The Numbers of Nature's Code

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Abstract
Assuming that Nature is described by a universal rewrite system and operates according to a process that we can refer to as Nature's code, we define certain numbers as being crucial indicators of how the code operates, whether in biology, chemistry or physics. We also show how these numbers originate in the most extraordinarily simple way from two numbers which correspond to the two distinct processes – conserve and create – within the system which can in turn be related to the properties of duality and anticommutativity. The numbers emerge in a number of distinct series which have distinct algebraic and geometrical representations. The geometrical structures translate easily from 3- to higher-dimensional representations, especially those of dimension 4 and 8, and also connect significantly with rotational symmetries and with Lie groups, up to E_8. The fundamental particles of physics are a classic case of the operation of the number series, in which all the significant numbers are represented, and no others. The structure leads to a complete classification of fermions and bosons within an overall E_8 representation. An almost parallel system emerges in the genetic code, leading to the processes of transcription and translation.
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1 Introduction

If Nature, at its most fundamental level, is described by a universal rewrite system [1-3] and operates according to a process that we can call Nature's code [4], then certain numbers generated by the rewrite structure become crucial indicators of how the code operates. These numbers have significant algebraic and geometrical manifestations, and appear in those aspects of physics and biology, and possibly also chemistry, in which the most efficient degree of information processing may be expected to occur. The crucial role of certain numbers, for example, 5, 8 and 24, is already a matter for discussion and sometimes related to special aspects of mathematics [5], but the deeper origins of these numbers as key aspects of fundamental natural processes have not been understood. The question remains: why are these particular numbers (and a number of others, such as 4, 32 and 64) particularly significant in Nature? In addition, we should like to know how many such numbers there are and how they relate to each other.

It is significant here that the 'numbers' we are concerned with are all integers, and that our argument has no direct connection with considerations of such non-integral numbers as the Feigenbaum number and the various dimensionless ratios that occur in physics, whose significance has been discussed by a number of authors, for example, Tipler [6]. The universal rewrite structure generates integers by a specific process, which simultaneously creates integral connections between algebra, geometry and group theory, and suggests their applications to biology and physics.

Here, we will propose that the integers of specific interest originate in the most extraordinarily simple way in the universal rewrite system, and that, ultimately, they all stem from just two numbers, 2 and 3, which correspond to the two distinct processes within the system. The number 2 emerges from the conserve process which requires duality at all times, and ensures that a doubling or bifurcation takes place at every new stage in the rewrite structure, while the number 3 emerges from the create process, which leads to anticommutative options being selected by default. All the other significant numbers emerge from the structure by these processes in combination.

2 Duality and Anticommutativity

The universal rewrite system exists prior to mathematics, but the characteristic mathematics it generates can be most easily expressed in an iterative series of the form:

$$
\begin{array}{l}
(1, -1) \\
(1, -1) \times (1, \boldsymbol{i}_1) \\
(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \\
(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \\
(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \\
(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots,
\end{array}
$$

where $\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{i}_3 \ldots$ are quaternionic units in which those with different subscripts are commutative to each other.

The rewrite system requires duality because of the need to maintain totality zero, but the zero is not unique, so we have many dualities. The dualities allow two apparently equal possibilities, commutative and anticommutative, but the only way of making these unique is to default on the anticommutative option. In effect, every alternate duality, after the first, completes a new anticommutative pairing. Dualities introduce doubling, so successive dualities (of any kind) require a series of powers of 2:

$$2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad \ldots \tag{1}$$

However, in the case of an anticommutative pairing, the two dualities (2×2) split into a $3 + 1$ combination, of which the 3 become the anticommutative part, while the 1 remains commutative. Specification of successive anticommutative triplets introduces successive powers of 3:

$$3 \quad 9 \quad \ldots \tag{2}$$

In effect, all the numbers that are most significant in nature at the fundamental level (in arithmetic, algebra, geometry, physics, chemistry, biology) have their origin in these two possibilities. Nothing else is as truly significant.

Now, though the rewrite system continues to infinity, a particularly significant stage is reached when 6 stages of duality have been introduced and the number 64 is reached, because at this stage we can create a closure with an immediate return to zero [3]. This also requires two sets of anticommuting triplets. Triplets, of course, can be combined with a series of dualities, as they simply represent the parts of a duality series with anticommutative components, i.e. when we split a 2×2 into a $3 + 1$. If we combine one triplet with powers of 2 representing duality, we get numbers such as

$$6 \quad 12 \quad 24 \quad 48 \quad 96 \quad 192 \quad \ldots \tag{3}$$

If we combined two triplets with the powers of 2, we get

$$18 \quad 36 \quad 72 \quad 144 \quad \ldots \tag{4}$$

These numbers arise merely from making the $3 + 1$ splits in series (1) explicit:

$$2 \quad 4 \quad 6+2 \quad 12+4 \ldots \tag{5}$$

The combinations in this series become increasingly complex, with the next two numbers, for example, being specifiable as $36 = 18 + 12 + 2$ and $36 + 24 + 4$.

The system does not require algebra but generates it and is easily understood in algebraic terms. If we begin with 1 and imagine the first six dualities that emerge, we obtain the 64 terms in the product:

$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$$

The terms $\boldsymbol{i}_1$, $\boldsymbol{j}_1$ and $\boldsymbol{i}_2$, $\boldsymbol{j}_2$ are units of independent (commutative) quaternion series whose third terms $\boldsymbol{k}_1 = \boldsymbol{i}_1\boldsymbol{j}_1$ and $\boldsymbol{k}_2 = \boldsymbol{i}_2\boldsymbol{j}_2$ will emerge when the complete product is taken; $\boldsymbol{i}_3$ is the first term of a third (incomplete) quaternion series. It is convenient to rearrange this information, so that, while $\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{k}_1$ remain a pure quaternion series, now identified as $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, the second set of quaternions are multiplied by $\boldsymbol{i}_3$ (now designated as pseudoscalar i) to become $\boldsymbol{i}_3\boldsymbol{i}_1$, $\boldsymbol{i}_3\boldsymbol{j}_1$, $\boldsymbol{i}_3\boldsymbol{k}_1$. These now have the property of being complexified quaternions or multivariate vectors, which can be given the labels **i**, **j**, **k**. The 64 possible combinations of the 8 base units ($1, i, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$, **i**, **j**, **k**) (or 16 including + and – values) then become:

