About a Theory of the Dual Relativity in a Universe With Three Time and Three Space Dimensions

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Abstract
Starting from a requirement of space-time symmetry, we propose several concepts of additional time dimensions, we study the consequences of the introduction of a multi-dimensional time in the special theory of Relativity and in relativist quantum theory.
We show that the mathematical process for generalizing the Dirac equation requires octonions and a 8 dimensional space-time with a privileged time direction. We also show that unlikely quantum correlations in pair of particles (as well as mechanical correlations in two coupled harmonic oscillators) imply a homogeneous 3-dimensional time due to the inseparability of space coordinates in the space continuum.
We then propose to define a 3-dimensional time with quantum hidden variables and we suggest that the scalar energy conservation is a consequence of the general parallelism of time flow at the macroscopic level.
Keywords: dual Relativity, Dirac equation, multi-dimensional time, quantum correlations, hypercomplex numbers.

1 Introduction

In previous communications, Daniel M. DUBOIS (1999) [1], Daniel M. DUBOIS and Gilles NIBART (2000) [2] have developed a theory of a dual Relativity derived from forward-backward space-time shifts. The continuous version of the discrete forward-backward space-time derivatives uses complex coordinates where imaginary components represent the space-time shift.

From this theory [2], the authors have deduced the equation of KLEIN [3], GORDON [4], and FOCK [5] in the 3+1 space-time; they have deduced the wave equation for photons and justified the use of plane waves in quantum theory as an interpretation of

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the mass. The theory of dual Relativity has superluminal solutions with an imaginary rest mass (tachyons), still in the 3+1 space-time.

Afterwards, the problem of space-time inversion has been raised from symmetry considerations between the subluminal Lorentz group SO(3,1,R) and the superluminal Lorentz group SO(1,3,R) and also from the inversion of the metric signature (+ – – –) at critical radius in the general theory of Relativity (e.g. the standard Schwarzschild metric has a pseudo-singularity[i] at its critical radius r_0 where the sign of gravitation potentials g_{00} and g_{11} is changed).

Remark: We use the same notation SO(p,q,E) to name a space-time having $p+q$ dimensions, a rotation group in this space-time, or the associated Lie algebra: p is the space dimension, q is the time dimension and E a set of numbers, usually R or C.
The notation SO(q,E) is the usual one in mathematics.

So there is no operator in the complex rotation group SO(4,C) which transforms a referential of the group SO(3,1,C) to one of SO(1,3,C), the space-time inversion is not possible. Let's say more simply that because t is a scalar and $\boldsymbol{r}$ is a vector they cannot be permuted. The solution needs to define a 3-dimensional time.

In this communication, we propose several concepts of additional time dimensions, we study the consequences of the introduction of a multi-dimensional time in the special theory of Relativity, in quantum theory, we develop a method for the generalization of the Dirac equation to a multi-dimensional time, and we discuss the problems of energy conservation and non-local correlations.

We show that in a pair of inseparable particles the symmetry of quantum correlations is represented by the transformation group SO(3,3,R) as in mechanical correlations of two coupled harmonic oscillators, and we deduce that this representation is done in a 3+3 space-time having a 3-dimensional time.

2 A Geometric Concept of a Three Dimensional Time

To introduce a 3-dimensional time, we replace the usual time coordinate:

$$x_0 = ct \tag{1}$$

by the 3 time coordinates defined as:

$$\begin{cases} x_{01} = ct_1 \\ x_{02} = ct_2 \\ x_{03} = ct_3 \end{cases} \tag{2}$$

The 3 time coordinates defined in equations (2) and the 3 usual space coordinates x_1, x_2, x_3 define a referential frame in an affine manifold which has a metric of signature (+ + + – – –). So its transformations belong to the group SO(3,3,R).

Although it easy to write 3 scalar time derivatives:

$$\frac{\partial}{\partial t_1}, \ \frac{\partial}{\partial t_2}, \ \frac{\partial}{\partial t_3} \tag{3}$$

[i] For $r = r_0$ it is called a « pseudo-singularity », and for $r = 0$ it is a true singularity.

similarly to the usual continuous time derivative, we have to consider all geometric postulates which are implicitly involved in this definition.

2.1 A 3-Dimensional Time With a Euclidean Geometry

If the 3 scalar time derivatives are considered as components of a vector, it is a "time gradient":

$$\vec{\nabla}_t = \frac{1}{c}\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right) \tag{4}$$

which belongs to a Euclidean geometry of 3 dimensions. The scalar product of the time gradient and a 3-dimensional time variation:

$$dt = \frac{\partial t}{\partial t_1} dt_1 + \frac{\partial t}{\partial t_2} dt_2 + \frac{\partial t}{\partial t_3} dt_3 \tag{5}$$

is the differential dt of the scalar time t which can be considered as the observed time t.

The differential dt has to be summed up along a time trajectory, so the resulting scalar time t is a function of 6 coordinates:

$$t = f(t_1, t_2, t_3; x_1, x_2, x_3) \tag{6}$$

2.2 A 3-Dimensional Time With a Curvilinear or Riemann's Geometry

Instead of the Euclidean gradient, we may consider the 3 covariant time derivatives such as:

$$D_{01} = \frac{D}{Dx^{01}}; \quad D_{02} = \frac{D}{Dx^{02}}; \quad D_{03} = \frac{D}{Dx^{03}} \tag{7}$$

where every covariant derivative of a vector V^i is classically defined [6] as:

$$D_{0k} V^i = \partial_{0k} V^i + \Gamma^i_{0k,h} V^h \tag{8}$$

Here the summing index h takes every time value 01, 02, 03, and every space value 1, 2, 3. If h takes only the time values, the time geometry would be independent of the space geometry, but we assume that space-time is a relativist continuum.

