Measuring Information Transfer in Online Adaptation Process of Recurrent Neural Networks

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Abstract
In this paper, we propose a simple model that focuses on the adaptation process of an agent to an unknown system in an online manner. The agent is equipped with a recurrent neural network, and by controlling the dynamics of the interacting system, it should predict its state in one-step prediction. To quantitatively characterize the interaction modality between the agent and the interacting system, we used transfer entropy. As a result, by varying the nonlinear parameter of the interacting system and the coupling strength, we numerically show that the adaptation dynamics can be distinguished between an agent-driven and a non-agent-driven dynamics.
Keywords : recurrent neural network, online learning, transfer entropy

1 Introduction

Anticipation, learning, and adaptation are inevitable features when we study biological systems [1][2]. Several studies attempted to show the diverse nature of these features by using recurrent neural network (RNN) learning. For example, J. Tani showed in his work that an RNN can learn a stochastic process by embedding chaotic dynamics [3]. In addition, T. Ikegami studied the diversity of our daily communication by coupling two RNNs and showed several cases where diversity originates due to the conflict between an autonomous anticipation and its impossibility [4]. In the context of online learning of an RNN, A. Saito has shown by analyzing the basin structures of an RNN that there exists some property of uncertainty that is qualitatively different from chaotic unpredictability, called "inaccessibility" [5].

In this paper, we study the adaptation process with RNN online learning, where the RNN performs the one-step prediction of a timeseries generated by its coupling system. The agent is equipped with an RNN and required to control the states of its coupling system. To characterize its interaction modality, we used transfer entropy [6]. Transfer entropy allows us to reveal the hidden causality between several dynamics which is usually difficult to extract only by looking at their dynamics.

This paper is organized as follows. In the next section, we explain our model in detail and show how our motivation is expressed in our model setting. In section 3, we observe some basic dynamic properties of our system and quantitatively show its interaction modality by using transfer entropy. And last, we summarize our results and discuss the possible line of work we can take in the future.

2 Model Description

By using an RNN, we deal with dynamical systems that adapt to the interacting systems in an online manner. The network is expressed as follows:

$$h_j = g(\textstyle\sum_i w_{ij} y_i + \theta_j), \tag{1}$$

$$o_j = g(\textstyle\sum_i u_{ij} h_i + \theta'_j), \tag{2}$$

$$g(x) = (1 + exp(-\beta x))^{-1}, \tag{3}$$

where y_i, h_i, and o_i represent the value of the input neurons, the hidden neurons, and the output neurons, respectively. In addition, θ_j and θ'_j represent the bias of the hidden neurons and the output neurons, respectively. w_{ij} (u_{ij}) represents the weight of connection from the i th input (hidden) neuron to the j th hidden (output) neuron. $g(x)$ is the sigmoid function, and β is the nonlinearity coefficient. In this paper, we set $\beta = 1.0$. The number of the input neurons, the hidden neurons, and the output neurons is set to 2, 2, and 1, respectively. Hence, we have only six connection weights, and they are set as $(w_{00}, w_{10}, w_{01}, w_{11}, u_{00}, u_{10}) = (-0.8, 0.9, -0.4, -0.7, 0.4, -0.5)$. Accordingly, the biases are set as $(\theta_0, \theta_1, \theta'_0) = (-0.9, 0.9, -1.0)$. o_0 is fed back to y_0 as a recurrence, and y_1 takes the value of its interacting system, explained later as an input.

The RNN is required to perform one-step prediction by learning from the current state value of its interacting system. For an online learning algorithm, we use the gradient descent method based on the current output error as follows:

$$E = \textstyle\sum_j \frac{1}{2}(o_j - t_j)^2, \tag{4}$$

$$\delta_j = o_j(1 - o_j)(o_j - t_j), \tag{5}$$

$$\delta'_j = h_j(1 - h_j)\textstyle\sum_i w_{ji}\delta_i, \tag{6}$$

$$\Delta u_{ij}(t+1) = -\gamma h_i \delta_j + \alpha \Delta u_{ij}(t), \tag{7}$$

$$\Delta w_{ij}(t+1) = -\gamma y_i \delta'_j + \alpha \Delta w_{ij}(t), \tag{8}$$

where t_j, E, δ_j, δ'_j, γ, and α represent the target value, the error function, the back propagated error to the output neurons, the back propagated error to the hidden neurons, the learning rate, and the momentum rate, respectively. In this paper, $(\gamma, \alpha) = (3.5, 0.3)$. For the target value, we use the well-known logistic map and define the input from the RNN to the map as follows:

$$x'_t = (1 - c)x_t + co_0, \tag{9}$$

$$x_{t+1} = ax'_t(1 - x'_t), \tag{10}$$

where a and c are the nonlinear parameter and the coupling strength, respectively. Note that this overall system is a closed dynamical system that has only two parameters, a and c. The initial states are set to $(x_0, y_0, y_1) = (0.5, 0.5, 0.5)$.

