Anticipatory Adaptive Control of Inspection Planning Process in Service of Fatigued Aircraft Structures

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Abstract
In this paper, a control theory is used for planning inspections in service of fatigue-sensitive aircraft structure components under crack propagation. One of the most important features of control theory is its great generality, enabling one to analyze diverse systems within one unified framework. A key idea, which has emerged from this study, is the necessity of viewing the process of planning in-service inspections as an adaptive control process. Adaptation means the ability of self-modification and self-adjustment in accordance with varying conditions of environment. The adaptive control of inspection planning process in service of fatigued aircraft structures differs from ordinary stochastic control of inspection planning process in that it attempts to reevaluate itself in the light of uncertainties in service of aircraft structures as they unfold and change. Thus, a catastrophic accident during flight can be avoided.
Keywords: Aircraft, fatigue crack, inspection planning, anticipatory adaptive control.

1 Introduction

In spite of decades of investigation, fatigue response of materials is yet to be fully understood. This is partially due to the complexity of loading at which two or more loading axes fluctuate with time. Examples of structures experiencing such complex loadings are automobile, aircraft, off-shores, railways and nuclear plants. Fluctuations of stress and/or strains are difficult to avoid in many practical engineering situations and are very important in design against fatigue failure. There is a worldwide need to rehabilitate civil infrastructure. New materials and methods are being broadly investigated to alleviate current problems and provide better and more reliable future services.

While most industrial failures involve fatigue, the assessment of the fatigue reliability of industrial components being subjected to various dynamic loading situations is one of the most difficult engineering problems. This is because material degradation processes due to fatigue depend upon material characteristics, component geometry, loading history and environmental conditions.

According to many experimental results and field data, even in well-controlled laboratory conditions under constant amplitude loading, crack growth results usually show a considerable statistical variability.

Fatigue is one of the most important problems of aircraft arising from their nature as multiple-component structures, subjected to random dynamic loads. The analysis of fatigue crack growth is one of the most important tasks in the design and life prediction of aircraft fatigue-sensitive structures (for instance, wing, fuselage) and their components (for instance, aileron or balancing flap as part of the wing panel, stringer, etc.).

2 Stochastic Modelling

To capture the statistical nature of fatigue crack growth, different stochastic models have been proposed in the literature. Some of the models are purely based on direct curve fitting of the random crack growth data, including their mean value and standard deviation (Bogdanoff and Kozin [1]). These models, however, have been criticized by other researchers, because less crack growth mechanisms have been included in them. To overcome this difficulty, many probabilistic models adopted the crack growth equations proposed by fatigue experimentalists, and randomized the equations by including random factors into them (Lin and Yang [2]; Yang et al. [3]; Yang and Manning [4]; Nechval et al. [5-7]; Straub and Faber [8]). The random factor may be a random variable, a random process of time, or a random process of space. It then creates a random differential equation. The solution of the differential equation reveals the probabilistic nature as well as the scatter phenomenon of the fatigue crack growth. To justify the applicability of the probabilistic models mentioned above, fatigue crack growth data are needed. However, it is rather time-consuming to carry out experiments to obtain a set of statistical meaningful fatigue crack growth data. To the writers' knowledge, there are only a few data sets available so far for researchers to verify their probabilistic models. Among them, the most famous data set perhaps is the one produced by Virkler et al. [9] more than twenty years ago. More frequently used data sets include one reported by Ghonem and Dore [10]. Itagaki and his associates have also produced some statistically meaningful fatigue crack growth data, but have not been mentioned very often (Itagaki et al. [11]). In fact, many probabilistic fatigue crack growth models are either lack of experimental verification or just verified by only one of the above data sets. It is suspected that a model may explain a data set well but fail to explain another data set. The universal applicability of many probabilistic models still needs to be checked carefully by other available data sets.

Many probabilistic models of fatigue crack growth are based on the deterministic crack growth equations. The most well known equation is

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = q(a(t))^b \tag{1}$$

in which q and b are constants to be evaluated from the crack growth observations. The independent variable t can be interpreted as either stress cycles, flight hours, or flights depending on the applications. It is noted that the power-law form of $q(a(t))^b$ at the right hand side of (1) can be used to fit some fatigue crack growth data appropriately and is also compatible with the concept of Paris–Erdogan law [12]. The service time for a crack to grow from size $a(t_0)$ to $a(t)$ (where $t > t_0$) can be found by performing the necessary integration

