A Particle Structure with Spatial Frequencies, and a Possible Hollow Mass

Gilles Nibart
Laboratoire de Physique Théorique Fondamentale de Paris
23 Boulevard Bessières, F-75017 Paris, France.
lab.phys.theo@club-internet.fr
gilles.nibart@club-internet.fr

Daniel M. Dubois
Centre for Hyperincursion and Anticipation in Ordered Systems,
CHAOS asbl, Institute of Mathematics B37, University of Liège,
Grande Traverse 12, B-4000 LIEGE 1, Belgium
http://www.ulg.ac.be/mathgen/CHAOS
Daniel.Dubois@ulg.ac.be

Abstract
In our previous paper [1], plane waves of any free massive particle were described with a transversal distribution that is defined by a new quantum number, the Bessel order β, which shows the possibility of a hollow structure.
The present paper will consider any free massive particle with spherical waves described by their radial distribution. The scale of different Bessel orders is defined and the properties and structure of the presence density are given. Then it is deduced that particles can have a hollow mass and can be made passive with an inefficient cross-section.
Finally, an application to Cosmology is proposed and it is suggested that antimatter of extremely high Bessel orders, in galaxy bulge and in star nucleus, may be a solution to the three problems of galaxy stability, dark matter and antimatter in the Universe.
Keywords: dark matter, antimatter, hollow mass, quantum theory, distribution.

1. Introduction

Quantum Theory has inherited the idea of "point-like particle" from the concept of "mass point" in Newton's mechanics and the concept of "point-like electric charge" from classical electrodynamics. The hypothesis of a point-like fermion leads mathematically to a divergence of its electric field at a null distance and a divergence of its self-energy (section 1.1 in ref. [1]), from the viewpoint of classical mechanics.

Nevertheless the concept of point-like electric charge has been justified by the problem of its stability: if the electric charge is expanded in space, the distributed charges should interact with repulsive forces [2]. Such a problem does not concern the mass, as gravitation is attractive.

The magnetic energy of a moving fermion leads to another problem: the magnetic field of the fermion requires a hollow structure (the section 2.3).

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An other reason is the difficulty to build a pertinent model of fermion: classical or semi-classical models [3] cannot be fully consistent with quantum theory as far as particles have an internal structure limited by a boundary of finite radius. These problems are consequences of the localization of a particle, as a point or a solid sphere in space, which might be avoided by models that are built on the wave nature of fermions (section 1.2 in ref. [1]). Therefore some authors have proposed models with internal waves, which are associated with the structure of the particle [4, 5, 6]. The internal waves are required to be superluminal waves [7, 8, 9].

Our model of particle is not a speculation about the so-called "internal structure" of a boson or a fermion, but it is a study of the distribution of waves in the outer space of the particle, as it can be deduced from the foundations of quantum theory. The wave nature of fermions is to be considered not only in the time domain with the well-known frequency ν of its phase wave:

$$h\nu = mc^2 \tag{1}$$

but also in the space domain, with spatial frequencies (the section 3.4).

In our previous paper [1] we have computed the transversal distribution of the plane wave of any free massive particle (fermion or boson) and we have shown that the plane wave associated with the energy-momentum $(E, \boldsymbol{p})$ of any mass particle:

$$\psi_k(\mathbf{x},t) = \psi_0 \cdot \psi_T(\mathbf{k} \wedge \mathbf{x}) \cdot e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} \tag{2}$$

must have a transversal distribution such as:

$$\psi_T(\mathbf{k} \wedge \mathbf{x}) = a\, J_\beta(r / r_0) \cdot e^{\pm i n \vartheta} \tag{3}$$

where r_0 is a scaling factor which is related to the rest mass m_0 as :

$$r_0 = \frac{\hbar}{m_0 c} \tag{4}$$

where β is a new quantum number, the Bessel order which is a positive real number :

$$\beta \in \Re \quad \beta \geq 0 \tag{5}$$

We have computed the transversal density of mass presence and drawn several figures of the transversal distribution of the density of presence of mass for several values of the Bessel order: β = 0, 0.33, 1/2, 1, 3/2, 2, 3 and 4 (figures 1 to 10 in section 2 of ref. [1]).

The present quantum theory does not predict any value of the Bessel order of each mass particle, as the Bessel order β is neither related to any kinetic variable, nor to any usual quantum number: it is a new quantum number. So a new law will have to be postulated.

In the present paper we recall that the general differential equation of waves have both solutions of plane waves with a transversal radial distribution (section 3.2.1) and solutions of spherical waves with a spherical radial distribution (section 3.2.2).

In the case of plane waves, we study further the transversal radial distribution of the presence density (section 3.2.1), and we deduce a few physical properties: a structure with spatial frequencies (sections 3.4, 3.5), a hollow structure of phase waves (section 3.6), an ineffective cross-section (section 3.7), a hollow structure of mass (section 3.8), and the presence density extended at the macroscopic scale (section 3.9).

We also propose some applications to Cosmology: a solution to the problem of antimatter (section 4.1) and dark matter (section 4.2), and a new approach to simulation of galaxies (section 4.3).

2. The Problem of Dimensionless Point Particles

2.1. Electric Energy Divergence of a Point-Like Electron

Can an electron be a dimensionless mass point and a dimensionless electric charge? Has an electron an infinite proper energy?

As we have shown [1], the total electric energy of an electron at rest is given by:

$$W = \frac{q^2}{8\pi\varepsilon_0 \, a} \tag{6}$$

so, when the radius tends to zero, the charged sphere would have an infinite energy:

$$a \to 0 \quad \Rightarrow \quad W \to \infty \tag{7}$$

therefore the electron cannot be a mass point.

2.2. About the Angular Momentum

The angular momentum of a little sphere of radius r and mass m is given by:

$$J = m\, r^2 \tag{8}$$

and a dimensionless mass point would have a zero angular momentum :

$$r = 0 \quad \Rightarrow \quad J = 0 \tag{9}$$

so the discrete values of a spin or orbital momentum predicted by quantum mechanics does not match the zero angular momentum of point-like particles. This also shows that a mass particle cannot be dimensionless and that there must be a spatial distribution of mass. The angular momentum has been excluded from the Principle of Correspondence of quantum mechanics, which is expressed in the coordinate representation as:

$$\begin{aligned} \hat{x}(x) &= x \times \\ \hat{p}_x &= -i\hbar \frac{d}{dx} \end{aligned} \tag{10}$$

and in the momentum representation as :

$$\begin{aligned} \hat{x}(x) &= i\hbar \frac{d}{dp_x} \\ \hat{p}_x &= p_x \times \end{aligned} \tag{11}$$

Because of the exclusion of spin and orbital momentum from the Principle of Correspondence, quantum theory has asserted that they are purely quantum numbers, although the angular momentum has been initially defined by classical mechanics. So quantum theory has to be enhanced with the introduction of a spatial distribution of mass for every type of particle and hopefully it will match the spin and orbital momentum operators.

2.3. Magnetic Energy of a Moving Electron

Starting from the well known Biot-Savard law:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \wedge \vec{u}_r}{r^2} \tag{12}$$

(with the notations used in most scholar books) Paul Marmet has computed the magnetic energy of a single moving electron [10] at constant velocity. He considered the definition of the electric current I:

$$I = \frac{dQ}{dt} \tag{13}$$

as a set of moving electrons of charge e^-:

$$I = \frac{d(N e^-)}{dt} \tag{14}$$

with a uniform velocity v:

$$I = \frac{d(N e^-) v}{dx} \tag{15}$$

He defined the density of magnetic energy u_m as the magnetic energy U_m per unit of volume V as :

$$u_m = \frac{U_m}{V} = \frac{B^2}{2\mu_0} \tag{16}$$

and deduced the differential of magnetic energy:

$$dU_m = \frac{\mu_0 (e^-)^2}{32\pi^2} \frac{v^2}{r^4} dV \tag{17}$$

Considering, as it is usual, that the electron has a radius, r_e, and integrating to the whole volume V, he obtained the total magnetic energy of a moving electron:

$$U_m = \frac{\mu_0 (e^-)^2}{8\pi} \frac{v^2}{c^2} \frac{1}{r_e} \tag{18}$$

so

$$U_m \sim \frac{1}{r_e} \tag{19}$$

where the magnetic energy of an electron is divergent at the zero radius, therefore the electron cannot be a point-like particle. Further in his paper [10] he demonstrated that the magnetic energy of the electron is equivalent, at the first order, to the kinetic part E_k of the relativist energy of the electron:

$$E_k = m_0(\gamma - 1)c^2 \tag{20}$$

if we take the value of the classical radius r_e.