Although our model resembles to the one proposed in [5], there are two clear differences. First, in [5], they used a second-order RNN. Second, since their focus was not on the adaptation process but on the on-line prediction, they did not introduce a coupling from the RNN to the logistic map. On this point, if we set the coupling strength c to 0, then the task setting in our model is the same as the one proposed in [5].

3 Simulation Results

3.1 Trajectory

To see the basic property of the system, we observed the trajectories of E, x_t, and o. We found that in the case of $0 < a < 3.5$, E rapidly converged to 0, and the trajectories of x_t and o tended to show fixed points. However, in the case of around $3.5 < a < 4.0$, $0.0 < c < 0.5$, E converged to 0 very slowly or never converged, and x_t and o frequently showed intermittent dynamics (Fig. 1).

3.2 Bifurcation and Convergence

We observed bifurcations of E, x_t, and o for regions whose convergence of E to 0 is very slow or does not occur. By varying the coupling strength c, we observed a chaotic regime, a periodic regime, and a fixed point. The common feature we observed by varying c is that in the large number of c, it tends to show the period 2 regime and then rests to the fixed point (Fig. 2).

To see the dependency of the convergence of E, we observed the average value of E in the a-c plane (Fig. 3). It showed that in the region of around $3.5 < a < 4.0$, $0.0 < c < 0.5$, the convergence speed tends to be very slow or never converged to 0.

3.3 Transfer Entropy

We have seen the basic dynamic property of the system we use in this paper. If E converges to 0, this means that the agent and the interacting system are perfectly

