Imaging the Iterons of Automata

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Abstract
We consider here the problem of imaging the iterons of automata. These are discrete counterparts of localized coherent structures from continues dynamical systems. Iterons emerge during iterated automata mappings performed over strings. They consist of filtrons (in serial processing) and particles (in cellular processing). The main problem with the visualization of iterons is their spreading over a medium that they propagate through. We propose here a solution to this problem, both for filtrons and particles; we identify *M*-segments or *G*-segments, respectively, which are determined by the activity of the underlying automata. Then we present various types of ST diagrams. Also, we present the new idea of embedding the observer into processing space. This entails the perceiving an event in various ways depending on the position of local observer (e.g. Doppler's effect). We show that Conway's glider (of basic period $p = 4$) in game of life cellular automaton can be seen as either $p = 3$ or $p = 5$ object depending on observer.
Keywords: coherent structures, discrete solitons, iterative computation, cellular automata, nonlinear dynamics, integrable systems, filtrons, particles, space-time diagrams.

1. Goal of the Paper

Three issues are concerned here with the so called coherent structures in discrete systems represented by nets of automata. First, we recall a general description of the fundamental mechanism that supports coherent structures in such systems. It is based on *iterations* of automata maps (IAMs), and thus justifies the usage of the term iterons of automata. Then, we describe a method to identify discrete coherent structures. This applies the concept of *activity* of automata which constitute the processing medium. At last, we present techniques of visualisation of these special objects, among which we show that commonly used space time diagrams may expose different images of an event depending on the *position* of observer.

2. Systems Where Iterons Occur

The spontaneous emergence of particular coherent structures has been recognized in numerous dynamical systems. Typically, these entities have the form of specially shaped disturbances that are moving throughout a medium with various velocities. Probably the best known example of such persistent structures is the solitary wave. They are periodic and exhibit peculiar behaviour during collisions. The colliding objects pass one another like ghosts—the collisions are nondestructive. These events are caused by a nonlinearity.

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Here we consider the phenomena of moving coherent structures in discrete systems. These objects have been encountered and described during the investigation of such different problems in discrete systems like:

- integrability of dynamical systems [29],
- basics of performing computations by means of streams of colliding objects [27],
- properties of solitary wave equations [1, 32],
- efficacy of standard and recursive digital filtering of discrete signals [30],
- properties of coherent structures in crystal models [6, 33],
- information processing that is equivalent to box-ball systems dynamics [9, 34, 33, 35],
- convergence of numerical procedures and complexity of combinatorial algorithms [12],
- behaviour of discrete solitons in cellular automata (CAs) [1, 3, 4, 10, 11, 13, 14], etc.

It is clear now that coherent structures represent typical behaviour, both for simple and for complex discrete systems. Especially, in a series of papers [17-29] it has been shown that some general and unified approach is possible and useful in describing the fundamental mechanism of emergence of all these structures. Namely, it appeared that all discrete systems described in literature which are capable of string processing and exhibit the mentioned moving entities, can be described and replaced by appropriate automata and their iterations.

Coherent structures emerge in such an approach as a result of *iterating process* performed by automata over the initial string of symbols. These structures are seen as moving periodic substrings of the string. This suggests that all known phenomena of coherent structures in discrete systems can be explained by iterated automata maps (IAMs). This is why a new general term, the iterons of automata, have been introduced to distinguish and describe such structures [20]. The iterons of automata seem to be a fundamental computational phenomenon.

The iteron of automaton M is a periodic structure that propagates like a disturbance throughout one-dimensional homogeneous automaton medium. The medium is formed by a pipeline of copies of automata M, so it operates like an IAM; we denote $M(a^t) = a^{t+1}$, where $t = 0, 1, \ldots$, counts the steps of iteration. The moving disturbance itself has the form of a periodic substring $a^t\_$ of the string a^t that passes throughout this line.

There are two types of iterons: particles and filtrons. The filtrons emerge in serial string processing that occurs within iterating net of copies of given automaton M. They are identified as M-segments of a^t that involve certain sequences of operations f_s associated with M. These sequences can be recognized as special paths on the automaton state diagram.

The particles appear during parallel string processing performed by cellular automata (CAs). To identify them we apply de Bruijn graph G of given CA, and show the particles within a^t as G-segments related to involved sequences of elementary rules (ERs) of CA.

Thus, in both cases iterons of automata are determined by some sequences of elementary operations; state-implied functions in serial IAMs and ERs in parallel IAMs. These sequences represent an active mode of automaton medium.

Having an identification tool one can exhibit the iterons of IAMs. We list some basic mechanisms of string processing and their equivalent automata and then we present various possibilities of imaging the iterons. There are various types of iterons behaviour [17-29]: multifiltron collisions, fusion, fission, and spontaneous decay or quasi-filtrons, bouncing filtrons, trapped colliders, complex filtrons and their collisions, cool filtrons, attracting and repelling objects, also the particles that are typical to some widely known CAs, such as 110, 54, and 30. Some of them will be illustrated.

Usually, TS (time-space) diagrams are applied to present the particles, and ST (space-time) diagrams to demonstrate the filtrons. These two cases are inherently associated with the position of observer; TS diagrams show the layout of symbols along the medium, while ST diagrams present the evolution of a string when it passes throughout the medium.

But there is also third possibility of viewing the substructures of strings, never presented in literature: this is when the observer is embedded into the net of automata, and the farther information needs a time to come and be registered by him. In this case the image perceived by the observer varies, depending on its position and velocity; we will show this phenomenon for linear cellular automata and using the Conway's game of life 2-d CA.

3. State Implied Functions f_s, M-Segments, and Filtrons of Automaton M

A Mealy type automaton M with outputs and an initial state is defined to be a system $M = (S, \Sigma, \Omega, \delta, \beta, s_0)$, where S, Σ and Ω are nonempty, finite sets of—respectively—states, inputs and outputs, $\delta : S \times \Sigma \rightarrow S$ is called the next state (or transition) function of M, and $\beta : S \times \Sigma \rightarrow \Omega$ is called the output function of M. Symbol $s_0 \in S$ denotes the initial state of M.

