The Wave Function of Rest Mass

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Abstract
For Donald C. CHANG, the rest mass of a particle is related to a transversal distribution of the amplitude of its wave function. We have computed the transversal distributions of the density of presence of a particle from the amplitudes of its wave function, we have drawn their surface graphs. As a consequence of the wave nature of particles, transversal distributions show a serie of maxima and minima which depend of a Bessel function Jn of order n. At the zero radius the density is a maximum for n=0 and it is a null minimum for n>0 which defines a hollow mass.
For John E. CARROLL, the rest mass of a particle should correspond to variations in a hidden transversal time of a 3+3 space-time. We have computed these variations and we have found that there is a photon correlation in the hidden time, and that the rest mass might correspond to oscillations for superluminal particles, but direct or inverse exponential variations for subluminal particles.

Keywords: rest mass, wave function, hollow mass, Bessel functions, hidden time.

1 Introduction

Historically the word "rest mass" appeared in the special theory of Relativity, as meaning the Newtonian mass, but it received no theoretical explanation other than a simple mechanical coefficient of inertia and an identical coefficient of gravitation, which is intrinsic of every particle. Moreover the mass is supposed to be attached to a dimensionless point of space, e.g. a gravity center, by classical mechanics and relativist mechanics.

International Journal of Computing Anticipatory Systems, Volume 17, 2006
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-03-2

1.1 Can an Electron be a Dimensionless Mass Point?

Has an electron an infinite proper energy?

Let's consider a sphere of radius a with an electric charge q. It produces an electric field $\vec{E}$, such as:

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r^2}\vec{u}_r \tag{1}$$

where $\vec{u}_r$ is a unitary vector in the direction $\vec{r}$. The field is associated with the energy density ϖ:

$$\varpi(\vec{r}) = \frac{\varepsilon_0 E^2(\vec{r})}{2} = \frac{q^2}{32\pi^2\varepsilon_0 r^4} \tag{2}$$

thus the total energy W in the space around the sphere:

$$W = \iiint \varpi(\vec{r})dV \tag{3}$$

is given by:

$$W = \frac{q^2}{32\pi^2\varepsilon_0} \int_a^{+\infty} \frac{dr}{r^2} \times \iint d\Omega \tag{4}$$

where Ω is the solid angle of the whole space. In integrating eq. 4, one obtains:

$$W = \frac{q^2}{8\pi\varepsilon_0 a} \tag{5}$$

so, when the radius tends to zero, the charged sphere would have an infinite energy:

$$a \to 0 \quad \Rightarrow \quad W \to \infty \tag{6}$$

therefore the electron cannot be a mass point. Although this problem led BORN-INFELD [1,2,3,4] and others [5,6] to build a theory of a nonlinear electromagnetic field, it was shown that the electron must have a finite radius [7].

A similar demonstration done with the gravitational field would show that the proper energy of a mass point is divergent. Therefore any mechanics of mass points is inconsistent.

As a solution of this problem, we show (in section 2) that any particle may have a hollow mass, which is described as a transversal distribution of the amplitudes of its wave function.

1.2 About the Wave Nature of the Electron

Classical models [8] and semi-classical models [9] of the electron try to forget the wave nature of the electron, as far as they postulate a well-defined radius of the electron. In the example of ref. [9] the electron is modeled as a torus with an eye radius r_e:

$$r_e = \frac{\lambda_c}{4\pi} \tag{7}$$

where λ_c is the Compton length:

$$\lambda_c = \frac{h}{m_e c} \tag{8}$$

The Nobel Prize, Louis de Broglie wrote [10]: "The electron can no longer be conceived as a single, small granule of electricity; it must be associated with a wave and this wave is no myth; its wavelength can be measured and its interferences predicted".

Diffraction experiments with electron beams have shown wave properties similarly to X-rays diffraction by crystals. The experiments have been done by RUPP, KIKUCHI, PONTE (authors cited in ref. 10).

The wavelength of an electron:

$$\lambda = \frac{h}{m|\mathbf{v}|} \tag{9}$$

is related with its velocity v which is understood as a "group velocity" defined by the Rayleigh equation:

$$\frac{1}{|\mathbf{v}|} = \frac{\partial(n\nu)}{\partial\nu} \tag{10}$$

where the frequency ν of the phase wave is related to energy by the relation:

$$h\nu = mc^2 \tag{11}$$

We recall that the phase velocity $\mathbf{v}_\phi$ in vacuum is:

$$\mathbf{v}_\phi = \frac{c^2}{\mathbf{v}} \tag{12}$$

In its "intrinsic" referential frame the electron is at rest, so its energy equation:

$$h\nu_0 = m_0 c^2 \tag{13}$$

defines the minimum frequency ν_0 of the electron as a function of the rest mass m_0, therefore a wave function seems to be associated with the rest mass of the electron.

1.3 Is the Rest Mass Associated With a Wave Function?

The equivalence between rest mass and energy is only known from fermions mass creation-annihilation, mass defect in atomic nuclei, disintegration energy of decay processes …etc. A mass defect or annihilation has its equivalent energy in γ photons, which have associated wave functions but are supposed to have a zero rest mass.

A relation connecting the rest mass and space separation of events in space-time continuum, has been studied by Arup ROY [11]. An experiment for measuring a very little rest mass of photon has been performed by Roderic S. LAKES [12]; it is based on MAXWELL-PROCA equations [13,14] where the Gauge invariance is lost. Another experiment, done by Jun LUO and al. [15], has been criticized by Jian Qi SHEN [16,17].

A true equivalence between rest mass defect or annihilation and energy of resulting photons should require the rest mass to be associated with a wave function.

