Mandate Game : Model of Anticipation Exchange and Decision-Making

Asaki Nishikawa
Graduate School of Science and Technology,
Department of Earth and Planetary Science, Faculty of Science,
Kobe University,
Nada, Kobe, 657,Japan
e-mail: asakiasakiasaki@yahoo.co.jp

Abstract
What types of organization structure are optimal, if cooperative acts must be achieved ? For resolving this problem, we introduced a game modeling characteristics of organizations, inspired by well-known economist K.J.Arrow in "The Limits of Organization". In this game, each agent gives a mandate to an adviser about his right for decision-making. He tells the adviser his fragments of an anticipation about cooperative acts. The advisers decide act of the agents who gave the mandate to him. The agents get a reward that was calculated by a distance from an optimal act. We used genetic algorithm for dynamics of this game. The network structures of delivering mandates between agents have evolved through three phases : 1) disconnected phase 2) connected phase 3) hub agent phase. Finally, we briefly discussed more abstract model that relates to anticipatory system formulation.
Keywords : Multi-Agents, Organization, Cooperation, Fragments of Anticipation, Decision-Making

1 Introduction

What types of organization structure are optimal, if cooperative acts must be achieved ? In other words, when agents cooperate and work with other agents, what kind of structure do they make? What kind of structure is chosen? It is the problem of this paper. (Fig.1) For considering this question, we propose an game based on the concept of "mechanism" of the game theory and the argument of "the limit of the organization" written by well-known economist K.J.Arrow[2].

1.1 K.J.Arrow's "Limits of organization"

In "Limits of Organization"[2], K.J.Arrow thought why decision-making by organizations is needed, The reasons are summarized as follows : 1) A necessity of co-operation of agents. They would gain less utility if they act in a disorderly way.

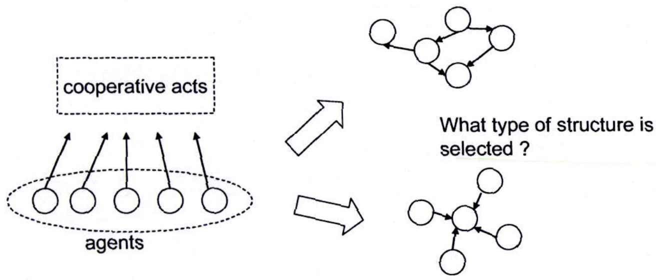

Fig. 1: The problem. When agents cooperate and work with other agents, what kind of organization structure they make? What kind of structure will be selected?

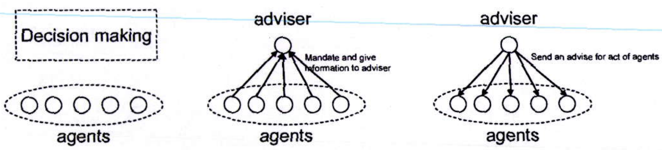

Fig. 2: Concept of mechanism. Agents mandate to an adviser his right of choice for his act. Each agent sends a message to the adviser. The adviser decides a optimal combination of behavior for all agents, based on the set of the messages sent to him.

2) Dispersion of information. Information for optimal cooperative acts is scattering among the agents. 3) Costs of distributing information. Because distributing information has costs, it is inefficient to distribute all information to all agents. Arrow deduces conclusions that the group of agents need the hub of some information for a quick decision-making. If there is no mechanism to collect knowledge about optimal acts of the other agents dispersed between them, all agents have to exchange their knowledge each other. It's inefficient from the viewpoint of the communication cost of the information. Arrow concluded that this was the reason that we simultaneously need "organizations" as well as markets without structure.

1.2 Concept of "Mechanism" in the Game Theory

"Mechanism" is a concept of the game theory[12]. It treats the situation which many agents make a decision by giving a mandate to an "adviser" about his right for the decision. It assumes many agents in a system. Each agent only has information about himself. Agents mandate his right of choice for his act to an adviser. Each agent sends a message to the adviser. The adviser decides a optimal combination of acts for all agents, based on the set of the messages sent to him. Each agent gets utility according to the combination of acts of agents. This kind of decision-making is called a mechanism in the game theory.

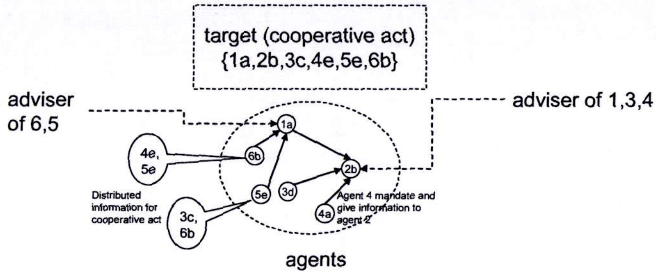

Fig. 3: The concept of the mandate game. In the mandate game, each agent mandates to the other agent his right of decision-making and sends the message about the optimal acts. In this figure, agent 1, 3 and 4 mandate their right to agent 2. However, it's agent 5 that has information on the suitable act about agent 3. The acts chosen are slightly different from a ideal act in this figure(For example c is a ideal act for agent 3, but d is chosen actually).

