linkage Fields

9.1. Presentation

In electromagnetic domain magnetic fields are usually induced for linking sets of electrical windings. These linkage fields may be produced by stationary waves or permanent wave trains.

9.2. Stability Effects

These field couplings bridge the windings by energetic channels. This process weaves an additional compensatory net to scatter the wave power through the whole set, when an aggressive wave is clashing with the (PTr)3. This injected power can spread into the whole system what allows a disturbance leveling over the complete net. This is an appreciable reduction of the instantaneous local perturbation. See (Fig.11).

This proves that the size increase of any system is an important stability factor. It is a logic effect because any large system integrates the energies of all its components into a resulting synthetic (PTr) whose power will be probably very high in face any disturbing one. Consequently in the large nets the wave power will oft be negligible.

10. Electromagnetic Modeling of Future

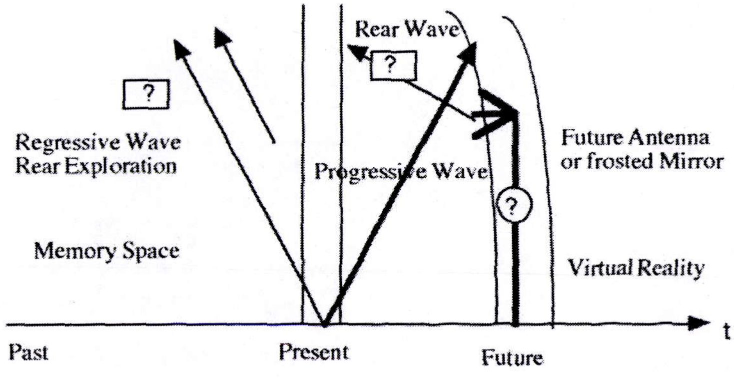

Fig.12: Regressive Wave to approximate the Future Rear Action Progressive Wave from Future sensors.

10.1. Composition of Future

To set up an anticipatory guidance it is necessary to obtain a few lightings about the probable disturbances which will attack our systems. But Future seems to behave as a very hazardous system. The sources to forecast the influences of Future may be the extrapolations of the past evolutions, by hoping that no important disturbance will suddenly appear to delete our present stability processes. Future may seem an unfinished building partly hidden by a smoke surrounding. Consequently any Future modeling is launched from our minds with our suppositions and feelings in accordance with the chosen targets. From these considerations it seems logic that any bridge to any selected Future has to be crossed by reactive bidirectional waves. It means: on the first way the progressive waves for searching any event, on the other side the regressive waves coming from Future to our Present. This would be similar to an echoic or radar phenomenon if our Future works as a reactance connected to an antenna; but with a weak reliability (Fig.12). It is important to note that electromagnetic frame seems very advantageous to set up these anticipation schemes. Therefore we have decided to choose a suited electrical machine to develop an anticipatory topography.

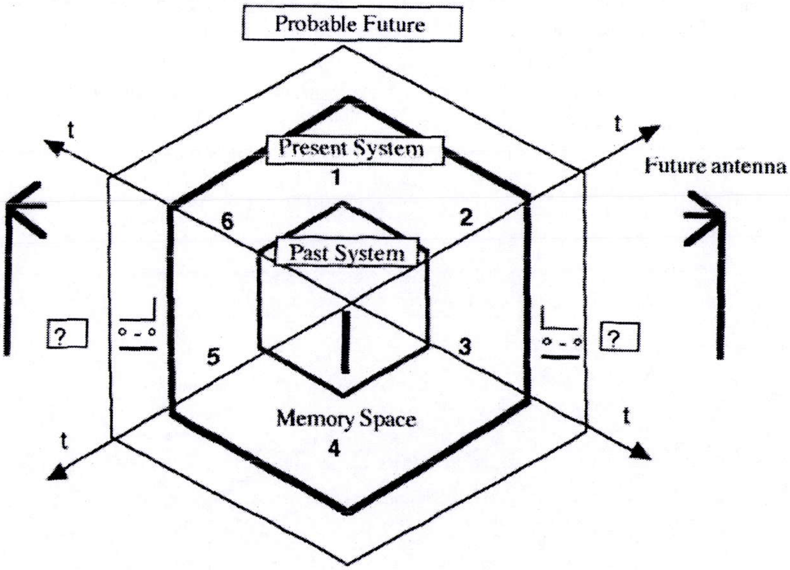

Fig.13: Time Evolution of a Compound System in a Hexagonal Configuration.

10.2. Electrical Machine as a Particular Simulation of Anticipation

The choice of the electromagnetic topology was suggested by a pair of strategic considerations which were developed along this study.

Firstly the inside of the electrical machine is a closed space with a lot of magnetic couplings where are situated the (n) electrical windings representing (n) elements. Here the windings may successively play the roles of inductor & induced parts. The conductive interconnections and the magnetic coupling construct a very stable macro system which can maintain its present dynamic working very firmly.

Secondly the polygonal structure selected as a synoptic analyzer of any compound system supports a configuration very similar to the electrical machine. These morphologic similarities are still more obvious when the future state of the system is put on the hollow polygon as represented in (Fig. 13).

10.3. Modified Electrical Machine as an Anticipation Tool

In our special electrical machine the elements of the present system are located on the rotor or armature (induced windings) analogous to the convex polygon, where each subsystem plays as a phase coil.