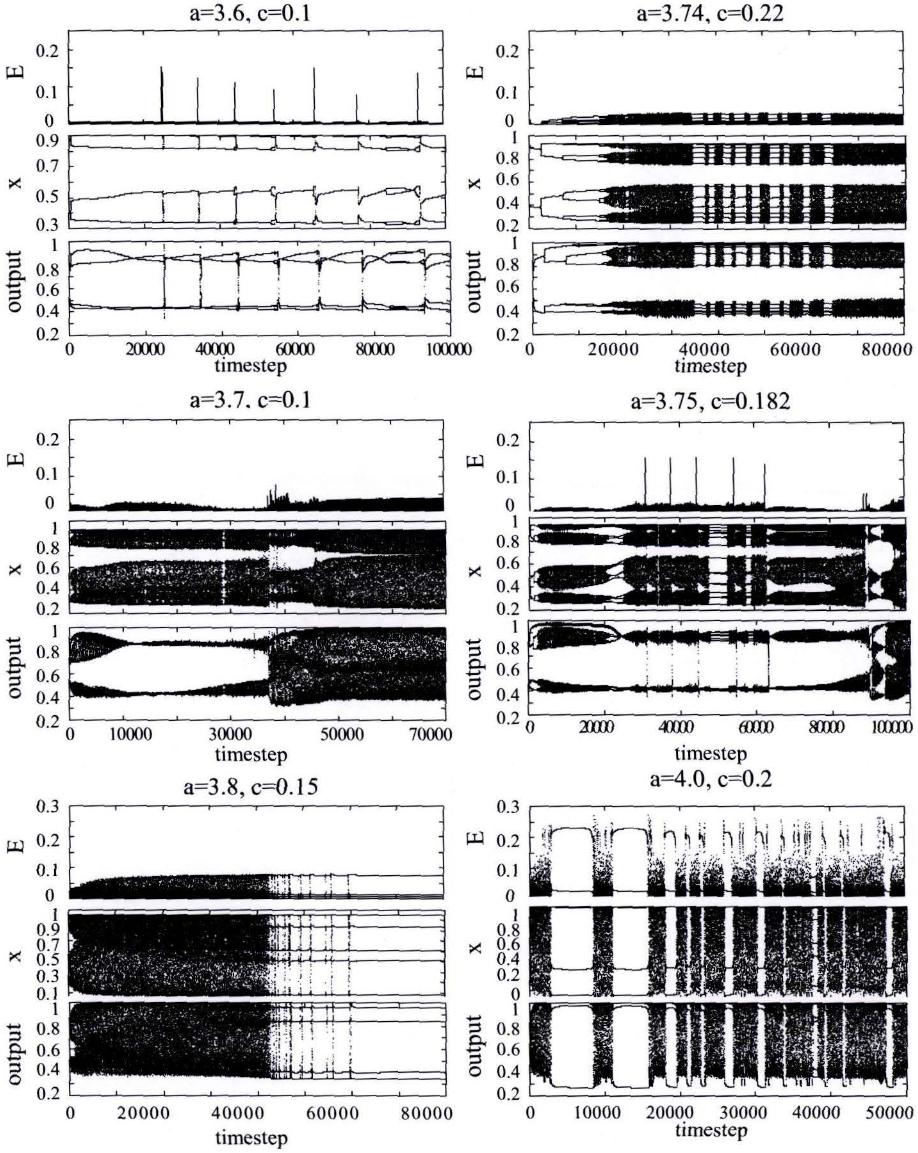

Fig. 1: Typical trajectories of E_t (upper line), x_t (middle line), and o_t (lower line). (a, c)= (3.6, 0.1), (3.7, 0.1), (3.85, 0.15), (3.74, 0.22), (3.75, 0.182), and (4.0, 0.2) are shown.

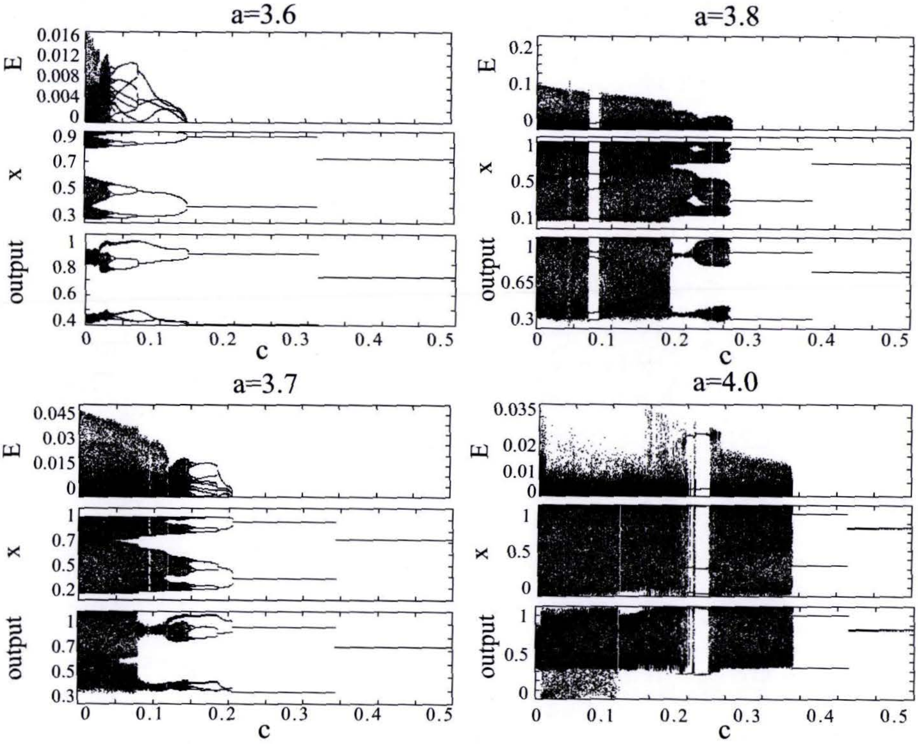

Fig. 2: Typical bifurcations observed in E_t (upper line), x_t (middle line), and o_t (lower line) by varying coupling strength c. a= 3.6, 3.7, 3.8, and 4.0 are shown. T is set to 10000, and the last 100 timesteps are plotted.

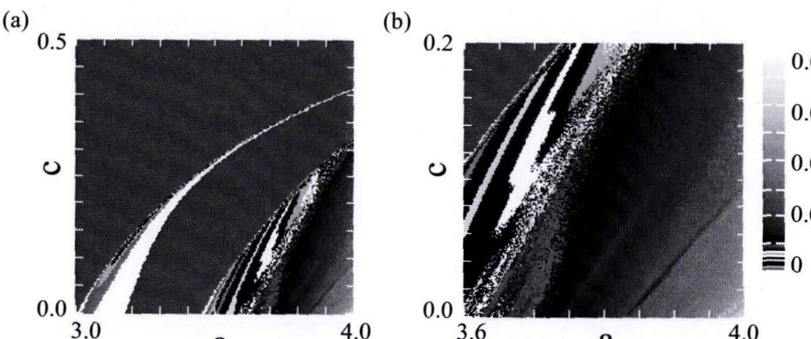

Fig. 3: Average of error shown in the a-c plane. The brighter the color, the larger the error. T is set to 1000, and the last 300 timesteps are averaged. (a) shows when $3.0 < a < 4.0$, $0.0 < c < 0.5$. (b) shows $3.6 < a < 4.0$, $0.0 < c < 0.2$.

assimilated. In this case, the agent does not need to perform prediction, and there is no information transfer between the agent and its interacting system. Our focus of interest is to quantify the adaptation process where the agent is constantly trying to adapt to its interacting system. We picked up the region of the a-c plane whose convergence of E to 0 is very slow or never converges (around $3.5 < a < 4.0$, $0.0 < c < 0.5$), and quantify its interaction regime. It is difficult to reveal the hidden causal relation with only the dynamics.