$$\int_{t_0}^{t} dt = \int_{a(t_0)}^{a(t)} \frac{dv}{qv^b} \tag{2}$$

to obtain

$$t - t_0 = \frac{[a(t_0)]^{-(b-1)} - [a(t)]^{-(b-1)}}{q(b-1)}. \tag{3}$$

For the particular case (when b=1), it can be shown, using Lopital's rule, that

$$t - t_0 = \frac{\ln[a(t)/a(t_0)]}{q}. \tag{4}$$

Thus, we have obtained the Exponential model

$$a(t) = a(t_0)e^{q(t-t_0)}. \tag{5}$$

The Exponential model is quite often used for calculation of growth of population/bacteria etc. The basic equation of it is

$$P_t = P_0 e^{rt}. \tag{6}$$

Rewrite (4) as

$$\tau_{j+1} - \tau_j = \frac{\ln[a(\tau_{j+1})/a(\tau_j)]}{q}, \quad j=0, 1, \ldots . \tag{7}$$

where τ_j is the time of the jth in-service inspection of the aircraft structure component, $a(\tau_j)$ is the fatigue crack size detected in the component at the jth inspection.

It is assumed, in this paper, that the parameter q is a random variable, i.e. $q \equiv Q$, which can take values within a finite set $\{q^{(1)}, q^{(2)}, \ldots, q^{(r)}\}$. However, in order to simplify the computation, at first we consider the case when only two values are chosen. Assume that, at any sampling time instant, the random parameter Q takes on two values, $q^{(1)}$ and $q^{(2)}$, with probabilities p and 1-p, respectively, and that the value of the probability p is not known. It takes on two values p_1 and p_2 with a priori probability ξ and $1-\xi$, respectively. Now (7) can be rewritten as

$$x_{j+1} = x_j + Qu_j, \quad j=0, 1, \ldots , \tag{8}$$

where

$$x_j = \ln[a(\tau_j)], \tag{9}$$

$$u_j = \tau_{j+1} - \tau_j \tag{10}$$

represents the interval between the jth and (j+1)th inspections.

3 Terminal-Control Problem

Let us suppose that a fatigue-sensitive component such as, say, upper longeron [13] (Fig. 1) has been found cracked on one aircraft at the time τ_0.

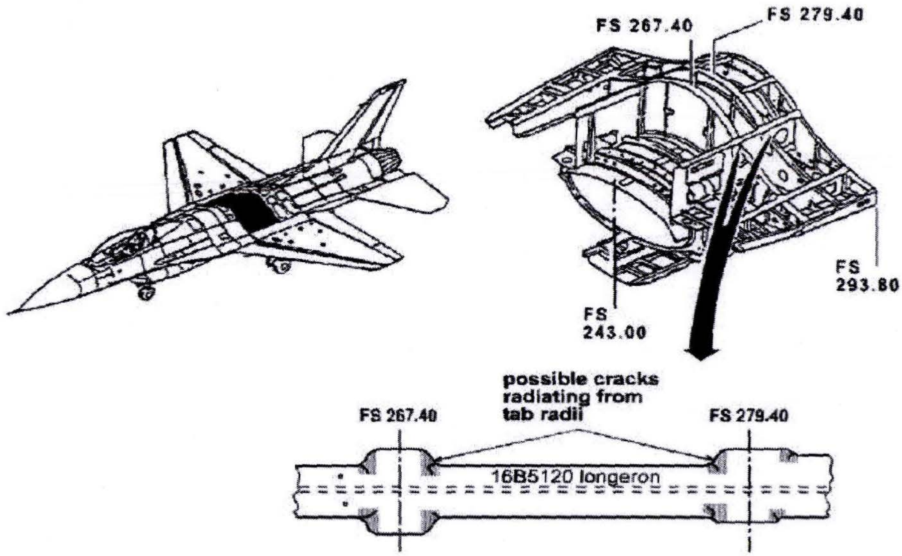

Fig. 1: Inspection points of the upper longeron of RNLAF F-16 aircraft.

The detectable crack length is $a_0=a(\tau_0)$. The maximum allowable crack length is $a^{\bullet}$ =4.75 mm (Fig. 2).

