# Thinking Machines: a Paraconsistent Evaluation of an Ab/use of the Gödel Theorems

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**Abstract**
Gödel's theorems have been used for various ends since their establishment in 1931. One of those ends is that of Robert Rosen in his defence of a new paradigm for biology, assimilating closed causal loops as among others a way to understand anticipatory systems. We will argue how this use of Gödel's theorems arise from Gödel's own Platonist interpretation of the theorems. Next, we will argue against that interpretation from the perspective of dialetheism, which is the statement that contradictions can simply be true. In order to do so, we will emphasize the analogy between the G-sentence in the Gödel theorems and the famous liar-paradox. Finally, we will outline the consequences of this reinterpretation for the argument of Rosen.
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## 1. Introduction: From Cantor's to Hilbert's paradise

It is famous that David Hilbert, the leading figure of formalism, has named Cantor's theory of transfinite sets "Cantor's Paradise". Ironically enough, the ordinal aspect of this theory already contained the global destruction in 1931 of Hilbert's own paradise, called "Gödel's theorems".

Those theorems have in their turn been at the root of a very broad literature, a remarkable part of which deals with the human mind. Thirty years later namely, in 1961, John R. Lucas has used the theorems to argue that the mind can't be simulated entirely by a computer (or a (Turing-)machine). In 1984, Robert Rosen has mentioned, again, the Gödel theorems in his famous book 'Anticipatory Systems' to defend the insufficiency of physicalist approaches to life in biology (Rosen, 1985: 57)[1]. Recently, Oxford professor of Mathematics of mathematics Roger Penrose has declared himself a follower of Lucas, in his use of the theorems to make a case against the defenders of hard artificial intelligence. Again it is argued that the human mind can't be represented algorithmically[2] (Penrose, 1990)

Globally, in Hilbert's imagery, one can speak of the Gödel theorems as cracks in the hedge around Hilbert's own paradise. Lucas, Rosen and Penrose have used these cracks to show something beyond them. There is something that escapes Hilbert's computable paradise according to these three authors, and that thing is related broadly to life (Rosen).

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1 His use of the theorems was only very limited at that time, and has clearly grown in later publications, especially (Rosen, 1999). See further.

2 Synonyms are: mechanically, recursively, computably or effectively.

Because of the limited scope of this paper, we will only treat Rosen's use of the Gödel theorems and argue how this use arises from Gödel's own Platonist interpretation of the theorems. Next, we will argue against that interpretation from the perspective of dialetheism, which is the statement that contradictions can simply be true. In order to do so, we will emphasize the analogy between the G-sentence in the Gödel theorems and the famous liar-paradox. Finally, we will outline the consequences of this reinterpretation for Rosen's argument.

## 2. Gödel: the Impossibility of Hilbert's Paradise.

### 2.1. Gödel's Construction Coding Formal Signs in Numbers.

Gödel found the crack in the formalist edifice after following Hilbert's mission. This mission was to find an absolute proof[3] for the consistency of arithmetic (Van Heijenoort, 1973: 349). This would finally allow to realize Hilbert's paradise: to reduce mathematics to syntax, which is the manipulation of signs, following completely deterministic rules. There should be no semantics involved in the sense that the meanings of the signs are not considered.

The prince of logic had to accomplish this mission within Hilbert's formalist frame of reference: the number-theoretical axioms[4] are contained in the formal system, and enriched by some rules of inference. So, for example, if we have two individuals (constants or variables) A and B and we have the rule "if we have two individuals, then we have their conjunction too", then we can infer from this premise and this rule of inference that we have the conjunction of A and B. There are no considerations about the meaning of these two entities involved here (we don't have to know what kind of individuals are involved to infer the conclusion). According to Hilbert, and this was his metaphorical paradise we mentioned above, the whole of mathematics could be reduced to proofs built by means of suchlike inferences. Every mathematical statement would be provable in it, or otherwise its negation would be[5]. Nothing would be left undetermined, which also implies a complete correspondence to truth, as we will see later further on (sections 2.&3.). In other words the set of statements, well formed according to mathematical standards, would be complete. A crucial element in this ideal set of proofs was the one establishing the consistency of the system. The reasons for the latter were the paradoxes that had appeared in mathematics at the turn of the century. The most famous one is the Russell paradox that appeared in the set theory of the father of

