Nature's Fundamental Symmetry Breaking

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Abstract
The question we are concerned with is: how can the highly ordered replicating state which we call 'life' form within a universe where the tendency of natural processes is towards a state of increasing disorder? The existence of a machine order code or driving process for life to emerge seems to be suggested by the generation of a universal rewrite system, with its own mathematical structure, from the single assumption of a zero totality universe; and it would appear that the successive stages which this system automatically generates correspond with the algebraic and geometrical structures which are fundamental in physics and biology in particular. The system has a number of significant aspects – cardinality, rather than ordinality, 'bifurcation' at each stage, and a key stage at which symmetry breaking first occurs. The progressive stages are worked out here in both algebraic and geometrical terms and illustrated through detailed applications to genetics and particle physics.
Keywords: genetic code, universal rewrite system, symmetry-breaking, DNA, codons, amino acids

1 Introduction

The question we have been concerned with in our previous presentations [1-6] is: how can the highly ordered replicating state which we call 'life' form within a universe where the tendency of natural processes is towards a state of increasing disorder? Life could not possibly emerge from a purely random arrangement of physical and chemical interactions. A simple calculation will show that the probability of this happening is zero. There must therefore be some driving process, and, to overcome the tendency to increased entropy and loss of information, it must be essentially a process which maximises information. It would seem that nature has an intrinsic information processing or machine order code, which probably determines the behaviour of all ordered systems, physical and chemical as well as biological, large scale as well as small scale. The existence of such a code seems to be suggested by the generation of a *universal rewrite system*, with its own mathematical structure, from the single assumption of a zero totality universe; and it would appear that the successive stages which this system automatically generates correspond with the algebraic and geometrical structures which are fundamental in physics and biology in particular.

The universal rewrite system has a number of significant aspects. One is that it is a series based on *cardinality*, rather than ordinality, i.e. a system in which change generates a new whole incorporating each previous cardinal (whole) state into a new cardinality – though, here, we have a cardinality of zeros, rather than of infinities. Each stage or 'alphabet' in the rewrite system is a totality, completely incorporating and extending the previous totality. Another is that each stage is a '*bifurcation*', splitting the previous zero totality 'universe' into two. (This 'bifurcation' should be regarded as a more general process than that which appears in chaos theory, which is effectively a special case for certain dynamic conditions.) Finally, a key stage is reached at order 64, where the complete 64-part algebra can be produced by a combination of just 5 of the terms used as generators, and where scalar values can be applied to make the combination of 5 terms or pentad into a *nilpotent*, squaring itself to zero, and so providing the required zeroing with the maximum efficiency. It would seem that this structure can emerge at more than one level, and the 5-fold nature suggests a connection with the Fibonacci sequence, which is ubiquitous in nature, and the Platonic solids leading up to the dodecahedron and icosahedron. It is significant that the point where the 5-fold nature becomes apparent is always accompanied by symmetry-breaking.

2 Double 3-Dimensionality

The algebra that emerges from the rewrite system requires nested 3-dimensional or anticommutative systems ($\boldsymbol{i}_1, \boldsymbol{j}_1$, and by implication $\boldsymbol{i}_1\boldsymbol{j}_1 = \boldsymbol{k}_1$), ($\boldsymbol{i}_2, \boldsymbol{j}_2$, and by implication $\boldsymbol{i}_2\boldsymbol{j}_2 = \boldsymbol{k}_2$), ($\boldsymbol{i}_3$ ...), which are independent of each other (commutative). In principle, the 3-D systems continue to infinity, but, at order 64, physical systems allow the truncation of the series and zeroing of further terms by creating a nilpotent structure, squaring to zero.

Table 1: The Rewrite Algebra

Group	Algebraic Units	3-D
Order 2	$(1, -1)$	$0 \times$ 3-D
Order 4	$(1, -1) \times (1, \boldsymbol{i}_1)$	$0.5 \times$ 3-D
Order 8	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$	$1 \times$ 3-D
Order 16	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$	$1.5 \times$ 3-D
Order 32	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$	$2 \times$ 3-D
Order 64	$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$	$2.5 \times$ 3-D

Now, physics is structured on 4 fundamental parameters, but, because 2 of them are dimensional, the 4 parameters require specification by 8 algebraic units:

	Time	Space	Mass	Charge
	↓	↓	↓	↓
units	1	3	1	3
	i	**i j k**	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$

Biology has much the same structure. This time, there are 4 bases, which, in the pairing between two strands, become equivalent to 8 algebraic units:

	T	A	C	G
	↓	↓	↓	↓
units	1	3	1	3

The algebra allows 64 possible combinations of the units:

$$\pm 1, \pm \boldsymbol{i}, \pm \boldsymbol{j}, \pm \boldsymbol{k},$$
$$\pm \mathbf{i}, \pm \mathbf{i}\boldsymbol{i}, \pm \mathbf{i}\boldsymbol{j}, \pm \mathbf{i}\boldsymbol{k}, \pm \mathbf{j}, \pm \mathbf{j}\boldsymbol{i}, \pm \mathbf{j}\boldsymbol{j}, \pm \mathbf{j}\boldsymbol{k}, \pm \mathbf{k}, \pm \mathbf{k}\boldsymbol{i}, \pm \mathbf{k}\boldsymbol{j}, \pm \mathbf{k}\boldsymbol{k},$$
$$\pm i, \pm i\boldsymbol{i}, \pm i\boldsymbol{j}, \pm i\boldsymbol{k},$$
$$\pm i\mathbf{i}, \pm i\mathbf{i}\boldsymbol{i}, \pm i\mathbf{i}\boldsymbol{j}, \pm i\mathbf{i}\boldsymbol{k}, \pm i\mathbf{j}, \pm i\mathbf{j}\boldsymbol{i}, \pm i\mathbf{j}\boldsymbol{j}, \pm i\mathbf{j}\boldsymbol{k}, \pm i\mathbf{k}, \pm i\mathbf{k}\boldsymbol{i}, \pm i\mathbf{k}\boldsymbol{j}, \pm i\mathbf{k}\boldsymbol{k}$$

5 combined units (pentad) are the simplest possible generators for the 64:

$$i\boldsymbol{k} \qquad i\mathbf{i} \;\; i\mathbf{j} \;\; i\mathbf{k} \qquad \boldsymbol{j}$$

Physically, these become the new quantities: energy (E), momentum ($\mathbf{p}$, 3 units, representing 3 spatial dimensions) and mass (m). Similarly, within the DNA translation process we see the 64 triplet codons generated from the 4 bases A, T, G and C (4^3). This process yields the 20 amino acids that are the constituents of proteins and here we see the 5-fold broken symmetry nature as well as that seen manifested within the icosahedral / dodecahedral symmetry of the DNA helix itself.