$\pm 1, \pm \boldsymbol{i}, \pm \boldsymbol{j}, \pm \boldsymbol{k}$,
$\pm \mathbf{i}, \pm \mathbf{i}\boldsymbol{i}, \pm \mathbf{i}\boldsymbol{j}, \pm \mathbf{i}\boldsymbol{k}, \pm \mathbf{j}, \pm \mathbf{j}\boldsymbol{i}, \pm \mathbf{j}\boldsymbol{j}, \pm \mathbf{j}\boldsymbol{k}, \pm \mathbf{k}, \pm \mathbf{k}\boldsymbol{i}, \pm \mathbf{k}\boldsymbol{j}, \pm \boldsymbol{k}\boldsymbol{k}$,
$\pm i, \pm i\boldsymbol{i}, \pm i\boldsymbol{j}, \pm i\boldsymbol{k}$,
$\pm i\mathbf{i}, \pm i\mathbf{i}\boldsymbol{i}, \pm i\mathbf{i}\boldsymbol{j}, \pm i\mathbf{i}\boldsymbol{k}, \pm i\mathbf{j}, \pm i\mathbf{j}\boldsymbol{i}, \pm i\mathbf{j}\boldsymbol{j}, \pm i\mathbf{j}\boldsymbol{k}, \pm i\mathbf{k}, \pm i\mathbf{k}\boldsymbol{i}, \pm i\mathbf{k}\boldsymbol{j}, \pm i\mathbf{k}\boldsymbol{k}$

All of the numbers (up to 64) in the series (1) to (5) can be identified within this product. Similar results could be obtained using the complex double quaternion algebra ($1, i, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, j, k$), in which ($i, j, k$) represent the unmodified ($\boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{k}_2$).

What is very significant here is that there is no place for the number 5 or any product of it. 5 and its powers and multiples do not fit naturally into these series. In fact, 5 is not a 'natural' number in the same way as 2 and 3, and yet it is ubiquitous in nature in very significant ways. The reason is that order 64 provides a particularly key stage in the rewrite structure, and 64 introduces 5 as a result of binomial combination.

The 64 combinations in the product of the 8 base units of the algebra are a group but the group does not require the 8 base units as generators. It is possible to select groups of 5 units from the 64 in many ways which will generate the entire algebraic system. Such pentads always have the same overall structure in which the symmetry of one of the two triplets ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$) is preserved and that of the other (**i**, **j**, **k**) is lost, for example,

$$ik \qquad \boldsymbol{i}\mathbf{i} \quad \boldsymbol{i}\mathbf{j} \quad \boldsymbol{i}\mathbf{k} \qquad j$$

Within the group of 64 it is possible to select 12 such pentads simultaneously (though many more are available in total), with only (± 1, ± *i*) not contributing.

This in itself might not be particularly significant, but for the fact that this kind of structure allows a level of closure and immediate return to zero by defining a *nilpotent* version of the combination which squares to zero. In physics, we have

$$ikE \qquad \boldsymbol{i}\mathbf{i}p_x \quad \boldsymbol{i}\mathbf{j}p_y \quad \boldsymbol{i}\mathbf{k}p_z \qquad jm$$

which is nilpotent as a sum, and represents conservation of energy-momentum in both classical and quantum physics.

With the nilpotent structure, we truncate the system by effectively applying its structural foundational principles to itself. We apply a zero total duality to the number of 3's and derive a 3 times attempt at duality this way from the 3-ness. The completion occurs when we do this mapping. The system encloses in on itself. However, the introduction of 5 always implies some degree of impurity, inhomogeneity and symmetry-breaking, as well as closure and uniqueness. It is also significant from the point of view of introducing the necessity of change in nature. In fact, 5 seems to be the *signature* of such things. The symmetry-breaking is most obviously manifested in one of the two component triplets (here ***i***, ***j***, ***k***). To make the 5 we can define the 3 to map onto as either changing or fixed (changing is characteristic of terms × by the *i* of the incomplete quaternion system) or as either observation or ontology (space / charge); we can even have two actual spaces, one changing and one fixed (again × by *i* to fix it). (The double 3-D becomes phase space.)

Of course, since the rewrite structure is universal and fractal, and can begin at any stage, higher combinations, in which the 5 becomes a new unit (i.e. a new number 1), also become possible. In addition to, say, 5 × 3, we can, for example, combine 5 with series (1)

$$10 \quad 20 \quad 40 \quad 80 \quad \ldots \tag{6}$$

and with series (3)

$$30 \quad 60 \quad 120 \quad 240 \quad \ldots \tag{7}$$

With the closure it introduces, 5 becomes a kind of boundary, and we can extend it by combination either from below or from above, using the 2's of duality and the 3's of anticommutativity. In addition 5 always divides into 4 + 1 and 3 + 1 + 1. It is a kind of 'artificial' construct representing a more fundamental and symmetrical 8 base units mapped onto a structure provided by one of the units. The closure is really that of the 8 base units (1, *i*, ***i***, ***j***, ***k***, **i**, **j**, **k**), which maintain a perfect group symmetry, and the significance of the 8 also appears in algebra with the creation of an 8-unit normed division algebra (octonions) to complement those of 1 unit (real numbers, 1), 2 units (complex numbers, 1, *i*) and 4 units (quaternions, 1, *i*, ***i***, ***j***, ***k***). The 7 imaginary (norm –1) units of octonions are a mathematical expression of the closedness of a system involving 8 (1 real, norm 1, and 7 imaginary, norm –1), and its special nature. In this case, just as quaternions double the complex algebra by introducing anticommutativity, so here we invent another property (antiassociativity) to double the quaternion algebra.