3 This is to be distinguished from a relative consistency proof that had already been given before Gödel's theorems. A relative consistency proof is a proof that relies on the supposed consistency of another system, obviously leading to a regressus ad infinitum of proofs. The only way to stop this regressus is to find an absolute inconsistency proof, that is one that does not result from the translation into another system. We will see how part of the genius of Gödel's proof lies in a way of expressing a proof about arithmetic in the language of arithmetic itself, establishing in this way an absolute proof (Van Heijenoort, 1973: 349).

4 Those are the Peano-axioms (see: Van Heijenoort, 1973: 350).

5 This is basically what the law of excluded middle states. Formally, this is stated as $A \vee \neg A$.

formalism, Gottlob Frege. Another one, and this is the one that showed Russell the way to his paradox, is the one Cantor himself found in his own set theory. An absolute consistency proof was to be found in arithmetic because the consistency of other parts of mathematics had already been established relative to that of arithmetic. So we could speak about Hilbert's paradise as the sum of complete formalisability and consistency.

A whole survey of Gödel's proofs would take us too far from our present purposes, so we will focus only on two important characteristics of the proofs: the arithmetic coding of formal proofs and the self-referential structure of the G-sentence.

One brilliant strategy of Gödel is the coding of formal signs[6], and the relations that hold between them, in numbers[7]. Important for our further discussion, is that there is a continuity between the axioms of arithmetic and the formal rules of inference. They both have the crucial characteristic of being algorithmic. This means that their application, with a sufficient input, always deliver a sound result in a finite number of steps[8]. All the signs used in the formulas representing those rules can also be coded into numbers. A whole formal system that allows to do inferences from the axioms of arithmetic can be represented exclusively by numbers and relations between them in that way. A proof is just a sequence of formulas that can be coded into numbers, so it can in its turn be coded into a number, using the numbers of the formulas constituting it. The coding is systematic and can be done in two directions: from the formulas to the numbers, and the other way around. So if we have a number, we can determine from which formula and/or proof it is the code. Let us name this coded formal system 'S'.

With all of these elements, there is an algorithmic way of deciding if, given the Gödel number of a proof and the Gödel number of a formula, the concerned formula follows from the proof. We just have to code back the Gödel number of the proof into the proof itself, and do the same with the formula. If the last formula of the decoded proof[9] is the same as the decoded formula, then this formula clearly follows from the proof. So we have an effectively decidable[10] function P that, for every two numbers a and b, we can determine if the relation of 'being a proof of' holds between them. If this is the case, the formula $a=P(b)$ holds, which means 'a is a proof of b'. Now, and this is a crucial point of Gödel's proof, this function P can be translated into Gödel's coded system. There is, in other words, a formula $Q(0^{(a)},0^{(b)})$ in the system such that it is provable if $a=P(b)$ and its negation, $\neg Q(0^{(a)},0^{(b)})$, is provable if $a \neq P(b)$. If we take $0^{(c)}$ as representing the Gödel number of $(\forall x)\neg Q(x,y)$, then we can construct the following (numeralwisable representable) sentence $(\forall x)\neg Q(x,0^{(c)})$. This sentence is the G(ödel)-sentence.

After this more technical part, we are going to step back and look at the construction from the perspective of its result. First of all it must be stressed, as Gödel himself

6 Formals signs, such as the individuals we saw above.

7 In arithmetic, the only inviduals are natural numbers. They can all be represented by the use of one constant, 0, and the successor function, ' (intuitively meaning, 'the successor of' or 'plus 1'). So, for example three, can be represented as 0'''.

