Mathematical Models Of Discrete Systems Goal-Directed Behavior Generating

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Abstract
In this paper the problem of discrete systems goal-directed behavior generating is considered. The finite-state machine represents a model of a discrete system. The problem of goal-directed behavior generating is solved by the theory of the universal finite-state machine. The class of finite-state machine simulated by a family of the polynomials is described. The using of this “numerical” model of behavior of finite-state machine allows to apply the algebraic methods to solve the problem of goal-directed behavior generating. For the described class analysis and synthesis problems of universal enumerator are solved.
Keywords: generating goal-directed behavior, the finite-state machine (FSM), the universal enumerator, the numerical model of FSM.

1 Introduction

In the field of systemology attention is traditionally paid to the problems of control and diagnostics of discrete systems behavior. The problem of goal-directed behavior generating (GDBG) is one of the same importance. Actually, generating goal-directed behavior demands consecutive solving of the following problems (both, the actual system behavior and the differing given one, are posed):

- determine, whether generating the given behavior is possible or not;
- determine the set of system transformations which allow generating the given behavior by the means of transforming its components and interrelations between them, changing the modes of functioning, supervision and outputs monitoring, etc;
- choose the optimum in the set of possible system transformations.

One of the basic features of the process of generating goal-directed behavior is the opportunity of getting additional information about the system, principles and means of its fail-safety. The achievement of the given mode of functioning can be carried out by different means, both internal and external. In the case of internal means, the system uses structural reservation rated, when designing the system. In the case of external means, the system uses external (according to its basic components) objects.

In the general case (that is, for an arbitrary discrete system) the problem of generating goal-directed behavior is algorythmicallly insoluble, but it can be solved by imposing certain restrictions on the system behavior.

We will deal with finite-state machine (FSM) as a model of a discrete system. As a rule, finite-state machines are considered as transformers, namely, system functioning is studied through consideration of the way input strings are transformed into output strings. However, sometimes it is even more important to determine the feedback – to find the group of input strings transformed into the given output string. It is so-called an inverse problem, in essence, it means determining an output string for the given input one - that is, the transforming form of behavior. In case the finite-state machine is represented as a set of generated output strings, the machine is considered to be an enumerator (and it has an enumerating form of behavior). Conceptually, the problem of generating goal-directed behavior in case when the structural redundancy is not available proves to be a transition from the transforming form of behavior to enumerating form. Actually, the transition from transforming to enumerating within the framework of the finite-state machines theory is complicated and time-consuming. A new, so-called "numerical" model of finite-state machine is represented which is based on its transition from transforming form of behavior to enumerating form, using several methods of algebra and a number theory.

2 Formal setting of the problem

The finite-state machine (FSM) $A=(S,X,Y,\delta,\lambda)$ is given, where
X is a finite set of input symbols,
Y is a finite set of output symbols,
S is a finite set of states,
$\delta: X\times S\rightarrow S$ is the transfer function,
$\lambda: X\times S\rightarrow Y$ is the output function.

Without loss of generality, we suppose that $S=Y$ and $\delta\equiv\lambda$, that is the output of A if its state. Hence, the initial finite-state machine is brought to the form

$A=(X,S,\delta)$. (1)

Let X^*, S^* be the sets of FSM input and states strings, respectively.

Definition 1.

The FSM $A=(X,S,\delta)$ realizes the family of finite-state mappings $\{\delta_p\}_{p\in X^*}$ of the form $\delta_p: S\rightarrow S^*$ and generates the set of states strings

$L(X^*)=\{s\,|\,(\exists\ s^*\in S)(\exists\ p\in X^*):\delta_p(s^*)=s\}$.

The transforming form of behavior of the finite-state machine A is represented with its family of finite-state mappings $\{\delta_p\}_{p\in X^*}$. The enumerating form of behavior of the finite-state machine A is represented with the set of states strings $L(X^*)$, generated by A. The set $L(X^*)$ is called the enumerable set of A.

The theory of universal FMS is the basis to solve the problem of GDBG. We cite a few main concepts of this theory.

Definition 2.

