Some Distributed Systems with Anticipation

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Abstract
Since the introduction of strong anticipation by D.M.Dubois the numerous investigations of concrete systems had been proposed. As concentrated (discrete, ordinary differential equations) as distributed (electromagnetic theory equations, cellular automata) systems with anticipation had been considered earlier. But further development of the theory of anticipatory systems depends on the investigations of new examples of distributed in space systems with anticipation.
So in proposed paper the new examples of distributed physical models with anticipation had been considered – namely the system of hyperbolic differential equations with special boundary conditions. It is proposed the mathematical formulation of problems, possible analytical formulas for solutions and interpretations of presumable distributed solutions. Complex behavior of such solutions is discussed. A list of further research problems is proposed.
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1. Introduction

Since the beginning of 90 – th in the works by D.M. Dubois - see (Dubois, 1992; 1997) the idea of strong anticipation had been introduced : " Definition of an incursive discrete strong anticipatory system ... : an incursive discrete system is a system which computes its current state at time t, as a function of its states at past times , ... , t-3, t-2, t-1, present time, t , and even its states at future times t+1, t+2, t+3, ...

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x(t+1), x(t+2), \ldots, p)$$

where the variable x at future times t+1, t+2, ... is computed in using the equation itself. Definition of an incursive discrete weak anticipatory system: an incursive discrete system is a system which computes its current state at time t, as a function of its states at past times , ... , t-3, t-2, t-1, present time, t , and even its predicted states at future times t+1, t+2, t+3, ...

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x^*(t+1), x^*(t+2), \ldots, p)$$

where the variable x* at future times t+1, t+2, ... are computed in using the predictive model of the system" (Dubois, 2001, p. 447).

Some first examples of such CA were 'fractal machines' (Dubois, 1997, 1998). Then many objects with anticipation had have been investigated – see the papers of D.Dubois. Special case of incursion had been founded when the possibilities of many solutions exists: "In the same way, the hyperincursion is an extension of the hyper recursion in which several different solutions can be generated at each time step" (Dubois, 1997, p.98). Some objects of CA with hyperincursion had been investigated and many ideas related to hyperincursion had been proposed: hyperincursion CA, anticipation logic, memory element, hyperincursion field (Dubois, 1997, 1998).

Most of such investigations had been concerned to discrete on time differences equations or systems of ordinary differential equations with anticipation. Of course, since the initial works by D.Dubois also some models consisting from coupling discrete time elements had been proposed (including cellular automata, for example (Dubois, 1992; Makarenko et al, 2008)). But such kind of systems is most useful for computer science (including computer architecture). Such models and systems and their investigations are important and very prospective (Dubois, 2008).

But the traditional for physics point of view on systems usually has as the basic type of models the partial differential equation with continuous time and space variables (as the examples see heat conduction equation, wave equation, hydrodynamic equations, equations for electromagnetic fields etc.). In case of such system we following the physical traditions will speak as about distributed physical systems or for shortening about distributed systems. It is evident that the considering of anticipation effects in such systems may be also very important (may be one of the first question on anticipation had been considered in electromagnetic theory with advanced and retarded potentials – see the works by R.Feynman). Some of anticipatory effects had been introduced and discussed in some papers by D.Dubois - on electromagnetic theory, relativity and quantum mechanics.

But it appears that the main investigations in the field of partial differential and integral equations with anticipatory property and interpretations of their solutions are the matter of future. It seems that the existence of relatively small number of such investigations lies in the lack of relatively simple examples of such kind of models and especially in absence of simple examples of analytical and easy visible solutions of model equations. So it is interesting to have at new examples of such mathematical problems with anticipation and their manifestation for such distributed system.