The automaton converts sequences of symbols (finite or infinite words). For each symbol σ_i read from an input string it responds with an associated output symbol ω_i which is a consecutive element of the resulting string. The input string is read sequentially from left to right, one symbol at each instant τ of time, in such a way that $\delta(s(\tau), \sigma(\tau)) = s(\tau+1)$ and $\beta(s(\tau), \sigma(\tau)) = \omega(\tau)$ for all $\tau = 1, 2, \ldots$.

Next state and output functions of automata are presented in tables or in a graph form that is called the state diagram of automaton. For any $s \in S$ and $\sigma \in \Sigma$ that imply $t = \delta(s, \sigma)$ and $\omega = \beta(s, \sigma)$ in Mealy model, there is a directed edge on the graph going from node s to node t, and labelled by σ/ω. In Moore model the output function is defined by $\lambda(s(\tau)) = \omega$ thus the outputs $\lambda(s)$ are attached to the nodes.

To allow automata iterations over strings we apply a unified set of input-output symbols $A = \Sigma = \Omega = \{0, 1, ..., m\}$. The successive strings $a^{t+1} = M(a^t)$, $t = 0, 1, \ldots$, are listed one under another, and form an ST (space-time) diagram. Sometimes we shift each output string by q positions to the left with respect to its input string.

We also describe the automaton's operation by (state-implied) functions $f_s : A \to A$. They depend on states and are such that $f_s(a_i) = \beta(s, a_i)$ for all $s \in S$ and $a_i \in A$. The succession of outputs of the automaton is given by *next* $[f_s(a_i)] = f_t(a_{i+1})$ with $t = \delta(s, a_i)$.

It is clear that the labelled path on state diagram of the automaton implied by any input string can be viewed as a sequence of operations f_s. Thus, any input string is related to the sequence of automaton state-implied functions.

The automaton medium represented by IAM can be either idle or active. The medium, at point M, is in its idle mode when the string of zeroes just passes M. It becomes excited when a nonzero segment passes. In this view the automata are substring recognizers. The idea is as follows. Suppose that M reads a string $a^t = \ldots a_1^t\, a_2^t \ldots a_{L_t}^t \ldots$. When a_1^t forces M to leave a fixed (starting) state, then it is marked as the beginning of a substring (M is activated). Next, when the symbol $a_{L_t}^t$ forces M to enter some fixed (final) state it is marked as the end of the substring (M is extinguished). The substring $a_1^t\, a_2^t \ldots a_{L_t}^t$ is said to be the M-segment.

We consider special M-segments. We assume strings $\ldots 0a_1 \ldots a_L 0 \ldots$ where symbol 0 represents a background; and $\delta(s_0, 0) = s_0$ for M. The initial state s_0 of M is chosen to be the starting state as well as the final state. In general case one can use another selection; e.g. some subsets of automaton states can play the role of starting states and/or of final states, or even these sets and their roles can evolve along the iteration time steps.

The filtron is defined as follows [18, 19]. By a p-periodic filtron $a^t\_$ of an automaton M we understand a string $a_1^t\, a_2^t \ldots a_{L_t}^t$ of symbols from A with $a_1^t \neq 0$, such that during the iterated processing of configuration $a^t = \ldots 0a^t\_0 \ldots$ by the automaton M the following conditions are satisfied for all $t = 0, 1, \ldots$:

- the string $a^t\_$ occurs in p different forms (filtron's orbital states), with $0 < L_t < \infty$,
- the string $a^t\_$ is an M-segment.

When a number of extinctions of given M occurs still before the last element of the string segment $a^t\_$ is read by M, we say that $a^t\_$ is a multi-M-segment string. Multi-M-segment strings lead to complex filtrons.

4. Elementary Rules, *G*-Segments, and Particles of 1-d Cellular Automaton *CA*

Now, let us consider cellular processing of strings. Linearly extended (or 1-d) CA is defined by $CA = (A, f)$ where A is a set of symbols called the states of cells, $f: A^n \to A$ is a map called the local function or the rule of CA, and $n = 2r + 1$ is the size of neighbourhood (or processing window) with r left and r right neighbours. Typically, especially when $|A| = 2$ and r is small, the rule is given by the number $\sum_{j=0}^{j=2^n-1} f(w_j)\cdot 2^j$; w_j denotes the neighbourhood state (contents of the window), with $w_{min} = (0, 0, \ldots, 0)$ up to $w_{max} = (1, 1, \ldots, 1)$. 1-d CAs convert the strings (configurations) of symbols in parallel. Thus, for a current configuration $a^\tau = \ldots a_i^\tau \ldots$, the next configuration $a^{\tau+1}$ is a result of updating simultaneously all the symbols from a^τ: for all $-\infty < i < +\infty$ we have $a_i^{\tau+1} = f(a_{i-r}^\tau, a_{i-r+1}^\tau, \ldots, a_i^\tau, \ldots, a_{i+r}^\tau)$. The resulting global CA map $a^\tau \to a^{\tau+1}$ is denoted by $\gamma(a^\tau) = a^{\tau+1}$.

One can specify the function f by the set of all $(n+1)$-tuples $(a_1, a_2, \ldots, a_{n+1}) \in A^{n+1}$, where $f(a_1, a_2, \ldots, a_n) = a_{n+1}$. Any such $(n+1)$-tuple is called here the elementary rule (ER) of CA. All possible sequences of adjacent ERs involved in CA processing of a string are identified by n-wide window sliding along the string, thus can be recognized on the de Bruijn graphs.