In historical terms, physics required 2 independent observable 3-D systems to understand physical processes involving change. To accommodate relativity (i.e. the fact that any 3-D system cannot be defined without an accompanying scalar) they each become '4-D' = 3 (+ 1), e.g. space (+ time). This pattern has been retained in later developments, and can be seen to be essential to the structuring of all fundamental information processing, whether physical, biological, or more abstract, and whether small-scale or large-scale. There are always at least 2 × 3-D systems required to specify a system in this way, one of which is nonconserved, and one conserved, and the combination – the variation of the nonconserved quantity against the background of the conserved one – gives us a description of the process of change.

Now, historically (in classical physics) the 3-D (or 4-D) systems were space (+ time), which was nonconserved, and momentum (+ energy), which was conserved, and the combination of space + momentum, which totally specified the dynamics of a system, was called 'phase space'. Both dimensional systems were in some sense observable. Quantum physics, however, shows that space and momentum are not truly independent; instead, the genuinely independent dimensional systems are space (+ time), which is nonconserved, and charge (+ mass), which is conserved, and only the first is observable. It was by combining the two in a 5-fold broken symmetry, that we could construct a second observable system, corresponding to the momentum (+ energy) that, historically, we had used to describe change.

It is because the second system, as used in measurement, is constructed partly from the first that the two 3D systems are not truly independent. In quantum mechanics, we say that the space (**r**) and momentum (**p**) operators, acting on the wavefunction (ψ) (an object varying in space and time) are not commutative. That is each does not act as though the other did not exist. Mathematically, the operation $(\mathbf{rp} - \mathbf{pr})\psi$ is not zero (but $i\hbar\,\psi$), and we cannot know both position and momentum accurately at the same moment – the well-known Heisenberg uncertainty. A similar condition applies for energy and time. On the large scale, with the combination of many quantum systems, the quantum coherence disappears, and the space (+ time) and momentum (+ energy) become independent; we then generate classical physics, which is the historical, but not logical starting point.

We may note that our two initial '4-D' systems would not be completely independent if they had the same scalar value, so one of the systems is separated from the other by being multiplied throughout by the 'pseudoscalar' *i*, which is like the first term in yet *another* 3-D system never completed. This extra term, through its incompleteness, seems to represent variation, and, in particular, variation with time. The passage of time seems to act like a 4-D system, in which structures are doubled to represent the old and the new as complete structures simultaneously in a higher space. This is the classic ' bifurcation or doubling process associated with the universal rewrite system, and in quantum mechanics, is something like the 'many worlds' interpretation in which the 'universe' bifurcates into two every time a measurement is made. There, however, it is assumed that the bifurcation is into two conserved 'universes'; here, by contrast, one is conserved, and one is not. In the universal rewrite system there is only a new cardinality or a new conserved state; the old one is lost.

3 The Emergence of the Icosahedral / Dodecahedral Symmetry

In Figure 1 is shown the geometric description of the universal rewrite system (URS) where in the parallel cases of physics and biology, the process begins with the URS order 2 producing mass and the two DNA base types (purine and pyrimidine). Subsequent rounds of doubling produce firstly the tetrahedron (order 4), time and the four DNA bases (2 purines and 2 pyrimidines); order 8 presents the star tetrahedron, the emergence of charge plus the formation of classical double stranded DNA; order 16, 2 star tetrahedra, space and the initiation of the process of DNA transcription (mRNA copy of one strand of DNA is made); order 32, 4 star tetrahedra, presents the mechanism of translation of the RNA code into amino acids within the ribosome (a system that requires a 4 way interaction), a geometric phase change into a 3^{rd} order tetrahedron. At this point symmetry starts to break. order 64 can be represented by two 3^{rd} order tetrahedrons forming a 3^{rd} order star tetrahedron and represents the full translation of all 64 triplet codons into 20 amino acids. Broken symmetry allows the formation of a 5-fold geometry which results from the addition of an extra 5^{th} star tetrahedron to produce the dodecahedron, the icosidodecahedron and the icosahedrons, representative of the formation of an amino acid chain (a protein) that can then form helical structures (alpha helices (5-fold nature) and beta pleated sheets 8-fold nature). The recently proposed binary system of DNA evolution from a biplet to a triplet code [7] is shown in grey within brackets.

Figure 2 shows the details of this process of breaking of symmetry. The doubling of order 16 (2 star tetrahedra) produces a complete set of 4 star tetrahedra (order 32). At this stage there are two possible geometric pathways; Firstly, a phase change which allows a 3rd order tetrahedron (10 tetrahedra representing 10 amino acids and 32 triplet faces upon 4 octahedra representing 32 of the 64 triplet codons of DNA with the 3rd order star tetrahedron completing the full set of 64 codons / 20 tetrahedra / 10 amino acids, 64 triplet codons represented by all 8 octahedra and their faces). Secondly, an extra star tetrahedron (perhaps borrowed from another order 32 set) allows the symmetry to break into a 5-fold system. Here we see the 5 star tetrahedra held within a sphere with star points equidistant to each. This automatically self constructs an internal icosahedron with 20 faces representative of the 20 amino acids, an internal icosidodecahedron and an external dodecahedron with 12 pentagonal faces that can represent 5 triplet codons each with a total of 60 for the whole object. There are 3 remaining star tetrahedra left over within the second set of star tetrahedra, i.e. there are 8 star tetrahedra in total with one group of 5 and one of 3 star tetrahedra.