8 An example of this is the conjunction mentioned above: any two individuals will certainly deliver their conjunction.

9 The proof is, as we emphasised before, algorithmically constructed.

10 Which is the same as 'algorithmic'; see footnote 3.

already indicated in the introduction to his proof, that the G-sentence has a self-referential structure similar to the Liar-paradox: just like the liar-sentence states of itself that it is false, the G-sentence states of itself that it is not provable. More specifically, it states that there is no proof of the fact that there is a proof of the sentence. The proof function Q is applied here to a pair of which one element refers to itself. This is the self-referentiality of the G-sentence. And this paradoxical structure leads to the contradiction that follows from the sentence.

If we assume that G is provable, this means that there is a proof of the fact that there is no proof of the formula coded as $0^{(c)}$ $((\forall x)\neg Q(x,0^{(c)}))$. Now, with the Gödelian construction, we can code this last sentence as $0^{(c)}$ (Van Heijenoort, 1973: 352) so we can conclude that there is a proof of the formula with code $0^{(c)}$: $((\exists x)Q(x,0^{(c)}))$ or, which is equivalent, $((\neg\forall x)\neg Q(x,y))$. This last sentence is precisely $\neg$G. From this we can conclude that if G is provable, then S is inconsistent. From this, it can be deduced that G is not provable[11]. But then, if we assume that G is not provable, and code G into $0^{(c)}$ like we did in the first part of the proof then we obtain the unprovability of $0^{(c)}$ $((\forall x)\neg Q(x,0^{(c)}))$[12]. The last sentence is indeed G.

We will now show the analogy with the liar paradox.

## 2.2. Gödel's Paradox.

We start from Graham Priest's[13] illuminating description:

'The paradox phenomenon starts with a set of *bona fide*, [sic] truths which are assertible. [...] Those that are left over we will call "the Rest'. The essence of the liar paradox is a particular twisted construction which forces a sentence, if it is in the *bona fide* truths, to be in the Rest (too); conversely, if it is in the Rest, it is in the *bona fide* truths. The pristine liar 'this sentence is false' is only a manifestation of this problem arrived at by taking the Rest to be the false.' (Priest, 2006: 23)

So in Priest's terms all provable truths are bona fide truths. Then we define an object in the extension of this property or this set (of bona fide truths), that is defined as an object in the rest or which does not satisfy this property. So this object is, when it is bona fide (in Priest's terms) or provable (in Gödel's terms), also an element of "the Rest" or unprovable.

If we look at it in formal logical terms we see that the liar paradox, a typical paradox departs from a definition. A sentence a is defined as its own untruth:

$\neg$T('a') a

This together with the T-scheme, which states that a sentence (a) is equivalent to its own truth (T('a')) delivers the paradox in an analogous way as in Gödel's argument. The clarifying value of this example is that it highlights a crucial step in the construction. In

11 This is the deductive rule 'reductio ad absurdum' that states: if you can deduce from a premisse its negation, then you have proven the negation of this premisse.

12 Technically, this involves ω-consistency because in establishing the inconsistency there is a number indicating a particular proof ($0^{(b)}$ in Van Heijenoort, 1973: 351) but not any proof (x). We have refrained from this distinction for reasons of clearness.

13 He is one of the fathers of dialetheism.

the formal translation of the liar-paradox, it is immediately clear how a sentence is defined as the non-truth of its name. In the Gödel-argument, this relation of naming between a and 'a' is replaced by the relation of numerical coding of $\forall x \neg Q(x,y)$ and $0^{(c)}$. So, analogous to the liar-paradox, we could rewrite Gödel's paradox as[14]:

$\forall x \neg Q(x,0^{(c)})$ $0^{(c)}$

or, which is the same:

$\forall x \neg Q(x,0^{(c)})$ $\forall x \neg Q(x,y)$

So a formula is being defined as its own unprovability, similar to the liar paradox. The merit of Gödel's proof has of course been to construct the sentence largely by arithmetical means, so in a way independently of semantics, unlike the liar paradox. This independency lies precisely in the fact that a predicate (being provable) that corresponds completely to a semantical one (being true)[15] is being translated into a (syntactical) function.