For any $CA = (A, f)$ with n-wide window, the Moore automaton $G_n = (A^n, A, A, \delta, \lambda, s_0)$, implied by the de Bruijn graph, is defined by the next state function $\delta((a_1, \ldots, a_n), a_{n+1}) = (a_2, \ldots, a_{n+1})$, and the output function $\lambda(a_1, \ldots, a_n) = f(a_1, \ldots, a_n)$. Automaton G_n mimics the CA operation over a string, and expresses the constraints on possible sequences of ERs involved in CA processing.

Let us use automaton G_n to detect the strings of ERs associated with particles. By a p-periodic particle $a^\tau\_$ of an automaton CA we understand a string $a_1^\tau a_2^\tau \ldots a_{L_\tau}^\tau$ of symbols from A such that during the iterated CA processing the configuration $a^\tau = \ldots u a^\tau\_ v \ldots$ occurs in p different forms: $a^\tau, a^{\tau+1}, \ldots, a^{\tau+p-1}$. Periodic strings u and v represent spatially regular areas. We choose the starting states of G_n to be related to area u, and its final states to area v. The roles of these sets can interchange in succeeding moments τ. The paths implied by $a^\tau\_$ on G_n, which lead from initial states to final ones, define G-segments similarly to M-segments. Note that one can use undirected graph G_n to identify G-segments. This is because all outputs of automaton G_n can be determined simultaneously for the entire configuration a^τ.

5. Models That Support Iterons

The first model supporting iterons—filter CAs—was introduced in 1986 [15]. This work founded a bridge between automata and nonlinear physics, presenting the discovery of discrete binary soliton-like entities emerging under particular IAMs. Since that time, a number of models that support discrete coherent structures have been introduced and studied in literature. Most of them presented the aspect of identifying and visualizing these objects to understand their dynamics. We list here some important specific models, and further show certain their iterons.

1. Iterating automata nets that perform numerical procedures [30].

2. Cellular automata and filter CAs [3, 10, 15].
3. Digital recursive filters and filter automata [15, 19, 20, 21].
4. Solitonic CAs [37].
5. CAs of higher order or homogeneous nets of cells with a memory [31].
6. Sequentially updated CAs (update schedule) [2].
7. Integrable CAs [9, 29].
8. CAs derived from the equations of motion [1, 33, 35].
9. Algorithmic procedures (so called fast rules) [12, 14].
10. Box and ball systems [9, 25, 34, 33, 35].
11. Crystal systems [6, 25].
12. Affine Lie Algebras [7].
13. Sets of algebraic equations.
14. Combinatorial models [7, 8, 12].
15. Invariants of shape (Young) tableaux [36].

Note, that most of them can be reduced to the IAM mechanism.

6. Forms of ST Diagrams

The first string-processing mechanism capable of supporting filtrons—PST model—was introduced in [15] as parity rule filter cellular automaton (PRFCA); (f_{PST}, r). It's binary coherent segments were called particles. For a given string a^t, its successor a^{t+1} is computed by updating function f_{PST} such that $a_i^{t+1} = f_{PST}(a_i^t, \ldots, a_{i+r}^t, a_{i-r}^{t+1}, a_{i-r+1}^{t+1}, \ldots, a_{i-1}^{t+1}) = 1$, if and only if $\sum_{j=1}^{r} a_{i-j}^{t+1} + \sum_{j=0}^{r} a_{i+j}^{t} \neq 0$ is even; r is called the radius of f_{PST}. It is assumed $f_{PST}(0, 0, \ldots, 0) = 0$.

Later, the PST model was generalized [17, 19] to a special family of automata, FA-family. The initial state s_0 of $M \in$ FA satisfy $\delta(s_0, 0) = s_0$. Also, the automata from FA convert strings in cycles of some operations. All cycles have the fixed length r+1. The first cycle starts at the nonzero element a_i: $\delta(s_0, a_i) \neq s_0$, and next cycles proceed until special substring $w \in A^{r+1}$ (called reset condition) occurs that coincides with the cycle. In PST model the alphabet $A = \{0, 1\}$, and the cycle of operations N (negate) and A (accept) has the form (N, A, …, A) = (NA^r) and reset conditions $\{w\} = \{*0^r\}$ where * denotes an arbitrary element.

The filtrons of automata M can be visualised in many ways. Here we present some forms of ST diagrams which are especially useful in exposing filtrons and particles of automata.

6.1. Text ST Diagrams

This is basic, natural form of showing the evolution of strings under IAMs. The consecutive strings of symbols are listed one under another in separate rows using their associated (standard) letters from alphabet A. Below is an example of this form. The IAM is performed by the automaton BBSC(6,4); its description is given further.

\\\	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	0	0	0	0	0	2	3	0	0	2	6	6	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	3	2	0	0	4	6	4	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	4	1	0	0	6	6	2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	5	0	0	2	6	6	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	4	0	0	4	6	4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	2	3	0	0	6	6	2	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	3	2	0	2	6	6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	4	1	0	4	6	4	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	6	6	2	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	0	2	6	6	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	0	4	6	4	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2	0	6	6	2	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	1	2	6	6	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	5	0	4	6	4	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	0	6	6	2	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	2	6	6	0

5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
.	2	3	_	.	2	6	6	_	.	.	.	.	.	.	.	.	.	.
.	.	3	2	_	.	4	6	4	_	.	.	.	.	.	.	.	.	.
.	.	.	4	1	.	.	6	6	2	.	.	.	.	.	.	.	.	.
.	.	.	.	5	_	.	2	6	6	_	.	.	.	.	.	.	.	.
.	.	.	.	1	4	_	.	4	6	4	_	.	.	.	.	.	.	.
.	.	.	.	.	2	3	_	.	6	6	2	.	.	.	.	.	.	.
.	.	.	.	.	.	3	2	_	2	6	6	_	.	.	.	.	.	.
.	.	.	.	.	.	.	4	1	.	4	6	4	_	.	.	.	.	.
.	.	.	.	.	.	.	.	5	_	.	6	6	2	.	.	.	.	.
.	.	.	.	.	.	.	.	1	4	_	2	6	6	_	.	.	.	.
.	.	.	.	.	.	.	.	.	2	3	_	4	6	4	_	.	.	.
.	.	.	.	.	.	.	.	.	.	3	2	_	6	6	2	.	.	.
.	.	.	.	.	.	.	.	.	.	.	4	1	2	6	6	_	.	.
.	.	.	.	.	.	.	.	.	.	.	.	5	_	4	6	4	_	.
.	.	.	.	.	.	.	.	.	.	.	.	1	4	_	6	6	2	.
.	.	.	.	.	.	.	.	.	.	.	.	.	2	3	2	6	6	_

Fig. 1. ST diagram in a basic text form; $A = \{0, 1, 2, 3, 4, 5\}$, and its enhanced version.