The actual self-construction of this five-fold structure starts within the intitial geometric URS. Here we have a single tetrahedron at order 4; 2 tetrahedra then produce a star tetrahedron which automatically constructs an octahedron internally, with the outer star points making the vertices of a cube (order 8); while 4 tetrahedra = 2 star tetrahedra plus 2 internal octahedra (order 16). The next stage is 4 star tetrahedra with 4 internal octahedra (order 32). At this point, two paths can be chosen; either these 4 star tetrahedra can undergo a phase change to a single 3rd order tetrahedron (order 32) and combine with a second 3rd order tetrahedron to form a 3rd order star tetrahedron (order 64); or the system can enter a broken symmetry state. Here a 5th star tetrahedron is borrowed from another set of 4 (say the –32 group) and after a readjustment to a position where all the star points are equidistant, the vertices will define the dodecahedron. For the dodecahedral symmetries (broken symmetry), the order becomes 5 star tetrahedra / 5 cubes (the cubes being defined by the outer points of each of the stars) / 5 octahedra (internally created within each star tetrahedron); these now, together, produce an internally constructed icosahedron and an icosidodecahedron (produced by the vertices of these 5 octahedra) and finally, the outer points of the star tetrahedra produce the vertices of an external dodecahedron.

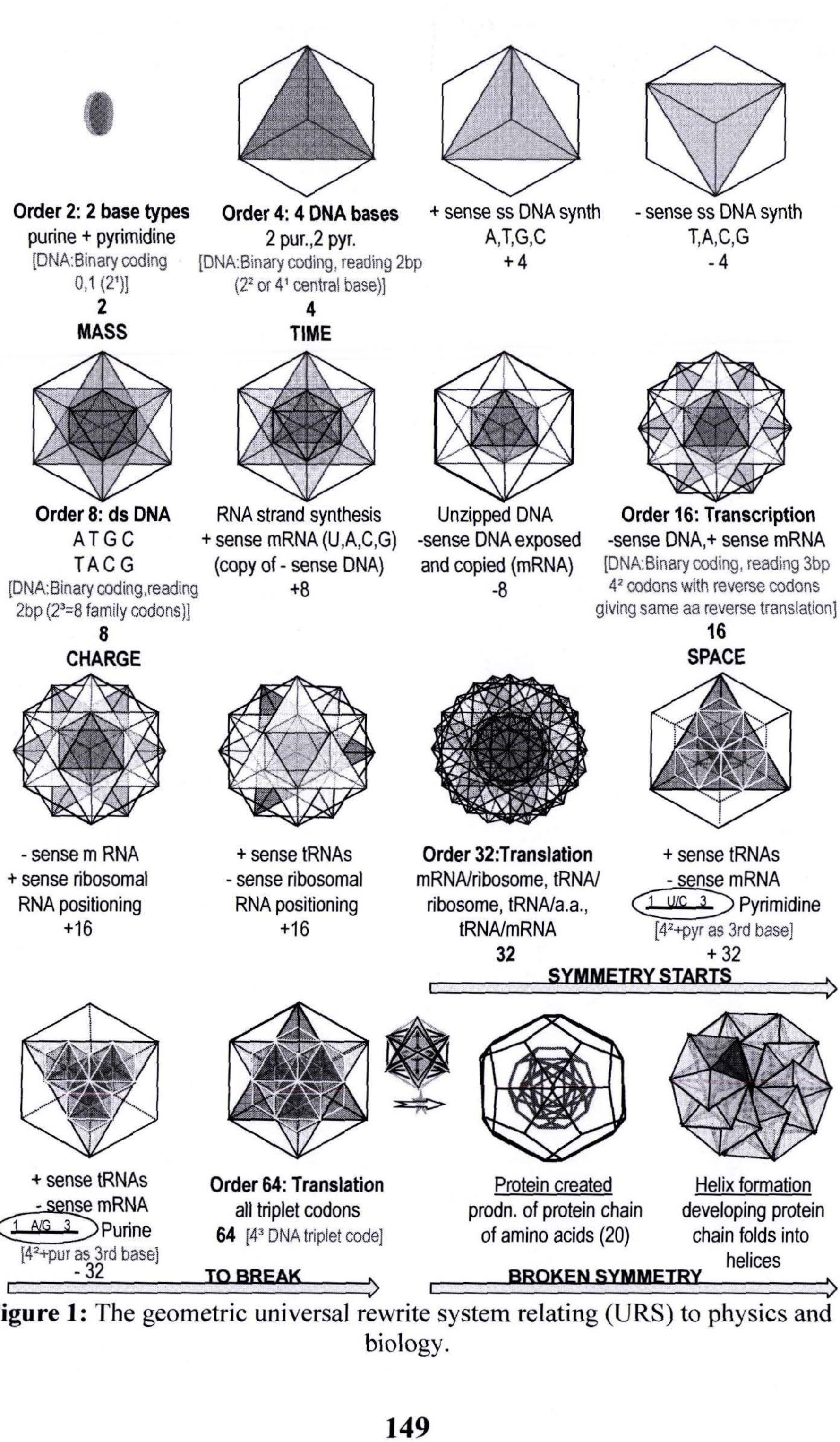


Figure 1: The geometric universal rewrite system relating (URS) to physics and biology.