Any person familiar with Gödel's theorems will have remarked by now that we have brought the argument in a different form from the original one. We have done this, partly for reasons of clearness, partly for the sake of our argument. The original proof namely turns the conclusions of the two last paragraphs around[16]: if S is consistent, then G is not provable in S, and if S is consistent, then ¬G is not provable in S. Those two sentences are equivalent to the ones we have derived. And they had the advantage of already preparing Gödel's conclusions and those of most of the logical community, that neither G nor ¬G are provable in S, because the premise that S is consistent can't off course be dropped. Instead of S being inconsistent, G is undecidable or, which is equivalent, S is incomplete. So Hilbert's paradise seems irrevocably cracked. It is not possible to completely formalise the truths that follow from a system.

It doesn't stop here. Let us look at Gödel's conclusion of the first part of his reasoning, that G is not provable if S is consistent. We can perfectly represent this in the language of S: C[17]$\rightarrow (\forall x)\neg Q(x,0^{(c)})$. This is easy to see, as G expresses its own uprovability. So it is a consequence of S that if C is provable then G is also. We remind the reader that if G is provable, then S is inconsistent. From the last two sentences we can deduce that if S is consistent, then its consistency is not provable in it. The importance of this second theorem, is that G gets explicitly connected to the non-provability of a central supposition (consistency) of arithmetic within arithmetic. With the second theorem, another blow has struck Hilbert's paradise: not only is every system necessarily incomplete. It also cannot prove its own consistency, and we remind the reader that this was Hilbert's other main concern.

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14 Priest has already rewritten Gödel's proof in a similar way, but in non-predicate logic (Priest, 2006: 239).

15 As we have remarked before, this is an implication of completeness (cf. supra). As we will see further, Platonism will deny this correspondence (see section 3.).

16 This done by the rule contraposition that states that if $A\rightarrow B$ then $\neg B\rightarrow\neg A$.

17 According to Van Heijenoort, C could be expressed in a variety of ways such as "there is no well-formed formula A such that both A and ¬A are provable; or "there is at least one well-formed formula A that is not provable". We will show that the latter is not a proper expression of consistency. (Van Heijenoort, 1973: 352.)

## 3. Trading-off (in)consistency and (in)completeness

The move of changing an inconsistency into an undecidability, as we showed at the end of the previous section, is a particular case of a more general strategy that has been discerned by Priest[18]. In his global defence of paraconsistency 'In Contradiction' he identifies some crucial elements in Gödel's argument and connects them in a clarifying way.

'(...) as the theory can prove its own soundness, it must be capable of giving its own semantics. In particular the T-scheme for the language of the theory is provable in the theory. Hence (...) the semantic paradoxes will all be provable in the theory. Gödel's “paradox” is just a special case of this.' (Priest, 2006: 47).

In Gödel's case, the semantics is the provability predicate[19]. This allows us to construct a self-referential sentence which is as such analogous to the paradoxes, as we have seen. As we have also seen, Gödel has turned the conclusion around into an equivalent sentence. This is a general strategy that one can apply to logical paradoxes. If one can deduce a logical paradox, then one can assume the truth of consistency, which is a premise of every (classical) reasoning, and deduce the undecidability of both conjuncts of a contradiction on the basis of consistency. Every contradiction deduced from a semantic paradox can be turned into an undecidability in that way. In syntactical terms this comes down to the exchangeability of the law of excluded middle (see note 6) and the law of non-contradiction[20].