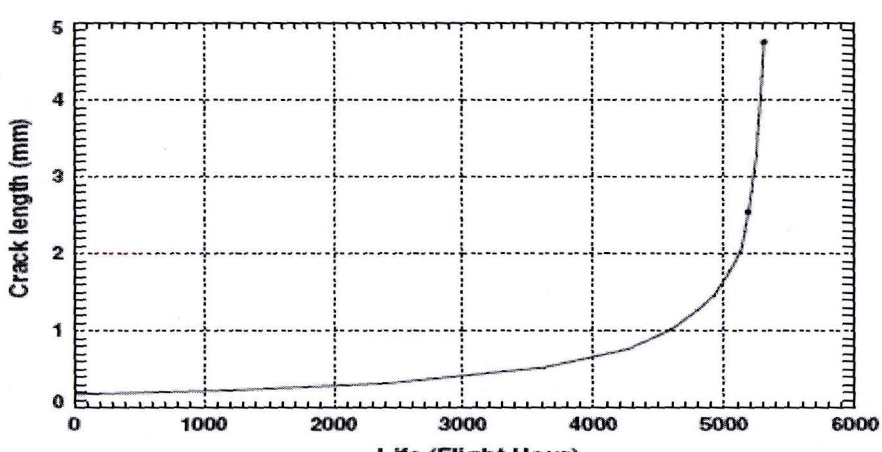

Fig. 2: RNLAF longeron mean crack growth curve. Functional impairment at 5310 flight hours. (assumed initial crack = 0.178 mm; critical crack length = 4.75 mm.)

We plan to carry out N in-service inspections of the above component and are in need to assign intervals, u_0, u_1, ..., u_{N-1}, between sequential inspections so that the performance index

$$I_N = E\{(x^{\bullet} - x_N)^2\}, \tag{11}$$

where $x^{\bullet} = \ln(a^{\bullet})$, is minimized.

4 Optimal Anticipatory Adaptive Inspection Planning Process

The design is initiated with the determination of the a posteriori probabilities

$$\xi_{1q^{(1)}} = \Pr\{p = p_1 | Q = q^{(1)}\} \tag{12}$$

and

$$\xi_{1q^{(2)}} = \Pr\{p = p_1 | Q = q^{(2)}\}. \tag{13}$$

By the Bayes theorem, it is found that

$$\xi_{1q^{(1)}} = \frac{\Pr\{p = p_1\}\Pr\{Q = q^{(1)} \mid p = p_1\}}{\Pr\{p = p_1\}\Pr\{Q = q^{(1)} \mid p = p_1\} + \Pr\{p = p_2\}\Pr\{Q = q^{(1)} \mid p = p_2\}\}}$$

$$= \frac{\xi\, p_1}{\xi\, p_1 + (1-\xi)p_2} \tag{14}$$

and

$$\xi_{1q^{(2)}} = \frac{\Pr\{p = p_1 \Pr\{Q = q^{(2)} \mid p = p_1\}}{\Pr\{p = p_1\}\Pr\{Q = q^{(2)} \mid p = p_1\} + \Pr\{p = p_2\}\Pr\{Q = q^{(2)} \mid p = p_2\}\}}$$

$$= \frac{\xi(1-p_1)}{\xi(1-p_1) + (1-\xi)(1-p_2)}. \tag{15}$$

Let the minimum of I_N be denoted by $f_N(x_0,\xi)$, where

$$x_0 = \ln(a_0), \tag{16}$$

a_0 is the initial crack length detected in the component. The minimum of I_N is a function of x_0 and the a priori probability ξ, and is given by

$$f_N(x_0,\xi) = \min_{(u_0, u_1, \ldots, u_{N-1})} E\{(x^{\bullet} - x_N)^2\}. \tag{17}$$

At any sampling instant $j + 1$, x_{j+1} takes on two values:

$$x_{j+1}^{(1)} = x_j + q^{(1)}u_j \tag{18}$$

and

$$x_{j+1}^{(2)} = x_j + q^{(2)}u_j. \tag{19}$$

For $j = 0$, x_1 takes on the value

$$x_1^{(1)} = x_0 + q^{(1)}u_0 \quad \text{with probability } p_0 \tag{20}$$

and the value

$$x_1^{(2)} = x_0 + q^{(2)} u_0 \quad \text{with probability } 1-p_0, \tag{21}$$

where p_0 is the expected value of p and is given by

$$p_0 = \xi\, p_1 + (1-\xi) p_2 . \tag{22}$$

Hence, for N=1,

$$f_1(x_0,\xi) = \min_{u_0} E\{(x^\bullet - x_1)^2\} = \min_{u_0} \left(p_0 (x^\bullet - x_1^{(1)})^2 + (1-p_0)(x^\bullet - x_1^{(2)})^2 \right). \tag{23}$$

For $N \geq 2$, invoking the principle of optimality yields

$$f_N(x_0,\xi) = \min_{u_0} \left(p_0 f_{N-1}(x_1^{(1)}, \xi_{1q^{(1)}}) + (1-p_0) f_{N-1}(x_1^{(2)}, \xi_{1q^{(2)}}) \right), \tag{24}$$

where $\xi_{1q^{(1)}}$, $\xi_{1q^{(2)}}$, $x_1^{(1)}$, and $x_1^{(2)}$ are defined in (14), (15), (20), and (21), respectively. As a result of the first decision, the process will be transformed to one of the two possible states $x_1^{(1)}$ or $x_1^{(2)}$ with probability p_0 or $1-p_0$. If the process moves to state $x^{(1)}$, the a posteriori probability $\xi_{1q^{(1)}}$ is computed. If the process moves to state $x^{(2)}$, the a posteriori probability $\xi_{1q^{(2)}}$ is determined.

