Mathematics and Physics as Emergent Aspects of a Universal Rewrite System

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Abstract
Mathematics and physics are shown to have a symbiotic relationship as emergent aspects of a universal rewrite system. In addition to explaining the 'unreasonable effectiveness' of mathematics in physics and the 'unreasonable effectiveness' of physics in mathematics, this emergent nature of both subjects makes sense of the distinction between syntactic and semantic approaches to logical reasoning. The system is also shown to generate constraints on the kinds of mathematics and physics that are possible, explaining, in particular, why symmetry is so significant in the subjects' foundations, and specifying which symmetries are most significant, as well as indicating their points of origin. Quantum mechanics emerges from this structure in a very specific form which enables us to understand the origin of symmetry breaking in physics and many other aspects of fundamental physical theory. Gravity also has special characteristics which explain its uniqueness among the four physical forces.
Keywords: universal rewrite system, emergence, symmetry breaking, quantum mechanics, gravity.

1 Introduction

Mathematics and physics have a symbiotic relationship. Eugene Wigner famously stressed the 'unreasonable effectiveness' of mathematics in physics [1], while, in more recent times, it has become clear that there is an equal degree of 'unreasonable effectiveness' of physics in mathematics [2]. We can only describe what we observe in the world external to us, and make abstractions from our observations, and so each of these two systems of thought must, in some way, reflect this world or our interpretation of it. But why are they different and why do they interact? The use of mathematics in physics is often represented as a 'convenience', but it is, in fact, anything but convenient. The mathematics used in physical theory (principally differential equations) is very different from the mathematics used in physical observation (essentially arithmetic), and theories expressed in differential form can usually only be related to potential observations by making approximations or inserting boundary conditions for particular cases. We must use mathematics in physics, not because it is convenient, but because it is the only way of expressing fundamental physical truths. On the other hand, mathematics, though abstract, cannot be created at the will of the creator; constraints always exist which, in some subtle way, seem to be related to physical experience, and which channel the mathematical development in almost predetermined directions.

The intimate relationship between mathematics and physics gives us a clue to establishing foundations for each. These foundations must lie in the points where they intersect in the most fundamental manner, the places where they emerge from a common origin. And it is most likely that the initial clue will come from physics. Mathematics appears to respond to structuring in a considerable variety of ways, but physics has had to survive the tests of observation and measurement made under the most exacting conditions and has evolved a mathematical structure which could never have been expected as the outcome of the process begun with classical physics several hundred years ago. We can regard this evolution as a kind of Darwinian selection of the most efficient way of processing information about the external world, and it is likely that those aspects of mathematics that are particularly important in physics lie closest to the foundations of the subject.

2 A Universal Rewrite System

A foundation for mathematics and physics has already been proposed using the computational idea of a universal rewrite system [3-5], though, in principle, the ideas go even beyond those of computation. At this stage, we have to imagine a natural process which cannot be characterised even by the kind of axiom set which is familiar in many foundational studies of mathematics. Certainly numbers cannot be assumed, not even the natural numbers, 1, 2, 3 ... which are already loaded with assumptions about discreteness as well as ordinality. If we begin at this point we will immediately generate the problems identified by Gödel's theorem. Nor can we assume algebra and algebraic symbolism, or formal logic. All these concepts must be seen as *emergent* from something which is almost beyond description.

However, it is not quite beyond description and the universal rewrite system provides exactly such a description. Because it has already been described in several publications, we will give only a brief outline here. Of course, because language is our only medium of communication, we are obliged to use terms that will later acquire special significance in mathematics and mathematical logic, but our usage should not imply that these are already assumed to exist. What we are able to provide by this method is a *fundamental description of process*, whose validity will become evident from its clear applicability to both mathematics and physics at the foundational level. The principal and only assumption is that the only thing that can be described at any time is a zero totality state, and this has no unique description, but is infinitely degenerate. Any deviation from the zero state, say a nonzero state, R, would necessarily incorporate an automatic mechanism for recovering the zero, say, a conjugate R^*, so that we have a totality of, say, (R, R^*), which is zero. However, zero totality, as we have said, is infinitely degenerate, and so this state does not define a *unique* zero. The universal rewrite system therefore incorporates a procedure for defining a new zero totality, with a recognisably new structure, which is not isomorphic to (R, R^*) because it defines the position of this structure within it, and the process continues indefinitely. In effect, we define a series of *cardinalities* based on zero, rather than infinity.

To separate the old zero cardinalities from the new, we suppose that a zero totality cardinality, otherwise undefined, which for convenience, though without any assumption as to form, we will describe as an 'alphabet', contains nothing new within it and a new zero totality outside it. These two aspects of the cardinality condition, which we can describe by the terms 'conserve' and 'create', and represent by the symbols $\rightarrow$ and $\Rightarrow$, are most conveniently displayed (though not defined) by a 'concatenation' or placing together, with no algebraic significance, of the alphabet with respect to either its components ('subalphabets') or itself. If the alphabet describes a cardinality or totality, then anything other than itself will necessarily be a 'subalphabet' and the concatenation will yield nothing new. The only other option will be concatenation with itself, which, to ensure that the cardinality is not unique, must yield a new cardinality or zero totality alphabet. That is, the condition of non-unique cardinality requires that

(subalphabet) (alphabet) $\rightarrow$ (alphabet) *i.e. there is nothing new*
(alphabet) (alphabet) $\Rightarrow$ (new alphabet) *i.e. the zero totality is not unique*

while the condition of zero totality will require that all terms in the alphabet are conjugated or dual, and, in principle, indistinguishable individually.