Some additional postulate is needed to define affine connection coefficients $\Gamma^i_{k,h}$ of a curvilinear "time manifold".

2.3 The Correlative 3-Dimensional Energy Vector

In relativist quantum theory the energy operator is the following time derivative:

$$E = i\hbar \frac{\partial}{\partial t} \tag{9}$$

so the 3-dimensional time derivatives in equation (3) result in 3 energy components.

In an Euclidean geometry, the time gradient (4) defines the 3-dimensional energy quantum operator **E**:

$$\mathbf{E} = i\hbar c \nabla_t \tag{10}$$

which can be called the "time momentum" $\mathbf{p}_t$:

$$\mathbf{p}_t = i\hbar \nabla_t \tag{11}$$

With both the time momentum $\mathbf{p}_t$ and the usual momentum operator $\mathbf{p}$:

$$\mathbf{p} = -i\hbar \nabla \tag{12}$$

we can build the 6-momentum operator:

$$\hat{\mathbf{p}} = i\hbar \left(\nabla_t, -\nabla \right) \tag{13}$$

in the 3+3 space-time having a metric $g^{\mu\nu}$ of signature (+ + + − − −) and we can write it with the usual tensor formalism:

$$\hat{p}^{\mu} = g^{\mu\nu} \frac{\partial}{\partial x^{\nu}} \tag{14}$$

So we can generalize the theory of Relativity to a 3-dimensional time using the transformation group SO(3,3,R).

In such a theory, any transformation of a referential will conserve the 6-momentum vector $\hat{\vec{\mathbf{p}}}$ while the Relativity conserves the momentum-energy 4-vector. At the classical limit, the 3-momentum $\vec{\mathbf{p}}$ and the energy vector $\vec{\mathbf{E}}$ are conserved, but the scalar energy, which is the length of the energy vector $\vec{\mathbf{E}}$, is not conserved.

Any interactions of *n* non relativist particles in a 3-dimensional time will conserve the sum of energy vectors:

$$\sum_{k=1}^{n} \vec{\mathbf{E}}_k = \text{constant} \tag{15}$$

The usual scalar energy in interactions of *n* non relativist particles:

$$\sum_{k=1}^{n} E_k = \sum_{k=1}^{n} \left\| \vec{\mathbf{E}}_k \right\| \tag{16}$$

will be conserved when the energy vector $\vec{\mathbf{E}}_k$ of the *n* particles are parallel, i.e. when all their time trajectories are parallel.

The isotropic multidimensional time leads to an energy vector with the same number of dimensions and the usual scalar energy conservation principle does not hold, except with a general parallelism of time flow. So we may give up the concept of isotropic multidimensional time, or we have to explain the general time flow parallelism.

3 The 3D Time From the Point of View of Electrodynamics and Relativity

After many attempts in unification of fields with the introduction of additional space dimensions, many authors have proposed to introduce new time dimensions in physics. For symmetry reasons many works consider a 3-dimensional time with the usual 3-dimensional space.

The symmetrization of Maxwell equation has led several authors [7,8,9,10,11,12] to develop a theory of the electromagnetic field in a 6-dimensional space-time. To match the relativist electromagnetic field in the usual 3+1 space-time, the authors have generalized the special Lorentz group into a 6-dimensional space-time, as it is also required by the bradyon-tachyon symmetry in theories [13,14,15,16,17] of superluminal particles.

Several authors [18,19,20,21,22,23] have raised some valid critics against the idea of an homogeneous 3-dimensional time, considering that the resulting energy vector is not consistent with energy conservation principle.

Physical properties of an isotropic[ii] multi-dimensional time have been studied by Vladilen BARASHENKOV, such as electrodynamics in multi-dimensional time [24], quantum field with 3D vector time, 6D transformations [25] and detection of rays in multi-time [26].
The introduction of arbitrary time trajectories in some classical particles would generate a faster-than-light behavior, like tachyons (see sect. 3 in ref. [27]) and it would turn light sources, even at rest, into invisibility for the observer after a very short time (see sect. 3 in ref. [27]). Moreover the internal coherence of waves and particles in matter requires a high level of time flow parallelism with the time flow of observers (see sect. 5 in ref. [27]).

According to Vladilen BARASHENKOV, the time flow parallelism is tightly related to the time arrow and energy conservation [28], and objects with different time trajectories are only possible at quantum level when energy conservation is violated in a quantum transition.

The possibility of predictions by observers have been analyzed by Max TEGMARK [29] in a p+q dimensional space-time, from mathematical properties of the space-time partial derivative equation of second order:

$$\left[\sum_{i=1}^{d}\sum_{j=1}^{d}\mathbf{A}_{\mathbf{ij}}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}+\sum_{i=1}^{d}b_i\frac{\partial}{\partial x_i}+c\right]u=0 \tag{17}$$

where $d = p + q$ is the number of space-time dimensions and $\mathbf{A}_{ij}$, b_i, c are parameters.
He has studied the different cases of the matrix $\mathbf{A}_{ij}$ corresponding to elliptic, hyperbolic and ultra-hyperbolic metrics. From L. ASGEIRSSON's theorem [30] Max TEGMARK has deduced that observers cannot do any prediction in multi-dimensional time directions (with 3D objects in a 3D space) because in an ultra-hyperbolic space-time "there are no space-like nor time-like hypersurfaces".

So, from the physical requirement of a time flow parallelism in our world, we have deduced that all natural observers have the same time trajectory and all technical devices built by natural observers have also the same time trajectory; but it does not prove that time is not multi-dimensional. As in this theory, scalar energy conservation is a consequence of the time flow parallelism, we may think that if we could master time trajectories of physical systems, we would start developing new technologies, finding new types of energy sources.