In a one-stage process, the optimum decision is found by differentiating (23) with respect to u_0 and equating the partial derivative to zero. This leads to

$$p_0 q^{(1)} (x^\bullet - x_0 - q^{(1)} u_0) + (1-p_0) q^{(2)} (x^\bullet - x_0 - q^{(2)} u_0) = 0. \tag{25}$$

Hence

$$u_0 = \frac{E\{Q\}}{E\{Q^2\}} (x^\bullet - x_0), \tag{26}$$

where

$$E\{Q\} = p_0 q^{(1)} + (1-p_0) q^{(2)} \tag{27}$$

and

$$E\{Q^2\} = p_0 [q^{(1)}]^2 + (1-p_0)[q^{(2)}]^2 \tag{28}$$

are functions of ξ. By defining

$$E_i\{Q\} = p_i q^{(1)} + (1-p_i) q^{(2)}, \quad i=1, 2, \tag{29}$$

it can readily be shown that $E\{Q\}$ can be written as

$$E\{Q\} = \xi\, E_1\{Q\} + (1-\xi) E_2\{Q\}. \tag{30}$$

Similarly, by defining

$$E_i\{Q^2\} = p_i [q^{(1)}]^2 + (1-p_i)[q^{(2)}]^2, \quad i=1, 2, \tag{31}$$

$E\{Q^2\}$ can be expressed in terms of ξ as

$$E\{Q^2\} = \xi\, E_1\{Q^2\} + (1-\xi) E_2\{Q^2\}. \tag{32}$$

The minimum for the one-stage process is given by

$$f_1(x_0,\xi) = G_1(\xi)(x^\bullet - x_0)^2, \tag{33}$$

where

$$G_1(\xi) = 1 - \frac{E^2\{Q\}}{E\{Q^2\}}. \tag{34}$$

By defining

$$h_0(\xi) = \frac{E\{Q\}}{E\{Q^2\}} \tag{35}$$

the optimum decision u_0 may be written as

$$u_0 = h_0(\xi)(x^\bullet - x_0). \tag{36}$$

It can be shown by mathematical induction that

$$f_k(x_0,\xi) = G_k(\xi)(x^\bullet - x_0)^2. \tag{37}$$

In view of (37),

$$f_k(x_1^{(1)},\xi_{1q^{(1)}}) = G_k(\xi_{1q^{(1)}})(x^\bullet - x_0 - q^{(1)}u_0)^2, \tag{38}$$

$$f_k(x_1^{(2)},\xi_{1q^{(2)}}) = G_k(\xi_{1q^{(2)}})(x^\bullet - x_0 - q^{(2)}u_0)^2. \tag{39}$$

The minimum for a $(k+1)$-stage process is

$$f_{k+1}(x_0,\xi) = \min_{u_0}\left(p_0 G_k(\xi_{1q^{(1)}})(x^\bullet - x_0 - q^{(1)}u_0)^2 + (1-p_0)G_k(\xi_{1q^{(2)}})(x^\bullet - x_0 - q^{(2)}u_0)^2\right),$$

$$k = 1, 2, \ldots, N-1. \tag{40}$$

From this recurrence relationship it is found that the optimum decision is given by

$$u_0 = h_k(\xi)(x^\bullet - x_0), \tag{41}$$

where

$$h_k(\xi) = \frac{E\{QG_k(\xi_{1Q})\}}{E\{Q^2G_k(\xi_{1Q})\}}, \tag{42}$$

$$E\{QG_k(\xi_{1Q})\} = \xi\, E_1\{QG_k(\xi_{1Q})\} + (1-\xi)E_2\{QG_k(\xi_{1Q})\}, \tag{43}$$

$$E_i\{QG_k(\xi_{1Q})\} = p_i\, q^{(1)}G_k(\xi_{1q^{(1)}})\} + (1-p_i)q^{(2)}G_k(\xi_{1q^{(2)}})\},\ i=1, 2, \tag{44}$$

$$E\{Q^2G_k(\xi_{1Q})\} = \xi\, E_1\{Q^2G_k(\xi_{1Q})\} + (1-\xi)E_2\{Q^2G_k(\xi_{1Q})\}, \tag{45}$$

$$E_i\{Q^2G_k(\xi_{1Q})\} = p_i\,[q^{(1)}]^2G_k(\xi_{1q^{(1)}})\} + (1-p_i)[q^{(2)}]^2G_k(\xi_{1q^{(2)}})\},\ i=1, 2. \tag{46}$$