The family of FSM $\{A_i = (X_i, S_i, \delta_i)\}_{i\in I}$ is given. The name of the family is identified with a set of indexes I. The FSM $A_I = (X, S, \delta)$ is called the universal FSM for the family I, if it is true that
$(\forall i \in I)(\exists \varphi_i : S_i \times X_i^* \to S \times X^*)(\forall s \in S_i)(\forall a \in X_i^*): \delta_i(s,a) = \delta(\varphi_i(s,a))$.

The FSM $A = (X, S, \delta)$ is called the universal enumerator for $\{A_i\}_{i\in I}$ of the family I (where $L(X_i^*)$ is enumerable set of $A_i, i \in I$), if the following condition is satisfied: $(\forall i \in I)\ L(X_i^*) \subseteq L(X^*)$.

Theorem 1 [5]

The FSM A is the universal FSM of the family of FSM $\{A_i\}_{i\in I}$ of the family I if and only if it is the universal enumerator for $\{A_i\}_{i\in I}$ of the family I.

Later on in this paper we will consider only *universal enumerators*.

Let's set a problem of GDBG in the terms of the theory of the universal automata.

Suppose that the FSM A models a desirable system behavior. Let I denote the class of possible behaviors of this system. For every possible system behavior $i \in I$ we consider the FSM A_i. Thus, we have the family of FSM $\{A_i\}_{i\in I}$. To solve the problem of GDBG of the system means that every FSM of this family can model somehow the behavior of A. In that way, the problem of GDBG is solvable if and only if every FSM of $\{A_i\}_{i\in I}$ is universal enumerators for A.

The problem of construction of the universal FMS A for the family I is called the synthesis problem of universal FMS. The inverse problem, the problem of construction of the family I, for which FSM A is universal, is called the analysis problem of universal FMS. Hence, the problem of GDBG can be solved by two ways. One way is to check that A is the solution of the analysis problem for each FMS from $\{A_i\}_{i\in I}$. Another way is to solve the synthesis problem for A, to construct the class of all universal FMS for A and to check that every FMS from $\{A_i\}_{i\in I}$ is a member of this class.

The construction problem of the universal enumerator is insoluble concerning the arbitrary family of FSM. Hence, the problem of GDBG is insoluble concerning the arbitrary system too. Therefore, now we try to isolate classes of FMS, for which this problem is soluble. One of such classes is the class of FMS simulated by the polynomials.

The states of the FMS should be enumerated with integers from 0 up to m-1, so that $S = \{0,1,\dots,m-1\} = GL(m)$ (it means that S coincides with the semigroup of remainders modulo m). The transition function of FSM can be considered as a substitution of the form:

$$\delta_x : \begin{pmatrix} 0 & 1 & \ldots & m-1 \\ s_0 & s_1 & \ldots & s_{m-1} \end{pmatrix}, x \in X \qquad (2)$$

Thus, the system behavior will be considered as a collection of substitutions (2) for every symbol from input set.

We denote $s = (0,1,\ldots,m-1)$, $s_x = (s_0, s_1, \ldots, s_{m-1})$. Let's consider the polynomial $f_x(s)$ as the function of vector s of the form:

$$f_x(s) = a_0 + a_1 s + a_2 s^2 + \ldots + a_l s^l \pmod{m}, a_k \in S, k = \overline{1,l}, x \in X, \qquad (3)$$

where the operations of addition, multiplication, raising to a power are the operations of ring of remainders modulo m.

Definition 3.

We say that behavior of FSM A (1) is simulated by the family of polynomials $\{f_x\}_{x\in X}$ (3), if $(\forall x \in X)\delta_x$ is represented by the polynomial fx, that is $f_x(s) = s_x$.

The degree of the polynomial simulated behavior the FMS, which has m states, is expressed by following formula.

$$l = \max(\alpha_0, \alpha_1, \ldots, \alpha_k) + p_1^{\alpha_1 - 1} \cdot \ldots \cdot p_k^{\alpha_k - 1} \cdot HOK([2^{\alpha_0 - 1}], p_1 - 1, \ldots, p_k - 1) - 1,$$

where $2^{\alpha_0} p_1^{\alpha_1} \ldots p_k^{\alpha_k}$ is factorization of m, $p_1, p_2 \ldots p_k$ are prime numbers.