6.2. Symbolic ST diagrams

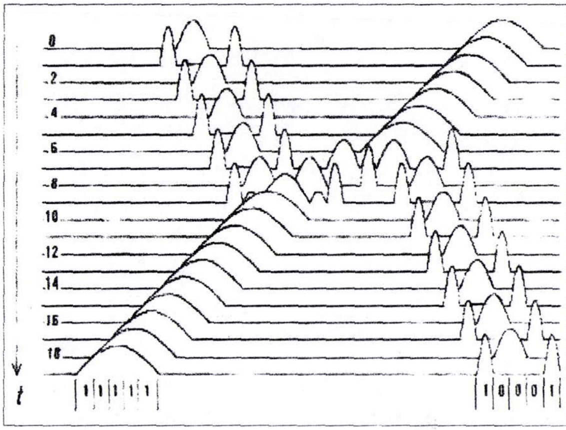
Symbolic or interpreted diagrams use special convention for letters associated with symbols from A. The aim is to expose and facilitate identification of M-segments encountered during processing of a string by any automaton M. The convention is as follows:

- symbols activating M are printed in bold,
- tail zeros, which are all consecutive zeros preceding immediately the extinction of automaton M, and which are read still during M activity, are denoted by the dash "-",
- all zero symbols read by the automaton M in its inactive mode are printed as dots "• ",
- all other symbols a from working alphabet A are shown unchanged.

It is especially easy to determine the M-segments of any string for automata M from FA-family. An example is shown in Fig. 2 a). The IAM is performed by PST automaton of r = 4. One can note that in rows 6, 7, 8 and 9 there is only one M-segment distinguished by a processing mechanism. All other rows contain two M-segments.

```
0  ....11---------.......11111-----.
1  ...10001-----.......11111-----..
2  .....11---------....11111-----...
3  ....10001-----....11111-----....
4  ......11--------.11111-----.....
5  .....10001-----.11111-----.....
6  .......1100000011111-------.....
7  ......100010011011001-----.....
8  ........110110010011--------....
9  .......101110100010001-----.....
10 .......11111-----..11---------...
11 ......11111-----..10001-----....
12 .....11111-----.....11---------..
13 ....11111-----.....10001------...
14 ...11111-----........11---------.
15 ..11111-----.........10001-----..
```

a)

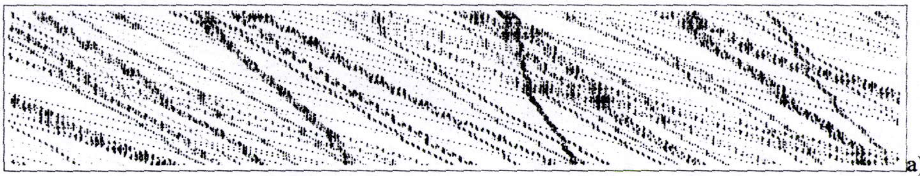


b)

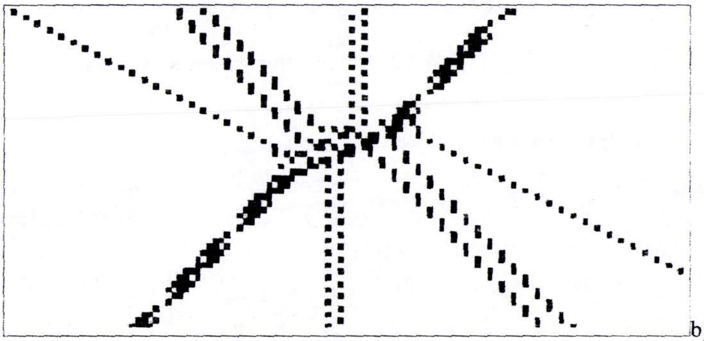
Fig. 2. Collision of two filtrons of PST automaton with $r = 4$; shift $q = 2$.

6.3. Graphical ST Diagrams

This kind of (pixel) images present all rows of diagrams using pixel codes (typically colour codes) of symbols from the basic alphabet A. The example of typical image is given in Fig. 3 a). Fig 3 b) and c) show chosen collision of four filtrons. The string's symbols are from alphabet $A = \{0, 1, 2, 3\}$. The automaton M_1 that supports this collision operates in cyclic manner (belongs to FA family). The cycle of operations is (h_j, f_1, f_2, f_3), and cyclic sequential processing stops when 0000 is encountered (reset word). State implied functions are: $h_0 = 0$, $h_1 = \begin{pmatrix} 0123 \\ 3022 \end{pmatrix}$, $h_2 = \begin{pmatrix} 0123 \\ 2301 \end{pmatrix}$, $h_3 = \begin{pmatrix} 0123 \\ 1230 \end{pmatrix}$, $f_1 = f_3$ $= \mathrm{id}$, $f_2 = \begin{pmatrix} 0123 \\ 0132 \end{pmatrix}$.



a)



b)

```
00 1-------.....10201-------..2003----...2231----.......
01 ..3-------....3000003-----...32------.2212----........
02 ....1-------....10201-------2001----.2122----.........
03 ......3-------..3000003-----.12000001322-----.........
04 ........1-------..10201000002003001223O3------........
05 ..........3-------3000003030330002333003-----.........
06 ............1-------10200023100203231O201-------......
07 ..............3000003030330002333003-----3--------....
08 ................10200023100203231O201-------1-------..
09 ................3030330002333003000003-----..3-------
10 ................200300122303----..10201-------..1-----
11 .................3201203032------.3000003-----....3---
12 ................201302002001----....10201--------....1-
13 ................1322----.12-------...3000003-----.....
14 ..............3323----.2003----......10201-------....
15 ..............3331----...32------.....3000003-----....
```

c)

Fig. 3. a), b) Pixel ST diagrams of solitonic collision of filtrons of automaton M_1. c) Symbolic ST-diagram shows details of the collision from b). The shift $q = 2$ in b) and c).