This, however, as the closure of 8 units demands, is the end of the line – there are no further normed division algebras. In addition, though the complex double quaternion algebra (1, *i*, ***i***, ***j***, ***k***, **i**, **j**, **k**) appears to be the algebra favoured by nature, its structure of 1 real + 7 imaginary units means that it can be mapped onto an octonion algebra with the same structure, especially as the aspects of the octonion algebra requiring antiassociativity appear to be the ones excluded by the more specific structure of (1, *i*, ***i***, ***j***, ***k***, **i**, **j**, **k**), and therefore to be without physical meaning [3]. Mapping onto the octonions also leads to the higher Lie group symmetries deriving from this algebra, such as E_8. However, these structures will automatically retain that breakdown into subgroups which ultimately derive from the structure imposed by the 'broken octonion' (1, *i*, ***i***, ***j***, ***k***, **i**, **j**, **k**).

3 Fundamental Particles

The fundamental particles constitute a classic example of the operation of number systems (1) to (7), in which all the significant numbers are represented (and no others). The different numbers arise as particles are classified and grouped together in different ways. The broken symmetries of the rewrite system are exactly matched by the various particle subgroupings. Because the symmetry-breaking is ultimately 3-dimensional in origin (and manifested, for example, in quarks and 3 particle generations), they map naturally onto geometries in 3-dimensional space, and the numbers tend to be those associated with aspects of the 3-dimensional Platonic solids. However, the higher groupings which collect together particles such as quarks and leptons, or fermions and bosons, also show relationships with structures in higher-dimensional spaces and groups connected with them. Here, as we would expect, the symmetries become less broken, culminating in an unbroken root vector structure in E_8, the highest group symmetry to emerge from the rewrite mathematics. Generally, the structures in the higher-dimensional figures carry with them the numbers associated with those from the lower dimensions.

In the Standard Model, the particles are divided into fermions (spin ½) and gauge bosons (spin 1), and the fermions are divided into quarks and leptons. There are 6 quarks arranged in 3 generations, each of which has 2 weak isospin states (up / down; charmed / strange; top / bottom), and each of which comes in 3 varieties of 'colour'. Corresponding to these are 6 leptons, again in 3 generations, each with 2 weak isospin states (electron neutrino / electron; muon neutrino / muon; tau neutrino / tau). There are no colours associated with the leptons, so each set of 3 coloured quarks and 1 lepton, in each isospin state in each generation, represents a kind of 4-dimensional structure, parallel to that of space and time. The total of real fermions in the Standard Model is therefore 24 (18 coloured quarks + 6 leptons).

In addition, each fermion has 2 possible spin states, and to every fermion state there is a corresponding antifermion state, making a total of 96 real fermionic + antifermionic states. The 2 spin states and fermion / antifermion options are an intrinsic aspect of the fermion's spinor structure, which in the nilpotent representation, which emerges from the rewrite mechanism, has the form ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*). Now, spin 1 bosons can be represented by ($\pm$ ***ik****E* $\pm$ ***i*****p** + ***j****m*) ($\mp$ ***ik****E* $\pm$ ***i*****p** + ***j****m*). In effect, 4 fermionic states are required to produce a boson, and the nilpotent formalism shows that 1 spin 1 vacuum boson (never seen, but still mathematically necessary) is created for every 4 real fermionic states, and 4 vacuum fermionic states (again never seen, but still necessary) are created for every real spin 1 boson.

Now, the number of real spin 1 gauge bosons in the Standard Model is 12 (8 gluons, $W+$, W^-, Z^0, γ), but virtually all Grand Unified theories (and certainly $SU(5)$) predict the existence of another 12 (6 X and 6 Y) to unify strong and electroweak interactions. (A GUT proposed by Rowlands and Cullerne achieves this unification at the Planck mass with the addition of an inertial pseudoboson, which is essentially a copy of the photon [3,7].) These are equivalent in number but are distinct from the 24 vacuum bosons which accompany the 96 real fermionic / antifermionic states in the nilpotent formalism, in the same way as the real fermionic / antifermionic states are distinct from the 96 vacuum fermion / antifermion states which accompany the 24 real bosons. A combination of 96 fermions / antifermions and 24 spin 1 gauge bosons creates a total of 120 real particle states. If there are an equal number of vacuum states, then the total becomes 240, the kissing number in 8 dimensions and also the number of root vectors in E_8.

Particles (i.e. fermions + gauge bosons) constitute a 5 which multiplies by 4 factor 2 dualities (spin up / down, isospin up / down, fermion / antifermion, particle / vacuum, symbolised respectively by S, I, A, V, and relating, respectively, to space, charge, time and mass) and 1 factor 3 triplet (generations, symbolised by G), and so (depending on the order in which the 4 factors of 2 and 1 of 3 are multiplied) generates all the number series from (1) to (7), up to a maximum number of 240. All products of 5 are equally artificial constructs, for example, linking fermions with bosons in the exceptional groups E_6 to E_8 through the fact that the last term in the 5 can be a scalar.

In effect, we invert the derivation of 12 structures from a 5-unit pentad, and map the fermions and bosons onto a new pentad structure, of which the pseudoscalar component (the iE term) is 24 leptons / antileptons, and the vector component (the $\mathbf{p}$ term) 72 quarks / antiquarks. Bosons are scalar particles, and scalars are the squared products of pseudoscalars and vectors, just as bosons are the squared products of fermions / antifermions. So if the 24 bosons occupy the *scalar* part of the pentad (the m term), then we can use nilpotency to group the 96 fermions (24 leptons and 72 quarks) with the 24 bosons into a single structure with 120 fermions plus bosons.