Of course, the nature of the new alphabet produced by $\Rightarrow$ will always be determined by the need to satisfy $\rightarrow$ in all possible cases. So, we can only find out what a new alphabet will look like when we have worked out all the ways in which concatenation with its subalphabets will yield only itself. Suppose, then, that our first zero totality alphabet has the form (R, R^*). Applying the conserve mechanism ($\rightarrow$) by concatenating it with its subalphabets should produce nothing new. So

$$(R)\,(R, R^*) \rightarrow (R, R^*)$$
$$(R^*)\,(R, R^*) \rightarrow (R^*, R) \equiv (R, R^*)$$

No concept of 'ordering' is required by concatenation, but each term must be distinct, so we can easily show that these concatenations lead to rules of the form:

$$(R)\,(R) \rightarrow (R)\,;\ (R^*)\,(R) \rightarrow (R^*)\,;\ (R)\,(R^*) \rightarrow (R^*)\,;\ (R^*)\,(R^*) \rightarrow (R)$$

Now we need to show that the zero-totality alphabet (R, R^*) cannot be unique, and that concatenation with itself (or 'create') must produce a new conjugated alphabet. Of course, this cannot be, say, (A, A^*), which, with the terms undefined, is indistinguishable from (R, R^*), and the only way of creating a new alphabet, which is distinguishable is by incorporating the old one. However, we must do this in such a way that the conserve mechanism still applies, that is, that the subalphabets yield nothing new. So we try

$$(R, R^*)\,(R, R^*) \Rightarrow (R, R^*, A, A^*)$$

Applying the conserve mechanism ($\rightarrow$) to this new alphabet, by concatenation with the subalphabets, produces

$$\begin{array}{rcl}
(R)\,(R, R^*, A, A^*) & \rightarrow & (R, R^*, A, A^*) \equiv (R, R^*, A, A^*) \\
(R^*)\,(R, R^*, A, A^*) & \rightarrow & (R^*, R, A^*, A) \equiv (R, R^*, A, A^*) \\
(A)\,(R, R^*, A, A^*) & \rightarrow & (A, A^*, R^*, R) \equiv (R, R^*, A, A) \\
(A^*)\,(R, R^*, A, A^*) & \rightarrow & (A^*, A, R, R^*) \equiv (R, R^*, A, A^*)
\end{array}$$

Here, to maintain an unchanged alphabet, and to specify that R, R^*, A, A^* remain distinct, we find that we have to arrange that each term 'cycles' into another. We also require that

$$(R)\,(A) \rightarrow (A)\,;\,(R)\,(A^*) \rightarrow (A^*)\,;\,(R^*)\,(A) \rightarrow (A^*)\,;\,(R^*)\,(A^*) \rightarrow (A);$$
$$(A)\,(A) \rightarrow (R^*)\,;\,(A^*)\,(A^*) \rightarrow (R^*)\,;\,(A)\,(A^*) \rightarrow (R).$$

Alternative rules, such as

$$(A)\,(A) \rightarrow (R)\,;\,(A^*)\,(A^*) \rightarrow (R^*)\,;\,(A)\,(A^*) \rightarrow (R^*)\,(A)$$

cannot be permitted because they would make R and A indistinguishable and the alphabet would not be extended.

If we now try applying the 'create' mechanism ($\Rightarrow$) to the new alphabet, it will quickly become clear that the result cannot be, say, (R, R^*, A, A^*, B, B^*), as the subsequent application of 'conserve' ($\rightarrow$) would introduce concatenated *terms*, such as AB, AB^*, which are not in the alphabet, and so we try the option of including the terms from the beginning, and this time we succeed:

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*).$$

However, a new complication arises when we successively perform the conserve operation with (R), (R^*), (A), (A^*), (B), (B^*), (AB), (AB^*), to leave the totality unchanged. The process is straightforward for the first six operations:

$$\begin{array}{l}
(R)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*) \\
(R^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R^*, R, A^*, A, B^*, B, AB^*, AB) \\
(A)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A, A^*, R^*, R, AB, AB^*, B, B^*) \\
(A^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A^*, A, R, R^*, AB^*, AB, B^*, B) \\
(B)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B, B^*, AB, AB^*, R^*, R, A, A^*) \\
(B^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B^*, B, AB^*, AB, R, R^*, A^*, A)
\end{array}$$