The class of FMS simulated by polynomials was investigated. We obtained the calculation method of the polynomial coefficients for given FMS-substitution under the condition of polynomial existence. The conditions of FSM simulation by the family of polynomials are also obtained. These results are given in articles [6,7].

Let's cite formal setting of synthesis and analysis problems for the class of FMS simulated by polynomials.

Synthesis problem

The FMS family $\{A_i\}_{i\in I} : A_i = (X_i, S, \delta_i)$ is given, where A_i is simulated by the family of polynomials $\{f_x^{(i)}\}_{x\in X_i}$. It is necessary to construct the FMS $A=(X,S,\delta)$, which is simulated by the family of polynomials $\{f_x\}_{x\in X}$ and is universal for family $\{A_i\}_{i\in I}$.

Analysis problem

The FMS $A=(X,S,\delta)$ simulated by the family of polynomials $\{f_x\}_{x\in X}$ is given. It is necessary to construct the family of FMS $\{A_i\}_{i\in I} : A_i = (X_i, S, \delta_i)$, where A_i is simulated by the family of polynomials $\{f_x^{(i)}\}_{x\in X_i}$, and A is universal for this family.

3 Construction method of enumerable set of the FMS simulated by the family of polynomial

As it was pointed out in 1, the problem of GDBG is equivalent to synthesis and analysis problems of the universal FMS. To solve these problems the method of construction of enumerable sets for considered class of FMS is necessary. Let's consider the mechanism of outcome of such sets.

First of all, we note that, if the transformation generated by input symbols x1,x2,…,xn is simulated by polynomials $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ respectively, then the transformation generated by input string $t=x_1,x_2,\ldots,x_n$, is simulated by polynomial $f_t(s) = f_{x_n}(f_{x_{n-1}}(\ldots f_{x_1}(s)))(\bmod m)$.

In fact, let $t = x_1x_2$, $f_{x_1}(s) = a_0 + a_1 s + \ldots + a_l s^l$, $f_{x_2}(s) = b_0 + b_1 s + \ldots + b_l s^l$.

Then

$$f_t = f_{x_1x_2}(s) = f_{x_2}(f_{x_1}(s)) = b_0 + b_1(a_0 + a_1 s + \ldots + a_l s^l) + \ldots + b_l(a_0 + a_1 s + \ldots + a_l s^l)^l =$$

$$= (b_0 + b_1 a_0 + b_2 a_0^2 + \ldots + b_l a_0^l) + (b_1 a_1 + 2b_2 a_0 a_1 + 3b_3 a_0^2 a_1 + \ldots + l b_l a_0^{l-1} a_1)s +$$

$$+ \ldots + (b_1 a_l + b_2(a_0 a_l + a_1 a_{l-1} + \ldots + a_l a_0) + b_3 \sum_{\substack{i,j,k=0 \\ i+j+k=l}}^{l} a_i a_j a_k + \ldots + l b_l a_0^{l-1} a_l)s^l + \ldots +$$

$$+ (b_l a_l^l)s^{l^2} = c_0 + c_1 s + c_2 s^2 \ldots + c_l s^l + c_{l+1} s^{l+1} + \ldots + c_{l^2} s^{l^2}.$$

Since the semigroup of remainders modulo m generated by element $s \in S$: $< s >= \{s, s^2, \ldots s^{r_0+m_0-1}\}$ is the cycle semigroup with the period m_0 and the index r_0 ($r_0 + m_0 - 1 = l$), values s^i ($i>l$) are repeated starting with degree l, that is $s^{r_0+i} = s^{m_0+r_0+i} = s^{2m_0+r_0+i} = \ldots = s^{km_0+r_0+i} = \ldots, i = \overline{0, m_0 - 1}$.

Thus, if we replace $s^{km_0+r_0+i}, i = \overline{0, m_0 - 1}, k > 0$ with s^{r_0+i} and group together coefficients of the polynomial, then we obtain the polynomial of degree l.