Mathematically, in fact, the only way of including bosons and fermions in the same representation is through the exceptional groups E_6, E_7, E_8, and these would seem to be represented by the stages 48 + 12 = 60, 96 + 24 = 120, 192 + 48 = 240. So, in a nilpotent system, we have a physical as well as mathematical reason for combining fermions and bosons in the same representation, and, in the nilpotent structure, a fermion is necessarily, in some sense, its own vacuum boson, and vice versa [3]. The 12 bosons we would put with the 48 are the 8 gluons, the 2 W and 1 Z boson, and the photon, that is, all except X and Y.

	quarks	leptons	bosons		fermions	bosons							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	S				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	S	I			
5	18	6	6	=	24	6	=	30	S				G
6	24	8	8	=	32	8	=	40	S	I	A		
7	36	12	12	=	48	12	=	60	S	I			G
8	48	16	16	=	64	16	=	80	S	I	A	V	
9	72	24	24	=	96	24	=	120	S	I	A		G
10	144	48	48	=	192	48	=	240	S	I	A	V	G

1	1 generation with 1 isospin and 1 spin
2	1 generation with 1 isospin and 2 spins
3	3 generations with 1 isospin and 1 spin
4	1 generation with 2 isospins and 2 spins
5	3 generations with 1 isospin and 2 spins
6	1 generation with 2 isospins and 2 spins + antiparticles
7	3 generations with 2 isospins and 2 spins
8	1 generation with 2 isospins and 2 spins + antiparticles + vacuum
9	3 generations with 2 isospins and 2 spins + antiparticles
10	3 generations with 2 isospins and 2 spins + antiparticles + vacuum

As we have defined 5 as a kind of boundary state, making up a new unit in the number series, it is significant that each fermionic state itself has a 5-fold structure, made up, similarly, of 4 + 1 = 3 + 1 + 1 components (***ip***, *ikE*, *jm*). So, from the point of view of *components*, numbers like 120, 144 and 240 would become, respectively, 600, 720 and 1200, which are numbers with special significance in the higher-dimensional geometries.

The particle structures outlined here are, remarkably, defined purely in terms of the integers generated by the universal rewrite system. There is, however, a fundamental physical system incorporated in the seemingly simple structures, for the non-Abelian Yang-Mills gauge theory through which the particle interactions are defined in the Standard Model can be shown to be a product of the original algebra that generated the integer series [3]. What we are doing here is showing that particle structures follow an algebraic scheme, which simultaneously generates the Yang-Mills theory. The basic reason why they do so is because of the dual 3-dimensionality of space and charge [3].

4 Some Significant Numbers

After 2, which represents duality in its many manifestations, 3 originates in anticommutativity. This is manifested in the dimensions of space and other vector quantities, and of angular momentum and other axial vectors; the 3 types of charge; the 3 symmetries *C*, *P*, *T*; the 3 generations of fermions and leptons; the 3 weak bosons; the

3 colours associated with quarks; the 3 physical quantities in the fermionic nilpotent (E, $\mathbf{p}$, m); the 3 bases in triplet codons; the orders of tetrahedron in rewrite geometry [4]; and the 3 quaternion systems introduced in rewrite structure before truncation. All of these can be shown to originate in anticommutativity [3].

All further numbers of significance have their origin in the two distinct processes represented by the numbers 2 and 3. The distinction between them can be seen in the two types of lattice in a 2-dimensional plane which have the most symmetry, the square lattice Z^2, with 4-fold symmetry, and the more densely-packed hexagonal lattice A_2, with 6-fold symmetry. The first is equivalent to the projection of a cube on to a plane from one face, while the second shows the cube projected with one vertex at the centre. In principle, the first makes no use of dimensionality (and therefore anticommutativity), only of orthogonality. The number 3 does not appear, and the 4-fold symmetry makes use only of the dualities of (1, −1) and (1, $\boldsymbol{i}_1$). The second, however requires the dimensionality and anticommutativity of (1, $\boldsymbol{i}_1$) and (1, $\boldsymbol{j}_1$), in addition to the (1, −1) duality.

When a set of two dualities, such as (1, $\boldsymbol{i}_1$) and (1, $\boldsymbol{j}_1$), incorporate anticommutativity, there is an immediate split of the 4 into 3 + 1. Space-time and mass-charge are characteristic expressions of this in physics, as are the divisions of fermions into 3 colours of quark with 1 corresponding lepton; the 3 + 1 electroweak bosons; and the real state + 3 vacuum states in the fermionic structure; and it may be that we can draw the A-T and C-G base pairings in DNA into a quasi-(3 + 1)-dimensional pattern.

The significance of the rewrite numbers in 3-dimensional geometry is especially evident in the Platonic and Archimidean solids, where we find 4 faces and 4 vertices in the tetrahedron; 6 edges in a tetrahedron, vertices in an octahedron, faces in a cube, square faces in a cuboctahedron; then 8 vertices in a cube, faces in a star tetrahedron, triangular faces in a cuboctahedron; and 12 edges in a cube and edges in an octahedron, as well as rotational / reflection symmetries of a tetrahedron. For these figures, the rewrite numbers are multiples of 2 and 3 only. At the same time, stacking up tetrahedra in the rewrite geometry, to build up increasingly complicated figures, allows us to keep doubling the order of the figures from 4 to 64.

However, when we reach 32 something significant happens. Because of the alternative definition of 32 as 2^5, 5-fold structures begin to appear and the Platonic and Archimedean figures begin to have characteristic descriptions incorporating powers of 5. So, there are 5 cubes in a dodecahedron; hexagonal planes in an icosidodecahedron; star tetrahedra in the rewrite geometry; 20 vertices in a dodecahedron; triangular faces in an icosahedron; triangular faces of an icosidodecahedron; 30 edges in a dodecahedron, vertices in an icosidodecahedron, rhombic faces in a triacontahedron; as well as 60 rotational symmetries of a dodecahedron and 60 edges in an icosidodecahedron. A factor of 10 also appears in the 6 great circle decagons of the icosidodecahedron. Significantly, both 30 and 32 occur in the descriptions of the icosidodecahedron (30 vertices, 32 faces) and its dual, the rhombic triacontahedron (32 vertices, 30 faces). The 30 faces of the rhombic triacontahedron incorporate 4 of the Platonic solids; to incorporate all the 5 Platonic solids we would need to construct a figure with 120 faces, for example by stellating the rhombic triacontahedron with 4-sided pyramids.