Finally he interprets the electron radius r_e "as the size of a central cavity with radius r_e in which there is no field, because this would require an amount of energy (and mass) which is not compatible with the electron mass". So Paul Marmet's work shows that the self magnetic field of the electron has a hollow structure.

Similarly we have shown in our previous paper [1] that the transversal distribution of the plane wave of a free particle may have a hollow structure at any Bessel order $\beta > 0$.

Further more this equivalence of the self-magnetic energy and the kinetic part of the relativist mass suggests that the electron might have a hollow mass.

3. Study of the Transversal Distribution of Particle Phase Waves

3.1. About the Wave Nature of the Electron

As we have recalled it in our previous paper [1] the electron cannot be conceived as an electrically charged point-like mass; it is a quantum of electricity that is associated to a wave that has a phase velocity and a group velocity. Electron interferences and electron diffractions can be predicted and experimented.

In its intrinsic referential frame the electron is at rest, so its energy equation:

$$h\nu_0 = m_0 c^2 \tag{21}$$

defines the minimum frequency ν_0 of the electron as a function of the rest mass m_0, therefore a wave function is to be associated with the rest mass of the electron.

3.2. The General Differential Equation of Waves

The general differential equation of waves:

$$\left(\frac{1}{c^2}\frac{\partial}{\partial t^2} - \nabla^2\right)\psi = 0 \tag{22}$$

has solutions with plane waves and solutions with spherical waves.

3.2.1. Solutions with Plane Waves

Plane waves represent translations of a free particle with a defined energy momentum 4-vector.

The case of plane waves has already been studied in our previous paper [1] and we recall the expression of its transversal distribution:

$$\psi_T(\mathbf{k} \wedge \mathbf{x}) = a J_\beta(r / r_0) \cdot e^{\pm i n \vartheta} \tag{23}$$

from which we have deduced the transversal radial density of presence d_T :

$$d_T = \psi_T \cdot \overline{\psi}_T \tag{24}$$

Resulting from a wave distribution function using a Bessel function of order $\beta \geq 0$, the transversal density of presence works as the square of a Bessel function of order β:

$$d_T(r) = a^2 J_\beta^2(r / r_0) \tag{25}$$

3.2.2. Solutions with Spherical Waves

Here we consider the case of spherical waves. They represent angular momentums, which may be an orbital momentum, a spin momentum or both.

Let's consider the orbital momentum operator $\boldsymbol{l}$, in spherical coordinates $\{r, \theta, \varphi\}$, the eigenfunctions of the squared momentum $\boldsymbol{l}^2$ and the projection operator $\boldsymbol{l}_z$ of the orbital momentum [11] can be expressed as:

$$\mathrm{f}(r, \theta, \varphi, l, m) = \Delta(r) \cdot \mathrm{Y}_{lm}(\theta, \varphi) \tag{26}$$

where l and m are the quantum numbers associated to the orbital momentum $\boldsymbol{l}$ and its projection $\boldsymbol{l}_z$ and $\Delta(r)$ is an « arbitrary » distribution function (SIC !).

In the usual quantum theory, this distribution function is not considered, as it is an arbitrary factor. It is then implicitly identified to the Dirac function, and we have:

$$\Delta(0) = 1 \tag{27}$$

for r = 0 on the trajectory of a point-like particle.

In the usual mathematical resolution, we search for the eigenfunctions $\mathrm{Y}_{lm}(\theta, \varphi)$, which are common to the two operators $\boldsymbol{l}^2$, and $\boldsymbol{l}_z$ and this way of computation implicitly require that angular variables θ, φ can be separable:

$$\mathrm{Y}_{lm}(\theta, \varphi) = \Phi_m(\varphi) \cdot \Theta_{lm}(\theta) \tag{28}$$

Obviously it is just a hypothesis of simplification. So the physical conditions of separability of angular variables θ, φ should have to be precisely explained by quantum theory. The eigenfunctions of the projection $\boldsymbol{l}_z$ of the momentum are written [12]:

$$\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{\, i m \varphi} \tag{29}$$

and the eigenfunctions of the squared momentum $\boldsymbol{l}^2$ are written [11] :

$$\Theta_{lm}(\theta) = (-1)^m \cdot i^l \cdot \sqrt{\frac{2l+1}{2} \frac{(l-m)!}{(l+m)!}} \cdot P_l^m(\cos\theta) \tag{30}$$

where $P_l^m(\cos\theta)$ is a Legendre polynomial associated to the polynomial $P_l(\cos\theta)$.

As particles cannot be dimensionless, a distribution function $\Delta(r)$ has to be introduced. We think that plane waves and spherical waves should be in a logical coherence. Plane waves propagate without any change in their cylindrical distribution, but spherical waves propagate with a spherical expansion in space, which has to satisfy the law in $1/r$ and the homogeneous spherical density has to satisfy the law in $1/r^2$ because it is related to a solid angle. So we postulate the following spherical distribution:

$$\Delta(r) = a' \frac{1}{r} J_\beta(r/r_0) \tag{31}$$

where β is the Bessel order.

The spherical distribution is formally related to the cylindrical distribution by:

$$\psi_T(r) = \frac{a}{a'} r \Delta(r) \cdot e^{\pm i n \vartheta} \tag{32}$$

although the radius vector **r** has 3 dimensions for spherical distributions and only 2 dimensions for cylindrical distributions. Consequently the presence density of the rest mass in 3 dimensions has to be expressed as:

$$D(r) = (a')^2 \frac{1}{r^2} J_\beta^2(r/r_0) \tag{33}$$

and so the spherical presence density is related to transversal presence density by :

$$D(r) = \left(\frac{a'}{a}\right)^2 \frac{1}{r^2} d_T(r) \tag{34}$$

3.3. Relations of the Scaling Factor to the Compton Length and the Wave Number

The Compton wavelength is the coefficient in the well-known equation of the Compton effect:

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\varphi) \tag{35}$$

So it has been introduced as:

$$\lambda_c = \frac{h}{m_0 c} \quad \text{or} \quad \bar{\lambda}_c = \frac{\hbar}{m_0 c} \tag{36}$$

From equations (4), (36) we deduce that the scaling factor r_0 is related to the Compton wavelength as:

$$r_0 = \bar{\lambda}_c = \frac{\lambda_c}{2\pi} \tag{37}$$

so the scaling factor is identical to the "reduced" Compton length.

Let us now consider the wave vector **k** of a mass particle defined by:

$$\mathbf{P} = \hbar \mathbf{k} \tag{38}$$

where **P** is the momentum, the wave number k of the phase wave of the particle or of a photon is deduced as :

$$mc = \hbar k \tag{39}$$

$$m = m_0 \gamma \tag{40}$$

where m is the relativist mass. So we have:

$$k = \frac{\gamma}{\lambda_c} = \frac{\gamma}{r_0} \tag{41}$$

The minimum frequency ν_0 of the electron (equation 21) corresponds to the minimum wave number:

$$\hbar k_0 = m_0 c \tag{42}$$

with the following relation:

$$\omega_0 = 2\pi \nu_0 = c\, k_0 \tag{43}$$

and finally we have:

$$k_0 = \frac{1}{r_0} \tag{44}$$

3.4. A Structure with a Spatial Frequency

All transversal distributions have a primary maximum at a smaller radius r_1 and several secondary maxima at greater radii $r > r_1$, which are each separated with a minimum of zero density. The Bessel order β and the scaling factor r_0, together, define the transversal distribution of the presence density of any particle.