Let's consider the semigroup $(F, \cdot)$, where F is the set of the polynomials (3) simulated behaviors of the FMS (1), $\cdot$ is substitution operator of functions: $f_x \cdot f_y = f_y(f_x)$. Later on, instead of $(F, \cdot)$ we will write in abbreviated form F and we will omit sign $\cdot$. Obviously, the cyclic semigroup generated by element f_x: $F_x = < f_x >$ is finite. Therefore, there are the whole positive numbers r and m (the index and the period of semigroup F_x respectively), that $F_x =< f_x >= \{f_x, f_{x^2}, \ldots, f_{x^{m+r-1}}\}$.

Let's denote $x^\alpha = \underbrace{xx\ldots x\ldots x}_{\alpha}$.

Consider the behavior of the FMS A if the stings x^α is input. If we input serially the symbol x several times, we will obtain the family of the substitutions $\delta_x, \delta_{x^2}, \ldots, \delta_{x^n} \ldots$, which are simulated by polynomials $f_x, f_{x^2}, \ldots, f_{x^n} \ldots$ respectively. Since it is true that

$f_{x^{\alpha+1}} = f_{x^r}$ for $\alpha=m+r-1$, where r is the index, m is the period of the semigroup $F_x=<f_x>$, the equality $\delta_{x^{\alpha+1}} = \delta_{x^r}$ is truly. Thus, if we input the symbol x α times, we will obtain all possible substitutions. To calculate number α we will use the following easy method.

Method 2

Enter: The FMS $A=(X,S,\delta)$, $|S|=m$, and the polynomial f_x (3) simulated behavior of A for input symbol $x\in X$.

Exit: The set of different polynomials $f_x, f_{x^2}, \ldots f_{x^\alpha}$ simulated behavior A for input strings, which consists of only x, and corresponding α.

Let $p=2$.

Step p.

a) Calculate the polynomial $f_{x^p} = f_x(f_{x^{p-1}})$. Let $j=1$.

b) Compare the polynomial f_{x^p} with the polynomial f_{x^j}.

If $f_{x^p}(s) = f_{x^j}(s)$, then $\alpha=p-1$. The method is finished.

If $f_{x^p}(s) = s$, then $\alpha=p$. (In fact, $f_{x^{j+1}}(s) = f_x(f_{x^j}(s)) = f_x(s)$, $j=r=1$). The method is finished.

Else, if $j<p-1$, we increase j by 1 and repeat the operation b). If $j=p-1$, then we increase p by 1 and execute step p.

Let's consider the semigroup F_A generated by polynomials $f_{x_1}, f_{x_2}, \ldots f_{x_n}$: $F_A = \langle f_{x_1}, f_{x_2}, \ldots, f_{x_n} \rangle$. The elements of this semigroup have the form $f_{x_{j1}^{\beta_{j1}} x_{j2}^{\beta_{j2}} \ldots x_{jk}^{\beta_{jk}} \ldots}$, where $0 \le \beta_{jj} \le \alpha_{ji}$. Obviously, F_A is finite. Let F_A^* be a set of different elements of F_A.

Let's denote $\alpha_i=m_i+r_i-1$ $(i=\overline{1,n})$, where m_i is the index and r_i is the period of semigroup $F_{x_i} = \langle f_{x_i} \rangle$, that is $f_{x_i^{\alpha+1}} = f_{x_i^{ri}}$. To obtain different elements of F_A, it is enough to consider the strings $t = x_{j1}^{\beta_{j1}} x_{j2}^{\beta_{j2}} \ldots x_{jk}^{\beta_{jk}}$, where the maximum number of x_i in t in succession is α_i.

In fact, if $t = t'x_i^{\alpha_i+1}t''$, then

$$f_t(s) = f_{t'x_i^{\alpha_i+1}t''}(s) = f_{t''}(f_{x_i^{\alpha_i+1}}(f_{t'}(s))) = f_{t''}(f_{x_i^{r_i}}(f_{t'}(s))) = f_{t'x_i^{r_i}t''}(s) = f_{t^*},$$

that is $\delta_t = \delta_{t^*}$.

Obviously, the set F_A^* represents in essence the set $L(X^*)$, the set of strings of states generated by the FMS A.

Let's determine what polynomials of form $f_{x_{j1}^{\beta_{j1}} x_{j2}^{\beta_{j2}} \dots x_{jk}^{\beta_{jk}} \dots}$ belong to F_A^*.