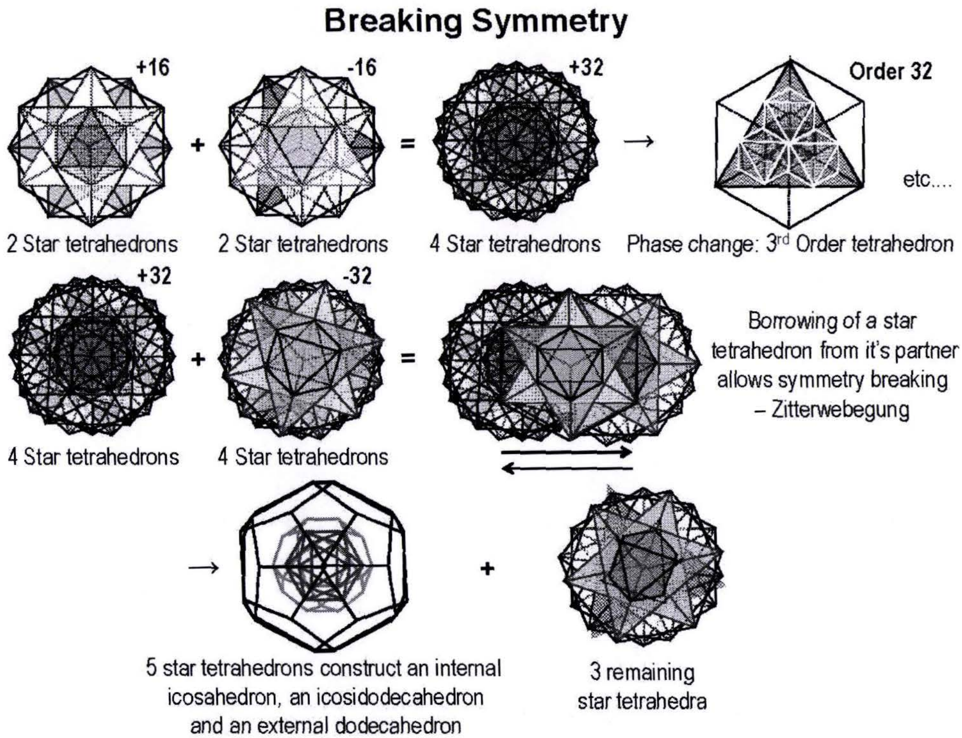


Figure 2: Breaking symmetry.

So, we now have 5 objects, i.e. star tetrahedra, and these can be ordered in 120 different ways to produce 120 different dodecahedral forms, which, in a physical example, could each represent a different fundamental particle. If we now consider the other chiral dodecahedra, these may represent the vacuum states and would produce another 120 to give a total of 240 particles.

This 5th star tetrahedron, as previously suggested, may perhaps be borrowed from the –32 set (4 star tetrahedra) required to complete order 64, and vice versa, in a process similar to the *zitterwebegung* of physics. (It is important here to recognise that nilpotent quantum mechanics provides a direct route to quantum field theory for all the fundamental interactions while preserving much of the Dirac apparatus, including the spinor structure and *zitterbewegung*, as fundamental components [3].) Of course this leaves 3 remaining star tetrahedra within our original –32 set, i.e. there is a total of 8 star tetrahedra within this process of the creation of a broken symmetry state and there is a possibility here of shuffling of different states, perhaps resulting in different particles. This process is also reminiscent of the hydrogen molecule and the sharing of an electron.

We have shown, previously, in a significant example of the doubling of 3-dimensionality in Nature [1-6], that the structure of DNA and translation of its code, via triplet codons, into proteins suggests an icosahedral / dodecahedral symmetry. The transformation of the icosahedron to dodecahedron (and vice versa) is effectively a transformation of the space, from privileging one 3-D to another 3-D, such as space to charge, or space to momentum space, lattice to reciprocal lattice, or inside space to outside space. The dodecahedron / icosahedron duality thus corresponds to that of the phase space of Heisenberg uncertainty, and needs a double 3-dimensionality; the structure can only be created through transforming one 3-D to another and this process can be seen to go via other, *Archimedean*, solids (which are composed, unlike the Platonic solids, of two or more types of regular polygons), e.g. the icosidodecahedron which has 32 faces. Ultimately, we need two 3-Ds to get uniqueness and variation; significantly, thermodynamics, which is a signature of a unique birth-ordering of events, comes from the vector or 3-D nature of the **p** term in quantum mechanics, and the decoherence which results from this; quantum mechanics is impossible without it. The symmetry which incorporates 2 separate 3-D structures is necessarily always a 5-fold one, with a broken symmetry, and this broken symmetry is manifested in many ways across a wide range of mathematics, for example, in the insolubility of the quintic equation, which relates directly to the symmetries that are important in biology and physics.

4 The Icosidodecahedron

A solid which seems to have great significance in the rewrite geometry is the icosidodecahedron (Figure 3). The 5 Platonic solids (2 dual pairs and the self-dual tetrahedron) group with 4 of the 13 Archimedean solids (2 dual pairs) to form the 9 edge-transitive polyhedra. These 2 Archimedean dual pairs include the cuboctahedron and its reciprocal, the rhombic dodecahedron plus the icosidodecahedron and its reciprocal, the rhombic triacontahedron. The icosidodecahedron has 30 vertices in total, 20 triangular faces and 12 pentagonal faces, making a total of 32 faces; it also has 60 edges; 6 great circle decagons; and 5 hexagonal planes of same side length as the pentagonal planes. The rhombic triacontahedron is its dual and any 2 vertices plus the centre make Golden triangles, with apex 36^{o}. Its symmetry group is the icosahedral group A_5. Within the previously described geometric construction of 5 equidistant star tetrahedra (concentrically placed within an imaginary sphere) we can see the emergence of a number of Archimedean solids by a progression from the internal icosahedron upwards through the icosidodecahedron, towards the dodecahedron.