To see the informational structure of this system, we used a measure that aims at extracting directed flow (transfer of information) between time series, called *transfer entropy* [6]. Given two time series x_t and y_t, the transfer entropy essentially quantifies the deviation from the generalized Markov process: $p(x_{t+1}|x_t) \approx p(x_{t+1}|x_t, y_t)$, where p denotes the transition probability. If the deviation from a generalized Markov process is small, then the state of Y can be assumed to have little relevance for the transition probabilities of system X. If the deviation is large, however, then the assumption of a Markov process is not valid. The incorrectness of the assumption can be expressed as follows:

$$TE(Y \rightarrow X) = \sum_{x_{t+1}} \sum_{x_t} \sum_{y_t} p(x_{t+1}, x_t, y_t) \log \frac{p(x_{t+1}|x_t, y_t)}{p(x_{t+1}|x_t)}, \tag{11}$$

where the sums are over all amplitude states, and the index $TE(Y \rightarrow X)$ indicates the influence of Y on X. The transfer entropy is explicitly nonsymmetric under the exchange of X and Y, and can thus be used to detect the directed exchange of information between two systems. The method is frequently applied in the field of sensorimotor coupling system research to quantify the informational structure over the redundant network architecture [7]. In this paper, we analyzed the following types of transfer entropy, (1) $TE(O \rightarrow I)$, (2) $TE(h0 \rightarrow x)$, (3) $TE(x \rightarrow h0)$, (4) $TE(h1 \rightarrow x)$, and (5) $TE(x \rightarrow h1)$ (Fig. 4). For (1), it shows the causal dependencies from the agent's output to input. If this value is high, it quantitatively means that the agent obtains information from the interacting system by stimulating it. For (2), (3), (4), and (5), they quantify the causal relations between the agent's and the system's internal dynamics.

See Fig. 5. It shows the values of transfer entropy plotted in the a-c plane. On (a), (c), and (e), we observed high values of transfer entropy when parameter a is around 3.5, and the value of parameter c is very small. Also, we observed relatively high values of transfer entropy when both a and c are large. On the other hand, in (d) and (f), they showed high values in all the regions where the value of E is not 0. Especially, if a is large and c is small, it tends to show a very high value. On (b), it shows that when parameter a is around 3.5 and the value of parameter c is very small, the interaction is agent-driven, and in the other region, the interacting system is significantly driving the agent's dynamics (Fig. 6).

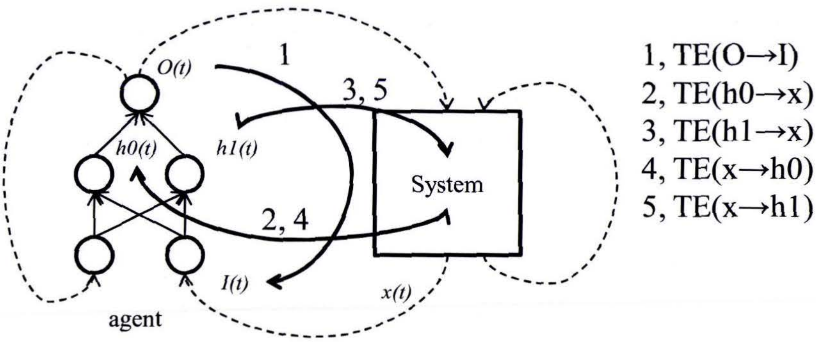

Fig. 4: A schematic diagram showing the information flow focused on this paper. See text for details.