2 Model

Using the scheme of a collective decision-making inspired by the concept of a mechanism, we modeled the situation proposed by Arrow. Features of our model are as follows. Like the Arrow's research, no agent has enough information for cooperative acts independently. Good exchanges of information let agents do optimal cooperative acts because each agent has partial information of the cooperative acts. From the notion of mechanism, we borrow the concept of "the adviser and the message". In the mechanism of the game theory, the adviser is the special agent distinct from other agents. However in our game, all agents can become an adviser of other agents. Being different from "mechanism" of game theory, each agent may have knowledge about other agents' optimal acts.

2.1 What is the Mandate Game ?

In this condition, the agents can get more utility than they act in disorderly fashion if they do good exchanges of knowledge. However, information for the optimal cooperative acts is scattering among the agents. Each agent can give a mandate to other agent about his right of decision-making. He follows the advice from the agent. Agents that received the mandates synthesize information as large as possible. They make advices for cooperative acts. What type structures of the mandate network are needed for gaining higher utility ? We show the model components more in detail below.

From here we assume there are N agents (Note that we will define a dynamics on M "individuals" each of which consists of N agents later).

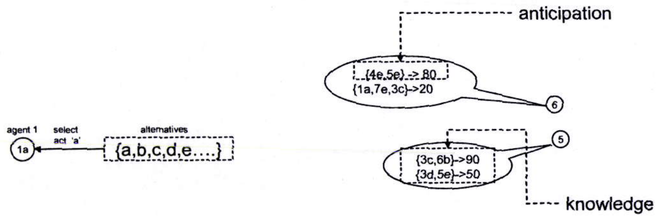

Fig. 4: Concept of anticipation. Each agent chooses an act from the set S of alphabets. Anticipation is a combination of more than one agents' acts and an expected utility for the acts. Knowledge is the set of the anticipations each agent has.

2.2 Anticipations

First of all, information that agent i has is modeled. Each agent selects his act from a set of alternatives. One agent has "anticipations". These consist of a combination of other agents' cooperative acts and its expected utility. Our definition of anticipation is related to both of Dubois' "weak" and "strong" anticipation[8][9]. It is "weak", because it has no relation with dynamics. Hence, it does not cause infinite self reference regression. However it's "strong", because it is made from true knowledge of targets, not from model. In the last section, we will discuss in more abstract form the relationship of mandate game to Dubois' formulation of anticipatory system. We call a collection of anticipations "knowledge".

Set of alternatives is represented by $S = \{a, b, c, \ldots\}$. An anticipation j that agent i has is expressed as $a_{ij} = (\{s_k\}_{k\in N, s_k \in S}, r_{ij})$. Here, r_{ij} is a positive finite real number, and represents expected utility as a reward of k acts $\{s_k\}$. For example, $(\{e_3, b_6\}, 80.0)$ or $(\{a_2, d_3, h_5, a_7\}, 100.0)$ are anticipations.

We give a alias "knowledge K_i to a_{ij} ", if we omit index j and regard them as a one set of anticipations.

2.3 Mandate, Synthesis, and Decision-making

Each agent can deliver a mandate to another agent about his right of decision-making. The agent transfers his knowledge to the mandated agent. If agent i mandates his right to agent m, knowledge K_m of agent m become $K_m \cup K_i$. This models mandate of a decision-making and one transfer of a message in the notion of mechanism. It's different from the mechanism of the game theory because agents i can mandate his right to an arbitrary agent m. If agent mandates his right to himself, he will make a decision by himself.

The mandated agents try to make larger anticipations by synthesizing the anticipations transferred from the other agents (See Fig.6). This models the merit of integrating collected informations and maintenance of consistency of acts. The way

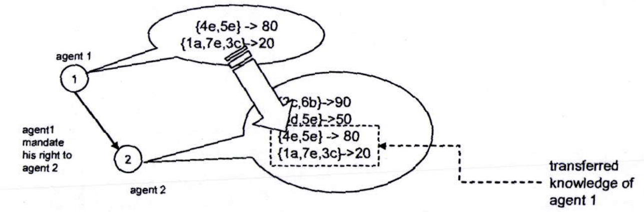

Fig. 5: The concept of mandate. Each agent mandates his right of decision-making to other agent and also transfers his knowledge as a message. Agent 1 mandates his right to agent 2, in this figure.

