Monologue and Dialogue, Under 'Soft Object'

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Abstract
In general, a formal system is based on reductionism. We, however in this paper, propose a formal system based on pseudo-reductionism. In the system which we propose, each object is variable, and can accumulate information by virtue of the variableness itself. The accumulated information can be exposed afterward. We apply this new concept to the transition of a directed graph, which can be regarded as monologue. In addition, also apply to dialogue-based society model which we already proposed. The dialogue in the model is based on monologues of autonomous agents.
Keywords: Monologue, Dialogue, Variable object, Implication, Reductionism

1 Introduction

We propose a formal system which does not have minimum objects in principle. All objects can vary their 'size', and their obviousness is deprived. In an ordinary formal system, especially in an axiomatic system, objects are irrefutable. However in this paper, we regard the irrefutability as a mere assumption, and argue about that.

Gunji et al. (2006) also deal in variable objects that change according to the situation in the name of skeleton. These problem establishments are, in a manner, denial of reductionism. Ordinary reductionism inevitably requires constituents that are irreducible to other constituents, and the description of the relations of those constituents is the description of the system itself. Namely, constituents are inviolable and absolute. In contrast, the description of a system consisting of soft objects is that of the relations of interim constituents that each observer can set up arbitrarily depending on standpoint and circumstances. Even if the size of a soft object varies as time proceeds, the 'inside' of the soft object changes to neither inviolable nor 'empty'. Each cannot observe the inside, and the hidden inside can be exposed afterward in some cases.

We evolve an idea of the irrefutability on a dialogue-based society model which we already proposed (Sawa and Gunji, 2007). If a dialogue between autonomous agents who have knowledge about the world is considered, it is natural that the agents do not necessarily have exactly the same view of the world. It follows that the agents hold the different view of objects one another. This is the reason why we treat of the dialogue. The dialogue is premised on and composed of monologues of each autonomous agent.

We introduce some measures of the world in order to argue about the soft object. In a formal system composed of ordinary objects and their implicational relations, the

consistency of a system is ensured by the transitivity law of implication. In contrast, the consistency is ensured by at least two aspects in a system based on soft objects. The first thing is the consistency in the light of relations between soft objects. The second is the consistency with respect to soft objects themselves. The former can be evaluated by the ratio of the part in which transitivity law is satisfied to the whole of the system, as well as the consistency of an ordinary system. The latter is measured by 'softness', which is defined by the number of arrows in a soft object. These consistencies do not necessarily consist together. We show the difference of results induced by the choice of the consistencies.

Viewing the model as transformation of graphs, the model is related to category theory (Mac Lane, 1998) and Paton (2002). From the standpoint of the emergence from dialogues, the model is similar to Infomorphism proposed by Barwise and Seligman (1997), Nowak (1999, 2000), Adamatzky (1998). The model as the emergence of logic is motivated by Szabo (1978), embodied mathematics by Lakoff and Núñez (2000), and Gunji and Higashi (2001).

2 Soft Object, and Measures of the Consistency of the World

In this paper, we treat of knowledge about causal relations. In order to introduce the variable object as described above into the scheme of causal relations, we prepare and define as below.

A directed graph (Harary, 1969) represents knowledge about causal relations. A node represents an object, and a directed edge (an arrow) corresponds to a causal relation. We do not consider a directed edge from a node to itself, and assume that there is only one directed edge between arbitrary two distinct nodes in the paper.

In an ordinary formal system, an object has the property of the identity, i.e. $X \to X$. If a cycle of causal relations (e.g. $X \to Y$, $Y \to Z$, and $Z \to X$) exists, there are causal relations between two arbitrary objects in the cycle. Therefore, we can consider that those objects are identified, in other words, the cycle itself can be regarded as one object. We call the set regarded as one unit, a soft object (see Fig. 1).

Definition 2.1 (*Soft object*). In a given directed graph, if a set of nodes in which there is at least one sequence of directed edges between every ordered pair of two nodes exists, we call the set soft object. Moreover, a node which is not identified with the other nodes (i.e. a 'singleton' from the standpoint of identification) is also called soft object.

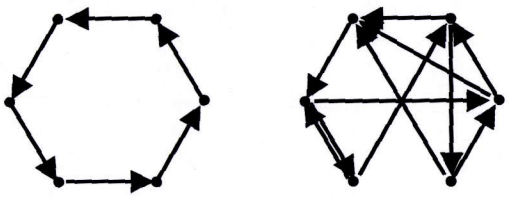

Figure 1: Examples of soft objects. Both are composed of 6 nodes. Left soft object is 'softer' than right one.

In a soft object consisting of n nodes, there are at least n directed edges, $n(n-1)$ at a maximum. We regard the density of directed edges as softness, since the transitive law does not necessarily hold in the scheme, and the number of order pair of nodes that are connected directly increases in proportion to the density.

Definition 2.2 (*Softness rate of a soft object*). Given a soft object s consisting of n_s nodes, the softness of the soft object is defined as

$$\mathrm{SR}(s) \coloneqq \begin{cases} 1 & (n_s = 1) \\ |s| \ / \ (n_s(n_s-1)) & (n_s \geq 2) \end{cases}, \tag{1}$$

where $|s|$ is the number of directed edges in s.

Definition 2.3 (*Softness of a directed graph*). Given a directed graph G consisting of n nodes, the softness of the whole graph is defined as

$$\mathrm{SR} \coloneqq \sum_{s \in G} n_s \mathrm{SR}(s) \ / \ n, \tag{2}$$

where n_s is the number of nodes of soft object s.