Obviously, the elements $f_{x_1}, f_{x_2}, \dots f_{x_n}$ are the members of this set. Let the equality $f_{t_p}(s) = f_{t^*}(s)$ is truly for some string t_p of length *p*, where $f_{t^*} \in F_A^*$, $|t^*| \le |t_p|$ ($|t|$ is the length of string *t*). Therefore, the polynomial $f_{t_p}(s)$ is not included into F_A^*. Besides, any polynomial simulated substitution, which includes string t_p as a prefix or as a suffix, is equal to some polynomial, simulated substitution, which is generated by strings shorter than t_p. Therefore, such polynomials are not included into F_A^* either. In fact,

$(\forall t') f_{t't_p}(s) = f_{t_p}(f_{t'}(s)) = f_{t^*}(f_{t'}(s)) = f_{t't^*}(s)$,

$(\forall t') f_{t_p t'}(s) = f_{t'}(f_{t_p}(s)) = f_{t'}(f_{t^*}(s)) = f_{t^* t'}(s)$.

If such polynomial f_{t^*} is not exist, then $f_{t_p}(s)$ is included into F_A^*.

Let the equality $f_{t_p}(s) = c = const$ ($c \in [0, m-1]$) is truly for some string t_p of length *p*. Hence, any polynomial simulated substitution, which is generated by the string included t_p as a suffix, equal to constant *c* and so, it is not included into F_A^* either.

In fact,

$(\forall t') f_{t't_p}(s) = f_{t_p}(f_{t'}(s)) = c$;

Let the equality $f_{t_p}(s) = s$ is truly for some string t_p of length *p*. Then any polynomial simulated substitution, which is generated by the string included t_p as a suffix or as a prefix equal to some polynomial simulated substitution, which is generated by strings shorter than t_p. Therefore, such polynomials are not included into F_A^* either. In fact,

$(\forall t') f_{t't_p}(s) = f_{t_p}(f_{t'}(s)) = f_{t'}(s)$,

$(\forall t') f_{t_p t'}(s) = f_{t'}(f_{t_p}(s)) = f_{t'}(s)$.

Let the set $SUF=\{t_{suf}\}$ be the set of strings of input symbols $\{x_1, x_2, \dots, x_n\}$, that $(t_{sup} \in SUF) \Leftrightarrow (\exists\, t^* : |t^*| \le |t_{suf}|, f_{t_{suf}}(s) = f_{t^*}(s))$.

We suggest the method of construction of all different elements of semigroup F_A.

Method 2

Enter: The FMS $A=(X,S,\delta)$, $X=\{x_1, x_2, \dots, x_n\}$, $|S|=m$ and the family of polynomials $f_{x_1}, f_{x_2}, \dots f_{x_n}$ (3) simulated behavior of A for input symbols $x_1, x_2, \dots, x_n$ respectively.

Exit: The set $L(X^*)$, set strings of states generated by A, that is enumerable set of A.

Step 1.

By using the *method 1* we calculate numbers $\alpha_i=m_i+r_i-1$ ($i=\overline{1,n}$), where m_i is the index and r_i is the period of the semigroup $F_{x_i}=\langle f_{x_i}\rangle$.

Let $F_A^*=\{f_{x_1}, f_{x_2},...f_{x_n}\}$. Let $T_1=\{x_1,x_2,\ldots,x_n\}$ be the set of string of length 1 in input set $\{x_1,x_2,\ldots,x_n\}$. For this step let $SUF=\{x_i^{\alpha_i+1}\}_{i=\overline{1,n}}$, $p=2$.

Step p.

(At the step p we construct the elements of the semigroup F_A $f_{x_{j1}^{\beta_{j1}} x_{j2}^{\beta_{j2}} \ldots x_{jk}^{\beta_{jk}}}$ such as $\beta_{j_1}+\beta_{j_2}+...+\beta_{j_k}=p$, that is the polynomials simulated the behavior of A for the strings of length p.)

a) Let's construct the set of the strings $T_p=\{t_p^i\}_{i\in I_p}$ in the following way. Construct strings of length p: add to the right every symbol from input set $\{x_1,x_2,\ldots,x_n\}$ to every string from T_{p-1}, that is $\forall k\in I_{p-1}$ construct string $t_p^i=t_{p-1}^k x_j$, $j=\overline{1,n}$.