But the last two concatenations seem to create ambiguities when we come to the operations of the concatenated terms, such as (AB) and (AB^*) on themselves and on each other. There are two clear possibilities for the concatenation $(AB)\,(AB)$, and we can regard these as the 'commutative' and 'anticommutative' options:

$$\begin{array}{lll} (AB)\,(AB) \rightarrow & (R) & (\textit{commutative}) \\ (AB)\,(AB) \rightarrow & (R^*) & (\textit{anticommutative}) \end{array}$$

However, it soon becomes clear that there is really no choice, because the commutative option ends up by making A and B indistinguishable and so does not extend the alphabet. So we are obliged to default on the anticommutative option, and the last two concatenations become:

$$\begin{array}{l} (AB)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB, AB^*, B, B^*, A, A^*, R^*, R) \\ (AB^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB^*, AB, B^*, B, A^*, A, R, R^*) \end{array}$$

This solves the problem for A and B, but it cannot be repeated to include new terms, such as (C), (D), etc., when the alphabet is extended at higher stages because some inconsistency will always reveal itself at some point in the analysis. Anticommutativity effectively produces a closed 'cycle' with components (A, B, AB) and their conjugates, and excludes any further C, D …-type term of anticommuting with them. (This could only be achieved by introducing the concept of antiassociativity into the structure when we have yet to require that the opposite property exists.) However, the separate successive cycles, say, (A, B, AB), (C, D, CD), etc., can be introduced commutatively into the structure, and this can be continued indefinitely. All the terms have a unique identity *because they each have a unique partner.*

The alphabets generated by the create process ($\Rightarrow$) thus lead to a regular series of identically structured closed anticommutative cycles, each of which commutes with all others. It is the structure which is familiar to us as the infinite series of finite (binary) integers of conventional mathematics. The closed cycles produce an infinite ordinal sequence, establishing for the first time the meaning of both the number 1 and the binary symbol 1 as it appears in classical Boolean logic as a conjugation state of 0, and the alphabets structure themselves as an infinite series of binary digits. In fact, by being forced to introduce anticommutativity, we simultaneously create the concepts of *discreteness* and *dimensionality* (specifically 3-dimensionality). Physically, 3-dimensionality requires discreteness, and discreteness requires 3-dimensionality. 3-dimensionality, or anticommutativity, is the ultimate source of discreteness in a zero totality universe.

3 The Emergence of Physics

Mathematics, we have seen, can be regarded as an emergent property of an ongoing rewrite process that has no defined starting point and that can be reconstructed endlessly in a fractal manner with self-similarity at all stages. Of course, once we have integers, the rest of the constructible number system follows automatically, together with arithmetical operations, such as addition and multiplication, and application of the constructed number systems to the undefined state with which the process begins indicates that, because it is not intrinsically discrete, it can be interpreted in terms of a continuity of real numbers in the Cantorian sense. In addition, the discrete anticommuting cycles, such as (A, B, AB), (C, D, CD), etc., introduce an algebra equivalent to an endless sequence of quaternion sets, with units ($\boldsymbol{i}_1$, $\boldsymbol{j}_1$, $\boldsymbol{k}_1$), ($\boldsymbol{i}_2$, $\boldsymbol{j}_2$, $\boldsymbol{k}_2$), etc., which are commutative with each other. If we treat the rewrite structure in arithmetical-algebraic terms using the mathematics that it generates, we can redefine conjugation as negation, and the concatenation process as algebraic multiplication, with the 'create' process becoming similar to a squaring operation, and write the successive units of the system in the form

$$\begin{aligned}
&(1,-1)\\
&(1,-1)\times(1,\boldsymbol{i}_1)\\
&(1,-1)\times(1,\boldsymbol{i}_1)\times(1,\boldsymbol{j}_1)\\
&(1,-1)\times(1,\boldsymbol{i}_1)\times(1,\boldsymbol{j}_1)\times(1,\boldsymbol{i}_2)\\
&(1,-1)\times(1,\boldsymbol{i}_1)\times(1,\boldsymbol{j}_1)\times(1,\boldsymbol{i}_2)\times(1,\boldsymbol{j}_2)\\
&(1,-1)\times(1,\boldsymbol{i}_1)\times(1,\boldsymbol{j}_1)\times(1,\boldsymbol{i}_2)\times(1,\boldsymbol{j}_2)\times(1,\boldsymbol{i}_3)\ \ldots,
\end{aligned} \tag{1}$$

though the attached scalar values for the equivalent R, A, B, C, D, etc., are not necessarily unitary, and may equally be real numbers. We see here that the natural algebra defined by the process is related to Clifford algebra, and incorporates real algebra, complex algebra (as incomplete quaternion sets), quaternion algebra and multivariate vector algebra (as the product of the complex and quaternion algebras), though not octonions. The mathematical frameworks generated, however, provide a syntactical logical structure, which once constructed, can be reconstructed from within to provide entirely new structures, as has always been possible with mathematics.