[ii] "isotropic" in the sense of physics, means "homogeneous".

A possible solution to the above problems due to the homogeneous multi-dimensional time may be to define a heterogeneous multi-dimensional time with the following postulates:

- the definition of the usual coordinate x_0 is unchanged (longitudinal time),
- additional time directions are not ruled by the theory of Relativity (transversal time).

Using the geometric algebra pioneered by HESTENES [31,32,33,34] continued by other authors cited by C. DORAN and A. LASENBY [35], John E.CARROLL [36,37] has studied the embedding of the usual 3+1 space-time in a symmetric 3+3 space-time, has deduced Maxwell's equations from this symmetry and he has found a "3D time which has a preferred direction for observed time" (cit. from ref. [37]).

In his formalism, the 6 coordinates are named:

$$x_{t1}, x_{t2}, x_{t3}, x_{s1}, x_{s2}, x_{s3} \tag{18}$$

where t and s are label for time and space. The metric has the signature $(+++---)$ and the equation of KLEIN [3], GORDON [4], and FOCK [5] is written:

$$\left(\frac{\partial^2}{\partial x_{t3}{}^2}-\frac{\partial^2}{\partial x_{s1}{}^2}-\frac{\partial^2}{\partial x_{s2}{}^2}-\frac{\partial^2}{\partial x_{s3}{}^2}\right)\boldsymbol{\Phi}+\mathbf{M}_0^2\boldsymbol{\Phi}=0 \tag{19}$$

where $\mathbf{M}_0$ is the "rest mass operator" defined by:

$$\mathbf{M}_0^2=\frac{\partial^2}{\partial x_{t1}{}^2}+\frac{\partial^2}{\partial x_{t2}{}^2} \tag{20}$$

and the usually measured time t is:

$$t=\frac{1}{c}x_{t3} \tag{21}$$

So the 2D transverse time is linked to the rest mass and the author states: "harmonic variations in transverse time give zero rest mass in free space for complex electromagnetic fields" (cit. from ref. [37]).

Xiaodong CHEN [38] has tried to introduce some additional time dimensions, which are compactified at the Planck scale, and he has built a modified version of the KALUZA [39] and KLEIN's [40] theory.

But his 6-dimensional metric (eq. 17 page 4 in ref. 38):

$$\hat{g}_{AB}=\begin{pmatrix} g_{\alpha\beta}(x^\alpha) & 0 & 0 \\ 0 & \psi(x^A,p^\alpha,m_0) & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad (\alpha,\beta=0,1,2,3 \quad A,B=0,1,2,3,4,5) \tag{22}$$

results in the geodesic differential (see eq. 19 page 4 in ref. 38):

$$ds^2=dx_\alpha dx^\alpha+\psi\, dx_4 dx^4-dx_5 dx^5 \quad (\alpha=0,1,2,3) \tag{23}$$

where ψ is the wave function of a given particle. Which one in the whole universe? For a free spinless particle he writes (see eq. 19 page 4 in ref. 38):

$$\Psi=e^{\frac{i}{\hbar}\left(p^\alpha x_\alpha-m_0x_5\right)} \tag{24}$$

So the potential g_{44} depends on the rest mass and momentum of one given particle and therefore $\hat{g}_{AB}$ is not a true metric tensor of a Riemann geometry.
The field potential g_{55} is negative so x_5 is a space coordinate and unfortunately the Xiaodong CHEN's theory has no additional time dimension.

In an other way, D.G. PAVLOV [41] has studied some peculiar Finslerian spaces of 2 to n dimensions, built on isotropic[iii] vectors, and he discussed the geometric shapes of equivalent light cones having additional time dimensions, from a mathematical viewpoint. The metric of his Finslerian spaces have the degree n so they are not quadratic forms, except in the 2-dimensional case, which is the metric of an Infinite Momentum Frame, i.e. a system of light-cone coordinates.

This suggests studying the problem of multi-dimensional time in the context of Infinite Momentum Frames, as defined by G. NIBART [42,43].

4 The 3D Time From the Point of View of Quantum Theory

4.1 General Considerations About Quantum Theory

As in the ZEEMAN's theorem [44], most objections to the multi-dimensional time presuppose implicitly that the causal order must always coincide with a linear time order, but such a macro-causality postulate is not compatible with the micro-causality principle, which has been expressed by the Bell's theorem [45].

4.2 Possible Extensions of Quantum Theory to Multi-Time Coordinates

Hitoshi KITADA [46] has introduced 3-dimensional time and 3-dimensional energy operators in quantum theory and he has deduced the usual uncertainty relation for the scalar time and the scalar energy. But unfortunately his time coordinates t_x, t_y, t_z, are not intrinsic coordinates: we criticize them further in section 5.

Starting from the generalized Dirac equation of PATTY and SMALLEY [47], BARASHENKOV and others [48,49] have proposed some interpretations of the relativist quantum theory in a 3+3 space-time. Because of the multi-dimensionality of time and energy, there is no energy gap between particles and antiparticles (figure 1 in page 42 of ref. [48]), so the author has only forbidden free transitions between particles and antiparticles by an additional postulate of microscopic time irreversibility.

The theory is not compatible with time reversal of the CPT transformation group, and the reinterpretation principle of STÜCKELBERG [50] and FEYNMAN [51] which is currently used in FEYNMAN's diagrams, does not hold in BARASHENKOV's theory.