6.4. Wave-like 2-d ST Diagrams

The filtrons are moving and periodic thus can be presented as wave phenomena. This kind of ST diagrams use special curves or graphical codes to visualise some segments of the filtrons. The aim is to expose their wave nature. An example of this kind of images is presented in Fig. 2 b). For a comparison a symbolic ST diagram of the same collision is given in Fig. 2 a).

6.5. Wave-like 3-d ST Diagrams

The filtrons can be interpreted as periodic sets of variables moving along a plane and changing their values. This gives a possibility of showing them as 3-d objects. Exemplary image of this type of ST diagram is given in Fig. 4. The automaton that supports this image belongs to FA class of automata; it performs the cycle of operations (NAAA) over A $\{0, 1\}$, and is reset at *000 segment.

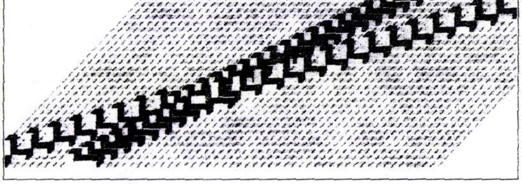


Fig. 4. A collision of two travelling filtrons of the automaton from FA class; (NAAA, *000).

6.6. Surface ST Diagrams

One can build a discrete surface basing on planar positions and the values of symbols. This technique is another possibility of building 3-d images. An example is presented in Fig. 5. The supporting automaton operates over $A = \{0, 1, 2, 3, 4, 5\}$. It's M-segments are associated with cycles $(h_0, f_1, f_2, f_3)(h_1, f_1, f_2, f_3)...(h_1, f_1, f_2, f_3)$ where the functions are: $h_0 = \begin{pmatrix} 012345 \\ 020004 \end{pmatrix}$, $h_1 = \begin{pmatrix} 012345 \\ 152403 \end{pmatrix}$, $f_1 = \begin{pmatrix} 012345 \\ 135420 \end{pmatrix}$, $f_2 = \begin{pmatrix} 012345 \\ 340521 \end{pmatrix}$, $f_3 = \begin{pmatrix} 012345 \\ 031452 \end{pmatrix}$.

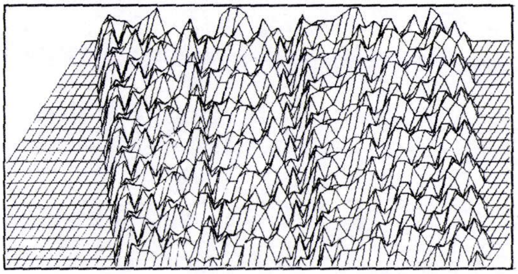


Fig. 5. Surface ST diagram of a filtron of period $p = 3$; $q = 1$.

Reset conditions are in the set {0000, 1300, 3000}. Text ST diagram of the filtron from Fig. 5 is given below.

```
0:   ...04332345121201045541202004133345523201241541300000000000...
1:   ...00451421323005122324115300354421224005523324411300000000...
2:   ...00004523514132550422433430402523010132355422235430000000...
3:   ...00000004332345121201045541202004133345523201241541300000...
```

6.7. Discrete ST Diagrams

They have a form of discrete charts shown in Fig. 6. This kind of image is useful, when one first chooses the shape of filtron and then looks for the automaton capable of supporting this filtron. Here, the state implied functions h_j and f_i that follow from the diagram are not completely specified; they have the form:

$$h_0 = \begin{pmatrix} 0123 \\ 00*1 \end{pmatrix},\ h_1 = \begin{pmatrix} 0123 \\ 1*31 \end{pmatrix},\ f_1 = \begin{pmatrix} 0123 \\ 0*** \end{pmatrix},\ f_2 = \begin{pmatrix} 0123 \\ 2301 \end{pmatrix} \text{ and } f_3 = \begin{pmatrix} 0123 \\ 0*** \end{pmatrix},$$

where * denotes arbitrary symbol. Thus a number of required automata exist in this case.

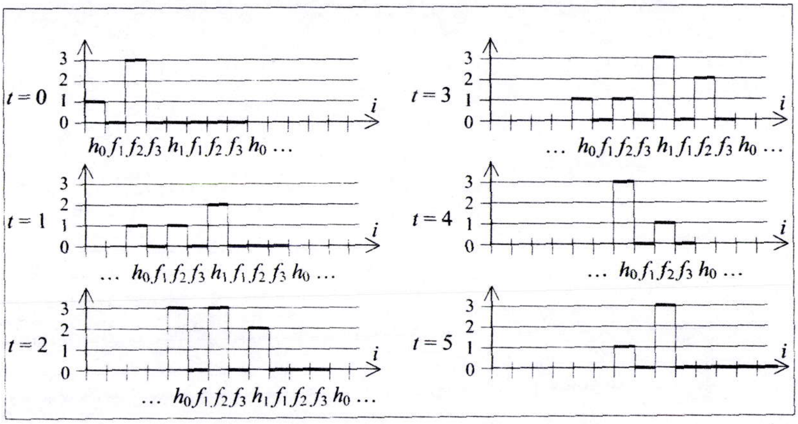


Fig. 6. Filtron of period $p = 5$ involves two cycles of operations $(h_0, f_1, f_2, f_3)(h_1, f_1, f_2, f_3)$.