At the same time, at the fifth level in the rewrite series, two 3-dimensionalities appear for the first time. It is significant that, at the point where factors of 5 begin to appear, we invariably have a broken symmetry in some respect, characterized by the appearance of Golden sections and the Fibonacci sequence. The broken symmetry is actually of one of the 2 component 3-dimensional structures while the symmetry of the other is kept intact. In physics, for example, the nilpotent fermionic structure ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{i}p_x$ + $\boldsymbol{i}\mathbf{j}p_y$ + $\boldsymbol{i}\mathbf{k}p_z$ + $\boldsymbol{j}m$), in reducing two 3-dimensionalities to 5 terms, breaks the symmetry of (***i***, ***j***, ***k***) while keeping that of (**i**, **j**, **k**) intact. The broken symmetries are apparent in the impossibility of tiling a plane with a repeating 5-fold structure – the uniqueness of the pattern in any part of the plane is reflected physically in the same localized uniqueness in quasicrystal structures, with their Golden sections and Fibonacci sequences. The uniqueness is removed when the quasicrystalline parameters are projected onto 6-dimensional axes, where the symmetries of *both* component 3-dimensionalities are preserved.

The Golden section or Golden mean, of course, has significance in dynamic as well as in static systems, as the KAM theorem demonstrates for chaos theory. The dynamic aspects are important in biology, which operates at the edge of chaos (and where the icosahedron provides the most stable form for a given volume), but the mathematical structure precedes both physical and biological realisations. The significance of the Golden mean in establishing stable conditions in a chaotic system implies that this idealised state has to be attained to reach the algebraic-geometric structure desired, not that creating stability in a chaotic system is a necessary requirement for finding physical significance in the Golden mean.

Another example of this symmetry-breaking is seen in the impossibility of solving quintic equations, where the inherent symmetries involve 5-fold components. The problem stems from the fact that the symmetries of regular 2-D n-gons, where n is not prime, are constructed from those of the p_i-gons, where the p_is are the primes from which n is constructed. In the nineteenth century, Galois linked this with the insolubility of the quintic equation (already established by Abel) by showing that the only algebraic equations that can be solved by a general method or formula are those whose underlying symmetry can be built up from such prime-sided shapes. Cubic equations require the symmetries of a triangle and quartic equations those of a tetrahedron; but quintic equations require the 60 rotational symmetries of a dodecahedron, which are indivisible and cannot be resolved into prime-sided shapes. The pentagonal faces of the dodecahedron cannot be rotated in combination with other shapes to produce these symmetries – a fact which is connected with the impossibility of tiling space with a 5-fold repeating structure, and the fact that 5-fold symmetry produces a breaking of symmetries structured purely from powers of 2. The 60 rotational and 60 reflectional symmetries of the dodecahedron, in fact, emerge from the possible 5! = 120 ways of ordering 5 objects, just as the 12 rotational and 12 reflectional symmetries of the tetrahedron emerge from the 4! = 24 ways of ordering 4 objects. Of course, where we have extracted the 5-fold aspect, in the dodecahedral / icosahedral structures, a 12 will always appear. Thus, we have 12 vertices of the icosahedron, 12 pentagonal faces of the dodecahedron and 12 pentagonal faces of the icosidodecahedron. (Further connections between the Galois group structure, fermions and the nilpotent universal rewrite system are demonstrated by Marcer and Rowlands [8].)

In terms of Platonic solid geometry, we can say that the 12 rotational symmetries of the tetrahedron represent only 1 set of vertices on a dodecahedron. This can be doubled because of the minor twin (star tetrahedron), leading to 24 'scalars' (bosons in the physical example) – the 'identity' set. Choosing another vertex set leads to 4 more, in a kaleidoscopic pattern, or a total of 120. So, we have 1 'identity' and 4 additional reflections. Then taking left-hand and right-hand rotations, we double again to 240. The dodecahedron itself has 60 rotation symmetries. It can be constructed from 5 cubes, each with a star tetrahedron. So there are 10 tetrahedra in total. Then, we have $12 \times 10 \rightarrow 120$ or $24 \times 5 \rightarrow 120$. Taking the two chiral dodecahedral forms gives us $2 \times 12 \times 10 \rightarrow 240$. (As we can see below, this can be related to the 24-cell, 120-cell and 600-cell.) Applying the same reasoning to genetics, we can suppose that doublings occur for left- and right-handed strands and codon / anticodon, or 2-stranded DNA + mRNA + tRNA, to produce $2 \times 2 \times 60 = 240$ total units.

A significant point in connection with particle representations is that the icosahedral number 20, just like the pentad and the number 60, has a double significance in this system. First of all, 4 pentads (fermion / antifermion, right- and left-handed) (or $\pm iE$, $\pm \mathbf{p}$) are needed to make a boson, so this is $4 \times 5 = 20$ units. Secondly, the particle themselves require a 20 for a quark / lepton / equivalent boson generation. The original 6 quarks / leptons are in three generations, each divided into two states of weak isospin (up / down). To write down a single line of this, say the first, we need 16 fermions – (3 colours of quark + 1 lepton) × 2 (for left- / right-handed) × 2 (for particle antiparticle) – plus 4 gauge bosons (in this case, 2 colourless gluons + Z^0 + photon). In a sense, everything else is a repetition of this pattern, multiplied by 6, or even 3 and a series of 2s.