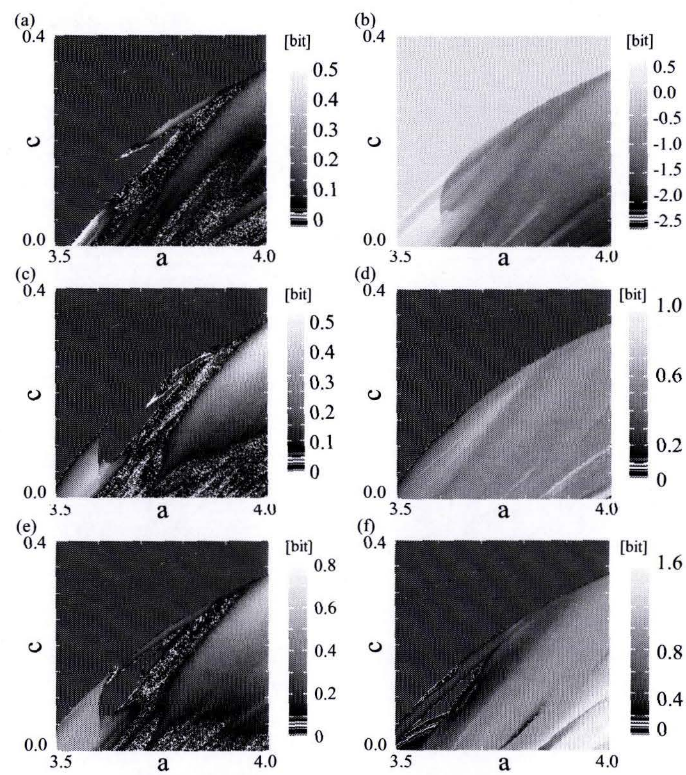

Fig. 5: The values of transfer entropy plotted in the a-c plane ($3.5 < a < 4.0$, $0.0 < c < 0.4$). For each plot, T is set to 3000, and the last 2000 timesteps are used to calculate the transfer entropy. The brighter the color, the larger the value. (a) $TE(O \rightarrow I)$, (b) $(TE(h0 \rightarrow x) + TE(h1 \rightarrow x)) - (TE(x \rightarrow h0) + TE(x \rightarrow h1))$, (c) $TE(h0 \rightarrow x)$, (d) $TE(x \rightarrow h0)$, (e) $TE(h1 \rightarrow x)$, and (f) $TE(x \rightarrow h1)$.

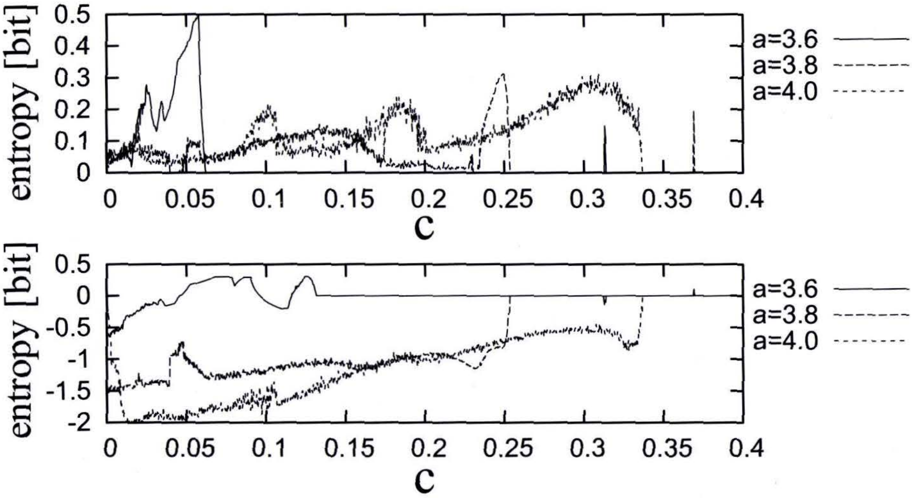

Fig. 6: The value of transfer entropy as a function of coupling strength c ($0.0 < c < 0.4$). The upper line shows the value of $TE(O \to I)$. The lower line shows the value of $(TE(h0 \to x) + TE(h1 \to x)) - (TE(x \to h0) + TE(x \to h1))$. The figure shows the cases of a=3.6, 3.8, and 4.0. For each plot, T is set to 3000, and the last 2000 timesteps are used to calculate the transfer entropy.

4 Conclusion

In this paper, we proposed a simple adaptation model and numerically showed its interacting regime accompanied by the adaptation process using transfer entropy. As a result, we showed that the adaptation dynamics can be distinguished between an agent-driven and a non-agent-driven dynamics in the a-c plane.

For future work, both dynamical systems analysis and information theoretic analysis of this system should be performed in detail. For example, we saw that, in the case of around $3.5 < a < 4.0, 0.0 < c < 0.5$, E converged to 0 very slowly or never converged. Actually, in our setting, we confirmed that, when T=1000000, the large part of this region still did not converged to 0 (e.g. $(a, c) = (3.83, 0.1)$). The analytical clarification of whether E converges to 0 or not in this region should be needed in the future.

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