Unfortunately the 8×8 Dirac matrix $\hat{\gamma}$ cannot be multiplied with the 6-vector $\hat{\nabla}$ in the equation (1) in page 38 of ref. [48] shown below:

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iii Mathematically, an isotropic vector is a null length vector, e.g. a vector on the light-cone surface (an equal combination of space and time components).

$$\left(i\hat{\gamma}\hat{\nabla}-m\right)\Psi=0 \tag{25}$$

and also the 4×4 Dirac matrix $\boldsymbol{\gamma}$ cannot be multiplied with a 3-vector $\mathbf{p}$ in the equation (2) in page 39 of ref. [48] shown below:

$$(\boldsymbol{\gamma}\,\mathbf{p}-\boldsymbol{\Theta}E+m)\Psi=0 \tag{26}$$

where E is the scalar energy and $\boldsymbol{\Theta}$ is a 8×8 matrix. So the BARASHENKOV theory is inconsistent. Moreover his "space- and time-independent eight-component spinor":

$$\Phi\left(\phi_1,\phi_2,\phi_3,\phi_4,\phi_5,\phi_6,\phi_7,\phi_8\right)^T \tag{27}$$

should be related to a spinor representation of the 3+3 space-time and to the SO(3,3,R) transformation group. This problem has not been mentioned by the authors.

4.3 Introduction of a Hidden Multi-Time in Quantum Theory

Xiaodong CHEN has proposed a new interpretation of quantum theory [52] where additional time dimensions are hidden variables. He has preferred a 3-dimensional time just to have the symmetry of space and time.

A relativist quantum theory with hidden time directions might be developed in the standard Minkowski space-time, so it does not need a modified KALUZA-KLEIN theory [38], and the author suggests that the 3+1 space-time may be a tangent space on the 3+3 space-time, like the HOPF bundle in a monopole theory [53]. We would rather prefer a theory using the Riemann geometry.

The basic concept of multi-dimensional hidden quantum time is the following: the multi-dimensional time flow is not a classical trajectory. The multi-dimensional time flow of Xiaodong Chen is "a set of quantum time paths" and every time path is possible with a given probability, which corresponds to a particular quantum state. Every quantum time path has a specific direction in the multi-dimensional time manifold and the direction of every time path can be represented by an angle. Any time angle is a hidden variable in the sense of the Bell's theorem [45].

In a 2D time manifold, polar coordinates (t,θ) are used, and in a 3D time manifold spherical coordinates (t,θ,ϕ) are used. So hidden variables are time path angles (θ,ϕ).

As clocks are macroscopic devices, any measured time is the mean length t of all probable time paths (clock) or the true length t of a collapsed state (detection device).

4.4 Wave-Package Collapse Conjecture on Multi-Dimensional Time

We can explicitly develop quantum axioms from the wave-package collapse conjecture of Xiaodong CHEN, as stated below:

- the set of time paths (with its hidden angles θ,ϕ) allow the distribution of all expected values of an observable within the multi-dimensional time manifold ;
- different expected values of an observable correspond to different time paths having different angles ;
- the whole distribution of time paths is modified by any time measurement ;

- the current time path of a detector (an observer or a measuring device) allows only one possible interaction in the multi-dimensional time manifold, and this interaction gives only one expected value of the observable as a measured value.

5 About the Definition of the 3D Time From a Velocity or a Momentum

JORGE and FRANCO [54] have defined a time vector with a geometric projection of the clock time t on the 3-space axis x_1, x_2, x_3, such as:

$$t_i = t \cos\alpha_i \quad (i = 1,2,3) \tag{28}$$

with a projection angle α_i defined by vector directions:

$$\alpha_i = (\vec{x}_i, \vec{\mathbf{v}}) \quad (i = 1,2,3) \tag{29}$$

where $\vec{\mathbf{v}}$ is a velocity. As the relativist mass is a scalar, an equivalent definition can be proposed with a momentum vector $\vec{\mathbf{p}}$:

$$\alpha_i = (\vec{x}_i, \vec{\mathbf{p}}) \quad (i = 1,2,3) \tag{30}$$

Hitoshi KITADA [46] has defined his time vector from the momentum of a test particle.

In that definition, the time vector $\vec{\mathbf{t}}$ is not a vector variable but a function and its components t_x, t_y, t_z, are not intrinsic coordinates, because they depend on a given velocity $\vec{\mathbf{v}}$ or a given momentum $\vec{\mathbf{p}}$ which has to be introduced to define the time vector.

The velocity $\vec{\mathbf{v}}$ or the momentum $\vec{\mathbf{p}}$ may be that of a moving body (a projectile) or the velocity $\vec{\mathbf{v}}$ may be that of a second referential frame.

If the velocity $\vec{\mathbf{v}}$ is that of a moving body, time components t_x, t_y, t_z, in equations (4) page 37 of reference [54] should be defined as:

$$t_i = \frac{x_i}{\dfrac{\sqrt{dx_1^2 + dx_2^2 + dx_3^2}}{dt}} \quad (i = 1,2,3) \tag{31}$$

As the time vector depends on a body velocity, we can not define a common vector time for two moving bodies. So it is not an intrinsic time of the observer.

If the velocity $\vec{\mathbf{v}}$ is that of a second referential frame as it is considered in the JORGE and FRANCO's paper [54], it means that the time components of the observer O depends on the direction of the second observer O'. Therefore the vector nature of time, according to these authors, requires two related observers {O; O'}.

However the JORGE and FRANCO's theory [54] of vector time is not consistent with the Relativity, because their vector Lorentz transformation (the 4 first equations at the top of page 43 in reference [54]) does not reduce to the well-known special Lorentz transformation for the angles:

$$\alpha = 0; \quad \beta = 0 \tag{32}$$

These authors ignores that after A. Einstein, a general Lorentz transformation has been established in the tensor formalism by S. Kichenassamy [55] and in the vector formalism by C. MØLLER [56] and M.-A. TONNELAT [57], and it is fully compatible with the Relativity.