Such string t is included into T_p if and only if it has not any strings, which is member of SUF, as a suffix. Hence, if a string is represented by form $t=t' t_{suf}$ ($t_{suf}\in SUF$), then it is not included into T_p. If $T_p=\varnothing$, then process of construction of F_A^*, set of all different elements of semigroup F_A, is finished. $L(X^*)=F_A^*$ Else we execute operation *b*).

b) For every string $t_p=t_{p-1}x_j\in T_p$ we calculate the polynomial $f_{t_p}(s)=f_{t_{p-1}x_j}(s)=f_{x_j}(f_{t_{p-1}}(s))$.

Let's compare the polynomial f_{t_p} with every polynomial of F_A^*. If $\exists t_k^*$ $1\le k\le p$, that $f_{t_p}(s)=f_{t_k^*}(s)$, then string t_p is included into SUF and is excepted from T_p. If $f_{t_p}(s)=const$, then t_p is included into SUF. Else the polynomial f_{t_p} is included into F_A^*. If $f_{t_p}(s)=s$, then polynomial f_{t_p} is included into F_A^*, and t_p is included into SUF and is excepted from T_p. Thus, after execution of this step F_A^* consist all different polynomials simulated behavior of A for strings of length i, $i=\overline{1,p}$. We increase p by 1 and execute step p.

4 Universal FMS Criterion

To solve the problem of GDBG (that is synthesis and analysis problem of universal FMS) it is necessary to have some criterion defining whether A is universal for the family $\{A_i\}_{i\in I}$. Such criterion is represented by the following theorem.

Theorem 2

The FMS $A=(X,S,\delta)$ simulated by the family of the polynomials $\{f_x\}_{x\in X}$ is the universal enumerator for the family of FMS $\{A_i\}_{i\in I} : A_i = (X_i, S, \delta_i)$, where A_i is simulated by the family of polynomials $\{f_x^{(i)}\}_{x\in X_i}$, if and only if $(\forall i \in I)(\forall x \in X_i)\ f_x^{(i)} \in L(X^*)$, where set $L(X^*)$ is constructed by using the *method 2*.

Example

The FMS $A=(X,S,\delta)$, $|S|=m=6$, $X=\{x_1,x_2,x_3\}$ simulated by polynomials $f_{x_1}(s)=2s^2, f_{x_2}(s)=5s, f_{x_3}(s)=1+2s^2$ and FMS $A_1=(X_1,S,\delta_1)$, $X=\{x_1,x_2\}$ simulated by polynomials $f_{x_1}^{(1)}(s)=4+2s^2, f_{x_2}^{(1)}(s)=5+4s^2$ are given. Let's define, whether A is the universal enumerator for A_1.

By using *method 2* we construct the enumerable set of A:

$L(X^*)=F_A^*=\{f_{x_1}, f_{x_2}, f_{x_3}, f_{x_1x_2}, f_{x_2^2}, f_{x_3x_1}, f_{x_3x_2}, f_{x_3^2}, f_{x_3x_1x_2}, f_{x_3^2x_2}\}$, where

$$f_{x_1}=2s^2, f_{x_2}=5s, f_{x_3}=1+2s^2, f_{x_1x_2}=4s^2, f_{x_2^2}=s, f_{x_3x_1}=2+4s^2, f_{x_3x_2}=5+4s^2,$$

$$f_{x_3^2}=3+4s^2, f_{x_3x_1x_2}=4+2s^2, f_{x_3^2x_2}=3+2s^2$$

$$f_{x_1}^{(1)}(s)=4+2s^2=f_{x_3x_1x_2}(s)\in L(X^*),$$

$$f_{x_2}^{(1)}(s)=5+4s^2=f_{x_3x_2}(s)\in L(X^*).$$

Therefore, A is the universal enumerator for A_1.

5 Solution of analysis problem of universal FMS

Let $L_A(X^*)$ be the enumerable set of A. Then, according to *theorem 2,* the FMS simulated by any subset of the polynomials of given set is enumerated by A and any FMS simulated by the polynomials, which are not the member of this set, is not enumerated by A. Thus we construct the family of the FMS $\{A_i\}_{i\in I} : A_i = (X_i, S, \delta_i)$, for which A is the universal enumerator by the following way.