Another kind of discrete ST diagram appears when successive charts are "stacked" one after another. We show such image in Fig. 8. The collision is supported by automaton BBSC(17,25). The class BBSC$(m, n) = (S, \Sigma, \Omega, \delta, \beta, s_0)$ is such [23, 33] that states $S = \{0, 1, ..., n\}$ and input/output symbols $\Sigma = \Omega = A = \{0, 1, ..., m\}$ are represented by the finite set of integers, and $s_0 = 0$ is the initial state. The next states are determined by $s' = \delta(s, \sigma) = s + \min(\bar{s}, \sigma) - \min(s, \bar{\sigma})$, while the outputs by $\omega = \beta(s, \sigma) = \sigma + \min(s, \bar{\sigma}) - \min(\bar{s}, \sigma)$, where $\bar{s} = n - s$ and $\bar{\sigma} = m - \sigma$.

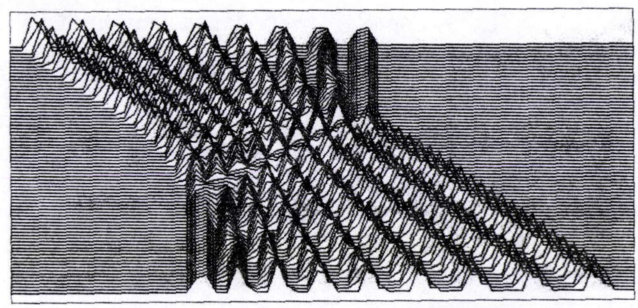


Fig. 7. Nondestructive collision of nine filtrons of automaton MBBSC(17, 25); $q = 1$, and $a^0 = \ldots$0G900F900E900D900C900B900A9009900890… .

6.8. Combinatorial ST diagrams

This kind of images follows from a description of filtrons by means of permutations. It has been pointed [36] that the so called invariants of coherent objects of soliton CAs can be characterized by a shape of pairs of standard Young tableaux. Processing of strings by soliton CAs (changing the positions of 1's) is governed by the stack principle (first in – last out), thus the form of an evolving string a^t can be described by a permutation π_t . Such permutation specifies the order of symbols that are released from the stack at given step t. An example is presented in Fig. 8.

```
0:  .1111.....11...1..................  4321657
1:  .....1111...11..1.................  4326571
2:  .........111..11.1................  3247651
3:  ............11..1.1111.............  2317654
4:  ..............11.1....1111.........  1327654
5:  ................1.11.....1111.....  1327654
6:  .................1..11........1111.  1327654
```

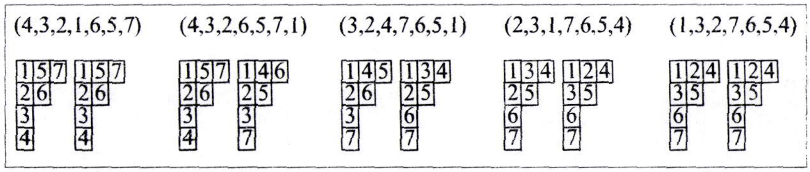


Fig. 8. Collision of 3 filtrons described by pairs of standard Young tableaux.

By Robinson-Schensted algorithm one can convert the permutation description of a collision to a description by means of pairs of standard Young tableaux. The shapes of these pairs are invariant under the IAM [36].

6.9. Filtered ST diagrams

One can obtain the images in this class when a primary text ST diagram of some CA is converted onto the image of elementary rules that were involved in processing in order to determine the G-segments. The result of such a conversion is shown in Fig. 9 b) where ERs (a_1, a_2, a_3, a_4) are printed at the position a_4 and indicated by their numbers (a_1, a_2, a_3) $\in$ {0, 1, ..., 7}. Two spatially periodic sequences of ERs: 5376 and 4012 represent two regular areas of strings from primary txt ST diagram (Fig. 9 a). This regular periodic areas, denoted in Fig. 9 c) by yyyy and xxxx, respectively, can be related (alternatively) to initial and final states of the supporting G_3 automaton. One can now identify the paths implied by strings on G_3 that lead from its initial states to final ones: they are represented by pairs of rules (2, 5), (6, 4), (5, 2) and (1, 3), and are seen in Fig. 9 b) as boundaries between yyyy and xxxx areas. These paths determine the G_3-segments of strings. Their localisation is shown on both: primary text ST diagram and filtered ST diagram in Fig. 9 c).

```
0   .111...1...1...111   012537653765376524   xx25yyyyyyyyyy52x
1   1...1.111.111.1...   537640124012401376   yyy64xxxxxxxxx13yy
2   11.111...1...111.1   240125376537652401   xxxx25yyyyyyy52xxx
3   ..1...1.111.1...1.   765376401240137653   yyyyy64xxxxx13yyyy
4   .111.111...111.111   012401253765240124   xxxxxx25yyy52xxxxx
5   1...1...1.1...1...   537653764013765376   yyyyyyy64x13yyyyyy
6   11.111.11111.111.1   240124012524012401   xxxxxxxx252xxxxxxx
7   ..1...1.....1...1.   765376537776537653   yyyyyyyy777yyyyyyy
8   .111.111...111.111   012401240001240124   xxxxxxxx000xxxxxxx
9   1...1...1.1...1...   537653764013765376   yyyyyyy64x13yyyyyy
10  11.111.11111.111.1   240124012524012401   xxxxxxxx252xxxxxxx
```