From (40) and (41) it follows that

$$f_{k+1}(x_0,\xi) = G_{k+1}(\xi)(x^\bullet - x_0)^2, \tag{47}$$

where

$$G_{k+1}(\xi)=E\{G_k(\xi_{1Q})\}-\frac{E^2\{QG_k(\xi_{1Q})\}}{E\{Q^2G_k(\xi_{1Q})\}}, \tag{48}$$

$$E\{G_k(\xi_{1Q})\}=\xi\, E_1\{G_k(\xi_{1Q})\}+(1-\xi)E_2\{G_k(\xi_{1Q})\}, \tag{49}$$

$$E_i\{G_k(\xi_{1Q})\}=p_iG_k(\xi_{1q^{(1)}})\}+(1-p_i)G_k(\xi_{1q^{(2)}})\},\quad i=1,2\,, \tag{50}$$

Equations (33), (34), (47), and (48) are recurrence relationships with which it is possible to evaluate the minimum for an N-stage process $f_N(x_0,\xi)$.

With the initial state x_0 and initial information ξ, the first optimum decision is

$$u_0=h_{N-1}(\xi)(x^{\bullet}-x_0), \tag{51}$$

where $h_{N-1}(\xi)$ is evaluated from (42) to (46) and (48) to (50), with $k=N-1$. The second optimum decision should be made after observation of the random variable Q in the first decision stage. If it is observed that $Q=q^{(1)}$, the a posteriori probability $\xi_{1q^{(1)}}$ and the new state

$$x_1^{(1)}=x_0+q^{(1)}u_0 \tag{52}$$

are used as the initial information for the remaining $N-1$ stages. The second optimum decision can be determined in similar manner and is given by

$$u_1=h_{N-2}(\xi_{1;q^{(1)}})(x^{\bullet}-x_1^{(1)})\,. \tag{53}$$

If the observed value of Q after the first decision is $q^{(2)}$, the a posteriori probability $\xi_{1q^{(2)}}$ and the new state

$$x_1^{(2)}=x_0+q^{(2)}u_0 \tag{54}$$

are used as the initial information and the initial state for the remaining $N-1$ stages. The second optimum decision is then given by

$$u_1=h_{N-2}(\xi_{1q^{(2)}})(x^{\bullet}-x_1^{(2)})\,. \tag{55}$$

Thus, after the first inspection, the computer must calculate the a posteriori probability $\xi_{1q^{(1)}}$ or $\xi_{2q^{(2)}}$, the new state x_1 and the second optimum decision u_1.

If the observed value of Q after the second decision is $q^{(1)}$, the a posteriori probability $\xi_{2q^{(1)}}$ and the new state

$$x_2^{(1)}=x_1+q^{(1)}u_1 \tag{56}$$

are used as the initial information and the initial state for the remaining $N-2$ stages, in particular, to determine the third optimum decision

$$u_2=h_{N-3}(\xi_{2q^{(1)}})(x^{\bullet}-x_2^{(1)})\,. \tag{57}$$

In (56), if $x_1=x_1^{(1)}$, then a posteriori probability is $\xi_{1q^{(1)}}$ and u_1 is given by (53); and if $x_1=x_1^{(2)}$, then a posteriori probability is $\xi_{1q^{(2)}}$ and u_1 is given by (55).

If the observed value of Q after the second decision is $q^{(2)}$, the a posteriori probability $\xi_{2q^{(2)}}$ and the new state

$$x_2^{(2)} = x_1 + q^{(2)} u_1 \tag{58}$$

are used to determine the third optimum decision, which is

$$u_2 = h_{N-3}(\xi_{2q^{(2)}})(x^{\bullet} - x_2^{(2)}). \tag{59}$$

By repeated observation and computation in the above manner, the optimum-inspection policy (u_0, ..., u_{N-1}) for the fatigue-sensitive component, which has been found cracked on one aircraft at the time τ_0, can be determined.

Each new optimum decision is made by using new information resulting from the observation of the random variable Q.

It will be noted that if the probability p is assumed to be known, then the minimum of (11) can be found as follows. Let the minimum of (11) be

$$f_N(x_0) = \min_{(u_0, u_1, \dots, u_{N-1})} E\{(x^{\bullet} - x_N)^2\}. \tag{60}$$

For N=1,

$$f_1(x_0) = \min_{u_0} E\{(x^{\bullet} - x_1)^2\} = \min_{u_0} \left(p(x^{\bullet} - x_1^{(1)})^2 + (1-p)(x^{\bullet} - x_1^{(2)})^2 \right)$$

$$= \left(1 - \frac{E^2\{Q\}}{E\{Q^2\}}\right)(x^{\bullet} - x_0)^2 \tag{61}$$

with

$$u_0 = \frac{E\{Q\}}{E\{Q^2\}}(x^{\bullet} - x_0), \tag{62}$$

where

$$E\{Q\} = pq^{(1)} + (1-p)q^{(2)} \tag{63}$$

and

$$E\{Q^2\} = p[q^{(1)}]^2 + (1-p)[q^{(2)}]^2 \tag{64}$$

are functions of p.