The Bessel function $J_\beta(x)$ with large arguments:

$$x >> \left| \beta^2 - \tfrac{1}{4} \right| \tag{45}$$

has an asymptotic form [13]:

$$J_\beta(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(x - \beta \frac{\pi}{2} - \frac{\pi}{4} \right) \tag{46}$$

so the resulting recurrence relation between its roots $z_{\beta,i}$ is:

$$z_{\beta,i} - z_{\beta,i-1} = \pi \tag{47}$$

Therefore the two successive zeroes of the transversal distribution of the phase wave are separated by half a period.

We can then express in its asymptotic form, the transversal distribution $\psi_T(r)$ of the phase wave as:

$$\psi_T(r) \approx a\sqrt{\frac{2}{\pi}\frac{r_0}{|r|}}\cos\left(\frac{r}{r_0}-\beta\frac{\pi}{2}-\frac{\pi}{4}\right) \tag{48}$$

and the transversal radial distribution $d_T(r)$ of the presence density as:

$$d_T(r) \approx a^2\,\frac{2}{\pi}\frac{r_0}{|r|}\cos^2\left(\frac{r}{r_0}-\beta\frac{\pi}{2}-\frac{\pi}{4}\right) \tag{49}$$

In both equations the successive zeroes match a similar recurrence relation:

$$r_{\beta,\,i}-r_{\beta,\,i-1}=\pi\, r_0 \tag{50}$$

So we see that the transversal distribution $d_T(r)$ of the presence density has a spatial period:

$$\Lambda_d=\pi\, r_0 \tag{51}$$

and that the transversal distribution $\psi_T(r)$ of the phase wave has a spatial period:

$$\Lambda=2\pi\, r_0 \tag{52}$$

which is identical to the Compton length:

$$\Lambda=\lambda_c \tag{53}$$

The spatial period depends only on the rest mass of the particle: it is independent of the velocity of the particle.

From the spatial period we can define a spatial frequency N as:

$$N=\frac{1}{\Lambda} \tag{54}$$

then from equation (44) and from:

$$N=\frac{1}{2\pi\, r_0} \tag{55}$$

we can introduce the spatial frequency Ω as:

$$\Omega=2\pi\, N=k_0 \tag{56}$$

So the transversal radial distribution of the phase wave has a wave nature in the space domain with a spatial frequency.

The presence density has the spatial frequency $2N$ and the spatial period $\Lambda/2$.

3.5. Spatial Frequencies of Some Particles

As we have shown, we can compute numerically the spatial frequency of some particles from its rest mass. We sum up the numeric results in the array below:

Data Array 1. Properties of transversal radial waves of some particles.

	electron	**proton**	**neutron**
rest mass m_0	$9.1093897\ 10^{-31}$ kg	$1.6726231\ 10^{-27}$ kg	$1.6749286\ 10^{-27}$ kg
scale factor r_0	$3.86159325\ 10^{-13}$ m	$2.10308932\ 10^{-16}$ m	$2.10019447\ 10^{-16}$ m
spatial period Λ	$2.4263106\ 10^{-12}$ m	$1.32140999\ 10^{-15}$ m	$1.3195911\ 10^{-15}$ m
spatial frequency N	$4.12148387\ 10^{11}$ m^{-1}	$7.56767396\ 10^{14}$ m^{-1}	$7.56767396\ 10^{14}$ m^{-1}

3.6. A Hollow Structure of Phase Waves

3.6.1. The Radii Value Depends on the Bessel Order

The radius r_1 is defined as the radius of the main maximum of presence density. The ratio r_1/r_0 is a function of the Bessel order β. At Bessel orders $\beta > 0$ radial distributions have a central hole due to the properties of Bessel functions. The radius r_2 of the central hole is defined as the shorter radius of half the main maximum of presence density. The ratio r_2/r_0 is a function of the Bessel order β. We have computed the main maximum, the ratio r_1/r_0 and the ratio r_2/r_0 as functions of the Bessel order β, numerically with the software MathCAD 6.0 SE, until the order 400. The results are given in the data array 2.

Data Array 2. Radius r_1 of the main maximum and radius r_2 of the hole.

Bessel order of the distribution	**maximum density of presence**	**r_1 / r_0 at the main maximum**	**r_2 / r_0 at half the main maximum**
0	1.000	0.000	N/A
0.1	0.747	0.464	0.009
0.2	0.633	0.677	0.074
0.3	0.558	0.855	0.169
0.4	0.503	1.015	0.273
0.5	0.461	1.166	0.380
0.6	0.428	1.309	0.488
0.7	0.400	1.447	0.595
0.8	0.376	1.581	0.703
0.9	0.356	1.712	0.809
1	0.339	1.841	0.916
2	0.237	3.054	1.961
3	0.189	4.201	2.989
4	0.160	5.318	4.010
5	0.140	6.416	5.026

Bessel order of the distribution	maximum density of presence	r_1 / r_0 at the main maximum	r_2 / r_0 at half the main maximum
6	0.125	7.501	6.040
7	0.114	8.577	7.051
8	0.105	9.648	8.062
9	0.098	10.711	9.071
10	0.092	11.772	10.079
20	0.059	22.219	20.138
30	0.046	32.525	30.178
40	0.038	42.786	40.200
50	0.033	52.924	50.225
60	0.029	63.187	60.246
70	0.026	73.367	70.264
80	0.024	83.543	80.279
90	0.022	93.640	90.294
100	0.021	103.775	100.307
200	0.013	205.237	200.318
300	0.010	304.468	300.345
400	0.008	406.243	400.516

3.6.2. The Hole Radius as a Function of the Bessel Order

The radius r_2 of the central hole (defined as the lower radius of half the main maximum) can be computed from the ratio r_2/r_0 as a function of the order β.
The function $r_2(\beta)$) has no known mathematical expression, so we have approximated it with the following linear function (using the software MathCAD 6.0 SE):

$$\frac{r_2(\beta)}{r_0} = A + B\beta \tag{57}$$

defined with coefficients A, B depending on the Bessel order, given in the data array 3:

Data Array 3. Coefficients A, B for intervals of Bessel orders

intervals of Bessel orders	***A***	***B***
0.1 to 0.9	-0.154	1.069
1 to 9	-0.034	1.010
10 to 100	0.035	1.0031
100 to 400	0.300	1.0004

Our applications to cosmology (in sections 4.1, 4.2) use very high Bessel orders (section 3.11.4). Our first approach may have accuracy less than 1% so we may consider that:

$$\frac{r_2(\beta)}{r_0} \approx \beta \tag{58}$$

3.7. The Concept of Ineffective Cross-Section

With strictly positive Bessel orders, the presence density has a null minimum at the centre of the particle. Moreover at higher Bessel orders the presence density of the particle is quite zero from the centre of the particle until very near a given radius r_2, and this results into a lower probability of interaction of a particle P_1 with a particle P_2. The presence density of a particle at Bessel orders 100 and 400 are shown with a zoom on the hole, respectively in figures 1 and 2 where the x coordinate represents the ratio r/r_0.