6 About the Relativist Space Time Entanglement

6.1 About the Symmetry of Coupled harmonic Oscillators

D. HAN, Y.S. KIM, and Marilyn E. NOZ [58] have built the O(3,3) group to describe the symmetries of two coupled harmonic oscillators and they have shown that the Lorentz group O(3,1) has a local isomorphism in O(3,3) depending on the choice of a time direction.

Considering any two coupled harmonic oscillators (eq. 2.1 in ref. [58]), there is a very particular referential where the two harmonic oscillators appear to be independent from each other, i.e. decoupled (eq. 2.12 in ref. [58]). It means that a particular observer does not perceive the space-time entanglement of some coupled harmonic oscillators.

Their "local isomorphism" between O(3,3) and SL(4,R) is truly an endomorphism of O(3,1) in O(3,3), which is defined by the choice of one of the 3 time directions (s,t,u) as there is a Lorentz group in every time direction.

In another paper, Y.S. KIM, and Marilyn E. NOZ [59] have explained that two coupled harmonic oscillators are entangled in the sense of quantum theory. They have shown that the wave equation of two coupled harmonic oscillators:

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp\left[-\frac{1}{4} e^{-2\eta}(x_1 + x_2)^2 + e^{+2\eta}(x_1 - x_2)^2 \right] \tag{33}$$

where η is a coupling parameter, and the wave equation the covariant harmonic oscillator:

$$\psi_\eta(z, t) = \frac{1}{\sqrt{\pi}} \exp\left[-\frac{1}{4} e^{-2\eta}(z + ct)^2 + e^{+2\eta}(z - ct)^2 \right] \tag{34}$$

are formally identical, and the substitution:

$$x_1 \to z \quad x_2 \to x_0 = ct \tag{35}$$

shows that space and time are entangled in the wave function of covariant oscillators.

6.2 A Relativist Model of a Pair of Particles

Starting from a 3-dimensional oscillator which is described in a 3+1 space-time and transformed within the SO(3,1) group, D. HAN, Y.S. KIM, and Marilyn E. NOZ [58] have shown that two coupled hannonic oscillators can be transformed within the SO(3,3) group.

Consequently two coupled harmonic oscillators may be described in a 3+3 space-time and the 3 space coordinates of the second oscillator are working as a 3-dimensional time for the first oscillator. And reciprocally, the 3 space coordinates of the first oscillator are working as a 3-dimensional time for the second oscillator.

In this way, Gilles NIBART [60] has already proposed a mathematical process eliminating time coordinates from the system of equations of a pair of particles.
We start from a relativist quantum equation of a particle A, such as:

$$f_A(ct_A, x_{A1}, x_{A2}, x_{A3}) = 0 \tag{36}$$

and the similar equation of a second particle B:

$$f_B(ct_B, x_{B1}, x_{B2}, x_{B3}) = 0 \tag{37}$$

where f is a given function, A, B are labels, and x_i are space coordinates.

As the correlations of the particles in a pair are defined at the same time t:

$$t = t_A = t_B \tag{38}$$

the correlations can be expressed by one or several equations with the time t, from conservation laws (e.g. energy, momentum, spin). So using equation (38) it is possible to eliminate all time variables from the system of equations (36), (37) which then simplifies into a unique equation, such as:

$$\xi\left(x_{A1}, x_{A2}, x_{A3}, x_{B1}, x_{B2}, x_{B3}\right) = 0 \tag{39}$$

where the 2 sets of space coordinates x_{A1}, x_{A2}, x_{A3} and x_{B1}, x_{B2}, x_{B3} are inseparable variables.

Every equation (36) or (37) is individually transformed within the SO(3,1) group, but the transformations of both correlated equations (36), (37) belong the SO(3,3) group.

The SO(3,3) group is related to the metric of the 3+3 space-time having the signature $(+++---)$. It means that if we consider x_{A1}, x_{A2}, x_{A3} as spacelike coordinates, x_{B1}, x_{B2}, x_{B3} are timelike, and reciprocally.

For an observer looking at the particle A having the position x_{A1}, x_{A2}, x_{A3} and a timelike (subluminal) velocity, the 3 coordinates x_{B1}, x_{B2}, x_{B3} will appear as temporal and the x_{B1}, x_{B2}, x_{B3} derivatives will appear as spacelike, because of the apparent superluminal correlations (E.P.R. paradox) but it does not mean that one of the particles of the pair has a superluminal velocity.

In its 3+3 space-time referential, the particle A has the time and space coordinates:

$$\{x_{B1}, x_{B2}, x_{B3}; x_{A1}, x_{A2}, x_{A3}\} \tag{40}$$

with the signature $(+++---)$ while in its referential the particle B has the time and space coordinates:

$$\{x_{A1}, x_{A2}, x_{A3}; x_{B1}, x_{B2}, x_{B3}\} \tag{41}$$

with the same signature $(+++---)$.

Consequently a 3-dimensional time emerges from the space-time entanglement in a pair of particles or two coupled oscillators. And the relativist explanation of the E.P.R. paradox and quantum correlations of a pair of particles just hold in the transformation group SO(3,3) not in the Lorentz group.

7 About the Generalization of Dirac Equation to More Dimensions

Although it is easy to extend the Minkowski space-time with 2 additional time dimensions and write a 3D time derivative operator with a metric of signature (+ + + – – –), we must be very careful to build the generalization of Dirac equations to the 3+3 space-time because it will include generalized Dirac matrix which have to be associated to the Lie algebra SO(3,3,C) and generalized Dirac spinors Ψ which have to be associated to a spinor representation of the transformation group SO(3,3,R).