In physical systems, the fifth component is always the symmetry-breaker, and at the same time partially redundant, a kind of artificial addition, because its values can, in principle, be determined from the other four. It also has more than one form with respect to the other components, and can even be reduced to a scalar. In physics, it provides the extra dimension (mass / electric charge) of Kaluza-Klein theory, exactly as in the fifth term of the energy-mass-momentum fermionic nilpotent, which also duplicates as a generator of weak-strong-electric charge [3]. In this sense, the 5 'dimensions' duplicate to 10, but, because of the nilpotency, 1 of the 5 dimensions or 2 of the 10 are always partially redundant. Ultimately, the nilpotency means that 10 dimensions can be viewed as 8, reflecting the 8 base units that emerge from the rewrite system, and that form a closed cycle in Clifford algebra. It is for the same reason that fermionic string theory describes the vibrations of a 2-dimensional string (the redundant part, in 1 dimension of space and one of time) over 8 degrees of algebraic freedom, and it is not unconnected that there are 10 generators for the Lorentz group. (The 'eleventh dimension' required for the extension of string theory to membrane theory is not really a true one, being merely another way of expressing the embedding infinite-dimension Hilbert space in which the '10-dimensional' nilpotents are described [3].)

The 5-fold symmetry seems to be essential for generating change in nature, and for distinguishing life from inert crystalline form. In biology, we note the significance of the icosidodecahedron in the genetic code; the 5-fold structure of the axis of DNA; the 5 dodecahedra in the DNA cycle and the 5 tetrahedra needed to produce 1 pentagonal disc in DNA. In addition, 32 is an important number in the structure of nuclei and the electronic structure of atoms (and possibly also 120), but the significance of this in terms of the rewrite structure has yet to be established. Of course, 64 also appears directly in biology, in the 64 codons of the genetic code, in the same way as it appears in physics in the 64 units in the Dirac (fermionic) algebra. A related 64 ($= 4 \times 4 \times 4$) also occurs in the indices for the affine connection, though there is a semi-redundancy because of the totality zero. We note here that the 32 and 30 that are significant for the icosidodecahedron readily duplicate to 64 and 60, numbers which are significant in both biology and physics.

5 4-Dimensional Representations

Though space is intrinsically 3-dimensional, the representation of a physical or biological system by a double set of coordinates, one of which is fixed and the other moving, or one conserved and the other nonconserved, can be structured using a 4-dimensional space. In some ways, this is close to the idea of time as a fourth dimension (and, mathematically, the time term doubles the algebraic units by adding imaginary versions to all the real terms), but a more exact way of thinking about it would be to say that the simultaneous visualisation of two places relating to an object in 3-dimensions gives the sensation of the passage of time. There are seventeen separate centres in the brain involved with visual perception, and it is specially significant that the perception of movement requires a totally different centre to that for the perception of an object as a body in 3-dimensional space, and that an individual can possess one facility without the other. There are also other ways of seeing 4-dimensionality as being intrinsic to physics and biology, particularly where the dimensionality is not necessarily directly spatial. Physics, for example, has a 4-dimensionality associated with the parameters space, time, mass and charge, and another (ultimately related) one involving the fundamental particles as quarks and leptons, while biology has a natural 4-dimensional 'space' associated with the 4 bases of DNA. Each also has a natural 8-dimensionality, with an intrinsic 6-dimensional component. The use of higher-dimensional 'spaces' brings in new aspects to the idea of Platonic solids and also connects them to other fundamental mathematical concepts, such as groups and the kissing number.

The kissing number is a way of describing the most efficient packing of space using identical spheres, or, in dimensions greater than 3, hyperspheres. In any dimensionality of space, it is the number of equally-sized spheres or hyperspheres that can simultaneously touch any central one. In one dimension, it would be 2, in 2 dimensions it is 6 (as in graphite, graphene, and the benzene and pyrimidine rings). In 3 dimensions it is 12, placed at the vertices of an icosahedron, and in 4 dimensions 24. Most of the exact kissing numbers for higher dimensions are not known, but for 8 dimensions it is 240, and for 5, 6 and 7, the respective numbers are believed to be 40, 72 and 126. The corresponding lattices for the kissing numbers from 1 to 8 are those of the Lie groups A_1 (= Z), A_3 (= D_3), D_4, D_5, E_6, E_7 and E_8. These numbers also bring into play another significant mathematical number, the Riemann zeta function, $\zeta(n)$, for, according to the Minkowski-Hlawaka theorem, lattices must exist in n dimensions with hypersphere packing densities greater than or equal to $\zeta(s) / 2^{s-1}$. However, the Riemann hypothesis projects an infinite number of values of 0 for $\zeta(s)$, when s is the complex number $a + bi$, with $a = ½$, and b seemingly random, which is suggestive of fermions with spin ½ and effectively random energy and momentum. Perhaps this means that, to take space to the limit of fermionic point-singularity (the only physical thing that can conceivably be defined in this way), with the complexity introduced by fermion spin, and zero minimum packing, we need to generate an infinite number of possible values of $\zeta(s) = 0$ by generating an infinite number of almost random possible states of energy and momentum.

The kissing numbers and groups for these dimensions are also closely related to the Platonic solids which they allow. Only in 3 and 4 dimensions are there more than three Platonic solids (though in 2 dimensions there are an infinite number of regular flat polygons, each of which is its own dual). Only in these dimensions are there solids with the 5-fold symmetries of the dodecahedron and icosahedron. Other dimensionalities have equivalents only of the tetrahedron, cube and octahedron. The 4-dimensional solids are a particularly interesting case, because here there are *six* Platonic solids – higher than in any other dimension – and one solid, the 24-cell, that is unique, and not even present in 3 dimensions, although there are some indications of a relationship with the cuboctahedron.