Several authors have discussed whether phase waves are truly physical waves or just interfering functions of statistical probabilities. As photons wave functions matches Maxwell electromagnetic field, the question concerns only particles with non-zero rest mass. A quantum-state tomography method has been experimentally used [18] to determine photon wave functions. It has been possible because photons are quanta of the electromagnetic field.

Hans de VRIES has studied the relativist kinematics of the wave packet [19] and he has found that any particle is moving with space-time shifts. He stressed that phase waves cannot propagate through a physical medium, for the phase wave of a particle at rest has an infinite velocity and an infinite wavelength.

On the contrary, Kiyoung KIM [20,21] has proposed a model of DIRAC vacuum [22] (with negative energy particles) where wave functions can propagate.

Daniel M. DUBOIS [23], Daniel M. DUBOIS and Gilles NIBART [24] have developed a theory of a dual Relativity derived from forward-backward space-time shifts. The continuous version of the discrete forward-backward space-time derivatives uses complex coordinates where imaginary components represent the space-time shift. From this theory the authors have justified the use of plane waves in quantum theory as an interpretation of the mass-energy conservation of quanta during their propagation.

In quantum theory, mass particles are associated to plane waves with the usual equation:

$$\psi_k = \psi_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{14}$$

where ψ_0 is a normalizing constant, the waves propagates in the direction of the vector $\mathbf{x}$, with no transverse distribution.

So this means that the energy of a particle is homogeneously spread over the whole plane of its associated waves. Thus, particles plane waves are as wide as the whole universe: it does not explain the Compton length in the Compton scattering.

As a given free particle has a propagation direction (given by the initial momentum) and a starting point (initial coordinates), its presence probability should not be homogeneous in the whole universe: it should then have a finite distribution of presence probability in transverse directions. Moreover, if the energy distribution is homogeneous in a plane wave, i.e. if the energy density is constant, the total energy in the whole plane would be infinite and this is impossible. So a non-uniform distribution of energy density is required in any plane waves.

2 The Transversal Distribution of Rest Mass in the Wave Function

Donald C. CHANG [25,26,27] has proposed "a slight modification from the traditional Copenhagen interpretation of the wave function" with a physical plane wave, which has a distribution of amplitudes in transversal directions.

Donald C. CHANG has postulated that (p. 4 in ref. [26]) "the wave function must have a cylindrical symmetry" and he has given the general expression of the wave function of a free particle, shown below:

$$\psi_k(\mathbf{x},t) = \psi_0 \cdot \psi_T(\mathbf{k}\wedge\mathbf{x}) \cdot \psi_L(\mathbf{k}\cdot\mathbf{x},t) \tag{15}$$

where the longitudinal component ψ_L is the usual wave function and the transversal component ψ_T is "the probability density of the particle in the transverse plane" (p.4 in ref. [26]). From the usual relativist equation propagation of the electromagnetic field:

$$\left(\frac{1}{c^2}\frac{\partial}{\partial t^2}-\nabla^2\right)\psi=0 \tag{16}$$

he has computed the transversal component as:

$$\psi_T(\mathbf{k}\wedge\mathbf{x})=a\,J_n(l\,r)\cdot e^{\pm in\vartheta} \tag{17}$$

where J_n is a Bessel function of first kind and order n, a is a normalizing constant, and **r**, θ are polar coordinates in the transversal plane. Donald C. CHANG has shown that (p.8 in ref. [26]) the number l is related to the rest mass as:

$$m_0=\frac{\hbar\,l}{c} \tag{18}$$

and thus we understand it as an inverted radius r_0 :

$$r_0=\frac{\hbar}{m_0 c} \tag{19}$$

which acts as a scaling factor of the variable r in the Bessel function $J_n(r/r_0)$.

The equation (16) is a Laplace-like equation in the 3+1 Minkowsky space-time, and the Laplacian operator ∇^2 in polar coordinates leads to the Bessel equation of the transversal component ψ_T.

Mathematically the solutions have not to be restricted to Bessel functions J_n of integer orders: n is not necessary an integer as Donald C. CHANG supposed it (p.5 in ref. [26]).

Bessel functions of positive half-integer order $n+½$ are also solutions of the Bessel equation, and they must be included.
As examples, the Bessel function $J_{\frac{1}{2}}$ of order ½ :

$$J_{\frac{1}{2}}(r/r_0)=\sqrt{\frac{2\,r_0}{\pi\,r}}\,\sin(r/r_0) \tag{20}$$

and the Bessel function $J_{\frac{3}{2}}$ of order 3/2 :

$$J_{\frac{3}{2}}(r/r_0)=\sqrt{\frac{2\,r_0}{\pi\,r}}\left[\frac{\sin(r/r_0)}{r/r_0}-\cos(r/r_0)\right] \tag{21}$$

are also solutions, although the Bessel function of negative order – ½:

$$J_{-\frac{1}{2}}(r/r_0)=\sqrt{\frac{2\,r_0}{\pi\,r}}\,\cos(r/r_0) \tag{22}$$

is excluded because its divergence at zero radius:

$$\lim_{r\to 0} J_{-\frac{1}{2}}(r/r_0)=\infty \tag{23}$$

Furthermore, Bessel functions of positive decimal order are also solutions, and they must be included (they can be computed with series).

Obviously the equation (17) has the cylindrical symmetry:

$$\psi_T(\mathbf{k} \wedge \mathbf{x}) = a\, J_n(r / r_0) \cdot e^{\pm i n \vartheta} \tag{24}$$

as the Bessel function $J_n(r / r_0)$ depends only on the radius r and the circular function $e^{\pm i n \vartheta}$ of the angle θ always corresponds to the unity radius. The three Donald C. CHANG surface graphs a, b, c in figure 1 of ref. [26] have been drawn from the complex function ψ_T at the Bessel orders n = 0, 1, 2 respectively, but only the graph a has the cylindrical symmetry required by the equation (17).