7.1 The Mathematical Process to Build Usual Dirac Equations

As the classical Hamiltonian function and the relativist Hamiltonian function are quadratic forms, from the basic quantum equation:

$$\mathbf{H}\Psi = i\hbar\frac{\partial}{\partial t}\Psi \tag{42}$$

they naturally led to quantum differential equations of second order in the space derivatives: respectively the Schrödinger equation and the equation of KLEIN [61], GORDON [62], and FOCK [63]. PAULI [64] has introduced the electromagnetic field potentials and his spin operator $\boldsymbol{\sigma}$ in the relativist Hamiltonian function, but his equation is still of second order in the momentum operator $\mathbf{p}$, i.e. in space derivatives.

DIRAC [65] has stressed that quantum mechanics requires equations which are linear in the wave function Ψ and in all space-time derivatives, so it requires differential equations of first order in space-time derivatives. Dirac applied a factorization method [65] to the equation of Klein, Gordon, and Fock, which led to the well-known Dirac equation.

The Dirac equation has been discussed by PAULI [66] and the algebra of Dirac matrix has been discussed by PAULI [67] and GOOD [68]. Later, the Schrödinger equation has also been linearized in space derivatives by LÉVY-LEBLOND [69].

The usual Dirac equation, in the spinor form is:

$$\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - \chi\right)\Psi = 0 \tag{43}$$

where χ is related to the rest mass with:

$$\chi = \frac{m_0 c}{\hbar} \tag{44}$$

Dirac matrix γ^{μ} are complex 4×4 matrix which works with relativist 4-vectors. They are super-matrix composed of Pauli matrix σ_i as shown below:

$$\gamma_0 = \begin{bmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{bmatrix}, \quad \gamma_n = \begin{bmatrix} 0 & \sigma_n \\ \sigma_n & 0 \end{bmatrix} \quad (n = 1,2,3) \tag{45}$$

Pauli matrix are expressions of infinitesimal rotation operators in spinor theory and the squares of Pauli matrix have the positive signature (+ + + +) as shown below:

$$\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_0 \tag{46}$$

where σ_0 is the identity matrix.

A fermion wave function Ψ is a bispinor:

$$\Psi \equiv (\psi_1, \psi_2) \tag{47}$$

which can be represented as a complex vector ψ with 4 vector components ψ^μ (μ=0,1,2,3). Thus the Dirac equation has a complex conjugate.

The transformations of bispinors Ψ belong to the group SL(2,C). Irreducible representations of the group SL(2,C) are equivalent to irreducible representations of the Lorentz group which is included in the transformation group SO(3,1,R). So SL(2,C) contains a spinor representation [70] of the Lorentz group.

The Pauli matrix σ_i are the generators of a Clifford algebra C(E_3) of 8 dimensions on the real set $\Re$, with the real vector basis:

$$\mathrm{C}(\mathrm{E}_3): \ \{\sigma_0, \sigma_1, \sigma_2, \sigma_3, i\sigma_0, i\sigma_1, i\sigma_2, i\sigma_3\} \tag{48}$$

where E_3 is the usual Euclidean space of 3 dimensions [71]. The Dirac matrix γ^μ are associated to the Minkowski space-time $E_{3,1}$ and are the basis of a Clifford algebra C($E_{3,1}$) of 16 dimensions on the real set $\Re$.

The quaternions algebra is isomorphic to the sub-algebra $C_+(E_3)$ generated by the following matrix:

$$\mathrm{C}_+(\mathrm{E}_3): \ \{\sigma_0, i\sigma_1, i\sigma_2, i\sigma_3\} \tag{49}$$

so Pauli matrix can be replaced by quaternions in quantum equations, such as:

$$\begin{cases} \mathbf{1} = \sigma_0 \\ \mathbf{i} = i\sigma_1 \\ \mathbf{j} = i\sigma_2 \\ \mathbf{k} = i\sigma_3 \end{cases} \tag{50}$$

and we stress that the product of two Pauli matrix is a quaternion:

$$\sigma_1 \cdot \sigma_2 = i\sigma_3 \quad \sigma_2 \cdot \sigma_3 = i\sigma_1 \quad \sigma_3 \cdot \sigma_1 = i\sigma_2 \tag{51}$$

Moreover the Minkowski space-time $E_{3,1}$ is isomorphic to the set **Q** of quaternions as shown below:

$$\mathbf{V} \leftrightarrow \mathbf{q} = ct\,\mathbf{1} + x_1\,\mathbf{i} + x_2\,\mathbf{j} + x_3\,\mathbf{k} \tag{52}$$

because the squares of quaternions units:

$$(\mathbf{1}^2, \mathbf{i}^2, \mathbf{j}^2, \mathbf{k}^2) = (\mathbf{1}, -\mathbf{1}, -\mathbf{1}, -\mathbf{1}) \tag{53}$$

match the signature ($+ - - -$) of the metric of the Minkowski space-time.

It has been possible to build the Dirac equation with Pauli matrix, from a Hamiltonian function of second order, because the set **Q** of quaternions is the factorization solution of the reduced positive quadratic form with 4 terms:

$$f(\mathbf{X}) = c^2t^2 + x_1^2 + x_2^2 + x_3^2 \tag{54}$$

which is the Minkowski metric multiplied by its own signature. The factorization uses the quaternions conjugates shown below:

$$f(\mathbf{X}) = (ct\,\mathbf{1} + x_1\,\mathbf{i} + x_2\,\mathbf{j} + x_3\,\mathbf{k}) \cdot (ct\,\mathbf{1} - x_1\,\mathbf{i} - x_2\,\mathbf{j} - x_3\,\mathbf{k}) \tag{55}$$

Remark: We may wish to factorize only the positive quadratic form of the 3-dimensional Eucidean space within the Minkowski space-time, but there is no hypercomplex numbers with 3 dimensions (see the HURWITZ'S theorem in section 7.2).