In general, the rewrite structure can be seen as providing an information processing system which is the most efficient possible because it is the most 'naturally' generated. Apart from being the most probable origin of mathematics, we can imagine that this system is the also one that is used where information processing is at a premium, in fundamental physics, in the emergence of life as a replicating structure, in the development of consciousness, in cosmology, and perhaps also in aspects of chemistry. However, its most obvious manifestation appears to be in fundamental physics. If there is a fundamental physical theory, it must emerge from 'Nature's' own processing in an even more constrained way than mathematics. Here, the syntactic logic of mathematics provides a necessary but not sufficient description of the zero totality alphabet. Physics, in accounting for a physical 'universe' in which everything is connected, has to describe the whole alphabet semantically, as well as syntactically; it must take account of the whole series of alphabet realisations at once in the same way as the Feynman path integral takes account of all possible paths in a quantum system.

At the fundamental level, physics requires information from just four fundamental parameters – mass, time, charge and space – and all aspects of physical law can be derived solely from their characteristics [5]. Here, mass refers to mass-energy, the source of gravity, not the unobserved quantity 'rest mass', while charge has three components referring to the sources of the weak, strong and electric interactions – the symmetry-breaking between the interactions is another emergent property. But what are these parameters and where do they come from?

The answer is that they are successive realisations of the zero totality alphabet as applied universally. They are successive 'cardinalities'. In the rewrite structure defined in (1), we see an emerging series of algebraic objects as we go through the first four realisations of the alphabet: real scalar (1), pseudoscalar or imaginary scalar ($\boldsymbol{i}_1$), quaternion set $(1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$, and multivariate vector set (or complexified quaternion set), constructed from $(1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$. It will be convenient to relabel these in the form: scalar (1), pseudoscalar (i), quaternion set ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$), and multivariate vector set ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$). We can recognise these also as the subalgebras of a Clifford algebra (strictly, the real Pauli Clifford algebra of Euclidean 3 space, of dimension $2^3 = 8$), though emerging in reverse order. In the Clifford algebra, the basic unit, the vector (our multivariate vector), generates in its product the bivector (our quaternion), and the vector and bivector generate in their product the trivector (our pseudoscalar). This is the complete algebra of Euclidean 3-dimensional space and it differs from ordinary vector algebra in that the vectors (our multivariate vectors or complexified quaternions, which are isomorphic to Pauli matrices) have a true algebraic product, which combines scalar and vector products in the same way as quaternions. The product of vectors **a** and **b**, for example, become:

$$\mathbf{ab} = \mathbf{a.b} + i\,\mathbf{a} \times \mathbf{b}.$$

In the language of the Clifford formalism, the minimum algebra needed to create the first four realisations of the alphabet is, successively scalar, trivector, bivector and vector. However, the trivector and bivector algebras also generate their own scalars, while the vector algebra generates the entire range of scalar, bivector and trivector. Each of these has a 'physical' realisation in one of the four parameters:

$(1, -1)$	1	scalar	mass
$(1, -1) \times (1, \boldsymbol{i}_1)$	$1, i$	trivector + scalar	time
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$	$1, \boldsymbol{i}, \boldsymbol{i}, \boldsymbol{k}$	bivector + scalar	charge
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$ $\times (1, \boldsymbol{i}_2)$	$1, i, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, \mathbf{i}, \mathbf{i}, \mathbf{k}$	vector + bivector + trivector + scalar	space

In other words, we can regard the four parameters as four successive realisations of the alphabet, four independent 'physical' representations of the 'universe', which, if physics is to be semantically true, must all exist at once.

The question we now have to ask is: why does physics seemingly only need these four realisations? Why do the representations not extend to infinity? The answer is that they do, but that physics has found a way of combining the first four in such a way that the higher realisations are all automatically zeroed. First of all, the realisations that we call mass, time, charge and space must exist independently of each other, i.e. be commutative. Since the algebras of mass and time are already commutative with all others, this means that the main criterion for the minimum algebra of combination is one in which the algebras of charge and space are commutative with each other. In effect, this means that the combination requires a double Clifford algebra or a tensor product of two Clifford algebras, each commutative with the other. In effect, this is the algebra of the sixth stage in the rewrite series represented in (1). This is a 64-part algebra, isomorphic to the Dirac algebra of relativistic quantum mechanics, a group of order 64 constructible from every conceivable product of the terms (1, i, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, $\mathbf{i}$, $\mathbf{i}$, $\mathbf{k}$), where 1, i, ($\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$) and ($\mathbf{i}$, $\mathbf{i}$, $\mathbf{k}$) are all commutative.