Fig. 9. Rule 54 CA; a) text ST diagram, b) sequences of ERs, c) positions of G_3-segments.

```
τ \ i  .2.4.6.8.0.2.4.6.8.0.2.4.6.8.0.2.4.   .2.4.6.8.0.2.4.6.8.0.2.4.6.8.0.2.4.
 0     111111.........1111111....11..11...   1.....2........1......2...1.6.5.6..
 1     11111.1........111111.1...11..1....   1....2..4......1.....2..4.1.6.5.4..
 2     1111.1.1.......11111.1.1..11.......   1...2....4.....1....2....45.6......
 3     111.1.1.1......1111.1.111.1........   1..2......4....1...2...3.1..4......
 4     11.1.1.1.1.....111.11111.1.1.......   1.2........4...1..2.3...2....4.....
 5     1...1.1.1.1....11111111.1.1.1......   1...1.......4..1.......2......4....
 6     .......1.1.1...1111111.1.1.1.1.....   .......1.....4.1......2........4...
 7     ..........1.1..111111.1.1.1.1.1....   ..........1...45.....2..........4..
 8     ............11.11111.1.1.1.1.1.1...   ............1.6.3...2............4.
 9     ............11.1111.1.1.1.1.1.1.1..   ............1.6.3..2..............4
10     ............11.111.1.1.1.1.1.1.1.1.   4...........1.6.3.2................
11     ............11.11.1.1.1.1.1.1.1.1.1   .4..........1.6.36.................
12     1...........11.1...1.1.1.1.1.1.1.1.   ..4.........1.6..4.1...............
13     .1..........1...1.....1.1.1.1.1.1.1   ...4........1.4.1.4...1............
14     1.1......................1.1.1.1.1.   ....4....................1.........
15     .1.1........................1.1.1.1   .....4......................1......
16     1.1.1..........................1.1.   ......4........................1...
17     .1.1.1............................1   .......4..........................1
18     ..1.1.1............................   ..1.....4..........................
19     .....1.1...........................   .....1...4.........................
20     ........1..........................   ........1.4........................
21     ...................................   ...................................
```

Fig. 10. Rule GKL CA: primary text ST diagram, and FIR filtered ST diagram.

Another usage of FIR filter over primary text ST diagram is presented in Fig. 10. Here, cellular automaton with GKL rule, defined by $\delta^{-1}(1) = \{1{*}10{*}{*}{*},\ {*}{*}{*}11{*}{*},\ {*}{*}{*}1{*}{*}1\}$, performs majority classification task. There are 18 symbols 0 and 17 symbols 1 in initial configuration, thus the result of processing, visible at $\tau = 21$, is the all zero configuration. At the right side of Fig. 10 the filtered ST diagram is shown, where a FIR filter used f: $A^4 \rightarrow \{0, 1, 2, 3, 4, 5, 6\}$ associated with automaton G_4 is given by: $f(0001) = 1$, $f(1110) = 2$, $f(1011) = 3$, $f(0100) = 4$, $f(1001) = 5$, $f(0110) = 6$ and $f(w) = 0$ for all others $w \in A^4$.

7. The Images Perceived by Local Observer

Space-time diagrams are very popular in presenting various phenomena especially in cellular automata. It is assumed usually that the observer has the *immediate access* at all moments to all positions of the ST diagram; thus such diagrams are like images seen by God. This assumption is so obvious that is not even clearly stated in any paper treating CAs. However, simple "gedanken experiments" and computer verifying simulations show that the events occurring in discrete spaces are not perceived in unique way when the observer is embedded into the processing space. In such case the information passing from distant cells to the observer needs a time, thus the description of an event depends heavily on the position and movement of such local observer. Also the "speed of light" assumed in CA model matters in this view. This idea of *local observer* being embedded in CA space seems to be not mentioned in literature yet (as far as we know).

We give two examples of this new and fascinating phenomenon. Fig. 11 shows a case of 1-d cellular automaton of 2nd order where two particles collide and local observer is embedded.

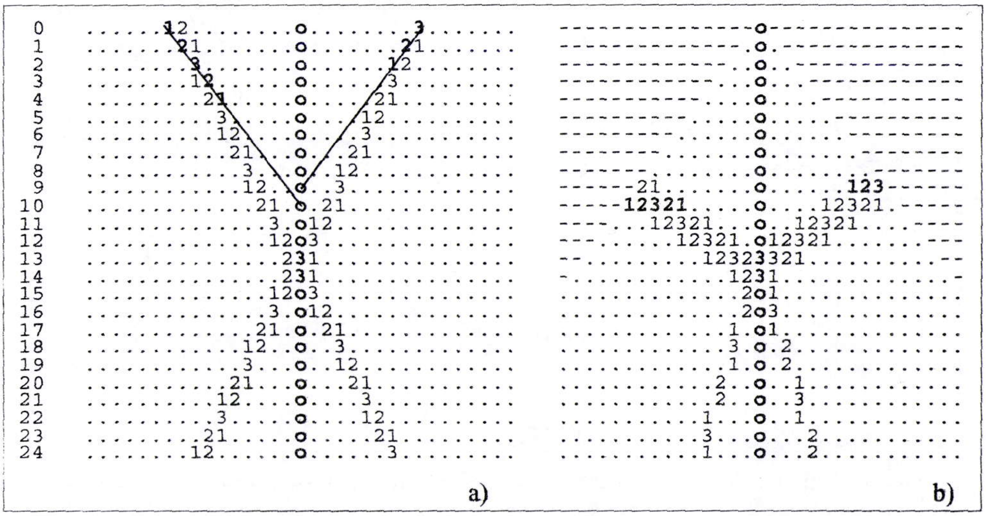


Fig. 11. a) Particles collide in a CA; b) the same collision seen by local observer "o".

Position of local observer is denoted by "o". Note that the knowledge about events from the environment increases with each clock step when information gradually flows down to local observer from farther consecutive "rings". This forms a sort of space-time cone. It is illustrated in the right diagram of Fig. 11 where dashed part is out of observer's horizon, and dotted part denotes a sphere of knowledge. It is seen that primary particles have the period $p = 3$, while those perceived by local observer look like still objects (when they approach the observer), or they have period $p = 5$ (when they escape out of the observer).