Because of all of this we propose in given paper some presumable examples of mathematical problems with anticipatory property in case of partial differential equation model. Of course presented paper is only one of the first steps in the investigations in such field and should be considered rather as the review of new research problems putting. So in given paper we propose mostly some possible formulations of mathematical problems, some presumable ideas on their solutions behavior and some presumable consequences for interpretations of such results. We will concentrate the discussion around only one such mathematical problem which allows to consider the anticipatory effects accounting for the case of partial differential equations. So at the second section of the paper we propose the description of one initial –boundary value problem for hyperbolic system with anticipation in boundary conditions. At the third section we consider some types of boundary conditions and some anticipatory difference equations with continuous time. Section four is devoted to presumable manifestation of anticipatory effects in such systems and to the discussion on further research problems.

2. Some hyperbolic systems with anticipation in boundary conditions

Here we describe one example of distributed mathematical model with anticipation. Considered object is the initial-boundary value problem for the system of two hyperbolic equations (remark that some initial-boundary problems for wave equation belong for such class). Such problems in classical case (without anticipation) had been obtained in considering oscillations of elastic string in bounded domain; oscillations of voltage and flows in transmission lines (see examples in (Sharkowsky et al, 1993; Romanenko&Sharkowsky, 1999; Sharkovsky, 2006)).

The basic classical problem has the next form. Let us consider the system of two hyperbolic equations of first order in bounded space domain

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0, \qquad (1)$$

$$\frac{\partial v}{\partial t}-\frac{\partial v}{\partial x}=0, \qquad (2)$$

$(x,t)\in\Omega(x,t);\ \Omega=[0,1]\times\mathbf{R}^+;\mathbf{R}^+=[0,+\infty);$

For definiteness of the mathematical problem the initial and boundary conditions should be added. For simplicity in this paper we will take the next initial conditions:

$$v(0,t)=\varphi_1(x),\ x\in[0,1]\ ; \qquad (3)$$

$$u(0,t)=\varphi_2(x),\ x\in[0,1]\ . \qquad (4)$$

Also for completeness the boundary conditions should be considered for $x=0$ and for $x=1\ \forall t\geq 0$. Recently very interesting results for such problem had been received with the special boundary conditions in the works of A. Sharkowsky with colleagues (Sharkowsky et al, 1993):

$$u(0,t)=v(0,t),\ t\in\mathbf{R}^+\ ; \qquad (5)$$

$$v(1,t)=f(u(1,t)),\ t\in\mathbf{R}^+\,. \qquad (6)$$

Of course such boundary conditions are of special form but such type of conditions allows complete mathematical investigations (Sharkowsky et al, 1993). The basic ideas in their works are the next. First of all from the theory of hyperbolic equations with constant coefficients the functions u and v remain constant along characteristics (solutions of equations)

$$\frac{dx}{dt} = 1 \ , \tag{7}$$

$$\frac{dx}{dt} = -1 \, . \tag{8}$$

The behavior of solution is well understandable with the help of the picture below.