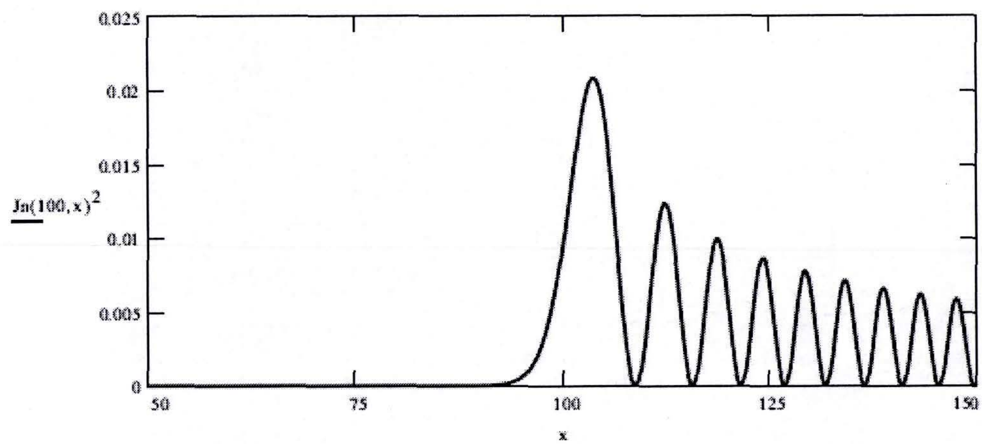

Figure 1. Transversal radial density of presence at Bessel order 100.

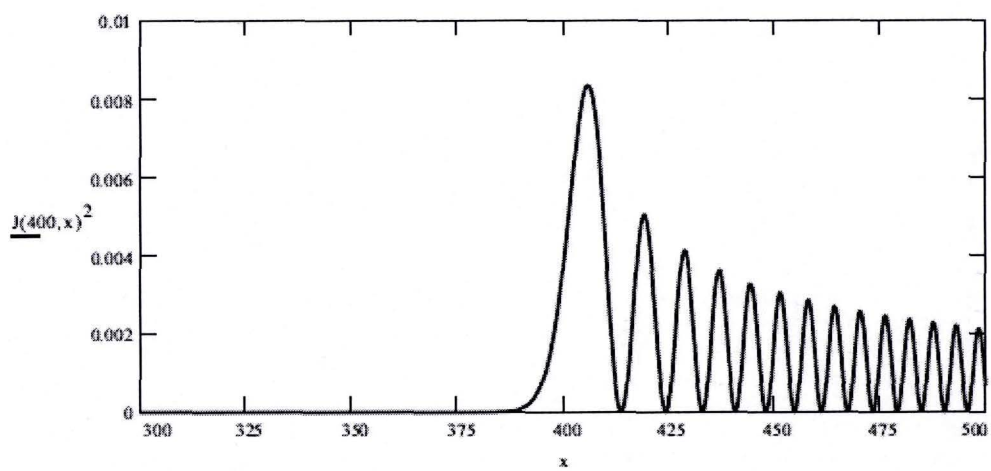

Figure 2. Transversal radial density of presence at Bessel order 400.

The figure 3 with a zoom at a very low density scale, shows that the presence density is quite zero from the centre of the particle until very near the radius r_2.

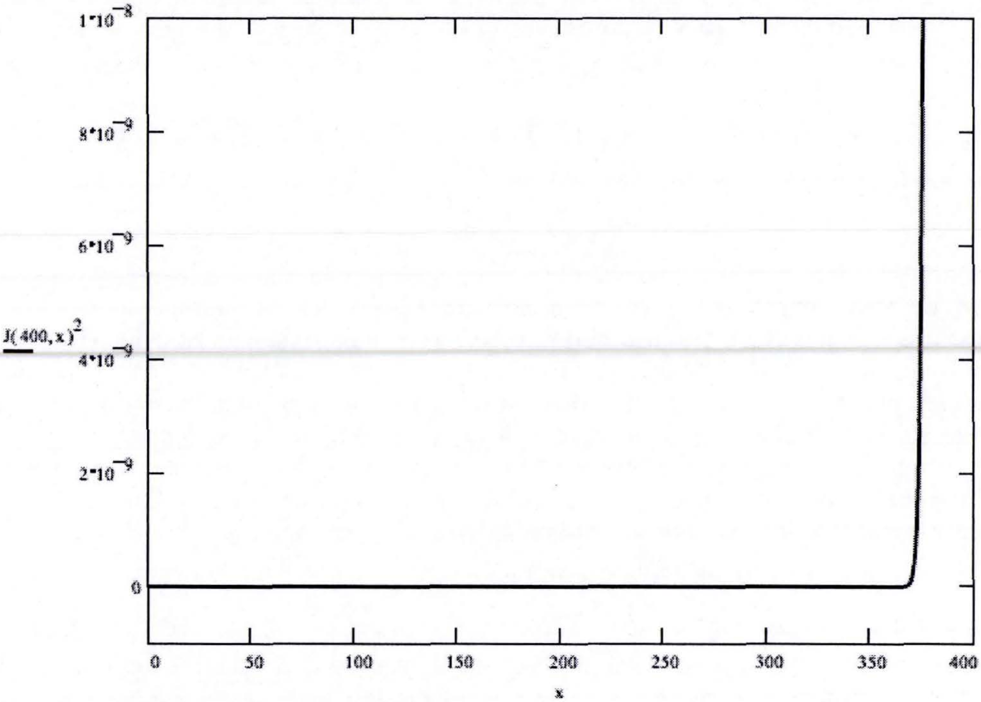

Figure 3. Zoomed graph of density of presence at Bessel order 400.

Let us consider two particles P_1 and P_2 (e.g. an electron and an anti-electron), they can interact only at a close enough distance which is experimentally defined by their efficient cross-section. If the particle P_1 has a high Bessel order, the radius r_2 of its hole is much greater than the efficient cross-section of the particle P_2 so they cannot interact, whatever their usual properties: therefore P_2 comes out of the scope of P_1. We then say that the particle P_1 has an inefficient cross-section.

3.8. A Hollow Structure of Rest Mass

Can the mass of a particle remain confined in a sphere of radius r_2 in which the presence density is zero? Obviously the answer is no. Therefore the hollow structure of waves implies also a hollow mass. Let us remark that in current quantum theory the normalization of wave functions, leading to probabilities, have discarded the absolute values of scalar coefficients, which represent the intensities, or amplitudes of waves in optics, i.e. the scalar ψ_0 in the plane wave equation (2):

$$\psi_k(\mathbf{x},t)=\psi_0\cdot\psi_T(\mathbf{k}\wedge\mathbf{x})\cdot e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{59}$$

In this way the normalization has also discarded the distribution of mass.

In our previous paper [1] we have emphasized that the plane wave of a mass particle is the wave function of its rest mass. The two major arguments are the following:

1. The frequency ν is related to the relativist mass and has the minimum ν_0 which is related to the rest mass by the relation (21):

$$h\nu_0 = m_0 c^2 \tag{60}$$

2. The transversal distribution of the plane wave depends on a scaling factor r_0 which is related to the rest mass by the relation (4):

$$r_0 = \frac{\hbar}{m_0 c} \tag{61}$$

So we have suggested that the transversal distribution of the presence density of a particle can represent the distribution of the rest mass, but it is true in only the 2 transversal dimensions. We have postulated a distribution function (31) in spherical waves, which is also in relation to the distribution of energy and mass for the same reasons as here above. And we have deduced a spherical presence density (33) which can represent the radial distribution $D(r)$ of rest mass in 3 dimensions. Consequently from the hollow structure of phase waves (section 3.6) we deduce a hollow structure of the rest mass which have the radial distribution $D(r)$.

3.9. The Presence Density in a Set of Particles

Although phase waves $\psi_k(\mathbf{x},t)$ defined in equation (2) are added to compute quantum interactions, transversal presence densities $d_T(r)$ defined by equations (24, 25) are added to compute the presence density of the rest mass in a quantum system of interacting particles. When spherical waves are considered, spherical presence densities $D(r)$ defined by equation (33) are added to compute the presence density of the rest mass. And in the case of the gravitation interaction spherical presence densities $D(r)$ are also added to compute the presence density of the rest mass.