In the Donald C. CHANG graphs b, c, in figure 1 of ref. [26] the cylindrical symmetry has been lost because the Math software draws a surface graph from the real component of the complex function ψ_T: we could reproduce these graphs with the software Mathcad 6.0 SE and we have seen that his graphs b, c of orders n = 1, 2 show rather the following function:

$$\psi_T(\mathbf{k} \wedge \mathbf{x}) = a\, J_n(r / r_0) \cos n \vartheta \tag{25}$$

which has not the cylindrical symmetry.

The transversal density of mass presence of a free particle is:

$$d_T(\mathbf{k} \wedge \mathbf{x}) = \psi_T \cdot \overline{\psi}_T \tag{26}$$

and thus

$$d_T(\mathbf{k} \wedge \mathbf{x}) = a^2\, J_n^2(r / r_0) \tag{27}$$

We have drawn (with Mathcad) the surface graphs of the mass presence density d_T for the Bessel orders $n = 0$, $n = 0.33$, $n = 1/2$, $n = 1$, $n = 3/2$, $n = 2$, $n = 3$ and $n = 4$.

An example with a decimal Bessel order is given in figure 4 and two examples with a half-integer Bessel order are given in figures 5 and 7.

For only $n = 0$, the mass density is maximum at the zero radius: $d_{r=0} = 1$, as shown in figure 1.

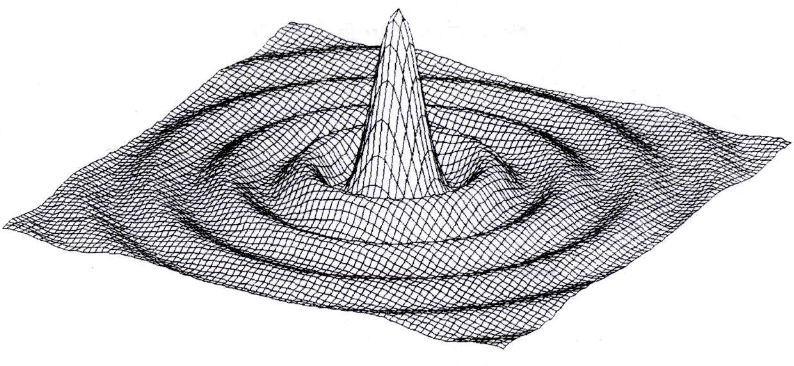

Figure 1: Transversal density of mass presence with the Bessel order zero.

For any value $n > 0$, the mass density is null at the zero radius $d_{r=0} = 0$ and is maximum at a radius r_1 which depends on the Bessel order n, therefore any mass particle possesses a hollow mass, as shown on the figures 4 to 10.

Since the decimal Bessel order $n = 0.1$, a hollow mass appears, as we can see it in comparing the figures 2 and 3, which are zoomed on the maximum density.

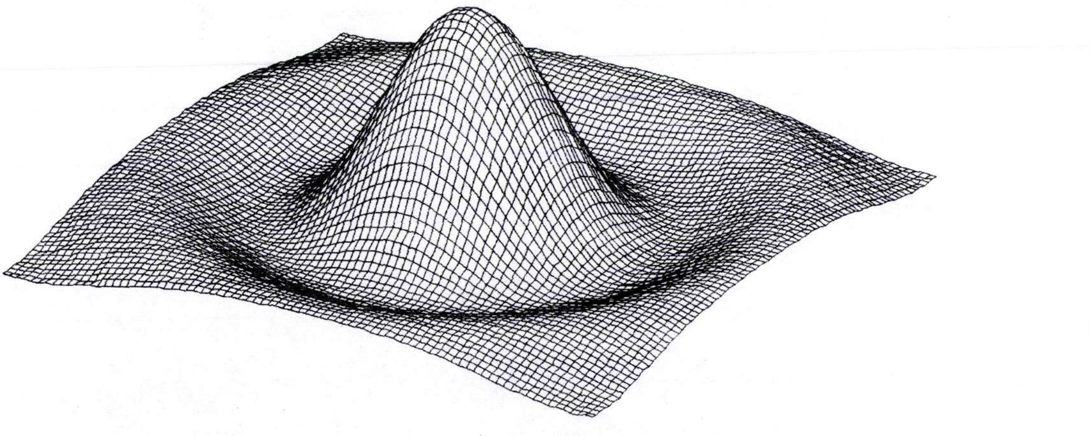

Figure 2: Zoom on the maximum density of mass with the Bessel order zero.

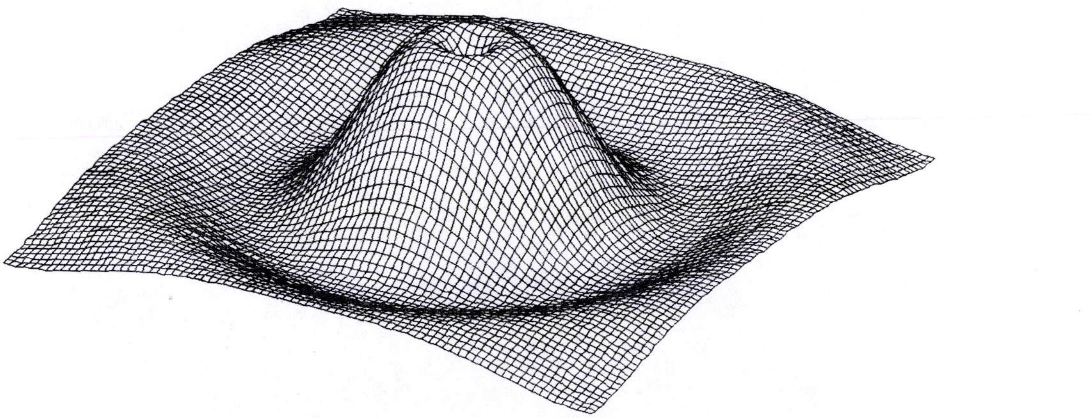

Figure 3: Zoom on the hollow mass with the decimal Bessel order 0.1.