## 4. Robert Rosen: The Two Options

Rosen departs from the observation that physicalist biology hasn't been able to account for some deadlocks in its theories. One of these deadlocks is the incapacity to reconstruct living systems on the basis of our scientific models of them (Rosen, 1999: 125). He sees as a reason for this the insufficiency of our mechanical models to describe living organisms (Rosen, 1999: 268). Globally, Rosen observes two options for mechanist science, when confronted with anomalies in theories, a conceptual and a technical one (Rosen, 1999: 74, 83) .

Either the global suppositions that lie behind physicalist biology such as algorithmicity have to be dropped and replaced by concepts that extend beyond them. Or the technical option, maintaining that the suppositions are sound, they have to be applied more

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18 As we can read in Mortensen (1995: 11) and in accordance with this, it has been the aim of paraconsistent logicians in Brazil (a.o. the pioneer of paraconsistent logic, Newton Da Costa) of dualising intuitionism, that assumes incompleteness in logic. There is also an analogy with topological duality between open and closed sets (Mortensen, 1995: 10).

19 As we have already emphasised, and this is a crucial characteristic of Gödel's argument, the provability predicate that corresponds completely to the semantical truth predicate is translated into a syntactical function, a characteristic function more precisely and, in this way, coded into a Gödel number. (Van Heijenoort, 1973: 351) Priest goes as far as stating that the predicate in Gödel's proof *is* a truth predicate, since $\exists x\ Q(x;y) \equiv y$ (Priest, 2006: 237).

20 This law states that a statement and its negation can't be jointly provable ($\neg$ (A & $\neg$ A).

thoroughly. More variables have to be supplied to the scientific theory so that it can resolve the anomalies.
Rosen defends the first option in biology, and his books contain extensive arguments against the second one, that is actually being applied in scientific practice. He also defends the first option in mathematics, though in a very specific way. The paradoxes are seen as such anomalies, and Rosen suggests that they show that global formalisation, of which completeness is an aspect, has to be abandoned (Rosen, 1999: 93-94).
First, let us argue against the first option in mathematics: there are no indications that the paradoxes are to be resolved by more refinement. This is clear in the globally accepted formal set theoretic system Zermelo-Fraenkel. The Gödel-sentence is undecidable in any model of ZF too (Penrose, 1994: 106). So Zermelo Fraenkel can indeed be seen as a 'refinement' in Penrose's and Rosen's sense, but as Penrose argues (Penrose, 1994, 106), an insufficient one. The problem of truth remains: a formalism that can't account for its own Gödel-sentence can't assure that it is sound (because it has to rely on external means for deciding the truth of the Gödel-sentence that follows from a central assumption, i.e. consistency, of the system).
Several logicians have tried to prove the consistency of arithmetic in several ways after Gödel, trying to get around Gödel's restrictions (Van Heijenoort, 354, 355). One way of doing this, has been by making use of informal arguments (Van Heijenoort, 1973: 354)[21]. Rosen knows about this option too, and uses it in his argument against reductionism in biology[22]. In his terms, formalists try to replace impredicatives by predicatives, but the result gets very poor in entailment. Rosen sees causality and inference as two modes of entailment and as such as analogous (Rosen, 1999: 88). According to him, life is characterised by closed causal loops that represent final causes. Likewise, formally, the G-sentence involves a self-referential construction. The attempt to deal with them in the constructive hierarchy can't succeed because self-referentiality isn't allowed.
As Rosen notes, the issue concerning paradoxes in set theory is still wide open (Rosen, 1999: 305). More specifically the paradoxes that are banished from the constructive

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21 A specific form of 'being informal' is 'being non-constructible'. This refers to the constructive hierarchy that was also established by Gödel. The constructive hierarchy is a set-theoretic construction that is in a sense quite analogous to a metalinguistic theory in semantics: it is built up in levels, indicated by an ordinal number. As such, it does not allow self-referential constructions, called 'impredicatives' by Rosen, following Russell (Rosen, 1999, 90). It has been proposed by Gödel as a model for set theory (Rosen, 1999: 91), and has been successful as such: the constructive hierarchy is the standard interpretation of ZF (Priest, 1995: 174). It is constructed not to allow selfreferentiality. This is done by the notion of constructibility, which states that every entity at a constructible level is contructed by applying one of the rules established at a previous level of the hierarchy. (Rosen, 1999: 91) This prevents the construction of the G-sentence. The solution is to add an informal, non-constructible theory to the constructive hierarchy (Van Heijenoort, 154-155).