Let us consider a set of electrons P_1, P_2, ... P_n. To simplify let us consider that they are on the same radial axis in given positions s_1, s_2, ... s_n. The total presence density $D(r)$ of rest mass at a great radial distance r on the axis is then given by:

$$D(r) = a'^2 \frac{1}{r^2} \sum_i J^2_{\beta_i}\left(\frac{r - s_i}{r_0}\right) \tag{62}$$

where β_i is the Bessel order of each electron. Squared Bessel functions are added with a difference in the spatial phase, which is due to s_i.

Linear combinations of squared Bessel functions result in a total presence density of rest mass where null minima disappear and microscopic fluctuations are more or less smoothed. Similar considerations apply to a set of anti-electrons, a set of protons or a set of neutrons.

3.10. The Presence Density at the Macroscopic Scale

At the macroscopic scale, we may neglect the microscopic fluctuations of the presence density of a particle in radial directions of space.

Obviously it is already done in usual quantum theory that never considers any transversal distribution.

We may define an average presence density. It may be $<D(r)>$ from equation (33) or $< d_T(r) >$ from equation (25) as:

$$< d_T(r) > = a^2 < J_\beta^2(r / r_0) > \tag{63}$$

Taking the approximation (46) for large arguments (45) we obtain:

$$< d_T(r) > \approx a^2 \frac{2}{\pi} \frac{r_0}{r} < \cos^2\left(\frac{r}{r_0} - \beta\frac{\pi}{2} - \frac{\pi}{4} \right) > \tag{64}$$

which simplifies into:

$$< d_T(r) > \approx a^2 \frac{1}{\pi} \frac{r_0}{|r|} \tag{65}$$

The above equation seems formally to be independent of the Bessel order, but it is not true: the mathematical validity condition of the approximation is:

$$\left| \frac{r}{r_0} \right| >> \left| \beta^2 - \tfrac{1}{4} \right| \tag{66}$$

and so physically we can take the condition:

$$|r| > r_0 \, \beta^2 \tag{67}$$

and therefore it excludes all radii r, which are shorter than r_2, i.e. it excludes all positions inside the hole.

Moreover at the cosmological scale, it not pertinent to consider microscopic fluctuations and the equation (65) can be used as an exact cosmological law:

$$d_T(r) = a^2 \frac{1}{\pi} \frac{r_0}{|r|} \tag{68}$$

which applies to the presence density of mass at distances r from the centre of a particle, which are greater than the hole radius r_2.

3.11. The Scale of Bessel Orders

From the plane wave solution (2) of the general wave equation (22) we have seen that the Bessel order β appears to be a new quantum number which has a continuous spectrum.

The distribution function has different properties at different values of the Bessel order; therefore we can build a classification of all possible Bessel orders.

3.11.1. The Bessel Order Zero

For $\beta = 0$, the main maximum is in the centre of the particle with the highest level:

$$d_{r=0} = 1 \tag{69}$$

The figure 1 in ref. [1] shows its surface graph. The distribution function is continuous, even at zero radius where the derivative takes the value zero.
This continuous distribution with $\beta = 0$ can replace the Dirac distribution which is often implicitly used in quantum theory. It is a continuous distribution with spatial frequencies.

3.11.2. The Low Bessel Orders

When the Bessel order $\beta > 0$ the presence density has a null minimum at the zero radius:

$$d_{r=0} = 0 \tag{70}$$

The main maximum stands at a radius r_1 where the derivative is always zero.
The radius r_1 is not null, so the distribution of the presence density has a hollow structure.
The diameter $2r_1$ of the hole depends on and increases with the Bessel order β.

"Low Bessel Orders" are mathematically defined as the following interval:

$$\beta \in]0, 3/2[\tag{71}$$

The derivative of the distribution function has not the value zero at the zero radius, and thus the presence density derivative is discontinuous at the centre of the particle, for every positive Bessel order less than 3/2.

The figures 3, 4, 5, 6, 7 in ref. [1] show surface graphs for the Bessel orders 0.1 (zoomed), 0.33, 0.5, 1.5 respectively.

3.11.3. Medium Bessel Orders

Medium Bessel orders are physically defined as giving a main maximum radius r_1 in the quantum scale.

3.11.4. High Bessel Orders

High Bessel orders are physically defined as giving a main maximum radius r_1 in the macroscopic scale. Extremely high Bessel orders are physically defined as giving a main maximum radius r_1 in the cosmological scale.

Let us consider a galaxy with a radius R_g of 10^5 light year, i.e. 9.46 10^{20} meters. Its ratio to the scale factor r_0 of the electron is:

$$\frac{R_g}{r_0} = 2.45 \cdot 10^{33} \tag{72}$$

and it shows that microscopic fluctuations of the presence density are not to be considered in the study of gravitation in a galaxy.

4. Applications to Cosmology

4.1. First Application: a Passive Antimatter in the Universe

4.1.1. The Signature of the Presence of Antimatter

It is commonly assumed that the Universe has emerged from vacuum with some big quantum fluctuations that have been frozen by an inflation phenomenon.

The assumed initial light was made of hot photons that created equal quantities of matter and antimatter: quarks anti-quarks pairs, electron anti-electron pairs, and proton anti-proton pairs.

And it is also assumed that the inflation has prevented pair's annihilations back to light, while all antiparticles have disappeared during this inflation period.

Can we believe that half of the mass of the Universe (antimatter) has been annihilated, while the other half (matter) has been prevented from annihilation?
How can it be possible during a so short time of 10^{-33} second?

The matter-antimatter imbalance requires the 3 Sakharov conditions [14]:

1. Baryon number B violation,
2. C-symmetry and CP-symmetry violation,
3. Interactions out of thermal equilibrium.

Actually, there is no experimental evidence of particle interactions where the baryon number conservation is perturbatively broken, nevertheless the Big-Bang Standard Model is assumed to violate the baryon number conservation non-perturbatively. Mathematically, the commutator of the baryon number B and the perturbative Hamiltonian H is zero:

$$[B,H] = BH - HB = 0 \tag{73}$$

So the baryon number violation is an unacceptable U(1) anomaly.

Moreover the baryon asymmetry of the universe has been naively estimated for the standard model. Observational results yield that the baryon asymmetry parameter η:

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \tag{74}$$

is between 2.6 10^{-10} and 6.2 10^{-10} as quoted in the literature.

Therefore only one antiparticle is cancelled when 10 billion pairs of particle and antiparticle are produced. Consequently it is hardly difficult to obtain a baryo-genesis at the large scale of the cosmos, which does not include a great quantity of antimatter.

A.D. Dolgov has reviewed the basic principles of baryo-genesis and the models of abundant creation of antimatter in the universe [15].

The question: "Why did cosmologists postulate that the Universe contains no antimatter?" has a very simple answer: "A presence of antimatter would produce annihilation of matter before the formation of stars."

Actually this postulate seems to be contradicted by recently known facts: we have detected some quantities of antimatter in our galaxy.

The European astronomical satellite "Integral" [16] has detected gamma photons of 511 keV from the centre of our galaxy [17]. Let's also mention an antimatter globular cluster with anti-helium in our galaxy "Milky Way" [18].

To preserve the postulate of antimatter absence, some authors have explained that the positrons may be created from gamma-ray bursts produced by some "mini-starbursts" [19], but this statement is not really convincing.

The Dapnia laboratory of Saclay's CEA has called it a fountain of anti-electrons, because the 511 keV photons flow from the center of our galaxy has been evaluated to 10^{+43} per second; it is 400.10^{+9} times less than from Sirius. Such observations showing a continuous annihilation of electron anti-electron pairs indicate the presence of huge quantities of anti-electrons inside the galaxy bulge.