A higher Bessel order generates a larger hollow, a lower maximum of density, and a mass density which is more widely spread out in space, as shown on the figures 4 to 10.

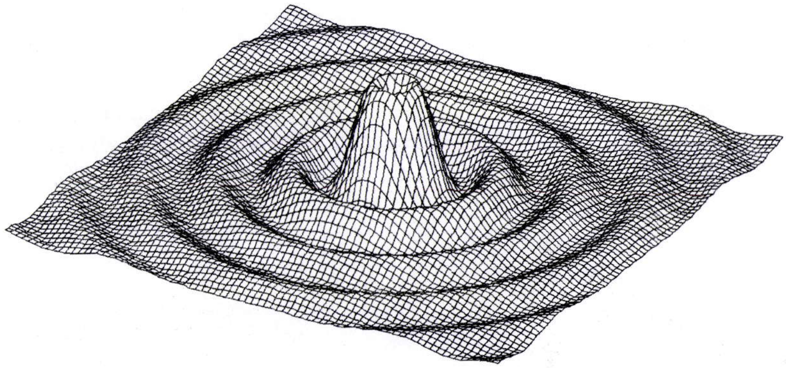

Figure 4: Transversal density of mass presence with the decimal Bessel order 0.33.

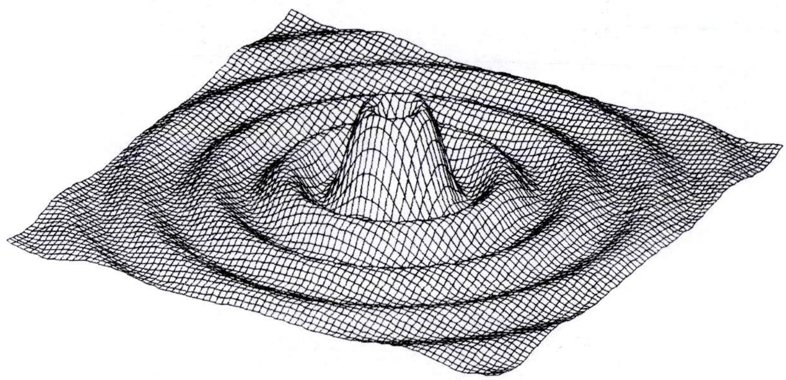

Figure 5: Transversal density of mass presence with the decimal Bessel order 1/2.

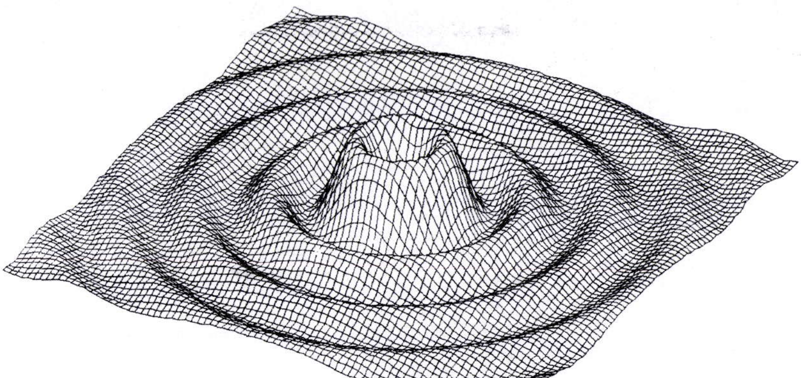

Figure 6: Transversal density of mass presence with the integer Bessel order 1.

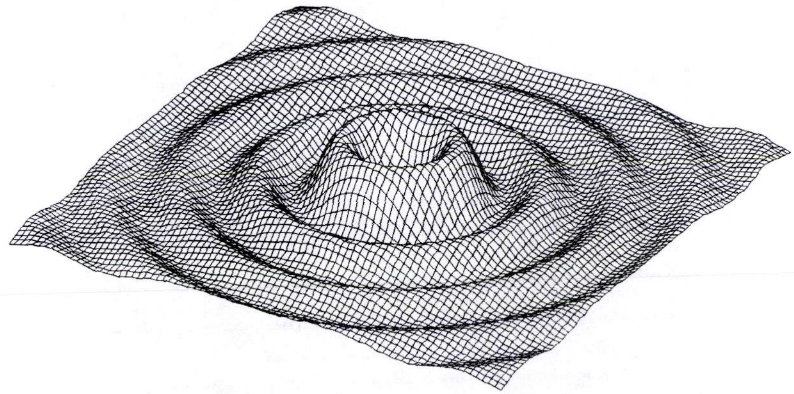

Figure 7: Transversal density of mass presence with the half-integer Bessel order 3/2.

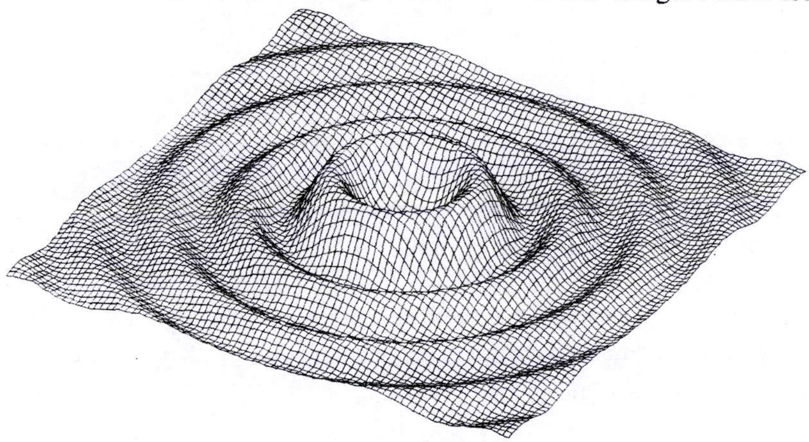

Figure 8: Transversal density of mass presence with the Bessel order 2.