However, the simplest way to define this group is not from the eight units (1, i, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, $\mathbf{i}$, $\mathbf{i}$, $\mathbf{k}$) but from five generators which are constructible only by breaking the symmetry of one of the two 3-dimensional sets, which is then used as the structural basis on which to arrange the other five units, for example:

Time	Space	Mass	Charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	
$\boldsymbol{k}$	$\boldsymbol{i}$	$\boldsymbol{j}$	

This process creates entirely new physical concepts at the same time as breaking the symmetry between the three components of charge, giving the respective weak, strong and electric components pseudoscalar (or trivector), vector and scalar properties:

Energy	Momentum	Rest Mass
$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$
weak charge	strong charge	electric charge
$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$
pseudoscalar	vector	scalar

The simplest packaging of mass, time, charge and space also presents us with a packaging of energy (E), momentum ($\mathbf{p} = \mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z$) and rest mass ($m$), with hidden information about charge structure, which is what we describe as the fermionic state. Of course, the scalar values involved with these terms are not determined, and it is possible to select values of E, p and m such that the object which combines these, ($i\boldsymbol{k}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$), is *nilpotent* or squares to 0. This will determine that the next and every subsequent realisation of the alphabet will be zeroed automatically, and thus truncate the series at this level (though also constructing a higher commutative algebra connecting the individual fermionic states, as in conventional Hilbert space [5]). Physics appears to have selected only these values to describe fermionic states.

4 Symmetry

The rewrite structure not only generates objects with the algebraic properties required by the physical parameters mass, time, charge and space, but also predicts their exact physical properties and establishes an astonishingly exact symmetry between them [5]. This symmetry, and, ultimately, all the significant symmetries in physics and mathematics, stem from the need to preserve zero totality by introducing duality. As they emerge from the rewrite structure, mass and time have no anticommutative components and no dimensional structure or discreteness. They are by default continuous and this property is responsible for time's irreversibility and mass's unipolarity. Charge and space, however, have anticommutative components and so are also discrete. The discreteness of charge is responsible for the definition of fermionic states as point-like objects or fundamental particles, and the special natures of the particular charges generated by the broken symmetry in the fermionic nilpotent determine which particle states are possible. The packaging of the four parameters using the discrete quantity charge is responsible for the emergent energy, momentum and rest mass being quantized. The discreteness of space is necessary to its status as the only parameter of measurement, but, because it is a nonconserved quantity, unlike charge, the units are not fixed. Ultimately, this means that space is most effectively represented by the constructible real numbers of nonstandard analysis, rather than the Cantorian real numbers applicable to mass.

A second property groups mass and space as having an algebra with positive norm ('real') and time and charge as having an algebra with negative norm ('imaginary'). A third property emerges from a peculiarity of the algebra required to incorporate all the required units. $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$ has an incomplete quaternion set, $(1, \boldsymbol{i}_3)$, which manifests itself physically as a complex algebra, with a pseudoscalar representing $\boldsymbol{i}_3$. The terms from this set with a pseudoscalar coefficient (time and space) are the ones which, physically, represent nonconserved quantities, whereas those with a scalar coefficient (mass and charge) represent the conserved ones. The incomplete quaternion seems to represent the dynamic aspect of the rewrite system. The realisation of the opposing statuses of mass and charge as conserved quantities, and space and time as nonconserved provides a classic case of physical compulsion generating mathematical structure, for it leads to a description of physical systems in terms of a differential calculus that has no intrinsic mathematical reason for existing. In fact, it leads to two such calculuses, standard and nonstandard, based respectively on limits and infinitesimals, and derived from the separate properties of time as continuous and space as discrete.

At the same time, the 'physical' properties of the fermionic state derive directly from the mathematical structure. Rest mass derives its conserved status from the conserved status of its parent quantities mass and charge, and the conserved nature of rest mass fixes the relationship between energy and momentum (and, consequently, that between time and space) within the nilpotent structure as that specified by relativity. In addition to this, the double Clifford algebra used to define the fermionic nilpotent necessarily requires it to generate a spin ½ angular momentum state, and the complete representation requires a 4-component spinor, which we have previously established must be a 4-component row or column vector with successive terms $(\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)$, $(\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$, $(-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ and $(-\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$ [5,6].

In terms of symmetry, it would seem that the rewrite structure fixes three sets of paired symmetrical relationships between the four parameters, which can be expressed in the form:

mass	scalar coefficient	norm +	commutative components
time	pseudoscalar coefficient	norm –	commutative components
charge	scalar coefficient	norm –	anticommutative components
space	pseudoscalar coefficient	norm +	anticommutative components

In physical terms, this becomes:

mass	conserved	real	nondimensional / continuous
time	nonconserved	imaginary	nondimensional / continuous
charge	conserved	imaginary	dimensional / discrete
space	nonconserved	real	dimensional / discrete

The symmetries in these tables are used directly in quantum, as well as classical, physics, in particular the requirement that a nonconserved aspect of a physical system (a variational property) always presupposes the existence of a conserved quantity (Noether's theorem). Quantum mechanics especially describes how knowledge of the fermionic state is derived from using the variational properties of time and space.