4.1.2. A Solution: the Presence of Passive Antimatter

Antimatter is necessarily present in great amount in the Universe because one of the 3 Sakharov conditions [14] is not met by baryons (section 4.1.1) and the presence of a great amount of positrons has been detected in galaxy bulge. So the following questions must be answered:

- How could stars be formed by accretion of matter without being annihilated by antimatter?
- How can a galaxy bulge contain so much antimatter and does not explode?

The only possible answer is that something in the Universe has prevented antimatter to interact with matter.

A first solution where matter and antimatter are geometrically separated in different sheets of the space-time manifold, has been initially proposed by A. Sakharov [20, 21], next expressed by J.P. Petit in a Newtonian framework [22, 23] and later developed by Robert Foot [24]. These theories are at a disadvantage, as they require a very peculiar cosmological model with a "twin Big Bang" derived from the standard model.

The second solution, which we propose here, is a direct consequence of the quantum theory extended with radial distributions. Antiparticles at high Bessel orders result in a passive state, which forbid any possible interaction with the corresponding particles, because of the inefficient cross-section (section 3.7). And reciprocally.

4.2. Second Application: a Solution to the Dark Matter Problem

4.2.1. The Puzzle of Dark Matter

Fritz Zwicky observed large clusters, such as the Coma cluster, and he found that the speed of galaxies is much too great to keep them gravitationally bound together unless their mass is one hundred times more than it was estimated from the number of stars [25]. According to astronomic computations of star velocities, about 90 % of the gravitational mass is an assumed "dark matter". Today the standard explanation is the presence of dark matter.

To explain dark matter, it has been proposed several hypothetical particles [26], which should play a key role in the nucleo-synthesis. So every candidate to dark matter can have several cosmological implications.

Some prospects for dark matter are the following: sterile, massive or extremely high energy neutrinos [27, 28, 29, 30, 31], WIMPs (Weakly Interacting Massive Particles) [32, 33] which are tracked with the Edelweiss experiment [34], neutralinos which are a particular type of WIMPs [35, 36, 37], axions (light neutral pseudo-scalar particles) [38, 39, 40], Higgs Dark Matter [41], matter unification [42], sub-quantum physics [43], and so on ...

A more fundamental question has been asked [44]: "Does the missing mass problem signal the breakdown of Newtonian gravity?»

Mordehai Milgrom has proposed an alternative [45, 46] to dark matter: a modified Newtonian dynamics (MOND). R.H. Sanders has studied X-ray emitting clusters of galaxies in the context of MOND [47]. A. Lue, G.D. Starkman have proposed cosmological scenarios built on MOND [48]. S. McGaugh has tested MOND predictions against WMAP data [49].

Another fundamental question can be asked: Does the missing mass problem signal the breakdown of point-like particle gravity?

A new solution is proposed here with a possible hollow mass.

4.2.2. Dark Matter: Particles at Extremely High Bessel Orders

Considering the transversal distribution of plane waves and the similar radial distribution of spherical waves, we have deduced that there is a distribution of the mass corresponding to the distribution of the presence density of any particle.

Can the mass of a particle remain confined in a sphere of radius r_2 in which the presence density is zero? Obviously the answer is no. Any mass particle of any Bessel order $\beta > 0$ will have a hollow mass, with correlative consequences on its inertia momentum and its angular momentum.

At extremely high Bessel orders, the hole radius r_2 of particles in a galaxy bulge can reach the radial distance of peripheral stars, and therefore the mass of such particles is mainly distributed far away from the centre of the galaxy. This is the expected property of dark matter. It means that a galaxy bulge should contain a specific matter, which produces a gravitational field only at large distances.

Considering the huge quantity of antimatter that is detected from the galaxy bulge, the dark matter effect should be produced by a great stock of passive antimatter at an extremely high Bessel order.

4.3. An Approach to the Simulation of a Galaxy

4.3.1. Recall About Newton Law

According to the Newton law, the orbital velocity $v(r)$ of a body around a central mass M is given by the well-known equation:

$$v(r) = \sqrt{\frac{GM}{r}} \tag{75}$$

where r is the radius of the orbit and G the gravitation constant, and the angular velocity $\omega(r)$ is given by:

$$\omega(r) = \frac{v(r)}{r} \tag{76}$$

and therefore the angular velocity is:

$$\omega(r) = \frac{\sqrt{GM}}{r^{\frac{3}{2}}} \tag{77}$$

as we currently observe it with the planets of our solar system.

4.3.2. Mass Distributions and Star Velocities in a Galaxy

Let us consider a model of galaxy with a continuous distribution of mass represented by the spherical density $D(r)$. For a star at a distance r from the centre of the galaxy, the gravitation is due to an equivalent total mass $\mathrm{M}(r)$ at the centre that is defined as:

$$\mathrm{M}(r) = \iiint_S D(r)dV \tag{78}$$

where S is a sphere of radius r, and it is usually computed with the equation:

$$\mathrm{M}(r) = 4\pi \int_0^r D(\rho)\rho^2\, d\rho \tag{79}$$

where ρ is an integration variable.

To have a galaxy stability, i.e. to allow a galaxy to rotate as a whole, all stars must have more or less the same angular velocity $\omega(r)$ and its condition can be expressed as:

$$\frac{d\omega(r)}{dr} \approx 0 \tag{80}$$

4.3.3. A Simple Method of Simulation of Star Velocities in a Galaxy

To simulate the star velocities in a galaxy, we have chosen a simple method of simulation: we have computed the angular velocity $\omega(r)$ from the equation (77) for an orbit of radius r around an equivalent central mass $\mathrm{M}(r)$ which depends on the mass distribution $D(r)$ as defined by the equation (79). The resulting figures of our simulations are not to be scaled as we have set the gravitation constant to $G = 1$.

4.3.4. Theoretical Star Velocities Around a Point-like Mass

In a gravitation field due to a point-like mass (a Schwarzschield field) $\omega(r)$ is computed from equation (77) with a constant mass which we have set to $M = 1$. The angular velocity decreases quickly with the radius, as shown in the figure 4.