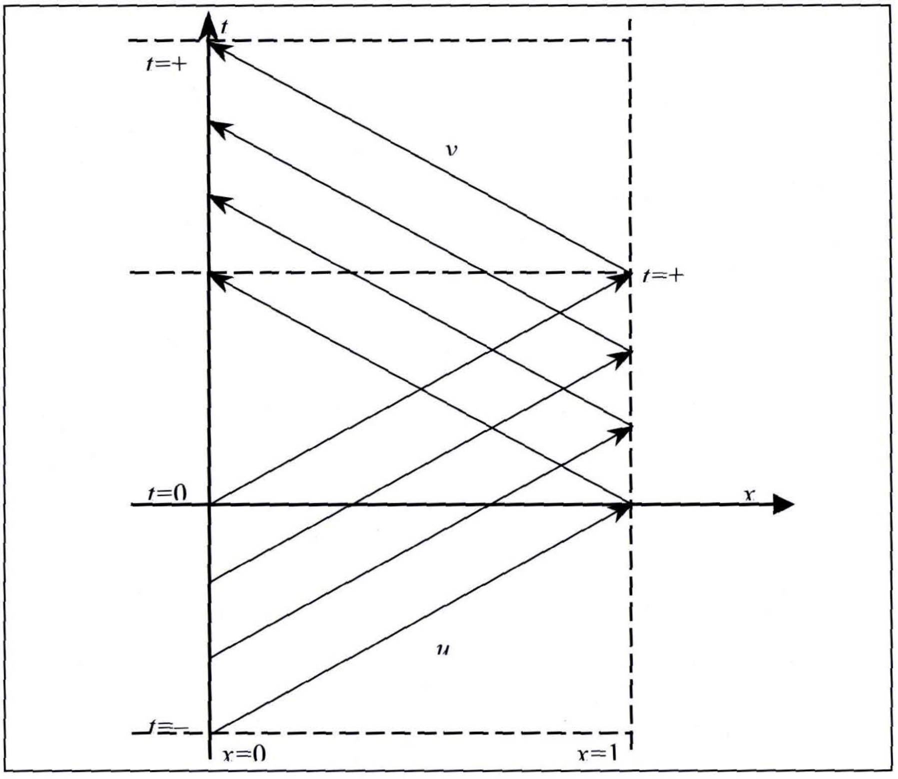

Fig. 1. The characteristics of the eqs. 1, 2.

That is the solutions in such case have the forms $u(x,t)=u(x-t)$; $v(x,t)=v(x+t)$. This corresponds to the properties of hyperbolic eqs.1, 2. So because the boundary conditions from eqs. 5, 6 we have

$$u(0,t+2)=v(0,t+2)=v(1,t+1)=f(u(1,(t+1))=f(u(0,t)),$$

and solution of problem eqs. 1- 4 has the form (Sharkovsky et al, 1993)

$$u(x,t)=y(t-x),$$

$$v(x,t)=y(t+x),\ (x,t)\in\Omega\ , \tag{9}$$

where $y(t)$ is the solution of difference equation

$$y(t+2)=f(y(t)),\ t\in[-1,\infty), \tag{10}$$

with the initial condition

$$y(t)\big|_{[-1,1)}=\varphi(t)=\text{def}\begin{cases}\varphi_1(-t),\ t\in[-1,0)\\ \varphi_2(-t),\ t\in[0,1)\end{cases}. \tag{11}$$

Thus the solution of partial differential eqs. 1, 2 had been related to the solutions of difference equations with continuous time eq. 10. Remark that one of the main examples of function which had been considered at (Sharkowsky et al, 1993) was

$$f(\omega)=\lambda\omega(1-\omega) \tag{12}$$

(eq. 12 is the logistic equation but with continuous time).

Many of important results had been received by A. Sharkovsky with colleagues. As one of the most interesting results we remark the limiting behavior of the solution eq. 10 when $t\to\infty$: relaxation oscillations; turbulent solutions and multivalued limiting function (Sharkovsky et al, 1993; Romanenko&Sharkovsky, 1999; Sharkovsky, 2006). Because of eq. 9 the properties of the solutions $u(x,t)$, $v(x,t)$ can be reconstructed from the behavior of solutions of difference eq. 10. Namely this property is crucial for investigation of the problem eqs. 1 – 4. Remark that also stochastic solutions had been found which had been named 'ideal turbulence' or 'dry turbulence' which has properties remembering real turbulence properties - see ((Sharkovsky et al, 1993; Romanenko&Sharkovsky, 1999; Sharkovsky, 2006) and pictures there.

Now let us try to extend the proposed initial-boundary value problem to the study of anticipatory effects and its manifestation in distributed systems.

We propose to introduce the anticipation into such model by considering the boundary conditions with anticipation (instead of eqs. 3, 4) and by using the properties of characteristics of eqs. 1, 2. Of course there are many presumable boundary conditions with anticipation accounting (including of integral type) but in this paper we will try to use for illustration of ideas the simplest possible (but remark that just such 'simplest' conditions doesn't means the simplicity during mathematical investigations).

3. Boundary conditions with the anticipation

For illustration of anticipatory effects into model above we introduce the anticipation into boundary conditions for eqs. 1, 2. Such conditions may have rather general form (or just more complex):

$\Phi\left(\{u(0,t_i)\},\ \{u(1,t_j)\};\ \{v(0,t_k)\},\ \{v(1,t_l)\}\right)=0$,
where $\left(t_i,t_j,t_k,t_l\right)$ - some moments of time.