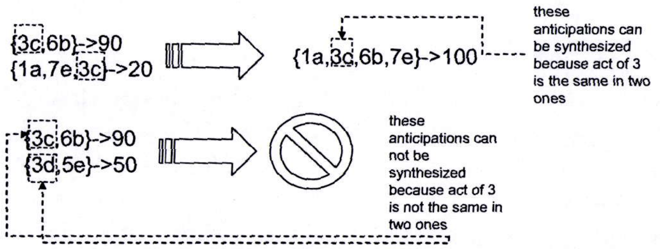

Fig. 6: The concept of synthesis. Each agent builds new anticipations by synthesizing anticipations. This figure shows examples of the both cases in which synthesis is possible and impossible.

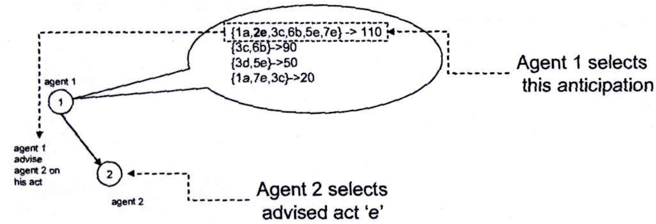

Fig. 7: The concept of decision-making. The mandated agents make advises based on the cooperative acts with the best expected utility. This figure is the example which agent 1 advises on the act of 2.

of synthesizing anticipations is below: First, the agent selects top k anticipations from his knowledge ordered by utility. Next, he synthesizes anticipations that met consistency conditions as below. All anticipations are "atomic", i.e., they can't be used partly. Two anticipations $a_{ij}, a_{ij'} \in K_i$ can be synthesized, when the following conditions were met. If $s_k \in a_{ij}$ and $s'_k \in a_{ij'}$ then $s_k = s'_k$. Or when two anticipations do not include acts inconsistent with the act of the same agent k, it's possible to synthesize these anticipations. When they are synthesized, new anticipation $a_{is} = (s_k \cup s'_k, f(r_{ij}, r_{ij'}))$ is added to the K_i. Here, f is a function which decides new utility r_{is} from two utilities. Refer to Fig.6 for an example in detail. In this paper, we decided f as follows.

1. For overlapped acts, total utility is not increased.

2. Utility after synthesizing won't be below the average of $r_{ij}, r_{ij'}$.

i.e., when $h = \frac{|a_{ij} \cap a_{ij'}|}{|a_{ij} \cup a_{ij'}|}$, we define f as $(r_{ij} + r_{ij'})(1 - \frac{h}{2})$.

Utility will be an average if the $h = 1.0$ (all acts are overlap.). It approaches average of two anticipations according to a degree of the acts' overlapping.

After the synthesis of anticipations, the agent selects a anticipation a_i^* that has best utility. Or, $a_i^* = \{a_{ik} | \text{for all } r_{ij} \in K_i, r_{ij} \leq r_{ik}\}$. The mandated agent advises agents about the act according to the cooperating act included in the selected anticipation. Agents select his act by the advise. i.e., agent m that mandates his right to an agent i selects s_m as his choice when a_i^* includes s_m. Therefore, if s_m is a choice of agent m, then $s_m^* = s_m$ (See Fig.7). When an act of mandating agent isn't included in the chosen a_i^*, we define that the agent chooses his act randomly.

2.4 Target and Dynamics

First, we give ideal cooperative acts as a target. After all agents have selected his act, we compare the target's acts with actual selected acts. We define reward utility of agents by following conditions: if a Hamming distance between target and the actual

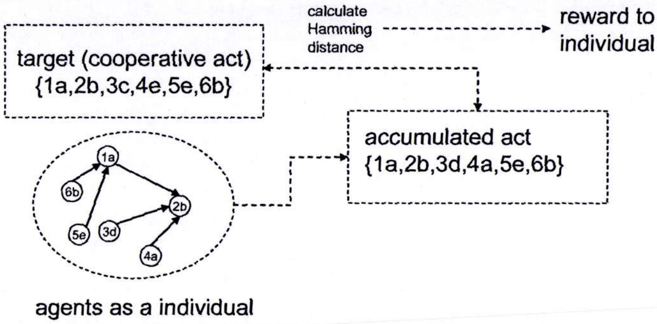

Fig. 8: The evaluation. After decisions by advices have ended, a Hamming distance between the cooperative acts of a given ideal target and the actual selected acts is measured. Acts of agent 3, 4 are different from the target, in this figure.