The self-dual pentatope (or pentachoron), the 4-dimensional analogue of the tetrahedron, has the numbers associated with the algebra of compacted fermionic structures. It is made up of 5 tetrahedral cells, with 10 faces, 10 edges and 5 vertices. Projection of the pentatope onto 3 dimensions shows 5 tetrahedra making one pentagonal disc (minus 7°12'). We have shown that 10 such discs complete one cycle of the DNA helix and when considered to be parts (top and bottom sections) of icosahedra with reciprocal internal dodecahedra, pinpoints the nucleotide positions viewed along the axis of DNA [4].

The tesseract, the 4-dimensional analogue of the cube (which can be simulated from the combination of a cube with another upon each face and connecting their vertices), and its dual, the 16-cell, the analogue of the octahedron, have the numbers associated with the uncompacted algebra. The tesseract has 8 cells, 24 faces, 32 edges and 16 vertices, while the 16-cell has 16 cells, 32 faces, 24 edges and 8 vertices. The 16 vertices may be represented by the 24 possible unit values of a quaternion of the form $a + bi + cj + dk$, where one of the terms a, b, c, d is either 1 or –1 and the others 0; they form a group under multiplication, always producing a product from inside the group. The representation of the tesseract as a projection of this 4-dimensional cube into 3 dimensions and put into motion via rotation, with one cube seemingly 'emerging' from the other, can be used to give a representation of 4 dimensions emerging from a double coordinate system.

The self-dual 24-cell, with no corresponding 3-dimensional figure, has the numbers associated with the possible types of fermions and bosons. This is important because the three colour-types of quark and the single type of lepton, combined, can be considered to make up a kind of 4-dimensional 'space'. The 24-cell has 24 octahedral cells (6 meeting at each edge), 96 triangular faces, 96 edges, and 24 vertices. (The 24 vertices again form a group under multiplication, being represented by the 24 possible unit values of a quaternion of the form $a + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}$, where the terms a, b, c, d are either all integers or all integers plus ½.) Now, as we have seen, there are 96 possible fermion states, made up of (3 colours of quark + 1 lepton) × (2 states of weak isospin in each generation: up / down) × (3 generations) × (2 for fermion / antifermion) × (2 for left- and right-handed), and 24 gauge bosons to carry their interactions (8 gluons, photon, W^-, W^+, Z^0, plus 6 X- and 6 Y-bosons for the strong-electroweak combination). The same is again true for the vacuum partners, which in effect double the total number of states: 4 real fermion states (fermion / antifermion, left- and right-handed) contribute to each vacuum boson state; and each real boson state requires 4 vacuum fermion states. So we can imagine the 96 fermions as being represented along, say, the edges and meeting at the vertices, whose number is also the kissing number in 4 dimensions. It may be that we could simultaneously see the 24 cells and the 96 faces as the vacuum partners. It is also significant that 72 of the fermions are quarks or antiquarks, and 24 are leptons or antileptons; and 72 is, appropriately, the kissing number for a 6-dimensional space, just as 24 is that for 4 dimensions.

The number 24, which provides the full rotational set of the tetrahedron and is the number of ways of ordering 4 objects, also occurs as the result of combining the two types of lattice in a 2-dimensional plane with the most symmetry, the square lattice Z^2, with 4-fold symmetry, and the hexagonal lattice A_2, with 6-fold symmetry. In a sense, this is like creating a 4-dimensional structure from two 3-dimensional ones (as we do when we simulate the construction of a tesseract from 2 cubes), in which only one of the 3-dimensionalities is made explicit. Now, we have 24 real spin 1 gauge bosons (which would be the 24 generators in an $SU(5)$ theory). It also happens (because of the $\mathsf{Z}^2 \times A_2$ lattice connection) to be the number of degrees of freedom available for a 2-dimensional bosonic string to vibrate, so leading to the 24 + 2 or 26-D bosonic 'string' theory. Because of the properties of the Riemann zeta function and the number 24, simultaneous vibrations of the 2-D string in 24 'dimensions' are required to give the correct ground state energy [5].

In our interpretation, the 2-D vibrations of the 'string' are real space and time, deriving from the holographic principle and nilpotent structure. The extra 'dimensions' are the 24 possible charge structures available to the bosonic states (equivalent to the creation of 24 bosonic types in 'charge space'). We may perhaps imagine bosons being created at boundary between two fermions, determined by the holographic principle as one dimension of space (spin axis) and one of time, and producing by this information the characteristic 24 boson structures in nilpotent (charge) space. (The inertial pseudoboson generated in the GUT proposed by Rowlands and Cullerne [3,6] could stand in for the time element in this 'space' and the Higgs for the 'embedding' space in which the 'vibration' is allowed.)

The 120-cell, which is the 4-dimensional analogue of the dodecahedron, and its dual, the 600 cell, which is the 4-dimensional analogue of the icosahedron, have the numbers associated with taking all the possible sets of fermion and boson states together on a single footing. They also incorporate all the significant numbers of the 3 previous structures, pentatope, tesseract and 24-cell. It is at this level that the 5-fold dodecahedral / icosahedral symmetry first emerges. The 120-cell is made up of 120 dodecahedral cells, with 4 at each vertex, the vertex figure being a tetrahedron. It has 720 pentagonal faces, 1200 edges and 600 vertices; 5 copies of the dual 600-cell can be inscribed in these vertices. The dual 600-cell is made up of 600 tetrahedral cells, 20 meeting at each vertex; there are 1200 triangular faces, 720 edges and 120 vertices, the vertex figure being an icosahedron. The edges of the vertices have length $1 / \phi$, and yet again form a group under quaternionic multiplication. (This time the members are of the form $a + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}$, where the terms a, b, c, d are all constructed from the so-called 'golden field', $x + \sqrt{5}\, y$, and x and y are rational numbers.) 24 are vertices of a 24-cell (16 being vertices of a tesseract and 8 of a 16-cell) and 96 are vertices of a snub 24-cell (which is made up of 120 regular tetrahedra and 24 icosahedra). The 96:24 or 4:1 fermion / boson ratio also reflects the division between the terms ***ii***, ***i*j**, ***i*k**, ***ik***, and the 'redundant' ***j*** in the nilpotent algebra; significantly, the ***j*** is attached to a purely scalar value, exactly as required for bosons. In DNA, the 120 can represent the 60 codons used for producing amino acids present on the sense strand and the 60 corresponding codons on the antisense strand. The total process always requires doubling. (The same kind of doubling occurs with fermionic spin, and Baez describes the 120-cell as 'a picture of the rotational symmetries of a 'spinor dodecahedron'' [5], which would have twice the $5 \times 12 = 60$ rotational symmetries of the dodecahedron itself.) On the basis of the third base often being in effect redundant, we can even divide the 120 codons / anticodons into those with first two bases nonidentical and those with first two bases identical, which divide in the ratio 96:24. It might even be possible to divide the 96 into 72 and 24, as in the analogous physics application, by, say, separating out the codons with two final bases identical. The 120-cell, projected onto 3 dimensions also shows remarkable similarity to a computer-generated scatttergraph of DNA viewed along its axis, showing the complete stacking of the 10 nucleotides as pentagonal discs.