For N≥2, the minimum of (11) is

$$f_N(x_0) = \min_{u_0} \left(pf_{N-1}(x_1^{(1)}) + (1-p)f_{N-1}(x_1^{(2)}) \right) = \left(1 - \frac{E^2\{Q\}}{E\{Q^2\}}\right)^N (x^{\bullet} - x_0)^2 \tag{65}$$

with u_0 given by (62).

For an illustration, one of the versions (for N=5) of adaptive minimizing the expected value of the performance index (11) for the upper longeron of RNLAF F-16 aircraft is plotted in Fig. 3.

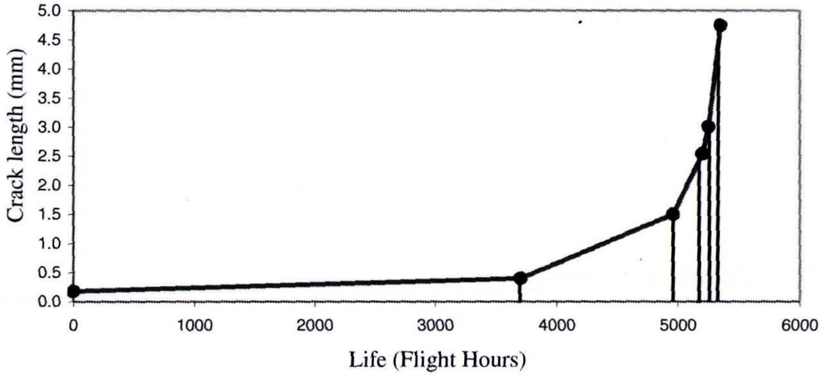

Fig. 3: Inspection schedule version for the upper longeron of RNLAF F-16 aircraft.

Fig.4 shows the deterministic inspection requirements for the RNLAF longerons [14].

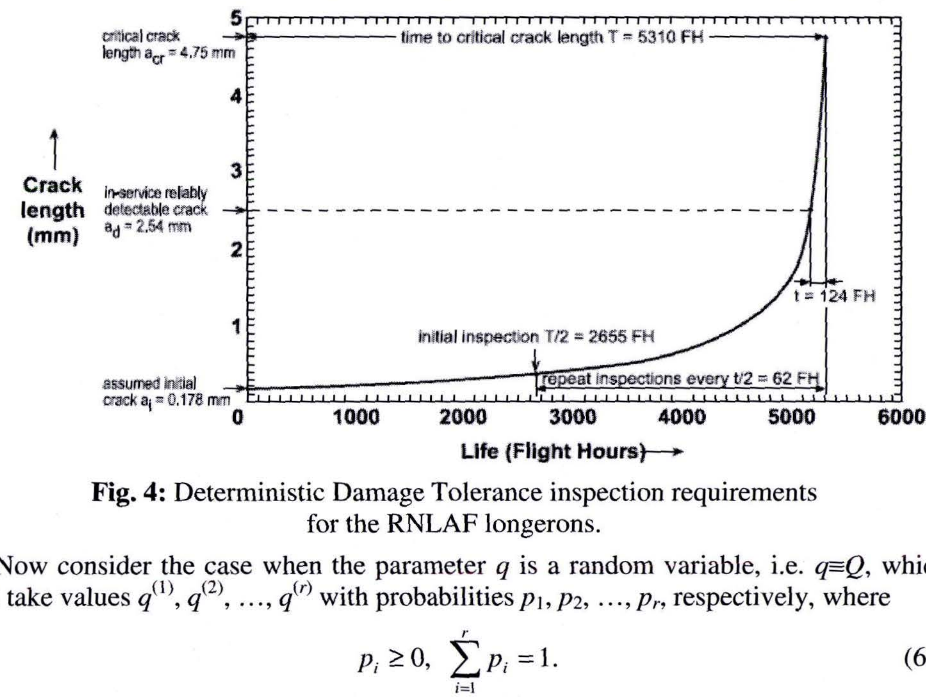

Fig. 4: Deterministic Damage Tolerance inspection requirements for the RNLAF longerons.