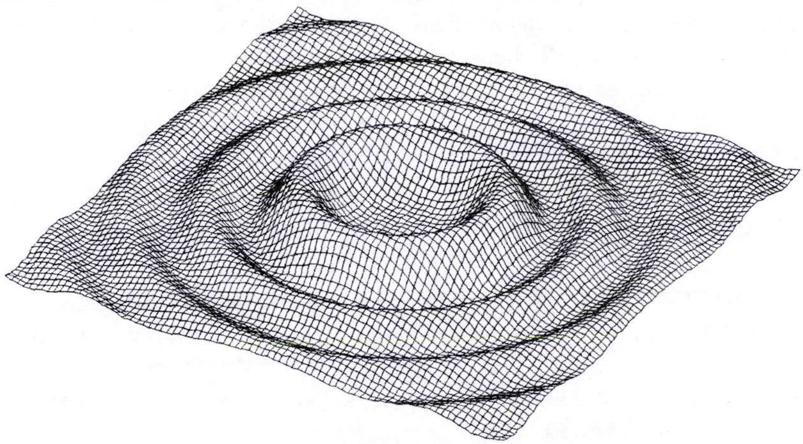

Figure 9: Transversal density of mass presence with the Bessel order 3.

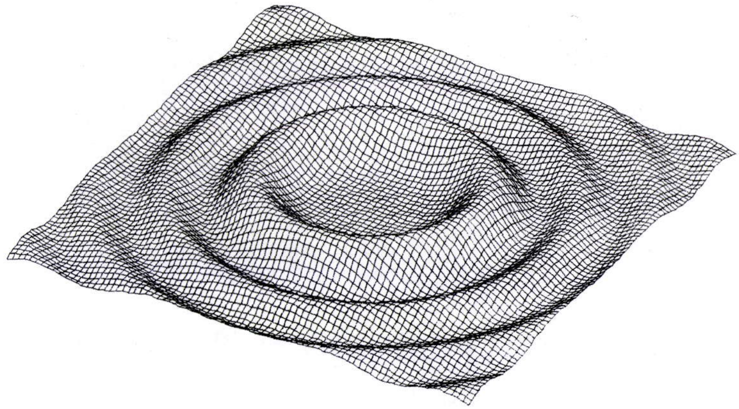

Figure 10: Transversal density of mass presence with the Bessel order 4.

All transversal distributions have a primary maximum at a smaller radius r_1 and several secondary maxima at greater radii $r > r_1$, which are each separated with a minimum of zero density, as shown in the figures 1 and 4 to 10. So the transversal distribution of mass has a wave nature with spatial frequencies.

The radius r_0, defined in equation (19), is not the radius of a particle, because it is smaller than the radius r_1 of the primary maximum:

$$r_1 > r_0 \tag{28}$$

(since the Bessel order ½ approximately) and because the electron mass is spread over to all secondary maxima. The usual plane wave is not the limit of this transversal distribution when the Bessel order becomes infinite.

Donald C. CHANG suggested (p.8 in ref. [26]) that the Bessel order n may be related to the helicity h of the particle with eigenvalues $n\hbar$, but he has not demonstrated such a relation. We think that the demonstration cannot be done because the Bessel order n is not related to the total magnetic momentum $\mathbf{l} = \mathbf{j} + \mathbf{s}$ and the factor $e^{in\vartheta}$ in equation (17) defines just the cylindrical symmetry of the transversal distribution of the mass density.

The Bessel order n and the radius factor r_0, together, define the transversal distribution of the mass presence density of any particle. Unfortunately the present theory does not predict the value of Bessel order of each mass particle.

As a consequence of Donald C. CHANG's theory, the relativist expression of energy (p.7 in ref. [26]) becomes:

$$E^2 = c^2\left(p^2 + \hbar^2 l^2\right) \tag{29}$$

or:

$$E^2 = c^2\left(p^2 + \frac{\hbar^2}{r_0^2}\right) \tag{30}$$

Finally his generalized wave function depends on the four parameters: ω, $\mathbf{k}$, r_0, n. The three parameters ω, $\mathbf{k}$, r_0 are respectively related with energy E, momentum $\mathbf{p}$, rest mass m_0 of a free particle, but the Bessel order n is not related to any kinetic variable.

Donald C. CHANG assumes the existence of spatial transversal oscillations (p.10 in ref. [26]), where the function $\psi_T(\mathbf{k} \wedge \mathbf{x})$ represents variations of the wave density in space, but there is no temporal oscillation in this theory.

The idea of temporal transversal oscillations can be found in the model of rest mass proposed by John E. CARROLL [28,29] with the equation (31) in section 3. We study this in the following section.

3 The Hidden Time Derivatives Equation of Rest Mass

J.E. Carroll has deduced classical Maxwell equations in the usual 3+1 space-time, from the symmetry required by geometric algebra [30,31,32,33,34] applied to a 3+3 space-time. His work results in a time which has 3 dimensions with components t_1, t_2, t_3, where the measured time $t = t_3$ is the 'principal' time t_3 related to the usual time coordinate $x_0 = c\, t_3$, while ct_1, ct_2 are purely theoretical coordinates as long as the author defines the velocity $\boldsymbol{v}$ as a derivative by the 'principal' time t_3: « the axes Ot_1, Ot_2 are all transverse to $\boldsymbol{v}$ ».