Our work on Nature's code has shown that the icosidodecahedron is fundamental to the rewrite geometry and has special significance within biology. This solid is interesting in that it has 32 faces reminiscent of order 32 – half the total number of DNA's triplet codons; 60 edges – the number of codons that actually code for an amino acid (missing the stop codons that produce no product amino acid), 20 triangular faces – the 20 amino acids that make up proteins; and 12 pentagonal faces each representing 5 triplet codons ($12 \times 5 = 60$ coding triplet codons). The 30 / 32 and 60 / 64 connections, observed in the geometry, seem to have a special significance in the rewrite structure.

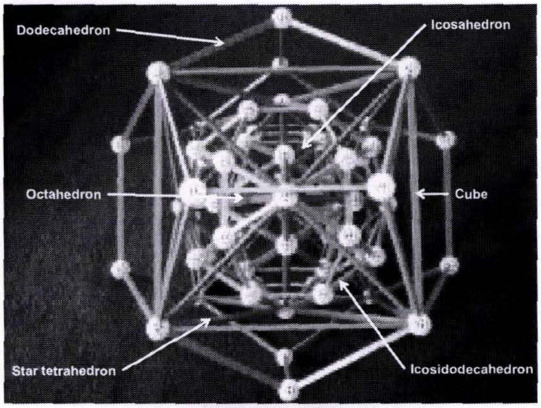


Figure 3: Left: A Zome tool model showing part of the broken symmetry-state geometry that realizes the 5 Platonic solids and the icosidodecahedron. Right: The icosidodecahedron.

The reciprocal of the icosidodecahedron, the rhombic triacontahedron, in a 2-D projection gives the same outline as half of the pentagonal outline shown within an X-ray crystallographic scattergraph of the DNA double helix as shown in Figure 4. Presumably, placing the 2nd chiral form would complete the picture, indicate the reverse strand of the DNA helix, and present a second set of 32 faces giving a total of 64 (total number of triplet codons).

Our previous work has suggested that the icosidodecahedron is the shape that corresponds, in principle, to the all-important order 64. If we have an icosidodecahedron, combining an icosahedron inside and a dodecahedron outside (or vice versa), then each (because of the intrinsic 5-fold symmetry) is a representation of a double 3-D (or, including the timelike variation between them, to a double 3-D, with the extra pseudoscalar). If we have a cuboctahedron, combining a cube outside and octahedron inside (or vice versa), each of its component shapes represents a single 3-D (with a timelike variation again providing an extra pseudoscalar). The two aspects could represent two 3-Ds, but not simultaneously. There is no instantaneous double 3-D and no 5-fold symmetry.

In the corresponding algebra, with its physical interpretation, we can reduce the 5-D nilpotent $ikE + i\mathbf{p} + jm$ to a 3-D nilpotent $ikE + ip + jm$ by removing the 3-dimensionality of **p**, so that it becomes scalar *p*. (Here, if a cube represents space, then its reciprocal octahedron represents momentum, or $E / p / m$.). Effectively, one fermion on its own behaves this way, but, in relation to other fermions, it constructs a 3-D space and a 3-D momentum, or 'physical space'. We see something like this in the geometrical diagrams related to the universal rewrite system where we can see the 6-sided cube repeating itself with increased inner structure [5].

Now, an icosidodecahedron incorporates all states at once, as do the icosahedron and the dodecahedron taken separately. However, unless there is variation between them, the picture is incomplete, because we don't see the extra pseudoscalar structure. The icosidodecahedron incorporates all possible states of itself at once, including the extremes of icosahedron and dodecahedron (which we could perhaps think about as like the **r**, *t* and **p**, *E* extremes of the physics), and all possible states in between – an infinite number of phases. We can imagine switching between phases or cycling through them, or, in quantum mechanical terms, as imagining that all happen at once.

We should think of this as a property of 'physical space', not of physics or biology, as such. In this context, when we talk about fundamental physical particles, we should realise that the 'charge space', which tells us how many physical particles we should have, is exactly dual with the momentum (+ energy) space, which constitutes quantum mechanics. The first tells us how space (+ time) combine with charge (+ mass) to transform them; the second tells us how charge (+ mass) combine with space (+ time) to transform them. The first is the conserved picture; the second the nonconserved.

When something happens, i.e. when some interaction occurs, one particular phase is selected, but only at that moment. In quantum mechanics, all possible states are equivalent (degenerate) until one is selected over all the others by making an observation. Gauge invariance (i.e. phase invariance) means all possible states are equal. Fixing means one particular state is selected. Famously in physics, the weak interaction fixes the gauge of the weak vacuum, privileging the real fermion state in all observable situations over the other three (vacuum states) in the Dirac wavefunction, and also the left-handed helicity over the right-handed. The predominance of matter over antimatter can be seen as a property of 'physical space', rather than a result of 'conditions during the big bang', as also the left- over right-handed chirality (a chirality fully established in the nilpotent theory [4]). It is a result of the production of the 5-fold state reducing the number of sign degrees of freedom. This is also reflected in biology where preferential chirality abounds, examples being the alpha helix of DNA, the D forms of dNTPS and ribose groups that make up the DNA and the L amino acids produced as a result of DNA translation.

The fermion, as structured in the Dirac wavefunction, always has four possible states – particle / antiparticle, right- / left-handed spin – one is 'real', and the other three are vacua. The free particle behaves as though it is continually switching between them, by a 'jittery motion' in and out of vacuum (*zitterbewegung*), and the mass and velocity of the particle are determined by the switching rate. When we observe the particle, of course, its behaviour is determined by which of the four terms represents its 'real' (observable) state. The icosahedron / dodecahedron switching is like *zitterbewegung*. We can see it as 'particle' / vacuum or space / momentum space, etc. Of course, there is also chirality of the shapes – left- and right-handed versions. The in-built chirality of biology can be seen as due to the chirality of physical space.

We can perhaps think of a real biological system, e.g. DNA / RNA, virus, protein, as having all the possibilities at once, or as near that as possible, and interacting when the possibility is selected which matches the one required by the interacting system. This must be true of chemical systems in general, as chemical systems are quantum mechanical, and biological systems, of course, are also interacting as chemical systems.