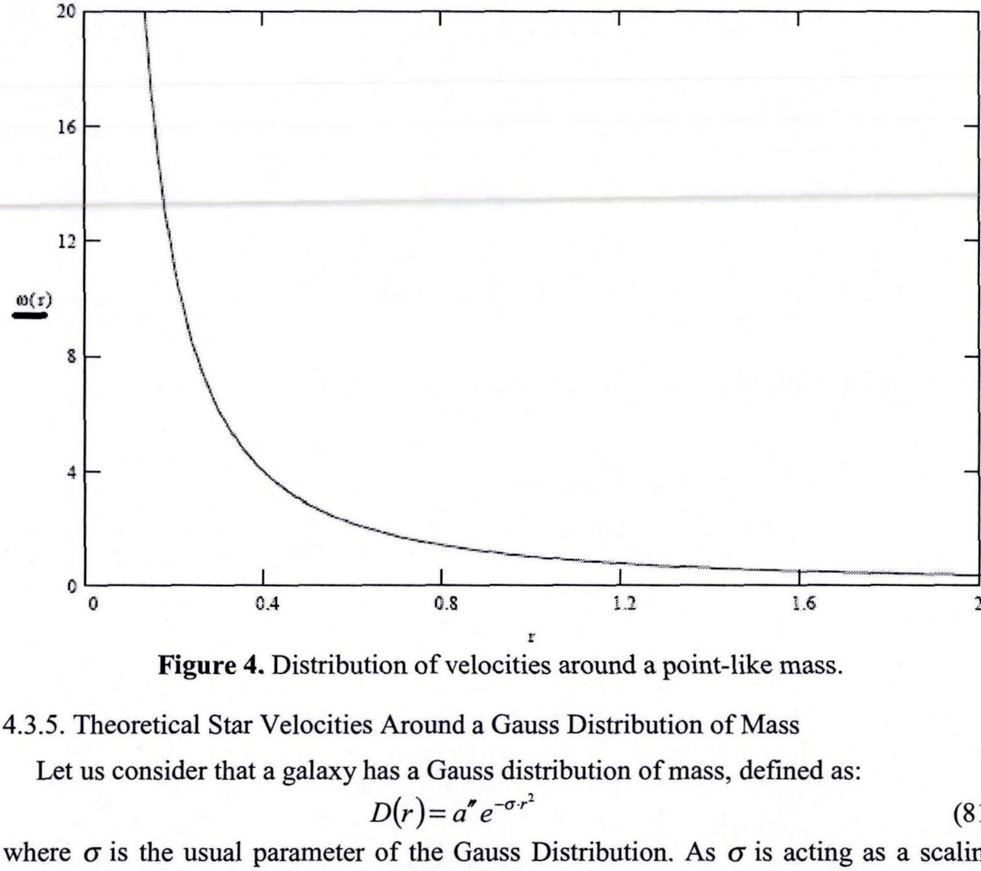

Figure 4. Distribution of velocities around a point-like mass.

4.3.5. Theoretical Star Velocities Around a Gauss Distribution of Mass

Let us consider that a galaxy has a Gauss distribution of mass, defined as:

$$D(r) = a'' e^{-\sigma \cdot r^2} \quad (81)$$

where σ is the usual parameter of the Gauss Distribution. As σ is acting as a scaling factor we have set it to $\sigma = 1$. In this simulation we have normalized the distribution by evaluating the coefficient a'' to have a total mass $M = 1$, so:

$$a'' = \frac{1}{M(\infty)} \quad (82)$$

is computed from the integral (79), which is always convergent.

Around a assumed Gauss distribution of mass the angular velocity decreases too quickly to enable the rotation of galaxy stars as a whole, as shown in the figure 5.

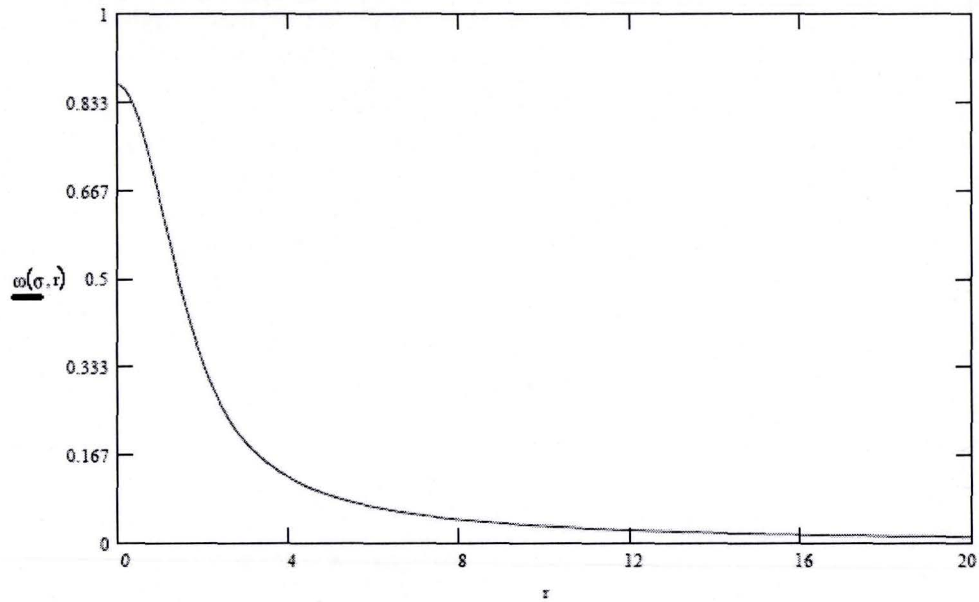

Figure 5. Distribution of velocities around a Gaussian distribution of mass.

4.3.6. Star Velocities Around a Galaxy Bulge of Very High Bessel Orders

In our first approach to the simulation of a galaxy, we study the effect of a set of particles (section 14), which have very high Bessel orders. Therefore we consider here that gravitation is only due to the mass in the galaxy bulge.

Obviously in a future accurate simulation of a galaxy we will consider the gravitation due to all stars, and this will contribute to lower the derivative of the angular velocity but as it is well known, this cannot reach the condition (80) without much added dark matter. However our simulation has reached the condition (80).

In this simulation, we consider the galaxy bulge as a set of particles of one kind (e.g. only electrons or anti-electrons), i.e. all particles have the same scale factor r_0. And for convenience we have set it to $r_0 = 1$.

The particles have several different Bessel orders. To simplify, only integer orders are considered and spherical densities are added as:

$$D(r) = a' \frac{1}{r^2} \sum_{\beta=0}^{N} A_\beta J_\beta^2(r / r_0) \tag{83}$$

and then integrated with equation (79). The coefficient a' has been set to $a' = 1$. The mass has not been normalized and the total mass has not been computed.

The coefficients A_β define the abundance of particles of the Bessel order β.

The simulation is fully able to approach the condition (80) by adjusting the coefficient A_β and it can even obtain a positive derivative:

$$\frac{d\omega(r)}{dr} > 0 \tag{84}$$

We have adjusted the coefficients A_β to have star velocities which decrease slightly with the radius r as it is observed in the arms of spiral galaxies. The chosen coefficients are the following:

$$A = (0.02;0.1;30;50;80;120;150;250;300;100;500;\\ 300;600;800;900;1200;1500;1000;1200;1500;1500) \tag{85}$$

where the line matrix A starts with the index $\beta = 0$.

The figure 6 shows the resulting distribution of star velocities. The graph has a slight negative slope after $\beta = 3.94$ until $\beta = 13.2$ and it has a positive slope in two intervals : after $\beta = 0.68$ until $\beta = 3.94$, and after $\beta = 13.2$ until $\beta = 17.4$.
With greater coefficients A_β at some given orders β we can obtain a positive slope since any order $\beta > 0$ as it is shown on the left of the graph.

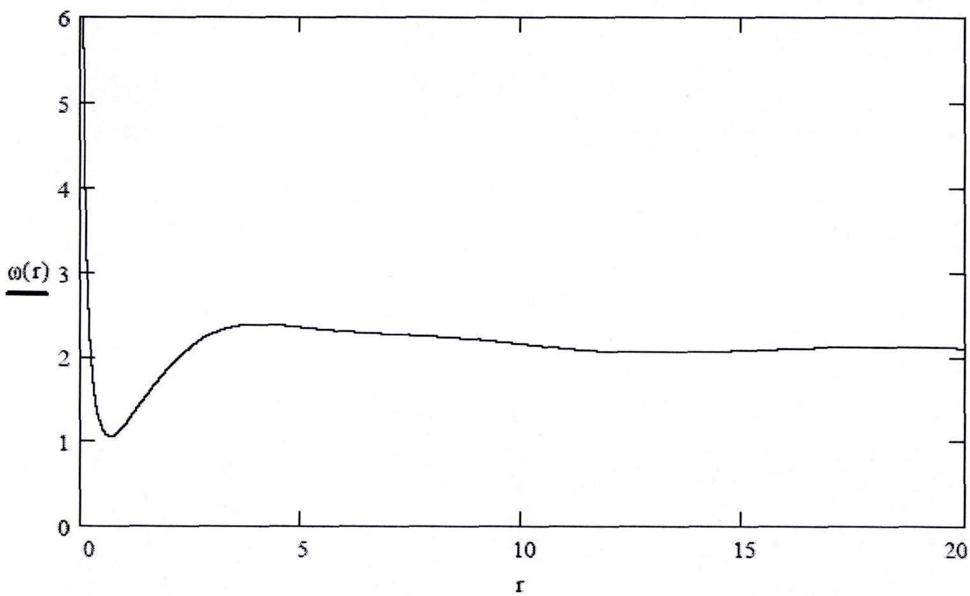

Figure 6. Distribution of velocities around a given set of hollow masses.