22 Another metaphor that Rosen uses, is one of size (see for example Rosen, 1999: 79, 123). This metaphor is largely due to Abraham Fraenkel (Hallett, 1984: xii). According to Hallett (idem: xiii), there is no intrinsic connection between size and the paradoxes. There could be a connection though with inaccessible cardinals (see Tiles, 1989: 180-181 and Drake, 1974: 67-68). To work this out would deviate us too much from our present purpose.

hierarchy can be represented in paraconsistent models of naive Cantorian set theory (for example the one presented in Priest, 2006: 256-257). Rosen doesn't take this into account. He makes the usually made (Priest, 2006: 33) mistake of confusing set theory with Zermelo Fraenkel, and leaving Cantorian set theory out of scope. We will come back to this later on.

The connection between Rosen's argument for non-reductionist biology on the basis of Gödel and Gödel's own interpretation of his theorem is the following: Rosen follows Gödel's turning around of the paradoxical conclusion: the G-sentence is unprovable, but he deduces an unbridgeable gap between formalisation and number theory from it (Rosen, 1999: 92). The reason therefore, as we saw before, is taken to be the impossibility of self-reference. So Rosen doesn't follow Gödel in his postulating transcendent truth beyond the formal system. Instead, he does the same with self-reference.

The problem with this, is that formalisability is used as equivalent to constructability. But this leaves the sense of formal system as used by Gödel in his argument out of scope. As we have mentioned in our outline of the argument, Gödel has shown that it is possible to put a formal system on a par with number theory through their shared algorithmicity. The formal system and number theory aren't simply separable as such. Everything that follows from both of them is simply algorithmic and everything that is algorithmic is formalisable. Non-formal statements of number theory are out of the question from this perspective (Rosen, 1999: 92).

## 5. Another Option: Paraconsistency and Self-reference

As we have already announced above, there is an alternative to Gödel's own interpretation of the theorems. In accordance with our exposition of the argument, it can be interpreted as a paradoxical reasoning, leading to a contradiction. In this interpretation, the G-sentence simply is provable and non-provable.

What is the problem with this conclusion? It is here that we encounter a dogma not only of formalism, but also of almost all of western scientific theories: ECQ or, in full, *ex contradictione quodlibet* which means a contradiction implies everything or contradictoriness implies triviality[23]. It is the merit of (Hilbert's) formalism to make this a central matter of its theory. As we have hinted at above, not only (total) formalisation was the central aim of formalism but also consistency[24]. Syntactically, the step from contradiction to triviality is made, among others, by the use of the rule disjunctive syllogism. If we have A, we can add any B to it as a disjunct (by the logical rule 'addition'). This gives A v B. B could be anything. Now if we also have ¬A, then we can drop A, and conclude (any) B. Because B can be anything, anything can be deduced in this way.

23 Triviality is often replaced by unsoundness in the litterature. An argument is sound if a correct conclusion follows from it. This implies true premisses and valid rules of inference. Triviality implies unsoundness.

24 In fact, a general difference between fregean-russellian logicism en hilbertian formalism, is that in the latter, consistency is identified with existence.