The symmetry incorporated in the tables has been tested to destruction over a period of greater than thirty years, and seems to be exact in every detail and successful in its predictive power [5]. As we have previously shown, the symmetric options are exact opposites, and so can be conveniently described by algebraic symbols, for example:

mass	x	y	z
time	$-x$	$-y$	z
charge	x	$-y$	$-z$
space	$-x$	y	$-z$

Conceptually, as well as algebraically, this represents zero totality (though the signs and symbols are of course arbitrary), and it constitutes a finite noncyclic group of order 4 (D2, Klein-4), in which element is its own inverse. We can generate group multiplication rules of the form:

$$x * x = -x * -x = x$$
$$x * -x = -x * x = -x$$
$$x * y = y * -x = 0$$

and similarly for y and z, to establish a group multiplication table of the form:

*	mass	charge	time	space
mass	mass	charge	time	space
charge	time	mass	space	charge
time	charge	space	mass	time
space	space	time	charge	mass

In this structure, mass has been selected as the identity element, reflecting its status as the first term to emerge from the rewrite structure, but, in mathematical terms, we could equally well have privileged charge, time or space. Many other representations of the symmetry are possible, including one using the H4 algebra (which is effectively equivalent to quaternions without + and – signs), with mass, charge, time, and space, respectively represented by the units 1, **i**, **j**, **k**.

*	1	**i**	**j**	**k**
1	1	**i**	**j**	**k**
i	**j**	1	**k**	**i**
j	**i**	**k**	1	**j**
k	**k**	**j**	**i**	1

We can also postulate a dual group, in which the real / imaginary (norm + / norm –) property is reversed, with components derived from the nilpotent structure. Here the mass dual becomes ***j****m*); the time dual becomes *i**k**E*; and the space dual ***i*****p**; with the charge dual as the (spin) angular momentum. Another duality which emerges is that between the quaternion (***i***, ***j***, ***k***) and vector (**i**, **j**, **k**) bases. It is this duality which allows us (in, for example, the holographic principle, or the derivation of the velocity addition law in special relativity) to switch between two orthogonal spatial units defining an area (say, **i**, **j**) and the quaternionic orthogonality defined by space and time. Both representations can be used to define the angular momentum associated with the fermionic state.

The symmetries between the parameters show that all the properties can be conjugated in one cycle of the algebra, which seemingly imparts a special meaning to an algebra of 8 units (1, *i*, ***i***, ***j***, ***k***, **i**, **i**, **k**), as is apparent in the mathematics. The algebras of the parameters are each subalgebras of a single Clifford algebra of this dimension, but because these subalgebras are commutative, the combination becomes equivalent to a double Clifford algebra, which, in mathematical terms, seems to allow the creation of an 8-component cycle on which it can map. Mathematics does this by creating another property, antiassociativity, which allows us to define an 8-unit algebra, octonions, on which we can map the units of mass, time, charge and space, giving the impression that this mathematical structure arises from a physical imperative. Significantly, if we retain the identities of the parameters in the mapping, the antiassociative aspects remain in the areas which have no physical meaning.

5 The Nilpotent Quantum Vacuum

Mathematics and fundamental physics both emerge from the universal rewrite system, but the special importance of the physical process is that it is compelled to be semantic as well as syntactic. The symmetry between the four parameters has an exactness which stems from the fact that it requires a zero conceptual, as well as zero physical, totality. The nilpotent quantum mechanics is only valid when taken over the whole universe seen by the fermionic state. Essentially, if a fermionic state represented by $(\pm ikE \pm \mathbf{ip} + jm)$ is created from *absolutely nothing*, then it leaves a 'hole' in the nothing represented by $-(\pm ikE \pm \mathbf{ip} + jm)$, which we may describe as 'vacuum'. The 'vacuum' is, in fact, the 'rest of the universe' as seen by the fermion – that is, the rest of the universe must be created at exactly the same time as the fermion, and it must be such as would allow that fermion in that state to exist. The superposition and the combination of vacuum and fermion must both be 0, both to maintain zero totality and at the same time extend it to all further realisations of the alphabet, i.e.

$$(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) - (\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$
$$-(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$

In this formalism, Pauli exclusion is an expression of nilpotency; every fermionic state, and every corresponding state of the 'rest of the universe', becomes an event in a unique birthordering. Locality is defined within the bracket $(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ and nonlocality outside it. Since fermions are only experienced through variations of spatial coordinates in time, the fermion can be defined by an operator, involving space and time variation, that acts on the rest of the universe to define the object represented by $(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, and can include any number of covariant or field terms to represent the external interactions. The operator finds, uniquely, the actual space and time variations that preserve the conserved quantities (the phase factor) and acts on it to produce the only amplitude that will square to zero. There is no need for a separate equation, and even the usual rest mass term can be eliminated, making the theory one of pure space and time variation. The full theory has been described in previous publications [5.6], and has many special features, including exact supersymmetry between fermions and *their own* vacuum bosons, and bosons and their own vacuum fermions, but the main point of interest here is that the 3 additional components of the spinor structure after the defining term effectively represent the vacuum 'reflections' of the fermion as seen through the weak, strong and electric interactions. The idempotent expressions $\boldsymbol{k}(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, $\boldsymbol{i}\,(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ and $\boldsymbol{j}(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ here represent the effective weak, strong and electric vacua, which act as discrete partitions of the continuous vacuum that represents the 'rest of the universe'. In addition the weak, strong and electric interactions are contained within the structure of the nilpotent operator itself. Only the gravitational interaction seems to exist outside the nilpotent structure, the reflection in the gravitational vacuum, $1(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, apparently being equivalent to the fermion's reflection in *the rest of the universe,* which is continuous by definition. This special nature of the gravitational interaction (which incorporates the gauge theory /

gravity correspondence associated mainly with string theory) can be understood through the origin of mass in the universal rewrite structure.