As the both time components t_1, t_2, can be neither measured nor observed, we say that the vector (t_1, t_2) represents a "hidden time". As the two continuous variables t_1, t_2 in the differential equation (31) are not related to any known and measurable variable, they seem to be a pure theoretical construct, and they cannot be subject to any experimental measurement. So the only possible experimental tests would be done with the temporal transversal variations of the generalized electromagnetic field.

In this theory, the time directions (t_1, t_2) define a rotation sub-group SO(0,2,R) which leaves the rest mass invariant, and the rotation sub-group SO(3,1,R) in the time direction t_3 which is the Lorentz group. Therefore the choice of the time direction t_3 as the observed time has introduced a symmetry breakdown in the group SO(3,3,R) which produces rest mass waves Ψ from a generalized symmetric field varying in a 3-dimensional time dimensions, including the hidden time (t_1, t_2).

In the present communication we resolve the hidden time derivative equation of rest mass (31) and we study whether the scalar mass M_0 has different 'components' in hidden time directions t_1, t_2.

In section 3.2, we consider the case of a one dimension mass, in section 3.3 the case of the two dimensions mass, and in section 3.4 we propose some assumptions about the nature of rest mass waves.

3.1 Postulates of the Hidden Time Derivatives Equation of Rest Mass

From a symmetric field propagation equation in a 3+3 space-time, J.E. Carroll has deduced the equation of KLEIN [35], GORDON [36], and FOCK [37] in the 3+1 spacetime and also the following wave equation of the rest mass (equation 1.1 in reference 28):

$$M_0^2\Psi = \left(\partial t_1^2 + \partial t_2^2\right)\Psi \tag{31}$$

where $\partial t_1^2 + \partial t_2^2$ represents partial derivatives of second order, and Ψ a wave function.

The author has done the following simplification:

$$c = \hbar = 1 \tag{32}$$

so we will rather write the rest mass wave equation with international units as follows:

$$M_0^2\Psi = \frac{1}{c^2}\left(\frac{\partial^2}{\partial t_1^2} + \frac{\partial^2}{\partial t_2^2}\right)\Psi \tag{33}$$

3.2 Solving the Rest Mass Equation for a 1-Dimensional Hidden Time

Let's suppose that the hidden time has only one dimension t_1 and let's write

$$dt_2 = 0 \tag{34}$$

in equation (33) we get the differential equation:

$$c^2 M_0^2\Psi = \frac{d^2}{dt_1^2}\Psi \tag{35}$$

which is a linear equation of the second order with constant coefficients. Its characteristic form is:

$$Ay'' + By' + Cy = 0 \tag{36}$$

where y is the unknown function Ψ. Its associated algebraic equation is:

$$Ar^2 + Br + C = 0 \tag{37}$$

so the differential equation (35) has solutions such as:

$$y(t_1) = e^{rt_1} \tag{38}$$

where r is a root of the equation (37). The equation coefficients are:

$$A = 1 \;\; B = 0 \;\; C = -c^2 M_0^2 \tag{39}$$

so the discriminant is:

$$B^2 - 4AC = 4c^2 M_0^2 \tag{40}$$

For mass particles $M_0 \neq 0$, the discriminant is not null and the equation (37) have the two roots:

$$r = \pm c M_0 \tag{41}$$

which are both real or both imaginary, depending on the sign of M^2. Therefore the general solution of the differential equation (35) can be written:

$$\Psi(t_1) = \alpha\, e^{M_0 c t_1} + \beta\, e^{-M_0 c t_1} \tag{42}$$

where the unknown coefficients α, β are constants or functions which are independent from the hidden time t_1.

3.2.1 About the Case of a Real Rest Mass

Usually $M_0^2>0$, the rest mass M_0 is real for subluminal particles. The rest mass wave is the sum of an exponential function and an inverse exponential function of the hidden time and it may also be written as:

$$\Psi(t_1) = A\, ch(\mathrm{M}_0\, ct_1) + B\, sh(\mathrm{M}_0\, ct_1) \tag{43}$$

where A, B are unknown constant coefficients.

The resulting law of rest mass wave is: when the rest mass wave is an exponential function or an inverse exponential function or their sum, $M_0^2>0$, so the particle is subluminal.

3.2.2 About the Case of an Imaginary Rest Mass

When $M_0^2<0$ the rest mass is an imaginary number, it is a superluminal particle [38, 39], i.e. a tachyon [40]. Let us write:

$$M_0 = im_0 \qquad m_0 \in \Re \tag{44}$$

The rest mass wave function is:

$$\Psi(t_1) = \alpha\, e^{i\, m_0 ct_1} + \beta\, e^{-i\, m_0 ct_1} \tag{45}$$

so it can be expressed as:

$$\Psi(t_1) = A\cos(\omega t_1 - \varphi) \tag{46}$$

where the unknown coefficient A is a constant or a function which is independent of t_1, and where φ is the wave phase. Thus it oscillates with the frequency ν such as:

$$\omega = 2\pi\nu = cm_0 \tag{47}$$

The resulting law is: when the rest mass wave is oscillating in the hidden time, $M_0^2<0$, so the particle is superluminal. We recall that the relativist energy decreases as an inverse exponential with the superluminal velocity [41].