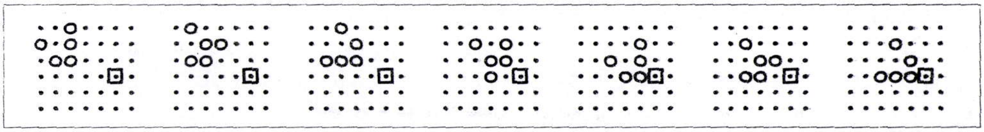


Fig. 12. Global view: glider of period $p = 4$ approaches observer □ localized at position $(\rightarrow,\downarrow) = (6,4)$.

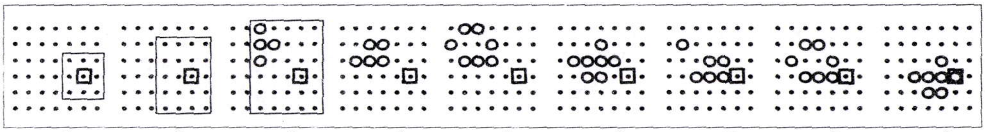


Fig. 13. Local observer view: glider emerges from beyond the horizon; now its $p = 3$.

Our second example shows the object which is called glider in famous Conway's game of life 2-d CA model. Glider is a periodic object with $p = 4$. The succession of its four orbital phases is shown in Fig. 12 ("o" denotes active cell). This is global (or God's) view. Now consider an observer which is inside of this space, the case when glider approaches. New image registered by this local observer is presented in Fig. 13. Now the same glider has different phase forms and different period $p = 3$. Next two figures allow to compare two images in the case when glider passed and the distance to observer increases. Fig. 14 shows God's view, and Fig. 15 shows local observer view. Now the glider seems to have 5 phases, its period is $p = 5$. This phenomena bear a likeness to Doppler effect.

Increasing space-time cones mentioned above are denoted in Fig. 13 and 15 at moments $t = 1$, 2 and 3 by enlarging framed areas. Consider the image at time $t = 3$ shown in Fig. 13. It is composed of 3 rings of data: the outer ring (with three active cells) has been taken from image at $t = 0$ in Fig. 12, the next (or middle) ring with one active cell comes from image at $t = 1$ in Fig. 12, and the last ring close to observer (no active cells are there) is from image at $t = 2$ in Fig. 12.

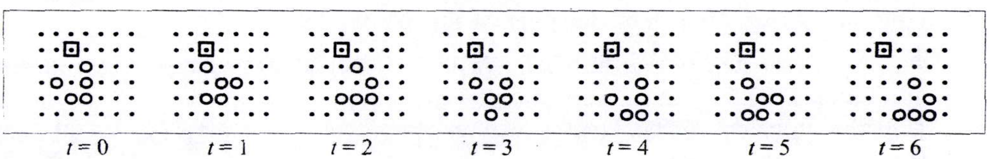


Fig. 14. Global view: glider ($p = 4$) escapes from observer at position (→,↓) = (3,2).

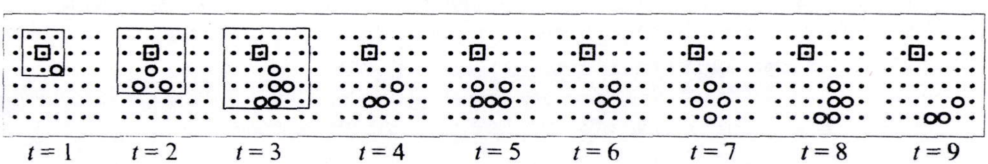


Fig. 15. Local observer view: glider emerges from beyond the horizon; now its $p = 5$.

8. Conclusions

In continuous and discrete dynamical systems there is a plethora of persistent localized objects. The most common are localized disturbances moving in physical media. There are also disturbances propagating in complex systems. In communication technology peculiar persistent components are known to assist signals despite digital IIR filtering; these are well shaped impulses propagating over long (electrical) lines. In logical nets hazard impulses are propagating in parallel to signal changes. Widely known are packets in streams of cars moving along roads. There are also some values migrating within numerical procedures and occurring in convergence acceleration algorithms. Special moving and interacting objects are seen in liquid and granular media. Many of these phenomena can be represented as solutions to some classes of NL wave equations (e.g. Painleve' classes), but for many of them the equations of motion are not known. This is especially the case in discrete systems. For discrete systems we proposed here the idea of iterons of automata that is based on the notion of active automaton medium. Also, some techniques of presenting these objects on various types of space-time diagrams have been given. Imaging of iterons of automata helps one to realize the realm of phenomena that they can represent. This may add new perspective to classical description of localized objects with wave equations.

It should be mentioned that various forms of ST diagrams are also used in many other domains associated with dynamical systems. Most of them are just special cases of the phase portrait [29].

Iterons, being a computational phenomenon associated with automata, represent general and unified approach to coherent structures. The realm of iterons is very reach.

Imaging the iterons of automata have given possibility of observing many new phenomena not shown in this paper. The most important of them are: trembling filtrons, quasi-filtrons, bouncing filtrons, trapped filtrons, orbiting filtrons, annihilating filtrons, decay of quasi-filtrons, repelling and attracting objects, comlex breathers, jumping over a bundle, and objects capable of changing the speed and period.

The images of iterons can be useful in understanding the behaviour of coherent structures and in using them in applications. Important potential applications arise when studying the collisions of coherent structures. Impressive example is given in [16], where new physical phenomena result when light interacts with a shock wave in specific transmitting media, namely in photonic crystals.

Another important issue in imaging of coherent structures of discrete systems is the role of observer in perceiving their behaviour. We showed that the images of an event depend on the position of the local observer. They depend on his movement, as well. Analytical tools and further research are needed to describe the images of coherent structures that are seen by local observer basing on the global views.

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