Let $|L_A(X^*)|=n$. We construct 2^n-2 subsets if given set, that is all subsets except for the null and the subset that equal to $L_A(X^*)$. We single out subsets $\{F_k\}_{k=\overline{1,n-1}}$, where F_k is set of subset of set $L_A(X^*)$ of cardinality k. We construct 2^n-2

FMS by following way: $\forall k, k=\overline{1,n-1}$ we construct the FMS $A_{ki}=(X_k,S,\delta_{ki})$, where $X_k=\{x_1,x_2,...x_k\}$ and the transfer functions are simulated by polynomials $\{f_{x_j}^{(ki)}\}_{j=\overline{1,k}}$, that $\{f_{x_1}^{(ki)}, f_{x_2}^{(ki)},..., f_{x_k}^{(ki)}\} \in F_k$. Let's remark, there are C_n^k (the binomial coefficient) FMS, that is $i=\overline{1,C_n^k}$.

Example

The FMS $A=(X,S,\delta)$, $|S|=m=3$, $X=\{x_1,x_2\}$ simulated by the polynomials $f_{x_1}(s)=s+s^2, f_{x_2}(s)=2s^2$ is given. Let's construct the family is the FMS $\{A_i\}_{i\in I}: A_i=(X_i,S,\delta_i)$, for which A is the universal enumerator.

We construct the enumerable set of A:

$L(X^*)=F_A^*=\{f_{x_1}, f_{x_2}, f_{x_1^2}\}$, where $f_{x_1^2}(s)=0$.

Since, it is true that $|L_A(X^*)|=3$, the family $\{A_i\}_{i\in I}$ contain $2^3-2=6$ FMS, namely $C_3^1=3$ FMS which has one-element input set and $C_3^2=3$ FMS which has two-element input set.

$A_{11}=(X_1,S,\delta_{11})$, $X_1=\{x\}$, $f_x^{(11)}(s)=s+s^2$;

$A_{12}=(X_1,S,\delta_{12})$, $X_1=\{x\}$, $f_x^{(12)}(s)=2s^2$;

$A_{13}=(X_1,S,\delta_{13})$, $X_1=\{x\}$, $f_x^{(13)}(s)=0$;

$A_{21}=(X_2,S,\delta_{21})$, $X_2=\{x_1,x_2\}$, $f_{x_1}^{(21)}(s)=s+s^2, f_{x_2}^{(21)}(s)=2s^2$;

$A_{22}=(X_2,S,\delta_{22})$, $X_2=\{x_1,x_2\}$, $f_{x_1}^{(22)}(s)=s+s^2, f_{x_2}^{(22)}(s)=0$;

$A_{23}=(X_2,S,\delta_{23})$, $X_2=\{x_1,x_2\}$, $f_{x_1}^{(23)}(s)=2s^2, f_{x_2}^{(23)}(s)=0$.

6 Solution of synthesis problem of the universal FMS

Let $\{A_i\}_{i\in I}: A_i=(X_i,S,\delta_i)$ be the family of FMS, where A_i is simulated by family of polynomials $\{f_{x_j}^{(i)}\}_{j=\overline{1,n_i}}$, n_i is cardinality of input set of A_i. Then, according to *theorem 2* the FMS A_F simulated by polynomials of set F, where $F=\bigcup_{i\in I}\{f_{x_1}^{(i)}, f_{x_2}^{(i)},..., f_{x_{n_i}}^{(i)}\}$, is the universal enumerator for the family $\{A_i\}_{i\in I}$. However, it is possible to construct the universal enumerator A of the family $\{A_i\}_{i\in I}$, which has cardinality of input set less than cardinality of input set of A_F.