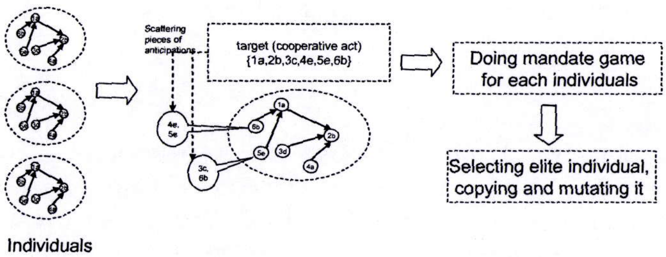

Fig. 9: Our implementation of dynamics. Regarding N agents group as one "individual", the mandate game is performed on individuals. As a result, individuals with different mandate networks get different utility, and an individual is replaced based on the utility.

acts is zero, then maximum utility is gained by agents. The target $T = (\{s_{ti}\}_{i \in N}, r_t)$ is expressed as the set with the same structure as the anticipation. s_{ti} includes acts of all agent i, and r_t is ideal utility obtained as a result of target acts. An actual utility is decided as follows. First, we define the set of the acts obtained as a result of the advices $s^* = \cup s_i^*$ (This may include random acts. Hence it may be not the same as the advices). Next, we define $d^*=$ "the Hamming distance between the $\{s_{ti}\}$ and the s^*". The function w which decides utility according to the Hamming distance is defined as follows in this paper. $w(d^*) = r_t$ if Hamming distance is 0. And $w(d^*) = 0$ when the Hamming distance is the maximum. There is arbitrariness in how to decide intermediate values. Here, a following way was adopted. When we call r_t divided by $|s_{ti}|$ "g", then $r_t - d^* g$ is w.

In dynamics of our model, we regard a group of N agents as an "individual" . We prepare M individuals, divide the ideal target information as many anticipations and distribute these to agents in the individual. We define one time step of our model's

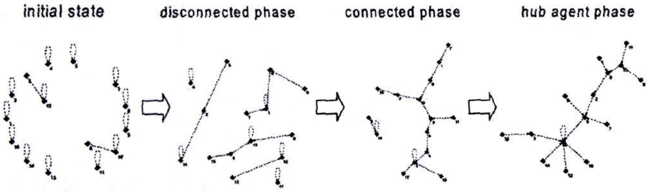

Fig. 10: The phase of an individual's mandate network. Extracting the individual from the population we chase the transition of it

dynamics as follows. 1) We do mandate game with individuals for the same target. 2) Copying the individual that gained best utility, and interchanging it with a least one. 3) When copying, edges of mandate network in the individual are randomly changed with probability p (mutation). A minimum genetic algorithm is included in this dynamics, hence networks evolve to structures which get more utility.

3 Results

In this configuration, like Arrow's consideration, we may have expected that an individual's gain maximize when its network structure is shape like a star. However actually such structure does not appear. Fig.10 shows a typical individual displaying the phase of transition with four stages from the population which is started with a certain knowledge distribution. In the initial condition, networks are not formed, and agents inside the individual do decision-making disjointedly. At a second stage, networks become like chains which some agents connect. At third stage, chains form one big chain. At last stage, the hub node which has big in-degree appears.

In Fig.11, two time series are compared in order to look at the relationship between utility and the fourth stage carefully. If we look at Fig.11, when the hub node whose in-degree was large appears, simultaneously we could see the big increase of average utility.

Furthermore in Fig.12, the change of the network in Fig.10 and the change of average utility were compared. After increasing loosely, utility was stagnant for a while until the chain-like structures appear. Becoming third stage (all chains are connecting), there was big increase. When the hub of fourth stage appeared, there was big increase again.

4 Conclusion

We simulated the situation that was considered by K.J.Arrow related to game theory's mechanism, by the mandate game. We have shown that the mandate networks' connectivity level contributes to growing up of average utility in the mid range degree of utility increase . The appearance of a hub agent contributes to them in

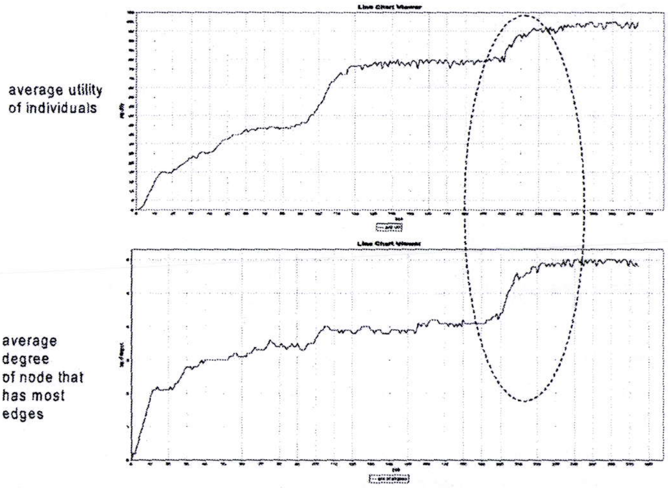

Fig. 11: The time evolution of utility and in-degree. Above figure shows average utility of all individuals. Figure below shows an average of in-degree of the node which has a largest one he time evolution of utility and centerity. Above figure shows average utility of all individuals.