A dodecahedral universe has been proposed, in the form of a hypersphere surrounded by 120 modified dodecahedra [9]. This would be bounded by reversed images of itself, each rotated by 36°. We might imagine this in a particle space, with the fermion reflecting itself in vacuum.

6 The Rewrite Algebra in Physics and Biology

Although 4 dimensions has a special role in producing the 'double 3-dimensionality' needed in both physics and biology, even higher dimensionalities have significance in representing aspects of the resulting structures. The significance of 5 dimensions is obvious from the generating algebraic units ***ii***, ***i*j**, ***i*k**, ***ik***, ***j***, and the projected kissing number of 40 has significance, for example, in respect of the dual process needed in the production of 20 amino acids.

In $n = 5$ dimensions and higher, the 'n-simplex' or hypertetrahedron has $(n + 1)$ faces, each of which is an $(n - 1)$-simplex; the 'n-cube' or hypercube has $2n$ faces, each of which is an $(n - 1)$-cube; and the 'n-dimensional cross-polytope' (equivalent to a hyperoctahedron) has 2^n faces, each of which is an $(n - 1)$-simplex. While the n-simplex is self-dual, the n-cube and n-dimensional cross-polytope are duals of each other. There are no other Platonic solids in these dimensions. The special nature of 4 dimensions results from the fact that the unit sphere is a group, as it also is in 1 and 2, and no other dimensionality. The unit sphere in 1 dimension can be thought of as the two points represented by the real numbers –1 and 1, whereas, in 2 dimensions, it becomes the sphere of the unit complex numbers, $e^{i\theta}$. In 4 dimensions the unit sphere becomes the group of unit quaternions or $SU(2)$, which is also the *double cover* of the rotation group in 3 dimensions, with 2 elements of the group corresponding to each 3-dimensional rotation. Because of this relationship, we can show that, in 4-dimensions, the 24-cell becomes the group of symmetries of the tetrahedron, while the 600-cell becomes the group of symmetries of the icosahedron. We can also see the 'double cover' aspect emerging in physical and biological applications, and as being ultimately generated in the 'doubling' process of the universal rewrite system.

8-dimensionality is a particularly interesting case because it is in many respects, and certainly in geometrical algebra, the end of the line before repetition. The kissing number for an 8-dimensional space is 240, and we already have an 8-dimensional aspect to the fundamental algebra, with its 8 fundamental units, and in the concept of fermion space-time plus vacuum space-time, or 4 fermions plus 4 antifermions. 8 is also the dimension of the highest division algebra, or octonions, the next (and last) stage up from quaternions; and the 8 fundamental units have many octonion-like properties. (They are not octonions, because they preserve associativity, but the nonassociative aspect can be mapped onto octonions in a way which avoids this [3].) In 8 dimensions, we have the Gosset polytope, which has 240 vertices, and which can be conveniently visualized by being projected onto a 3-dimensional space. This has 240 vertices (which could represent particle states plus vacuum states), and these can also be associated with the 240 root vectors of the rank 8 exceptional group E_8 (which have an icosahedral / dodecahedral structure). This is the highest group generated from the symmetry of the octonions, and has long been considered as a possible candidate for the group that would unify all the particles and interactions. Its subgroups include the other exceptional groups, derived from the octonions, E_7, E_6, F_4 and G_2, in addition to $O(16)$, $SU(8)$, and the groups considered to be important in particle physics, $SU(5)$, $SU(3)$, $SU(2)$ and $U(1)$. Because it is an exceptional group, it can incorporate both fermions and bosons in a single representation [10], and, in addition, because it has 8 additional dimensions, in addition to the 240 root vectors, it can also incorporate mass and gravity. The total process of transcription and translation in DNA also involves 240 units, 60 for codons on the sense strand of the DNA and 60 for the codons on the antisense strands, in addition to 60 for mRNA and 60 for tRNA. Here, we note that four individual units are needed for each codon / anticodon combination, in the same way as four fermion /

antifermion units are needed for each boson. In addition, if we include the four extra (mostly STOP) codons in DNA, we produce the number 248 associated with the E_8 *group*, rather than the root vector lattice.

It is significant, that, in 8 dimensions, there is no need for the icosahedron / dodecahedron structure and for 5-fold symmetry breaking, because all 8 dimensions can be seen at once. The 5-fold symmetry breaking is only apparent in 3 dimensions. The nonperiodic icosahedral or 5-fold structure of quasicrystals, for example, is removed when the spatially observed data is plotted onto a 6-D space, with axes drawn so as to connect opposite vertices of the icosahedron.

7 Conclusion

The numbers that appear to have special significance in physics and biology appear to be derivable in a relatively simple way from the various combinations of the numbers 2 and 3, whose origins lie in the respective 'conserve' and 'create' processes of the universal rewrite system. The number 5, which emerges as a result of binomial combination, has a significant role as the breaker of symmetry.

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