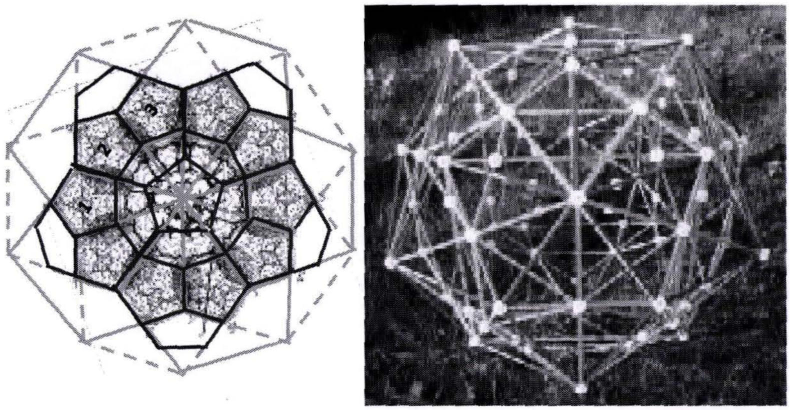
It appears that biological systems are very near chaotic. This would make sense in terms of the need to produce as many possibilities as possible to find the one which 'works'. Chaos occurs when the continual bifurcation of the system (which we regard as one of a number of possible approaches to the idealised general case required by universal rewrite) goes at too fast a rate. The defining of new possibilities in the universal rewrite system can be seen as a continual bifurcation, and the rate which just stays inside chaos will be the most efficient possible that maintains structural order.

5 The Icosidodecahedral Phase

We have previously seen the uniqueness of each DNA profile as probably being in some way analogous to Pauli exclusion in physics. Now Pauli exclusion in nilpotent theory has two forms. First, if $\psi_1 = (\boldsymbol{i}\boldsymbol{k}E_1 + \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$ squares to 0, then the combination state with $\psi_2 = (\boldsymbol{i}\boldsymbol{k}E_2 + \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$ would be 0 unless ψ_1 is different from ψ_2. But, conventionally, Pauli exclusion is done by defining the combination state as $(\psi_1\psi_2 - \psi_2\psi_1)$, and then saying that the wavefunctions ψ_1 and ψ_2 are antisymmetric, which means that, if we exchange ψ_1 and ψ_2, then the combination state is the negative of what it was before. So $(\psi_1\psi_2 - \psi_2\psi_1) = -(\psi_2\psi_1 - \psi_1\psi_2)$. Now, if you apply this definition of combination state, i.e. $(\psi_1\psi_2 - \psi_2\psi_1)$, to ψ_1 and ψ_2, using nilpotents, you find that after all the subtractions, only one thing is left, and that is $8i\mathbf{p}_1 \times \mathbf{p}_2$. This is certainly antisymmetric, as required, but it also has a deep significance. $\mathbf{p}_1 \times \mathbf{p}_2$ only has a nonzero value when $\mathbf{p}_1$ and $\mathbf{p}_2$ are in different directions – it doesn't matter about their scalar magnitudes. In other words, we can only tell if ψ_1 and ψ_2 are different by the instantaneous relative directions in 3-D of their momentum operators $\mathbf{p}$. So, each is uniquely specified by this alone. We can also imagine projection of the magnitudes of E, $\mathbf{p}$ and m along three axes being unique for each ψ, and the spin (momentum) vector carrying the information about E (handedness), $\mathbf{p}$ (direction), m (magnitude). This would link in with the first way of seeing Pauli exclusion. Now, the only things that specify an Archimedean solid are the magnitude, handedness / chirality, and instantaneous direction in 3-D determined against some arbitrary fixed axis. So if the identity of a living organism's DNA is uniquely defined by the relative 'position' of its defining solid (presumably uniquely determined by its sequence of bases) or direction of its defining axis with respect to some fixed reference, then this (plus size and handedness) is the same set of information as we use in the E-$\mathbf{p}$-m of the fermion, and we could backwards project the solid onto E-$\mathbf{p}$-m to characterize it. (Significantly, if we look at the particle states, they are in principle all connected as though they were versions of each other.)

We can imagine the area between the inner icosahedron and the outer dodecahedron as in constant flux, with the icosahedron, in effect going up and down. The solid would then be expanding and contracting up and down the 3 axes at once, migrating through states that are actually Archimedean solids, i.e. passing through each 'phase' up to the dodecahedron. This process starts with the icosahedral triangular faces and, as the solid increases in size up the 3 axes, it transmutes into other forms as the pentagonal faces start to appear and the triangular ones start to span out and reduce in size until we end up with the flat dodecahedral pentagonal faces alone. The previously described geometric progression shows the construction of an icosidodecahedron (where we can also realise a reciprocal form of the rhombic tricontrahedron with relevance to the top-down view of a scattergraph of the DNA helix). If you view from outside this fluctuating object from a random view point – say a vertex of an icosahedron and its 5 associated triangles – you will see the pentagonal unit of 5 triangles effectively start to flatten and spread out and the pentagonal faces start to appear as the object increases in size from the icosahedron. This pentagonal area will effectively appear to the observer, to turn and expand in a spiral on its way up to the pentagonal face of the outer dodecahedron. The direction this turns will depend on the chirality of the final dodecahedron.

The genetic information for each living entity can be considered as being in one of an infinite number of phase states between the icosahedron and dodecahedron, with each state representing a different codon usage determined by what faces are effectively formed at a specific point. Hence we can have, at one extreme, say, the thermophiles, which have 70% GC content and therefore make heavy use of GC biased codons for an amino acid, and, at the other extreme, AT rich organisms. In addition, everything again becomes unique within a species as there are always tiny mutations introduced at every round of DNA replication. As long as these mutations are within the boundaries set by the phase, no new species will be formed. Perhaps stable species in evolutionary terms are those whose genetic information can be represented by Archimedean solids and have greater symmetry characteristics. We can start to consider whether the actual Archimedean solids are states that are more stable positions and that things in between tend towards one or other of them and a species hovers near these more stable forms. This would be something like resonance in 3 dimensions. An interaction of one microorganism or living system with another in close proximity could allow a new combined resonance state to be set up which would shift codon usage in each, with a subsequent mutual information exchange occurring to allow deeper and deeper interaction. This may be observed initially, as a symbiosis and eventually as a full integration, as proposed for mitochondria within eukaryotic cells.