Now consider the case when the parameter q is a random variable, i.e. $q \equiv Q$, which can take values $q^{(1)}, q^{(2)}, \ldots, q^{(r)}$ with probabilities $p_1, p_2, \ldots, p_r$, respectively, where

$$p_i \geq 0, \ \sum_{i=1}^{r} p_i = 1. \tag{66}$$

These probabilities are unknown. Suppose the parameter Q is observed n times. Let n_i = number of occurrences of $Q = q^{(i)}$. Clearly

$$\sum_{i=1}^{r} n_i = n. \tag{67}$$

Then, the likelihood function is given by

$$L(n_1,\dots,n_r \mid n, q^{(1)},\dots,q^{(r)}, p_1,\dots,p_r) = \binom{n}{n_1,\dots,n_r} p_1^{n_1}\cdots p_r^{n_r}, \tag{68}$$

a multinomial distribution.

The convenient prior distribution to use over the p_i's is a member of the multidimensional Beta family, i.e.,

$$\xi(p_1,\dots,p_r) = \frac{1}{\mathrm{B}(m_1,\dots,m_r)} p_1^{m_1-1}\cdots p_r^{m_r-1}, \tag{69}$$

where $\mathrm{B}(m_1, \dots, m_r)$ is the generalized Beta function defined by

$$\mathrm{B}(m_1,\dots,m_r) = \prod_{i=1}^{r}\Gamma(m_i)\Big/\Gamma\left(\sum_{i=1}^{r} m_i\right) = \mathrm{B}\left(m_1, \sum_{i=2}^{r} m_i\right)\mathrm{B}\left(m_2, \sum_{i=3}^{r} m_i\right)\cdots \mathrm{B}(m_{r-1}, m_r), \tag{70}$$

$$\Gamma(m) = \int_0^\infty x^{m-1}e^{-x}dx. \tag{71}$$

For a positive integer m, $\Gamma(m)=(m-1)!$

To verify that (69) is in fact a frequency function, we note that

$$p_r = 1 - \sum_{i=1}^{r-1} p_i. \tag{72}$$

Restricting to the case of three random variables ($r-1=3$) for convenience, and recalling that

$$\sum_{i=1}^{3} p_i < 1, \tag{73}$$

we have to show that

$$[\mathrm{B}(m_1, m_2+m_3+m_4)\mathrm{B}(m_2, m_3+m_4)\mathrm{B}(m_3,m_4)]^{-1}$$

$$\times \int_0^1 dp_1 \int_0^{1-p_1} dp_2 \int_0^{1-p_1-p_2} p_1^{m_1-1}p_2^{m_2-1}p_3^{m_3-1}(1-p_1-p_2-p_3)^{m_4-1}dp_3 = 1. \tag{74}$$

Using repeatedly the relation

$$\int_0^a p^{m-1}(a-p)^{n-1}dp = a^{m+n-1}\mathrm{B}(n,m), \tag{75}$$

the integral in (74) is readily seen to equal unity. This result is easily generalized to any number of variables. Thus (69) is a frequency function.

From our choice of likelihood function and prior distribution it follows directly that the ensuing posterior distribution will be a new member of the same multidimensional Beta family (a consequence of the judicious choice of the prior family).

The new parameters (m_i'') are easily obtained from the old ones (m_i') and the observed data (n_i) by means of the following rule: $m_i''= m_i'+n_i$. The prior parameters, (m_0, …, m_r) have to be selected. If the decision-maker has prior beliefs it is logical to select the parameters to reflect these.

If we integrate (69) over all p_i except p_j we obtain the marginal prior distribution of p_j,

$$\xi(p_j) = \frac{1}{B(m_j, \sum_{i \neq j} m_i)} p_j^{m_j - 1} (1 - p_j)^{\sum_{i \neq j} m_i - 1}, \quad 0 \leq p_j \leq 1. \tag{76}$$

The prior probability of $Q = q^{(j)}$ is given by

$$\Pr\{Q = q^{(j)}\} = p_{0j} = \int_0^1 p_j \xi(p_j) dp_j = \frac{m_j}{\sum_{i=1}^{r} m_i}. \tag{77}$$

Thus, with the determination of the a posteriori distributions

$$\xi_{1q^{(j)}}(p_1, \ldots, p_r \mid Q = q^{(j)}) = \frac{1}{B(m_1, \ldots, m_j + 1, \ldots, m_r)} p_1^{m_1 - 1} \cdots p_j^{m_j} \cdots p_r^{m_r - 1}, \quad j = 1, \ldots, r, \tag{78}$$

the marginal a posteriori distributions

$$\xi_{1q^{(j)}}(p_i \mid Q = q^{(j)}) = \frac{1}{B(m_i, \sum_{s \neq i,j} m_s + m_j + 1)} p_i^{m_i - 1} (1 - p_i)^{\sum_{s \neq i,j} m_s + m_j + 1}, \quad 0 \leq p_i \leq 1, \tag{79}$$

and the a posteriori probabilities

$$p_{1i}(j) = \int_0^1 p_i \xi_{1q^{(j)}}(p_i \mid Q = q^{(j)}) dp_i = \frac{m_i}{\sum_{s=1}^{r} m_s + 1}, \quad i \neq j; \quad p_{1j}(j) = \frac{m_j + 1}{\sum_{s=1}^{r} m_s + 1}, \tag{80}$$

we obtain the following.