Let's construct the enumerable sets for the family $\{A_i\}_{i\in I}$: $L_{A_i}(X_i^*)=\{f_{x_1}^{(i)}, f_{x_2}^{(i)},...f_{x_{n_i}}^{(i)}, f_{t_{n_i+1}}^{(i)},..., f_{t_{p_i}}^{(i)}\}$, where the subscript of polynomials $f_{t_j}^{(i)}, j=\overline{n_i+1,p_i}$ means that these polynomials are constructed for strings of length t with number j; and p_i is cardinality of input set of $L_{A_i}(X_i^*)$. If $(\exists i^*,i',k)(i^*\neq i')$, ($k$ is the subscript of element in some set $L_{A_i}(X_i^*)$) that $f_{x_j}^{(i*)}(s) = f_k^{(i')}(s)$, then we have't need to include polynomials $f_{x_j}^{(i*)}$ into the set of polynomials simulated the behavior of the universal FSM A. So, A is constructed by following way: the polynomial simulated behavior of A_i, is the polynomial simulated the behavior of the universal FMS A if and only if it can not be generated by the polynomials $f_{x_1}^{(j)}, f_{x_2}^{(j)}, \ldots f_{x_{n_j}}^{(j)}, j \in I, j \neq i$, this is it doesn't equal to any polynomial from set $L_{A_j}(X_j^*)$ $(j \in I, j \neq i)$.

Example

The family of FMS $\{A_i\}_{i=1,2}$: $A_i=(X_i,S,\delta_i)$, $|S|=m=3$, $X_1 = X_2 = \{x_1, x_2\}$ simulated by the polynomials $f_{x_1}^{(1)}(s) = 2s^2, f_{x_2}^{(1)}(s) = 1 + s + 2s^2, f_{x_1}^{(2)}(s) = 2 + s^2, f_{x_2}^{(2)}(s) = 2 + 2s^2$ is given.

The FMS $A=(X,S,\delta)$, $|S|=m=3$, $X = \{x_1, x_2, x_3, x_4\}$ simulated by the polynomials $f_{x_1}(s) = 1 + s + 2s^2, f_{x_2}(s) = 2 + s^2, f_{x_3}(s) = 2 + s^2, f_{x_4}(s) = 2 + 2s^2$ is universal for $\{A_i\}_{i=1,2}$.

Let's construct enumerable sets of $\{A_i\}_{i=1,2}$:

$L_{A_1}(X_1^*) = \{f_{x_1}^{(1)} = 2s^2, f_{x_2}^{(1)} = 1 + s + 2s^2, f_{x_1x_2}^{(1)} = 1 + s^2, f_{x_2x_1}^{(1)} = 2\}$,

$L_{A_2}(X_2^*) = \{f_{x_1}^{(2)} = 2 + s^2, f_{x_2}^{(1)} = 2 + 2s^2, f_{x_1^2}^{(2)} = 2s^2, f_{x_1x_2}^{(2)} = 1 + s^2, f_{x_2x_1}^{(2)} = 0, f_{x_2x_1x_2}^{(2)} = 2\}$

Note that $f_{x_1}^{(1)}(s) = f_{x_1^2}^{(2)}(s) = 2s^2$. From this it follows that the FMS $A=(X,S,\delta)$, $|S|=m=3$, $X = \{x_1, x_2, x_3\}$ simulated by the polynomials $f_{x_1}(s) = 1 + s + 2s^2, f_{x_2}(s) = 2 + s^2, f_{x_3}(s) = 2 + 2s^2$ is universal for $\{A_i\}_{i=1,2}$ too.

We can propose the others universal FMS for the given family, for example $A=(X,S,\delta)$, $|S|=m=3$, $X = \{x_1, x_2\}$, simulated by the polynomials $f_{x_1}(s) = 1 + s, f_{x_2}(s) = s^2$. However, we can draw this conclusion only after we construct the enumerable set of given FMS. Thus, by using this method it is impossible to construct all universal FMS for given family.

7 Conclusions

In this research, we isolated the class of finite-state machines for which the problem of discrete systems goal-directed behavior generating can be solved. To solve this problem we applied the new approach to modeling discrete systems behavior. This approach allows to apply the well known algebraic methods to solve the problem of denumerability of finite-state machines. As a result, the following tasks were solved:

1. The «numerical» FSM model was investigated. This «numerical» model uses polynomials to simulate transfer functions of finite-state machines.
2. The class of finite-state machines simulated by polynomial families, was described.

3. For the described class the problem of discrete systems goal-directed behavior generating was solved.

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