A B

Figure 4: The geometry of the DNA helix. A: The outline of the rhombic tricontrahedron overlaid upon a computer generated scattergraph of DNA. B: The Archimedean solid the rhombic tricontrahedron, the reciprocal of the icosidodecahedron.

6 The Rewrite Algebra in Physics and Biology

The 30 / 32 or 60 / 64 connection seen in the previously described Platonic solid structures also appears in the rewrite algebra. Of the total of 64 generators in the rewrite algebra, as used in physics, 60 can be arranged into 12 nilpotent pentads, with the remaining 4 being the nondimensional units 1, –1, ***i***, – ***i***. One way of arranging the 12 pentads is as follows:

ii ij ik ik j	**–ii –ij –ik –ik –j**
iii iij iik ik j	**–iii –iij –iik –ik –j**
ji jj jk ii k	**–ji –jj –jk –ii –k**
iji ijj ijk ii k	**–iji –ijj –ijk –ii –k**
ki kj kk ij i	**–ki –kj –kk –ij –i**
iki ikj ikk ij i	**–iki –ikj –ikk –ij –i**

Each pentad here is a fermion, with the first term representing the energy operator, the next 3 the momentum operator, and the last term the mass operator. So these structures (if we treat them symbolically) can be seen as representing 12 fermions, say, 6 quarks and 6 leptons, or 6 quarks / leptons and 6 antiquarks / antileptons. The total becomes $2 \times 12 = 24$ if we include left- and right-handed states (the parity P duality); and $2 \times 2 \times 12 = 48$ if we include fermion and antifermion states (the charge conjugation C duality), in addition to quarks and leptons.

generation		isospin	
1	electron neutrino	up	$\boldsymbol{i}\mathbf{i}$ $\boldsymbol{i}\mathbf{j}$ $\boldsymbol{i}\mathbf{k}$ $i\boldsymbol{k}$ $\boldsymbol{j}$
	electron	down	$i\mathbf{i}\boldsymbol{i}$ $i\mathbf{i}\boldsymbol{j}$ $i\mathbf{i}\boldsymbol{k}$ $i\mathbf{k}$ $\mathbf{j}$
2	muon neutrino	up	$\boldsymbol{j}\mathbf{i}$ $\boldsymbol{j}\mathbf{j}$ $\boldsymbol{j}\mathbf{k}$ $i\boldsymbol{i}$ $\boldsymbol{k}$
	muon	down	$i\mathbf{j}\boldsymbol{i}$ $i\mathbf{j}\boldsymbol{j}$ $i\mathbf{j}\boldsymbol{k}$ $i\mathbf{i}$ $\mathbf{k}$
3	tau neutrino	up	$\boldsymbol{k}\mathbf{i}$ $\boldsymbol{k}\mathbf{j}$ $\boldsymbol{k}\mathbf{k}$ $i\boldsymbol{j}$ $\boldsymbol{i}$
	tau	down	$i\mathbf{k}\boldsymbol{i}$ $i\mathbf{k}\boldsymbol{j}$ $i\mathbf{k}\boldsymbol{k}$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antielectron-neutrino	up	$-\boldsymbol{i}\mathbf{i}$ $-\boldsymbol{i}\mathbf{j}$ $-\boldsymbol{i}\mathbf{k}$ $-i\boldsymbol{k}$ $-\boldsymbol{j}$
	antielectron	down	$-i\mathbf{i}\boldsymbol{i}$ $-i\mathbf{i}\boldsymbol{j}$ $-i\mathbf{i}\boldsymbol{k}$ $-i\mathbf{k}$ $-\mathbf{j}$
2	antimuon-neutrino	up	$-\boldsymbol{j}\mathbf{i}$ $-\boldsymbol{j}\mathbf{j}$ $-\boldsymbol{j}\mathbf{k}$ $-i\boldsymbol{i}$ $-\boldsymbol{k}$
	antimuon	down	$-i\mathbf{j}\boldsymbol{i}$ $-i\mathbf{j}\boldsymbol{j}$ $-i\mathbf{j}\boldsymbol{k}$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitau-neutrino	up	$-\boldsymbol{k}\mathbf{i}$ $-\boldsymbol{k}\mathbf{j}$ $-\boldsymbol{k}\mathbf{k}$ $-i\boldsymbol{j}$ $-\boldsymbol{i}$
	antitau	down	$-i\mathbf{k}\boldsymbol{i}$ $-i\mathbf{k}\boldsymbol{j}$ $-i\mathbf{k}\boldsymbol{k}$ $-i\mathbf{j}$ $-\mathbf{i}$