6 Gravity and the Nature of Mass

The universal rewrite system shows that the 'universe' cannot evolve separately from the laws of physics. The laws of physics are not emergent properties of an evolving physical universe which change with time, but an emergent aspect of an abstract zero totality system from which time itself is an emergent component. The fundamental physical phenomena of cosmological redshift and cosmic microwave background radiation must be part of the unchangeable laws of physics, not an accident of initial conditions which could have led to a different outcome. They may conceivably determine the initial conditions or the cosmological evolution (and some discussion of this has taken place in terms of the rewrite mechanism [5,7]), but they are not accidental products of it.

Mass, as derived from the universal rewrite system, is a continuous quantity. This is precisely what we know from such manifestations as the Higgs field, the zero-point energy, the cosmic microwave background radiation, even ordinary fields. It is also conserved, acting as a completely fixed (though unobserved) physical standard. Unlike charge, it has no negative manifestation, and, again unlike charge (which should be regarded as a discrete quantum number rather than as a measurable quantity), it does not change its effective value under different conditions. Rest mass, of course, can be created or destroyed, but this is only one manifestation of mass, and no particle's mass is completely determined by it. There is good reason to suspect, in fact, that the amount of mass is the same at every point in space (perhaps the expectation value of the Higgs field) and that it changes only in its manifestation because of the presence of charges of each kind.

In the nilpotent theory, the vacuum associated with mass would appear to be of the form $1(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, which is, in fact, equivalent to the 'rest of the universe'. In other words, mass is, in principle, nonlocal, and we should expect the same for gravitation. Gravity has all the characteristics required of the carrier of nonlocal quantum correlations. It may have the $U(1)$ symmetry characteristic of a scalar gauge coupling, but this does not require it to be quantized. Apart from the fact that a successful theory of 'quantum gravity' is not yet forthcoming, the value of cosmological constant that it predicts is 10^{123} times too high to fit experimental data! If we assume that there are 10^{123} possible bits of data in the observable universe [8,9], then the prediction is about as wrong as it could possibly be, and the reason can be pinpointed immediately. If the process occurs with respect to the universe as a whole (nonlocal), rather than to one quantum state (local), then the answer would be correct. In fact a correct prediction of dark energy was made two decades before the experimental discovery exactly on this basis [5,10-12].

Now, we never really observe the action of gravitational force, but rather the inertial reaction to it from discrete matter. There is every reason to believe that the inertial reaction is local, and can be quantized. It is, however, a fictitious force, dependent on observation within a noninertial frame of reference. Many efforts have been made to explain it in more fundamental terms using Mach's principle (which would, incidentally, numerically cancel the inertial mass-energy of the universe with the negative potential energy of gravity, so fulfilling a requirement of the first stage of the rewrite structure if the gravity is a vacuum or nonlocal effect). It is, in fact, possible to integrate such concepts into a coherent structure based on the idea that, while gravity is 'real' but nonlocal, inertia is fictitious and local.

Essentially, we assume that gravity is instantaneous and nonlocal, but that physical observation of any kind requires local, time-delayed interaction involving the speed of light. In this case, the Lorentzian space-time used for local coordinate systems would be inappropriate for gravity and would effectively create a noninertial frame for the gravitating system, manifesting itself in the form of fictitious inertial forces. Relativistic effects would be observed (including general relativistic ones) but they would be effects due to the choice of coordinate system rather than ones intrinsic to gravity, epistemological rather than ontological. Gravity would appear to have a 'magnetic' inertial component and an acceleration-dependent inductive force analogous to that which occurs in electromagnetic theory, say:

$$F = \frac{G}{c^2 r} m_1 m_2 \sin\theta \frac{dv}{dt}.$$

Now, in 1953, Sciama [13] considered the possibility of explaining Mach's principle using such an inductive force (though in his model it was a real one). In this theory, the inertia of a body of mass $m = m_1$ would be attributed to the action of the total mass $m_u = m^2$ within a c-defined event horizon, specified by radius m_u, so making the inductive force equation equivalent to $F = Kma$, with K a constant. But, let us also suppose that the continuous mass-field required by the universal rewrite mechanism provides a standard by which we can define a unit inertial mass nonlocally for the entire universe, in the same way as the near-constant gravitational field **g** provides a way of defining a unit mass at the Earth's surface. We imagine that mass m_u defines a radial inertial field of constant magnitude from the centre of a local coordinate system, and, at the same time, use the principle of equivalence, to equate this to the gravitational field (Gm_u / r_u^2), which, independently of the local coordinate system, defines a unit of gravitational mass within the same event horizon. If we use isotropy to remove the angular dependence, we obtain:

$$\frac{Gm_u}{c^2 r}\frac{dv}{dt} = \frac{Gm_u}{r_u^2}.$$

The important thing here is that we obtain an acceleration

$$a = \frac{dv}{dt} = v\frac{dv}{dr} = \frac{c^2 r}{r_u^2}$$

which can be integrated with respect to r, to give

$$v = \frac{cr}{r_u} = H_0 r$$

where H_0 is the Hubble constant, and the acceleration is now

$$a = \frac{v^2}{r_u} = H_0^2 r.$$

If gravity is nonlocal, then this acceleration is a fictitious one, which describes the effects on the coordinate system produced by using a Lorentzian space-time to model the instantaneous interaction. To calculate the equivalent vacuum density, we combine it with the gravitational force due to total mass m at any distance, to give

$$F = \frac{G}{r^2} - H_0^2 r = \left(\frac{4}{3}\pi G\rho - H_0^2\right) r.$$

The equivalent Poisson-Laplace equation becomes:

$$\nabla^2\phi = 4\pi G\left(\rho - \frac{3H_0^2}{4\pi G}\right) = 4\pi G\left(\rho + \frac{3P}{c^2}\right) = 4\pi G\left(\rho - 3\rho_{vac}\right),$$

where the vacuum density is

$$\rho_{vac} = \frac{H_0^2}{4\pi G},$$

which is equivalent to a 'dark' energy density or negative pressure

$$-P = \frac{H_0^2 c^2}{4\pi G},$$

and cosmological constant

$$\lambda = 8\pi G\rho_{vac} = 2H_0^2.$$

If we define the critical density for a 'flat' universe as

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

we find that

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{2}{3},$$

which is within the limits of current observations, though it was obtained twenty years before the first measurements [14-15].

This calculation suggests that the so-called 'dark energy' is analogous to a centrifugal force, and a corresponding 'Coriolis' effect might be considered for rotating systems. In addition, an argument can be made for suggesting that the cosmic background radiation might be a vacuum effect generated by the radiation produced by the inertial acceleration [5]. Such explanations are, at least in intent, physical in origin, whatever their cosmological consequences.

Another way of looking at the problem is to observe that the local space-time of the Dirac equation is naturally skewed because it is describing a singularity. The skewing is manifested in the spin, which comes from the vector properties of **p** (i.e. space), which is the object in the nilpotent that retains its own 3-D integrity (the variable one in physics) when everything else is mapped onto the other one (the conserved 3-D or charge). The double 3-D structure can be connected with the Berry phase of π which results from the fermion singularity defining its own multiply connected space, and hence requiring a spin ½ state [5]. (Berry phase [16] generally defines the connection between the two commutative 3-dimensionalities in a system.) There is also a generic connection with the Penrose twistor, defined by a complex 4-dimensional space, though that space lacks the intrinsic double 3-D substructure of the fermionic nilpotent, and finds its definition in terms of the massless photon rather than the massive fermion [17,18]. Gravity, however, does not require the twist because, as a nonlocal effect, it does not recognise charge and singularity. It also recognises only one 3-dimensionality, whereas a combination of two is required to produce spin. But if the twist is already in the space we are using for observation, then we need to reverse the effect, to project back from to an unskewed space, and so generate centrifugal- and Coriolis-type forces, presumably centred on a chargelike pseudosinguarlity. (This may connect with the discussion of space-time torsion and its effects, particularly in gravitating systems, by Haramein and Rauscher [19].)

A discussion of rotation produced by fictitious forces links in with the idea of curvature in general relativity. Here, it is relevant to note that the affine connection uses $64 = 4 \times 4 \times 4$ indices, like the Dirac nilpotent, but once again some of the information (the last factor 4) is redundant because of the use of a zero totality [20]. The affine connection comes about because we are using space, the last physical construct of the rewrite system (after mass, time and charge), and carrying with it the properties it inherits from being the *second* 3-D structure – even though we don't actually need the first (charge) – because physically it is defined through the first. So, while the Dirac equation only has a single redundancy, due to nilpotency, because the spin ½ is not redundant, the affine connection has a double redundancy because of the combined effect of no second 3-D term (and hence no intrinsic spin) and zero totality. Mathematically, we can say that, in nonlocal gravity, space and time are commutative with each other; but in the Dirac equation, they are not, because of the charge quaternion (or γ matrix) connection, and this is what introduces both the spin and the relativistic connection in the nilpotent structure.

7 Conclusion

Any fundamental explanation of physics must also explain the origin of mathematics and the symbiotic relationship between the two subjects. In fact, both can be seen as emergent aspects of a universal rewrite system, based on the idea of zero totality, and the difference between them can be attributed to the fact that physics has to apply semantic, as well as syntactic, reasoning, in effect to explain everything at once. The zero-totality condition ensures that symmetry is built into both subjects from the beginning, and a particular symmetry is seen to operate in physics at the fundamental level and be responsible for many of the most significant physical facts. Quantum mechanics emerges in a very specific form which enables us to understand, among other things, the meaning of vacuum and the origin of symmetry breaking. The uniqueness of gravity is emphasized through the way that mass originates, and this leads us to significant predictive consequences.

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