The stator, usually the main inductor, plays here as the future system. But this special stator, analogous to the hollow polygon has to move along the radii what considerably reduces its inductor efficiency because the air-gap increasing. This last specificity is a logic fitting because forecasting is always a hazardous endeavour. The influences of Future are indicated by the reactive waves emitted from the stator and through the variable air-gap; showed in (Fig. 14). *In this device it is possible to simulate different* future dynamics by regulating the current and radial displacement of the stator and by detecting the correspondent variations of currents and voltages in the armature windings. This may give idea to develop such further synoptic anticipatory tools.

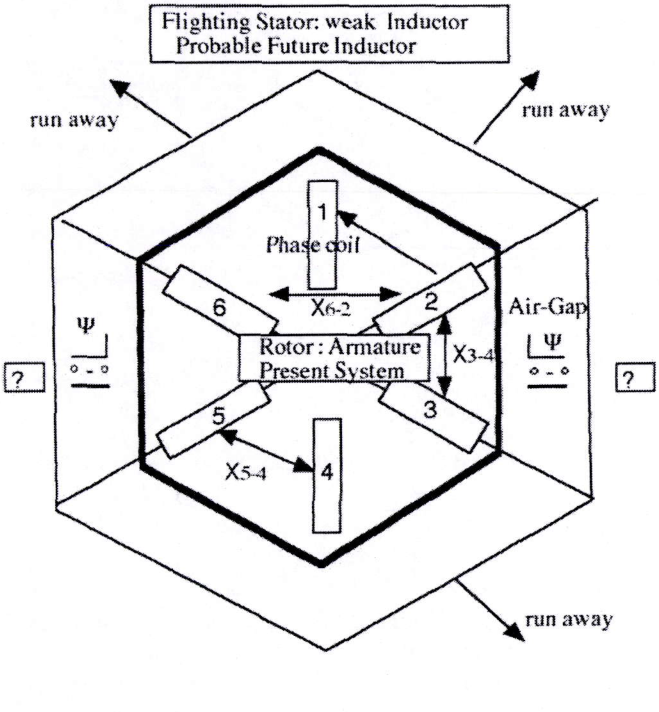

Fig.14: Electrical Machine Structure : special anticipatory Picture.

The winding connections are drawn in (Fig.1b).To reveal the effects of Future it is useful to produce armature rotation of velocity **v** for cutting the stator magnetic flow.
The basic explanatory equations (8&9) are particular forms of (4&5)
Voltage balance in (k) subsystem:

$$V_k = R_k I_k + \mathbf{\Psi} \mathbf{v} + \mathbf{j}[X_k I_k + \Sigma l\, X_{kl} I_l] \qquad (8)$$

Power balance in (k) subsystem:

$$\mathbf{S}_k = (VI)_k = R_k\,(I_k)^2 + \boldsymbol{\Psi}\mathbf{v}(I_k) + \mathbf{j}[X_k\,(I_k)^2 + \Sigma l X_{kl}\,(I_l)^2] \tag{9}$$

The terms containing X_{kl}, I_l and $X_{kl}\,(I_l)^2$ are due to the couplings along the armature of Present state Incursivity of Future: $\boldsymbol{\Psi}$ is considered as the magnetic field from the stator which is playing as the fuzzy inductor of Future. Consequently the terms $\boldsymbol{\Psi}\mathbf{v}$ in (8) and $\boldsymbol{\Psi}\mathbf{v}(I_k)$ in (9) bring in the present state the influence of Future. These terms are proportional to the armature velocity and therefore develop mechanical power: $\boldsymbol{\Psi}\mathbf{v}$ is the dynamic voltage, $\boldsymbol{\Psi}\mathbf{v}(I_k)$ the torque applied on the armature. We underline that in this configuration the record of Future influences would be valuated by means of easy measures.

12. Conclusion

This report highlights the advantages of symbolic geometry and of electromagnetic domain to scan and regulate any dynamical system.

12.1. Benefits of Geometry

Along this communication we have noted the synoptic characteristics of geometric configurations as a help for understanding the system working. The circular polynomial of (n) degree [$P^n(\theta)$] in parallel with (n)gone act as vector space generators in their plane. Note the similarity between star spaces and multi window display, which act as synoptic entropy reducer. The (Ptr)s. play strategic roles to describe the power balances in transformations. They are universal graphical tools, with a simple configurations easy to suit.

12.2. Electromagnetic Modeling

Here, we have to underline: flexibility and large impact of the phasors where the (Ptr)s are introduced. Electromagnetic waves and (BG) show and explain with easiness the power transfers, distributions, system couplings, any delayed influence and correlations in space and time. The waves are unavoidable trains of any communication and transfer procedures and consequently they act as necessary vehicles for all exchange.

The adapted electrical machine is a very strong coupled universe, consequently very stable, where we can represent the components of a compound system by means of armature windings. Here a moving stator is used to bring the effects of different Future horizons which are carried by waves through the air-gap. This will probably yield new tools for Future exploration. To complete this trip between geometry and electromagnetism, note the similar configurations of (n)gones and electrical machines.

Symbolic geometry in association with complex tensors & (BG) allows an appreciable thought time saving.

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