For N=1, the minimum of (11) is

$$f_1(x_0, \xi) = \min_{u_0} E\{(x^\bullet - x_1)^2\} = G_1(\xi)(x^\bullet - x_1)^2 = \left(1 - \frac{E^2\{Q\}}{E\{Q^2\}}\right)(x^\bullet - x_0)^2 \tag{81}$$

with

$$u_0 = h_0(\xi)(x^\bullet - x_0) = \frac{E\{Q\}}{E\{Q^2\}}(x^\bullet - x_0), \tag{82}$$

where

$$E\{Q\} = \sum_{i=1}^{r} p_{0i} q^{(i)} \quad \text{and} \quad E\{Q^2\} = \sum_{i=1}^{r} p_{0i} [q^{(i)}]^2. \tag{83}$$

For $N \geq 2$, the minimum of (11) is

$$f_N(x_0, \xi) = \min_{u_0} \left(\sum_{i=1}^{r} p_{0i} f_{N-1}(x_1^{(i)}) \right)$$

$$= G_N(\xi)(x^\bullet - x_0)^2 = \left(E\{G_{N-1}(\xi_{1Q})\} - \frac{E^2\{QG_{N-1}(\xi_{1Q})\}}{E\{Q^2 G_{N-1}(\xi_{1Q})\}} \right)(x^\bullet - x_0)^2 \tag{84}$$

with

$$u_0 = h_{N-1}(\xi)(x^\bullet - x_0) = \frac{E\{QG_{N-1}(\xi_{1Q})\}}{E\{Q^2G_{N-1}(\xi_{1Q})\}}(x^\bullet - x_0), \tag{85}$$

where

$$E\{QG_{N-1}(\xi_{1Q})\} = \sum_{i=0}^{r} p_{0i}q^{(i)}G_{N-1}(\xi_{1q^{(i)}}),\ E\{Q^2G_{N-1}(\xi_{1Q})\} = \sum_{i=0}^{r} p_{0i}[q^{(i)}]^2G_{N-1}(\xi_{1q^{(i)}}). \tag{86}$$

5 Optimal Number of In-Service Inspections

By plotting $f_N(x_0,\xi)$ versus N the optimal number of in-service inspections N^* can be determined as

$$N^* = \arg\inf_N [c_f f_N(x_0,\xi) + c_N N], \tag{87}$$

where c_f and c_N represent the specified weight coefficients. Fig. 5 illustrates the graphical method of finding the optimal number of in-service inspections.

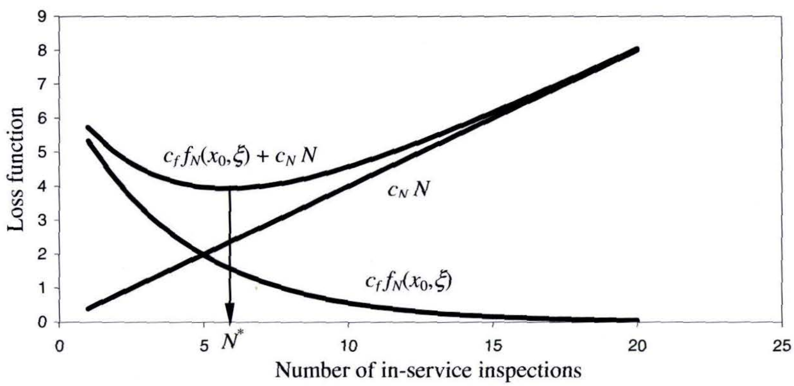

Fig. 5: Graphical method of finding the optimal number N^* of in-service inspections.

6 Conclusions

An analytical solution to the terminal-control problem is generally not easy to derive, and numerical procedures should be followed. In this paper, the design of adaptive and learning control processes is considered. The design of such processes is carried out on the basis of the Bayes theorem and the functional-equation approach of dynamic programming. An inspection planning process of cracked aircraft structure component is used to illustrate the design procedure.

Acknowledgments

This research was supported in part by Grant No. 06.1936, Grant No. 07.2036 and Grant No. 09.1014 from the Latvian Council of Science and the National Institute of Mathematics and Informatics of Latvia.

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