But for the discussion on presumable effects of anticipation accounting we restrict the consideration at this paper by considering the next boundary condition at the right point of the interval $(x=1)$

$$v(1,t+1)=f(u(1,t+1),\ u(0,t+2)). \tag{13}$$

The eq. 13 replaces the eq. 6. This leads (just as for the eqs. 1 – 6) to the study of equation

$$u(0,t+2)=f(u(0,t),u(0,t+2)) \ . \tag{14}$$

So the counterpart for problem eqs. 10 – 11 with continuous time is the investigation of equation

$$y(t+2)=f(y(t),y(t+2)) \ . \tag{15}$$

With the initial condition on the interval $[-1,1)$ of the form

$$y(t)\big|_{[-1,1)}=\varphi(t)=\begin{cases}\varphi_1(-t), & t\in[-1,0)\\ \varphi_2(-t), & t\in[0,1)\end{cases}. \tag{16}$$

Let us suppose that we can find (by any of precise methods, or in some cases numerically) the solution $y(t)$ of the problem eqs. 15, 16. Then the solution of the problem eqs. 1 – 5, 13 has the form $u(x,t)=y(t-x)$, $v(x,t)=y(t+x)$ which is very useful for qualitative understanding of solution components $u(x,t)$, $v(x,t)$ behavior.

The eq. 15 with the initial condition eq. 16 may be very complex mathematical object. Because of the presumable multi-valuedness of solution the strict mathematical formulation of results will be forthcoming.
But some initial considerations may be posed now.
The set of possible values of the solution of the problem eq. 15, 16 may be infinite. So if the possible values $y(t)$ is the closed interval I, then $y(t) \in 2^I$, where 2^I is the set of all subset of interval I

$$y(t): R^+ \to 2^I, \quad . \tag{17}$$

Remark that usually $I = [0,1]$ had been considered.
One of the most interesting qualitative (and quantitative) problems is the asymptotic behavior at $t \to \infty$ and the dependence of the properties of the solution under the values of initial conditions

$$x(t) = \varphi(t), \ \ \varphi(t) \in 2^I, \ t \in [0,1].$$

Remark that just if f is single-valued function, at further moments of time the solutions may become multi-valued (Romanenko&Sharkovskiy, 1993), so it is useful to consider the general multi-valued case from the beginning.
The detailed investigation of such multivalued dynamical systems may constitute the subject of further mathematical investigations.
Especially interesting is the problem of limiting behavior as $t \to \infty$: limit solutions, attractors of such solutions, probability properties of limiting objects.
But also the new problems arrows: observability of solutions, selection the single-valued trajectories of the system, its limiting objects complexity and such complexity measures, searching adequate mathematical spaces for considering the problems and solutions.
Just the problems of adequate definition of periodic behavior (and moreover of dynamical chaos), Lemeray's staircase, Poincare's bifurcation diagram, ergodicity, mixing (Moon, 1990; Shuster, 1992) are interesting for multivalued case.

4. Examples of anticipatory boundary conditions

The number of the boundary conditions with anticipation accounting may be very large. Remark that searching correct and important for interpretations boundary conditions constitutes separate research problems.
But just now some already investigated discrete time equations with anticipation exist which may serve as the prototype for the continuous time difference equations.
Here we pose the list of such examples and description of presumable effects induced by such difference equations.

3.1 Variants of logistic equations

The classical logistic equations with discrete time has the form:

$$x_{n+1} = \lambda x_n (1 - x_n) \ , \tag{18}$$

$$t = 0,1,2,\ldots \ ,$$

where $x_0, x_1, x_2, \ldots, x_n, \ldots$, $x_i = x(i)$ is the values of unknown function in discrete moments of time $t = 0,1,2,\ldots,n$, λ – parameter. Having initial condition $x_0 = a$ the solution at the next discrete moments of time can be calculated.