What is left out of consideration because of the dogma of ECQ, are the formal systems to which this dogma doesn't apply. So they are systems in which inconsistency doesn't imply triviality nor unsoundness[25]. There are several ones of them[26]. The most appropriate one in this context is a many-valued paraconsistent logic, such as LP[27]. This because semantics is extended here with a third value, next to 'true' (or $\{1\}$) and 'false' (or $\{0\}$) , which is 'true and false' (or $\{1,0\}$). The G-sentence, in such a system, simply gets assigned the value 'true and false'. In this way, a paraconsistent approach to arithmetic can overcome the incompleteness Gödel deduced from his argument. The G-sentence is indeed decidable: it is true and false. Accordingly, this logic can serve to model naive, Cantorian set theory and which allows, as we have mentioned before, self-referential paradoxes (Priest, 2006: 256-257).

We do agree with Rosen's first option in mathematics, the paradoxes show that a supposition of mathematics has to be dropped, but it is not the global edifice of formalisation, it is only the supposition of non-contradicticity that has to be dropped.

Before looking at the consequences Rosen of this new premise, it is important to remark that this new option of paraconsistent logic has clarified the hidden dilemma in Hilbert's project, which is in the end one between (complete) formalisability and consistency. If one chooses the first, one has to drop the second and inversely. Robert Meyer, who uses another paraconsistent logic than the one described above, relevant logic, has used this to call for a revived Hilbert program based on paraconsistent logic (see: Mortensen, 1995: 19). The question is, off course, to which degree one can still speak of Hilbert's program when consistency is dropped.

## 6. Lessons: Back to Hilbert's (Cracked) Paradise.

After showing the other interpretation of Gödel's theorems, we can turn back to Rosen. The first one of his two options, the global change of suppositions, is shown to be divided in two possible choices in its turn. These are the same as the two possibilities of the dilemma in Hilbert's project. There is the one that Rosen defends, of leaving global formalisation and there is the one of leaving global consistency. This has been shown. We don't want to argue for that extensively but Rosen's own design for biology pleads for leaving global consistency. After all, he has pleaded on several places for an objective approach to complexity. Complexity, that is according to Rosen, "systems which can accommodate impredicatives, or closed loops of entailment" (Rosen, 1999: 94). Paraconsistent systems could allow to model closed causal loops and to accept the contradictions that follow from their impredicative structure, without being unsound or trivial. So this could be another possible piste for researchers that want to put Rosen's intuitions into practice. For dialetheism itself, life could be another argument for dialetheism in relation to the external world[28]. In this way, the paraconsistent systems

25 That is why Van Heijenoort's equating of consistency with untriviality (see note 18) is wrong.

26 For a short and clear overview, see: Priest, 2004.

27 In full, this is 'logic of paradox'. This system was first proposed by the Argentinian logician F. G. Asenjo in 1966 and later popularized by Priest and others. (see for example Priest, 2006 221-228).

28 As such, it could be added to the three areas that are discussed by Priest (2006: 159-204): change, motion and legal context.

allowing complete formalisation can be an enrichment of the tools to describe the external world. This, contrary to the blame of Rosen on contradiction that formalism is an attempt to reduce the external world to the formal system (Rosen, 1999: 78).

Connected to this is Rosen's rejection of the incorporation of semantics into syntax, which we have described as central to Gödel's argument (Rosen, 1999: 268). It is necessary to stress a remarkable fact about the meaning ascribed to semantics in the treated literature. This is solely considered as referential. Rosen subscribes such a concept of semantics. He supposes that the paradoxes arise from an incapacity to fully express semantics into syntactical symbols. In other words, semantics is too rich to be incorporated into syntax (similar to the richness of living organism for reductionist approaches). The logical paradoxes show a different picture though. The liar paradox doesn't involve the translation of semantics into syntax like in Gödel's construction. So the ground of the paradoxality that is shown by the similar Gödel theorems is to be found elsewhere than in the relation between semantics and syntax. This different conclusion results from a different approach to semantics than the one being handled by Rosen though. Semantics isn't exclusively considered as referential, but also and mainly rule-governed. Once we have this more systematic and explicit look at semantics, we see that the paradoxes can't just be located in the insufficiency of syntax in relation to semantics.

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