generation		isospin	
1	up quark	up	$\boldsymbol{i}\mathbf{i}$ $\boldsymbol{i}\mathbf{j}$ $\boldsymbol{i}\mathbf{k}$ $i\boldsymbol{k}$ $\boldsymbol{j}$
	down quark	down	$i\mathbf{i}\boldsymbol{i}$ $i\mathbf{i}\boldsymbol{j}$ $i\mathbf{i}\boldsymbol{k}$ $i\mathbf{k}$ $\mathbf{j}$
2	charmed quark	up	$\boldsymbol{j}\mathbf{i}$ $\boldsymbol{j}\mathbf{j}$ $\boldsymbol{j}\mathbf{k}$ $i\boldsymbol{i}$ $\boldsymbol{k}$
	strange quark	down	$i\mathbf{j}\boldsymbol{i}$ $i\mathbf{j}\boldsymbol{j}$ $i\mathbf{j}\boldsymbol{k}$ $i\mathbf{i}$ $\mathbf{k}$
3	top quark	up	$\boldsymbol{k}\mathbf{i}$ $\boldsymbol{k}\mathbf{j}$ $\boldsymbol{k}\mathbf{k}$ $i\boldsymbol{j}$ $\boldsymbol{i}$
	bottom quark	down	$i\mathbf{k}\boldsymbol{i}$ $i\mathbf{k}\boldsymbol{j}$ $i\mathbf{k}\boldsymbol{k}$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antiup-quark	up	$-\boldsymbol{i}\mathbf{i}$ $-\boldsymbol{i}\mathbf{j}$ $-\boldsymbol{i}\mathbf{k}$ $-i\boldsymbol{k}$ $-\boldsymbol{j}$
	antidown-quark	down	$-i\mathbf{i}\boldsymbol{i}$ $-i\mathbf{i}\boldsymbol{j}$ $-i\mathbf{i}\boldsymbol{k}$ $-i\mathbf{k}$ $-\mathbf{j}$
2	anticharmed-quark	up	$-\boldsymbol{j}\mathbf{i}$ $-\boldsymbol{j}\mathbf{j}$ $-\boldsymbol{j}\mathbf{k}$ $-i\boldsymbol{i}$ $-\boldsymbol{k}$
	antistrange-quark	down	$-i\mathbf{j}\boldsymbol{i}$ $-i\mathbf{j}\boldsymbol{j}$ $-i\mathbf{j}\boldsymbol{k}$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitop-quark	up	$-\boldsymbol{k}\mathbf{i}$ $-\boldsymbol{k}\mathbf{j}$ $-\boldsymbol{k}\mathbf{k}$ $-i\boldsymbol{j}$ $-\boldsymbol{i}$
	antibottom-quark	down	$-i\mathbf{k}\boldsymbol{i}$ $-i\mathbf{k}\boldsymbol{j}$ $-i\mathbf{k}\boldsymbol{k}$ $-i\mathbf{j}$ $-\mathbf{i}$

These particle states can be represented either by an icosahedron, with triangular faces (4 × 5 × 3), or a dodecahedron, with pentagonal faces (4 × 3 × 5). Another alternatative, is the icosidodecahedron with 32 faces, 20 of which are triangular (4 × 5 × 3) and 12 pentangular (4 × 3 × 5).

The biological system becomes algebraically equivalent to the physical one through the A-T and C-G pairing becoming algebraically equivalent to space-time and charge-mass. Also, both the four parameters of physics and the four bases of genetics can be represented by a tetrahedron. (In the case of physics, this leads to a representation of *C*, *P* and *T* symmetries.) The tetrahedron has 12 rotation symmetries, corresponding to the 12 pentads. In biology, each face of the tetrahedron is a triplet codon. Each pentad is a unit on a DNA strand that highlights a triplet codon (Figure 4A) and 10 defines a double helix of DNA and aids in the placement of the dodecahedra: 3 pentagonal faces (1.5 dodecahedra) give placement of 3 dNTPs or a triplet, and back faces would represent the opposite DNA strand 3 dNTPs. The total can become 2 × 12 = 24 if we consider the opposite chiral form of the dNTPs and DNA and hence a mirror form of the dodecahedron.

64 possible combinations are manifested in the triplet code (4 × 4 × 4), and the triplet code is created because of the *need* to have the 64 combinations. 60 of these actually code, the others being STOP mechanisms, etc. This separation of the number of algebraic units into 60 + 4, and the further creation of 48 units within the 60, can be accomplished by some such algebraic classification of the codons as:

48	2 first bases different
12	2 first bases the same
4	3 bases the same

(There are further ways of dividing up the 60 into combinations such as 36 + 12 + 12.). The third order star tetrahedron includes all 64. The total doubles again for physical particles if we include the *3-dimensionality* of the quarks (as we did originally for space and charge), or 3 + 1-dimensionality for quark-fermion (as with space-time and charge-mass). Then we have 3 colours of quark and 2 = 1 + 1 becomes 4 = 1 + 3, as previously, meaning that we have 2 × 2 × 2 × 12 = 96 possible fermionic / antifermionic states – these are the ones known to exist.

In the corresponding genetics, the 60 translated codons can be represented by the 12 pentagonal faces of a dodecahedron; the dodecahedron, with 12 pentagonal faces, can be taken as equivalent to 12 pentads, each representing 5 triplets. A dodecahedron incorporates 5 positive and 5 negative tetrahedral within (that constitute the 5 star tetrahedra), each of which can rotate 12 times, again dividing the 60 into 12 pentads (× 2, 1 positive the other negative). 48 + 12 would further emerge from 4 dodecahedra + a fifth, completing a cycle of 5. It is possible that the amino acids can be grouped into 5 yet to be determined groups represented by one face of a dodecahedron in a dodecahedral arrangement of codons (5 per face) and also a star tetrahedral arrangement – 64 triplets (8 octahedra – each face representing a triplet) plus 20 tetrahedra (representing 20 amino acids). This 60 is also reiterated again in the system of the pentameric disks of 5 tetrahedra that create the double helix and the highlighting of a triplet codon. The 12 pentads in genetics can be seen in the dodecahedral arrangement of the codons (5 triplets per face) on 12 faces (minus stop codons) as shown in Figure 4A.

7 Conclusion

We have suggested that a universal rewrite system, based on a zero totality universe, acts as a driving process, maximizing information, and determining the behaviour of ordered systems, physical and chemical as well as biological, large scale as well as small scale. The system automatically generates geometrical and algebraic structures, in progressive stages, which we have shown, using many detailed examples, correspond to those relevant to genetics and particle physics. The key to significant change in Nature has been identified as the symmetry-breaking that occurs when 5-fold structures appear in the geometric and algebraic representations.

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