Furthermore, we know [72] that the subset $\mathbf{Q}_1$ of quaternions of unity length is a Clifford sub-group of $C(E_3)$ which can be identified to the matrix of the group SU(2) which represent infinitesimal rotations in the Eucidean space E_3. Therefore the algebra Q(2) of quaternion matrix is the Clifford algebra $C(E_{3,1})$ of the Minkowski space-time $E_{3,1}$ and the algebra $C(E_{3,1})$ is isomorphic to the algebra of Dirac matrix γ^{μ} [71]. Therefore quaternions, Pauli matrix, Dirac matrix and rotation operators are very tightly related to the 3+1 space-time structure, as G. NIBART [73] have shown it.

Further, the Dirac equation has been re-written as a "nilpotent Dirac equation" by Peter ROWLANDS [74,75] in a quaternion multi-vector formalism.

7.2 A Mathematical Process to Generalize Dirac Equations

To do the generalization of Dirac equations to an extended metric having more time or space dimensions, we will have to do the linearization of the positive quadratic form associated to the extended metric, because relativist quantum theory requires covariant equations of first order in space-time derivatives.

Obviously, we can easily write space-time derivatives in an extended Minkowski space-time or an extended Riemann manifold, having additional dimensions (see section 2). For example we can define a 3-dimensional time derivative as the vector:

$$\vec{\nabla}_t \equiv \frac{1}{c}\frac{\partial}{\partial \mathbf{t}} = \frac{1}{c}\left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2}, \frac{\partial}{\partial t_3}\right) \tag{56}$$

associated to a 3+3 space-time which has the following metric:

$$dS^2 = dx_{01}^2 + dx_{02}^2 + dx_{03}^2 - dx_1^2 - dx_2^2 - dx_3^2 \tag{57}$$

of signature $(+++---)$, and which have a symmetry between space and time within the transformation group SO(3,3,R).

Although it seems to be easy to write a generalized Dirac equation by substituting the scalar time derivative in the usual Dirac equation, by a vector time derivative, as shown below:

$$\frac{1}{c}\frac{\partial}{\partial t} \to \vec{\nabla}_t \tag{58}$$

the generalized Dirac equation has to be consistent with classical mechanics, with the theory of Relativity and with basic quantum theory. So it is more difficult to define correctly extended Dirac matrix and extended spinors in an extended Minkowski space-time (or in an extended Riemann manifold) because the new Clifford algebras and the new spinor representation of the new metric has to be consistent with the new transformation group, e.g. SO(3,3,C).

The consistency requires that the reduced positive quadratic form with 6 terms:

$$f(\mathbf{X}) = x_{01}^2 + x_{02}^2 + x_{03}^2 + x_1^2 + x_2^2 + x_3^2 \tag{59}$$

associated to the extended Minkowski metric, can be decomposed into a product of hypercomplex numbers **h**. So we may imagine a set **H** of hypercomplex numbers defined as:

$$\mathbf{h} = x_{01}\mathbf{h}_{01} + x_{02}\mathbf{h}_{02} + x_{03}\mathbf{h}_{03} + x_1\mathbf{h}_1 + x_2\mathbf{h}_2 + x_3\mathbf{h}_3 \tag{60}$$

which factorize the positive quadratic form such as:

$$x_{01}^2 + x_{02}^2 + x_{03}^2 + x_1^2 + x_2^2 + x_3^2 = \mathbf{h}\mathbf{h}^* \tag{61}$$

where $\mathbf{h}^*$ is the hypercomplex conjugate of **h**:

$$\mathbf{h}^* = x_{01}\mathbf{h}_{01} - x_{02}\mathbf{h}_{02} - x_{03}\mathbf{h}_{03} - x_1\mathbf{h}_1 - x_2\mathbf{h}_2 - x_3\mathbf{h}_3 \tag{62}$$

The squares of hypercomplex units must have the following properties:

$$\mathbf{h}_{01}^2 = +1, \mathbf{h}_{02}^2 = -1, \mathbf{h}_{03}^2 = -1, \mathbf{h}_1^2 = -1, \mathbf{h}_2^2 = -1, \mathbf{h}_3^2 = -1 \tag{63}$$

so the hypercomplex metric has the signature $(+-----)$ which is different from signature $(+++---)$ of the extended Minkowski metric.

The positive term $\mathbf{h}_{01}^2$ defines the "real" component $\mathbf{h}_{01}$ of an hypercomplex number **h**, therefore there is a privileged time direction.

The consistency requires also to build an irreducible spinor representation of the extended space-time, e.g. the 3+3 space-time manifold. We must not forget that "the geometries in N-dimensional Euclidean spaces can be described by Clifford algebras that were introduced as extensions of complex numbers" [76]. And extended Dirac matrix should be built from the same set of hypercomplex numbers.

The needed set **H** of hypercomplex numbers has to be a real division algebra, with the following important properties:

- it has a multiplicative identity element $\mathbf{h}_0 = \mathbf{1}$,
- every non-zero element **h** has a multiplicative inverse $\mathbf{h}^{-1}$ on both sides,
- every element **h** has an hypercomplex conjugate $\mathbf{h}^*$, similarly to quaternions $\mathbf{q}$, $\mathbf{q}^*$.

After the quaternions, we just know the octonions [77] because division algebras of hypercomplex numbers exist only for the dimensions 1, 2, 4, 8 according to the HURWITZ theorem [78] which has been independently proved by KERVAIRE [79], Raoul BOTT and MILNOR [80]. The theorem has been generalized by HOPF [81]: all division algebras have the dimension 2^n.