3.3 Solving the Rest Mass Equation for a 2-Dimensional Hidden Time

Now let us consider the case of the rest mass equation (33) with a 2-dimensional hidden time:

$$M_0^2\Psi = \frac{1}{c^2}\left(\frac{\partial^2}{\partial t_1^2} + \frac{\partial^2}{\partial t_2^2}\right)\Psi \tag{48}$$

3.3.1 Introduction to the 2-Dimensional Hidden Time Rest Mass Equation

The general solution (42) of the rest mass equation (35) with a 1-dimensional hidden time is a particular solution Ψ_1 and Ψ_2 of the rest mass equation (48) in every time direction t_1 and t_2, as written below:

$$\Psi_1(t_1) = \alpha_1\, e^{M_1 ct_1} + \beta_1\, e^{-M_1 ct_1} \tag{49}$$

$$\Psi_2(t_2) = \alpha_2\, e^{M_2 ct_2} + \beta_2\, e^{-M_2 ct_2} \tag{50}$$

Because the rest mass equation (48) is linear in Ψ we have the additive solution:

$$\Psi(t_1,t_2)=\Psi_1(t_1)+\Psi_2(t_2) \tag{51}$$

which is studied in section 3.3.2 but we can also have the multiplying solution which is studied in section 3.3.3:

$$\Psi(t_1,t_2)=\Psi_1(t_1)\times\Psi_2(t_2) \tag{52}$$

3.3.2 The Case of Additive Rest Mass Waves

In the case of additive rest mass waves, the rest mass wave has two components in t_1 and in t_2, with the same exponent value cM_0. From equations (48), (49), (50) and (51) we obtain the derivative:

$$M_0^2\Psi(t_1,t_2)=M_1^2\Psi_1(t_1)+M_2^2\Psi_2(t_2) \tag{53}$$

where c^2 has been eliminated, and we deduce that squared mass components are equal:

$$M_0^2=M_1^2=M_2^2 \tag{54}$$

i.e.:

$$M_0=\pm M_1=\pm M_2 \tag{55}$$

and thus the additive rest mass wave is:

$$\Psi(t_1,t_2)=\alpha_1 e^{M_0ct_1}+\beta_1 e^{-M_0ct_1}+\alpha_2 e^{M_0ct_2}+\beta_2 e^{-M_0ct_2} \tag{56}$$

where the unknown coefficients α_1, α_2, β_1, β_2 are constants or functions which are independent from the hidden time t_1 and t_2.

For a subluminal particle, the rest mass wave is the sum of exponential functions and inverse exponential functions of t_1 and t_2 and it may also be written as:

$$\Psi(t_1,t_2)=A_1\,ch(\mathrm{M}_0\,ct_1)+B_1\,sh(\mathrm{M}_0\,ct_1)+A_2\,ch(\mathrm{M}_0\,ct_2)+B_2\,sh(\mathrm{M}_0\,ct_2) \tag{57}$$

where the unknown coefficients A_1, A_2, B_1, B_2 are constants or functions independent from the hidden time t_1 and t_2.

For a superluminal particle, the rest mass is imaginary:

$$M_0=i\,m_0 \tag{58}$$

then the rest mass wave can be expressed as:

$$\Psi(t_1,t_2)=\alpha_1 e^{im_0ct_1}+\beta_1 e^{-im_0ct_1}+\alpha_2 e^{im_0ct_2}+\beta_2 e^{-im_0ct_2} \tag{59}$$

It is a 2-dimensional oscillator:

$$\Psi(t_1,t_2)=A_1\cos(\omega t_1-\varphi_1)+A_2\cos(\omega t_2-\varphi_2) \tag{60}$$

where the unknown coefficients A_1, A_2 are constants or functions independent from the hidden time t_1 and t_2, and where φ_1, φ_2 are the wave phases. It oscillates with the same frequency ν in the both time directions t_1, t_2:

$$\omega=2\pi\nu=cm_0 \tag{61}$$

3.3.3 The Case of Multiplying Rest Mass Waves

In the case of multiplying rest mass waves, the rest mass M_0 has two dependent components M_1 and M_2. From equations (48), (49), (50) and (52) we obtain the derivatives:

$$\frac{\partial^2}{\partial t_1^2}\Psi(t_1,t_2) = M_1^2\,\Psi_1(t_1)\Psi_2(t_2) \tag{62}$$

$$\frac{\partial^2}{\partial t_2^2}\Psi(t_1,t_2) = M_2^2\,\Psi_1(t_1)\Psi_2(t_2) \tag{63}$$

then from equations (48) and (52) we get:

$$M_0^2\,\Psi_1(t_1)\Psi_2(t_2) = M_1^2\,\Psi_1(t_1)\Psi_2(t_2) + M_2^2\,\Psi_1(t_1)\Psi_2(t_2) \tag{64}$$

and we deduce the rest mass components relation:

$$M_0^2 = M_1^2 + M_2^2 \tag{65}$$

and thus the multiplying rest mass wave is:

$$\Psi(t_1,t_2) = \left(\alpha_1 e^{M_1ct_1} + \beta_1 e^{-M_1ct_1}\right)\left(\alpha_2 e^{M_2ct_2} + \beta_2 e^{-M_2ct_2}\right) \tag{66}$$

where the unknown coefficients α_1, α_2, β_1, β_2 are constants or functions which are independent from the hidden time t_1 and t_2. The mass components M_1 and M_2 also are constants or functions, which are independent of t_1 and t_2.

An interesting particular case is the oscillation of rest mass components in the observed time t_3:

$$\begin{cases} M_1 = M_0 \cos\omega_3 t_3 \\ M_2 = M_0 \sin\omega_3 t_3 \end{cases} \tag{67}$$

with an unknown frequency ν_3:

$$\omega_3 = 2\pi\nu_3 \tag{68}$$

For a subluminal particle, the rest mass wave is the product of sums of exponential functions and inverse exponential functions of t_1 and of t_2, which may be written as:

$$\Psi(t_1,t_2) = [A_1\,ch(\mathrm{M}_1 ct_1) + B_1\,sh(\mathrm{M}_1 ct_1)][A_2\,ch(\mathrm{M}_2 ct_2) + B_2\,sh(\mathrm{M}_2 ct_2)] \tag{69}$$

where the unknown coefficients A_1, A_2, B_1, B_2 are constants or functions independent from the hidden time t_1 and t_2.