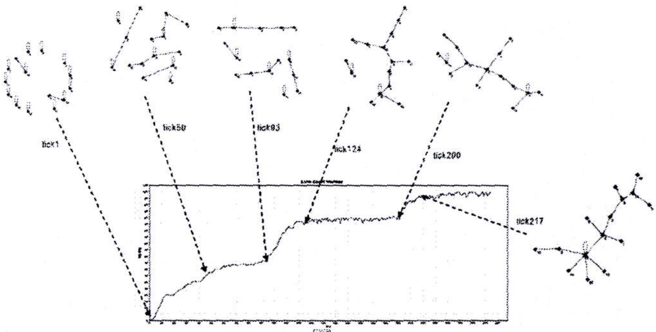

Fig. 12: Phase of mandate networks and time evolution of their average utility. As for utility we took an average of all individuals. As for network, extracting a certain individual we chased its transition.

the range of high degree utility. Although network of star structure is optimal, the network with coexistence of high and low degree nodes did not appeared. If we use a relatively low number of agents in an individual (usually 5-10), then star-like structures appear. However, if we run a relatively high number N simulation (30-60) then even the hub agents disappear. The reason of these facts may be as follows: If we use the low N individuals, then star-like structure is achieved by random mutations. They become dominants in an early stage of the evolution dynamics. If we use the high N setting, probability of star-like structure's appearance becomes low before other types of structures achieve high utility

Are these features of our game negative ? If we think this model as a model of origins of a central agent in the system, we think so. However, if we think this as a model of integration of some central agents predefined in the system these features have positive meanings. This model shows effects of consistency imposed on the system that consists of several incomplete central agents like patients of multiple personality disorder[13] or like Planaria with multiple brains[5]. From these features, we may make a hypothesis that our usual integration of personalities (or brains) comes from the constraint of the consistency. In another paper, we show a model of an origin of central agents gathering other agents information in large population[3]. We can think the agents in the mandate game as the centering agents in the above paper, because they have a partial microcosm of their own system from their own view. Our game does not show origins of the centers but shows the effects of imposing consistency on them.

Many research concerning the autonomous distributive system is done (typically [6]). What we discussed is an extreme case in which centralization of information becomes necessary. There is a model in which an agent controls cooperations between the subordinate agents [11]. However, for the fixed layered structure, changes of structure are not handled in it. In addition, [7] shows the solution which has the structure of a pyramid in the system of the Lotka-Volterra equation with N kinds exists. His model is different from our model as follows: It presupposes the vertical food chain. It only treats a quantitative relationship, consistency is not concerned. [4] and [1] model the leadership in a social network. [4] lacks the viewpoint of emergence. Both of them only focused on a quantitative relationship as [7]. We ignored costs of information transfers, recursive mandate cases, and multiple targets effects, in this paper. If we consider these points we may obtain another conclusion from our game.

Finally, by abstracting our game model,we can imagine an anticipatory oscillators system as below like Dubois' formulation[9]. For example, we can write a formula as an abstraction of our game as follows.

$$x_i(t+1) = f(x_i(t), x_i(t+1), o(x_j(t+1))) \tag{1}$$

where o is observation function.$j \in J_i(t)$ is a subset of oscillators that obeys some kind of consistency condition. For example,

$$\sum_{j \in J_i} o(x_j(t+1)) = 1 - x_i(t+1) \tag{2}$$

If we think o as a modeling function, then $x^* = o(x)$.i.e.,it means Dubois' "weak" anticipation. If o is identity function, then this formula is "strong" anticipation one.

This system consists of oscillators which try to gather other oscillators' future dynamics that meet the consistency laws as large as possible. It combines network structure dynamics, and concept of consistent anticipations. In fact, there exists a model of brain function including transfer of predictions between several brain areas [10]. To make brain activities more coordinative, there also should be some kind of consistency conditions that impose unity on gathered predictions.

This abstraction of our game will become a reference of future research about consistency and multi agents' decision-making with anticipations.

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