Consequently the extension from SO(3,1,R) to SO(3,3,R) or to SO(4,4,R) requires to use octonions (also named Cayley's numbers). Octonions have 8 dimensions on the real set $\mathfrak{R}$ and the resulting extended Dirac matrix $\hat{\gamma}$ will be equivalent to 8×8 complex matrix, but the multiplication of octonions is not associative, so the product of 3 extended Dirac matrix may not be associative.

In this way, PATTY and SMALLEY [47] have proposed a generalization of Dirac equation with some 8×8 complex matrix $\hat{\gamma}$, but their theory is not consistent in the 3+3 space-time, as we have explained it in section 4.2.

Since the HURWITZ theorem [78], an extended Dirac equation needs 8 space-time dimensions to have a dimensional coherence between extended Dirac matrix, extended Dirac spinors and extended space-time derivatives. To have a space-time symmetry we might think of a 4+4 space-time with one additional space dimension and a 4-dimensional time.

8 About an Extended Minkowski Space With Complex Coordinates

Several attempts have been done to replace real coordinates of the usual 3+1 space-time by complex coordinates. Such a complex space-time corresponds to the group SO(3,1,C) and it is considered to have 8 real dimensions in the group SO(6,2,R).

In a previous communication, Gilles NIBART [82] has introduced complex coordinates from the definition of a complex velocity: the real component is the usual velocity related to kinetic energy and the imaginary part is related to potential energy.

Elizabeth A. RAUSCHER and Russel TARG [83] have defined a 8-dimensional space-time with 4 additional imaginary coordinates. From 4 complex coordinates Z^{μ} they have defined an extended interval dS^2 which has always real values:

$$dS^2 = \eta_{\mu\nu}\, dZ^{\mu} dZ^{*\nu} \tag{64}$$

where $\eta_{\mu\nu}$ is a complex metric tensor, beside the usual relativist interval ds^2:

$$ds^2 = g_{\mu\nu}\, \mathrm{Re}\left(dZ^{\mu}\right) \mathrm{Re}\left(dZ^{\nu}\right) \tag{65}$$

The authors have defined two generalized light hyper-cones in the 8-dimensional space-time. The usual light cone in the real space-time defines the mechanical causality in the sense of the ZEEMAN's theorem [44], and the light cone in the complex space-time defines a generalized causality with anticipation properties.

After Jean E. CHARON [84], Gerald KAISER [85] has proposed "a new synthesis of Relativity and quantum mechanics through the geometry of complex space-time". In KAISER's theory the complex space-time is a relativist extension of classical phase space, and according to the latter author it resolves the problem of localization of quanta.

9 Overview of Several Postulates of Multi-Dimensional Time

Presently we might build several different theories of multi-dimensional time, so we have to postulate pertinently the basic properties of multi-dimensional time.
Its possible properties are the following (non exhaustive list):

- time may be isotropic[ii] or not isotropic,
- the multi-dimensional time may be a time vector in a given geometry, or not,
- the time vector may be associated to a scalar energy or an energy vector, depending on the definition of the measured time,
- the Lorentz group and the Minkowski space may be extended to a 3D time, or not,
- every time direction may be compactified or uncompactified,
- every time direction may be an observable or a hidden variable in quantum theory.

As the measured time is the proper time of a clock which is a macroscopic device, the measured time might be:

- the length along a time trajectory,

- the length along a preferred time direction,
- a geometric projection on the time direction of an observer or a detector,
- an eigenvalue of the multi-dimensional time observable,
- or the result of the collapse of a hypothetic multi-dimensional time wave.

10 Conclusion

The symmetric generalization of the Minkowski space-time to the 3+3 space-time and the special Lorentz group to the SO(3,3,R) group, which is needed by the development of some theories of electromagnetism and the development of a reciprocal theory of tachyons-bradyons, leads to introduce a 3-dimensional time into the theory of Relativity and into the relativist quantum theory.

We have shown that an isotropic[ii] multi-dimensional time does not preserve the causal order (local macro-causality principle) and also produces a general instability of quantum systems and cosmological bodies, unless there is a general time flow parallelism related to the time arrow.

And the scalar energy conservation principle does not hold with an isotropic multi-dimensional energy vector, unless there is a general time flow parallelism. So the scalar energy conservation is a direct consequence of the general parallelism of time flow.

So we may think that if we could master the time trajectory of physical systems, we would start developing new technologies, to produce new types of energy sources.

We have shown that the generalization of Dirac equation to more than 3+1 space-time dimensions, with the correlative extension of Dirac 2-spinors and Dirac 4×4 matrix, requires a hypercomplex algebra of dimension 8 on the real set (octonions) which introduces a privileged time direction.

As the extended 8× 8 matrix cannot multiply with 6-vectors of the 3+3 space-time, we will have to define an affine manifold with 4+4, 3+5 or 5+3 space-time dimensions.

Some authors have proposed to add compactified time dimensions in a Kaluza-Klein-like theory, or to add hidden time variables to quantum theory, thinking that it would provide a physical explanation of quantum inseparability and non-local correlations: in a quantum theory with hidden time variables there is no classical time trajectory, but a quantum distribution of time paths.

We have shown that quantum correlations in a pair of particles can be represented in a 3+3 space-time with symmetries in the SO(3,3) group, as the mechanical correlations in two coupled harmonic oscillators have symmetries in the SO(3,3) group. Any correlated pair generates a space-time entanglement, which has an intrinsic entangled homogeneous 3-dimensional time.

Although the scalar energy cannot be conserved in interactions between separated particles having energy vectors in different multi-dimensional time trajectories, a 3-dimensional time emerges from space-time entanglements in correlated particles, which have a scalar energy.

We then think that we should develop a new concept of multi-dimensional time having a distribution of quantum paths with hidden variables related to space-time entanglements at the quantum scale and also a strong parallelism of time flow at the macroscopic level.

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