Logistic equation is the source for proposing of many other difference equations. Starting from eq. 18 some equations with anticipation had been proposed

$$x_{n+1} = \lambda x_n (1 - x_{n+1}) ; \tag{19}$$

$$x_{n+1} = \lambda x_{n+1} (1 - x_{n+1}) ; \tag{20}$$

$$x_{n+1} = \lambda x_{n+1} (1 - x_n) \ . \tag{21}$$

The eqs. 19, 21 are less interesting because of linearity. The eq. 20 is more interesting because for some range of parameters it has two different values of solutions at each time moment (Dubois, 1998).

$$x_{n+1}^{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{\left(1 - \tfrac{4}{\lambda} x_n\right)},$$

Starting from eqs. 19 – 21 we may propose the difference equations with continuous time

$$x(t+1) = \lambda x(t)(1 - x(t+1)) ; \tag{22}$$

$$x(t+1) = \lambda x(t+1)(1 - x(t+1)) ; \tag{23}$$

$$x(t+1) = \lambda x(t+1)(1 - x(t)) \ ; \tag{24}$$

The most interesting is eq. 23 which supplies the possibilities of emerging of two values for given time moment. Taking eqs. 22 – 24 the initial conditions have the form

$$x(t) = \varphi(t), \ t \in [-1,0] \ . \tag{25}$$

Then it is easy to calculate the solutions of the eqs. 22 – 24, and their limiting behavior for $t \to \infty$.For example in numerical calculations we can calculate the solutions step by step for time intervals

$$(0,1],\ (1,2],\ (2,3], \ldots ,(n,n+1] \quad (26)$$

3.2 Example with cubic nonlinearities

Some modification of eqs. 19 – 21 included some auxiliary nonlinearity (Dubois, 1998; Krivy, 2008).

$$x_{n+1} = \lambda x_n^2 (1 - x_n) \ ; \quad (27)$$

$$x(t+1) = \lambda x^2 (t+1)(1 - x(t)) . \quad (28)$$

3.3 Weighted logistic equation with anticipation

Other kind of logistic equation is the equation

$$x_{n+1} = \alpha \lambda x_n (1 - x_n) + (1 - \alpha) \lambda x_{n+1} (1 - x_{n+1}) , \quad (29)$$

where $\alpha \in [0,1]$ is some parameter (weight of anticipation). The case $\alpha = 1$ corresponds to the absence of anticipation and $\alpha = 0$ to the case of full anticipation. Remark that such kind of anticipation corresponds to the accounting some mean characteristics of the system and had been used for example in considering some cellular automata (Makarenko et al, 2008).

3.4 Two-step model with anticipation

Such equations had been proposed in (Makarenko&Stashenko, 2005). The equations at discrete moments of time t have the next form:

$$\begin{cases} p_{t+1} = \alpha \cdot m + (1-\alpha) p_t - \alpha \cdot f(p_{t+2} - p_{t+1}) \\ p_{t+2} = \alpha \cdot m + (1-\alpha) p_{t+1} - \alpha \cdot f(p_{t+2} - p_{t+1}) \end{cases} . \quad (30)$$

The function $f(x)$ depends on the parameter α and has the following expression:

$$\begin{cases} f(x) = 0, & x \le 0 \\ f(x) = \alpha \cdot x, & x \in (0, 1/\alpha] \\ f(x) = 1, & x > 1/\alpha \end{cases} . \quad (31)$$

The solutions of these equations are especially interesting because its behavior remember behavior in the case of transition to chaos (at least the existence of periodic solutions with increasing of period length had been found in the case of multi-valued solutions). One of possible counterparts of eq. 30 has the same form but the values of time t are continuous.

All the examples from subsection 3.1 – 3.4 may be used as the examples of investigation of problem with eqs. 1, 2 (of course with slight modification for relating values of moments t and t+2 (which is useful for investigation the problem eqs. 1 – 5, 14 of distributed models)).

The detailed analysis of proposed mathematical problems will be the subject of future investigations. But just now it may be proposed some presumable consequences of described mathematical problems. So in the next section we pose some comments and presumable interpretations of results.