For a superluminal particle, the rest mass is imaginary:

$$M_0 = i\,m_0 \quad m_0 \in \Re \tag{70}$$

Let's write its imaginary components as:

$$M_1 = im_1 \quad M_2 = im_2 \quad m_1, m_2 \in \Re \tag{71}$$

the rest mass wave can be then expressed as:

$$\Psi(t_1,t_2) = \left(\alpha_1 e^{im_1ct_1} + \beta_1 e^{-im_1ct_1}\right)\left(\alpha_2 e^{im_2ct_2} + \beta_2 e^{-im_2ct_2}\right) \tag{72}$$

It is a 2-dimensional oscillator:

$$\Psi(t_1,t_2) = A\cos(\omega_1 t_1 - \varphi_1)\cos(\omega_2 t_2 - \varphi_2) \tag{73}$$

where the unknown coefficient A is a constant or a function which is independent of the hidden time t_1 and t_2. The oscillator is composed of two frequencies ν_1, ν_2:

$$\omega_1 = 2\pi\nu_1 = cm_1 \tag{74}$$

$$\omega_2 = 2\pi\nu_2 = cm_2 \tag{75}$$

which are both dependent as shown below:

$$\omega_1^2 + \omega_2^2 = c^2 m_0^2 \tag{76}$$

The wave function of a superluminal particle with rest mass oscillations in the observed time t_3 can be deduced from equations (67) and (71) as:

$$\Psi(t_1,t_2) = A\cos[m_0\cos(\omega_3 t_3)ct_1 - \varphi_1]\cos[m_0\sin(\omega_3 t_3)ct_2 - \varphi_2] \tag{77}$$

with the unknown frequency ν_3:

$$\omega_3 = 2\pi\nu_3 \tag{78}$$

3.4 About the Physical Nature of the Rest Mass Waves

We should discuss about the physical nature of the wave Ψ, which is associated to rest mass. If we suppose that the rest mass wave Ψ has a known physical nature: the theory of the rest mass wave Ψ could then be tested experimentally.

3.4.1 Components of the Zero Rest Mass Wave of the Photon

Usually electromagnetic fields are known to propagate and oscillate only within the observable time $t = t_3$ with the usual Maxwell propagation equation of the photon wave Ψ_0:

$$\left(\frac{\partial^2}{c^2\partial t^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2}\right)\Psi_0 = 0 \tag{79}$$

but the zero rest mass wave Ψ_0 may have opposite variations in the hidden time (t_1, t_2) with the following correlation:

$$\left(\frac{\partial^2}{\partial t_1^2} + \frac{\partial^2}{\partial t_2^2}\right)\Psi_0 = 0 \tag{80}$$

Consequently the two components of the zero rest mass wave Ψ_0 in the t_1 and t_2 dimensions may be different from zero and thus they might have to be related with the mass of a pair of particle-antiparticle. The equation (80) will be called the "photon correlation in the hidden time".

3.4.2 Rest Mass Wave of Mass Particles

We might suppose that the rest mass wave Ψ of mass particles have a gravitational nature, but we may rather think that the rest mass wave Ψ has an electromagnetic nature.

Let's assume that the Caroll symmetric field propagating the 3+3 space-time coordinates:

$$\Psi(ct_1, ct_2\, ct_3, x_1, x_2, x_3) \tag{81}$$

is physically an "extended electromagnetic field" in the physical 3+3 space-time: it propagates within the 3-dimensional time, including the hidden time. From this viewpoint, an electromagnetic beam Ψ with very specific variations in the hidden time, might interact with the mass wave Ψ of a mass particle, making a perturbation of the observed mass.

If we could produce an electromagnetic beam Ψ with specific exponential or inverse exponential variations, it would be able to increase or decrease the gravitational mass of an interacting particle. If we could produce an electromagnetic beam Ψ which oscillates in the hidden time, with correct resonant frequencies ν_1, ν_2, ν_3, the mass wave Ψ of an interacting particle will start to oscillate in the hidden time (t_1, t_2) too, then the mass would become imaginary and the particle would jump over the "light barrier".

4 Conclusion

On the one hand, from Chang's work on rest mass, we know that the usual wave function of a mass particle should be modified to introduce the spatial distribution of amplitudes in transversal directions.

We have drawn several possible surface graphs of the transversal distribution of mass density, using Bessel functions of positive real orders, and we have shown that it is consistent with the wave nature of mass particles.

The transversal distribution of mass density is related to rest mass through a scale factor r_0 and it depends on a new quantum number: the "Bessel order" which is not necessary an integer n. As far as its values are not predicted by the theory, the Bessel order may have a continuous spectrum.

To resolve the divergence of mass point particles at radius zero, we have proposed to introduce the transversal distribution of wave functions amplitudes with a zero or positive Bessel order. We have shown that any mass particle of positive Bessel order has a hollow mass.

On the other hand, the rest mass operator introduced by J.E. Caroll defines variations in hidden time of an electromagnetic field, which is generalized to a 3+3 space-time. We have studied these variations in hidden time and we have shown that they are only oscillations (i.e. waves) for superluminal particles.

We have suggested that the photon correlation in the hidden time might have something to do with particle-antiparticle pair correlations, and we have suggested that the field variations associated to rest mass might have the same physical nature as a generalized electromagnetic field which propagates in a 3+3 space-time.

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