To obtain a nearly flat curve in a wide interval of radii and to approach the condition (80), it has been necessary to give a low value to the coefficient A_0. This shows that particles of Bessel order zero must be in a limited quantity in the galactic bulge.

Actually it is possible with anti-electrons because they are annihilated when their Bessel order β decreases to zero giving γ-bursts.

The figure 7 shows the resulting radial distribution of mass densities.

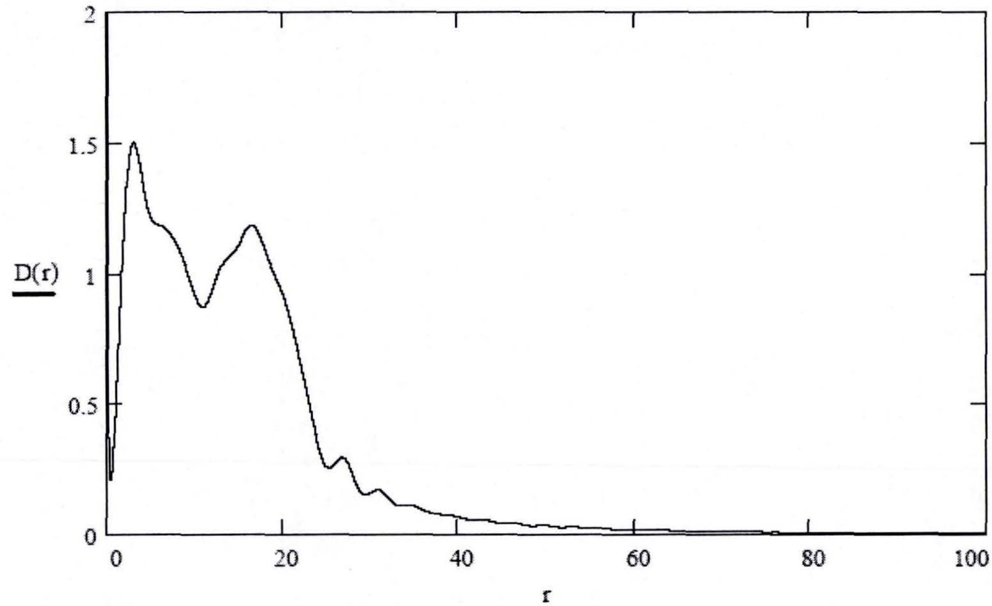

Figure 7. Distribution of mass densities due to a set of hollow masses.

The equivalent central mass M(r) computed from the integral (79) increases with the radial distance r as it is shown in the figure 8, the equivalent central mass M(r) continues to increase and seems to reach a maximum at greater distances, but we have not yet proven that the integral (79) is convergent with the distribution (83).

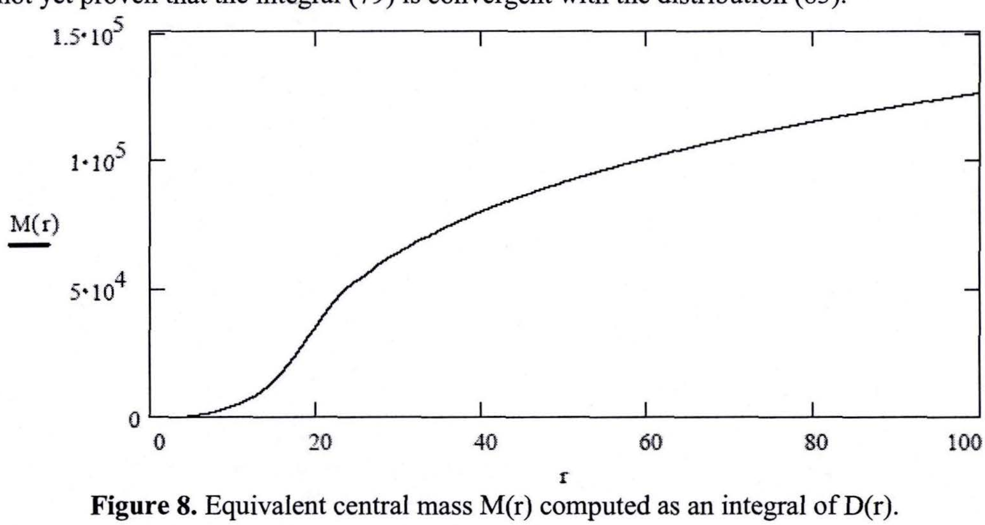

Figure 8. Equivalent central mass M(r) computed as an integral of D(r).

5. Conclusions

5.1. Plane Waves and Spherical Waves

A transversal distribution has been introduced in the equation of plane wave of any free massive particle (fermion or boson).

We have postulated a spherical radial distribution of spherical waves, which is mathematically coherent with the cylindrical radial distribution of plane waves.

The asymptotic distribution function at a large distance shows a spatial structure of waves with a main spatial frequency. Thus the so-called "particles" should be considered as a system of spatial vibrations, rather than a corpuscle, which might have an internal structure, limited by a given radius.

The radial distribution of the presence density of spherical waves has been interpreted as the presence density of the rest mass, because the parameter r_0 of the distribution is directly related to the rest mass.

5.2. A new Quantum Number with new Properties

The Bessel order is a new quantum number, which has a continuous spectrum; therefore quantum theory has to be enhanced with transversal radial distributions and spherical radial distributions. The present quantum theory cannot predict any values of the Bessel order for any known particles, but its applications to cosmology is now opening to a new knowledge.

With the Bessel order zero, the presence density has a maximum at the center of the particle which then matches point-like particle predictions at best.

When the Bessel order is not zero the presence density has a null minimum at the center of the particle and the transversal or the spherical distribution has a hollow structure, which implies a hole in the presence density and thus a hollow mass.

At high Bessel orders, the presence density is zero from the center of the particle until a given radius r_1, and it results into a lower probability of interaction.

At extremely high Bessel orders, the presence density is quite zero from the center of the particle until a large distance, and it results into an “inefficient cross-section” with an extremely low probability of interaction. As an example, an electron of Bessel order zero cannot annihilate with an anti-electron of extremely high Bessel order: the anti-electron is then considered as being passive.

5.3. Applications to Cosmology

We have proposed two applications to Cosmology. As we know, the presence of antimatter in a galaxy bulge is actually proven by γ-bursts of 511 keV. Simulations by many authors have already shown that a galaxy stability requires dark matter and the presence of a dark matter is usually assumed after observations of star velocities in a galaxies.

We have proposed a common solution to these two problems: antimatter (mainly composed of anti-electrons) is made passive by extremely high Bessel orders and stored in the galaxy bulge (maybe also in star nucleus). And this passive antimatter has a hollow mass with a radius that can reach the radius of the galaxy. Thus it behaves as the required dark matter.

In other words, dark matter is hidden in the galactic bulge, but it produces a gravitational mass in long distances. The quantum structure of particles with a radial distribution which depends on their Bessel order can have a gravitational effect upon stars running at very great distances.

We think that the Bessel order is always zero in most terrestrial experiments, but it may take extremely high values for some particles or antiparticles, under some physical conditions: extremely strong gravitation field and / or extremely strong magnetic field in galaxy bulges and in star nucleus.

5.4. Towards a new Big Bang Scenario

The concept of a non point-like particle and solutions of the wave equation with a radial distribution is logically leading to a new Big Bang scenario.

The assumed inflation period has not destroyed antimatter but another mechanism involving hollow mass particles has made the antimatter passive. Moreover the universe is using hollow antimatter of very high Bessel orders as a central engine of galaxies.

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