4. Presumable manifestations of anticipation in distributed systems

At previous sections of the paper we described formulations of one class of distributed systems with anticipation. The detailed investigation of such object is task of further studies. But just now we can discuss some interesting possibilities. First of all such presumable solutions of distributed models can help in understanding of anticipatory effects manifestation. At the level of the discrete equations with continuous time t we may received the multivalued counterparts to the complex single-valued solutions from (Sharkowsky et al, 1993). The analog to relaxation oscillations – multivalued relaxation oscillations may looks as single –valued with elementary structures replaced by multi-valued. The turbulent oscillations in multivalued case consist with multivalued counterparts to single-valued elements of solutions. In the case of multivalued oscillations the characteristic length of multivalued structures will tend to 0 as $t \to \infty$. This follows to considering the limits of multivalued functions. Remember that already in single-valued case such limit is multivalued function. One presumably useful problem for such case is the investigation of Koh's fractals posed on the unit interval with decreasing support of basic structure and increasing number of such structures at unit interval.

At the level of distributed system which is described by system of hyperbolic equations all peculiarities of solutions of difference equations will determine the properties of distributed system solutions because of eq. 9. So first of all interesting are the properties multivalued solution (especially presumable stochastic properties). It may be proposed that the solutions of the case of multivalued turbulent oscillations may lead to new type of turbulence when we can receive the complex behavior of multivalued functions. We can name such presumable case as 'anticipatory turbulence'. The relation to Young's measure-valued solutions may be interesting in such problems.

Other problem for investigation is the problem of measurement (observation) of such multivalued stochastic solutions - that is the problem of receiving the singlevalued

selection of such solution and investigation of its properties (including complexity, degree of stochasticity etc).
Such studies (and some others) are more or less evident consequences of proposed mathematical problems. But some absolutely new problems and relations may arise. First of all remark that the wave equation (and just electromagnetic field equations) has the type of eq. 1, 2. So some questions may be proposed for such objects. In electromagnetic theory implicitly retarded and advanced waves and potentials are present. So presumably the anticipated boundary conditions may serve also for electromagnetic theory for choosing advanced and retarded potentials and its relation.
The interpretations and origin of anticipated boundary conditions like eq. 13 may be interesting problems. From the physical point of view such conditions may correspond to existing extra channels relating the values at different time moments (or to the channels of transmission information about values at different time moments). Also such questions may be investigated in the frames of hidden space-time dimensions or hidden parameters. Remark that such issues remember some theories of brain activities with considering extra dimensions.
Next possible interesting problem is some kind of unusual inverse problems. Let us suppose that we can observe the solutions of the problem of eqs. 1-5 with anticipatory boundary conditions eq. 13 only on the restricted space subinterval [a, b] of [0, 1]. Suppose that we don't know the boundary conditions at all (but know only the form of eqs. 1, 2. How can we reconstruct the type, form and details of boundary conditions from the observed behavior of the solution (especially from the multivalued properties or just from the properties of measured singlevalued selection). Remark that also the values of initial conditions may be unknown (non-observable) and we may have the observation of solution at finite time interval [T1, T2] from $[0, \infty)$, T1>0. Remark that this problem remembers some problems in cosmology and may be related to the problem of origin of quantum-mechanical properties.
So recently it is evident that many problems may be proposed for investigation. Here we only remark that the familiar problems (with anticipatory boundary conditions) may be proposed for other distributed systems (for arrays of coupled oscillators or other elements, neural networks, other systems of partial differential equations, stochastic and integral equations etc.). Remark that some simple variants of boundary conditions had been considered for CA in earlier works by D.M. Dubois.

5. Conclusions

Thus in given paper we propose to extend the studies of anticipatory effects on some distributed systems which have classical description by partial differential equations. The anticipatory conditions for given hyperbolic system in bounded space domain allows transforming the studies of distributed in space system to studies of other new object – difference equations with anticipation with continuous time. This allows posing new mathematical problems on investigation of such new objects and its solutions. Just first studies allow to found new properties and to propose new research problems.

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