Note that the denominator of SR(s) is the number of directed edges in a complete graph consisting of n_s nodes. SR is weighted average of all SR(s), using the number of nodes in each soft object as the weighting factor.

SR is a measure of the consistency of the world in the light of respective things themselves. On the other hand, TR (transitivity rate) proposed in (Sawa and Gunji, 2007) is a measure of the consistency from the standpoint of relations among things. Moreover in this paper, we define a measure of the transitivity among soft objects.

Definition 2.4 (*Transitivity rate*). Given a directed graph G, TR is defined as

$$\mathrm{TR} \coloneqq |G| \ / \ |G'|, \tag{3}$$

where $|G|$ is the number of directed edges in G, and G' is the graph transformed from G, in which the transitive law holds completely by adding requisite directed edges.

Definition 2.5 (*Graph induced by soft object*). Given a directed graph G, we define a new graph G_s as follows: nodes, all soft objects in G; directed edges, ordered pairs of soft objects such that there exist one or more directed edges from constituents of one soft object to those of another in G.

Definition 2.6 (*Transitivity rate among soft objects*). Given a directed graph G, STR is defined as

$$\mathrm{STR} \coloneqq |G_s| \ / \ |G_s'|. \tag{4}$$

For instance, the value of TR, SR, and STR of the directed graph in Fig.2 are 0.79, 0.39, and 0.80, respectively.

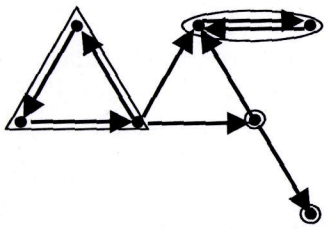

Figure 2: An example. There are four soft objects, which are framed by a triangle, an oval, and circles respectively.

Definition 2.7 (*Inner-arrow and Inter-arrow*). Given a directed graph, we call an arrow (directed edge) in a soft object, inner-arrow. Meanwhile, an arrow between two soft objects is called inter-arrow.

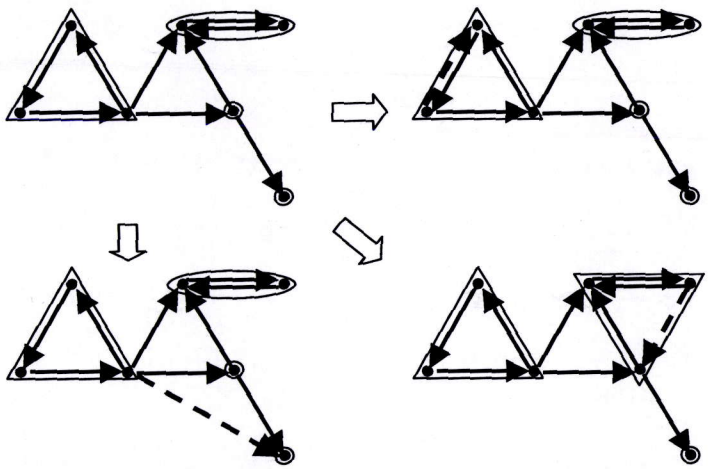

Figure 3: Examples of adding an inner- or inter-arrow (dashed line). For a given directed graph (upper left), if an inner-arrow is added, SR and TR increase (upper right). Adding an inter-arrow, STR and TR increase (lower left). On the contrary, a new soft object is composed and the values of all measures decrease (lower right).

If an agent intends to improve the consistency of the description in soft objects, the agent is faced with the choice between such two consistencies (SR and STR) that do not necessarily consist together. Roughly speaking, adding inner-arrow is related to the increase in SR; adding inter-arrow invokes the increase in STR. In this regard, however, addition of inter-arrow makes a new soft object in some cases, and it leads to the reconsideration of the view of the world. Figure 3 shows an instance.

3 Monologue Under Soft Object

Due to two partially incompatible consistencies, the improvement of knowledge described in soft objects is in diverse ways, as seen in the example of Fig. 3. Thus we classify the way of the improvement of knowledge represented by a directed graph as follows.

Definition 3.1 (*Classification of improvement of a directed graph*). When we transform a directed graph as its TR increases or does not change, we call this way of transformation TR-oriented way. Same applies to SR, or STR.

We regard the transition of a directed graph as the monologue of an agent. Here we compare monologues caused by the various ways of improvement. A directed graph, in which some soft objects consisting of multiple nodes exist, is added one directed edge, and then one directed edge which is randomly selected is removed in every units of time. The number of consecutive transitions is 200. See Table 1 and Fig. 4.

Table 1: Results by the various ways of improvement, starting with a same directed graph consisting of 20 nodes. Each value is the average. Random way makes one soft object consisting of all 20 nodes, hence the value of STR is indeterminate.

	Random	TR-oriented	SR-oriented	STR-oriented
TR	**0.32**	**0.86**	**0.54**	**0.33**
SR	**0.77**	**0.99**	**1.00**	**0.77**
STR	**Indeterminate**	**0.87**	**0.65**	**0.90**
Number of soft objects	**14.02**	**19.62**	**18.43**	**14.13**

In a monologue caused by the SR-oriented way, soft object tends to split into smaller ones. Therefore the number of soft objects increases. The tendency toward the split can be explained from the number of a complete graph, which is the denominator of SR(s). Similarly, the number of soft objects increases, if TR-oriented way is adopted. A cycle of arrows need more arrows than a sequence of the same number of arrows in order that the value of TR is 1. As it were, the 'cost' for keeping a cycle is high, hence the cycle tends to be dissolved.

In contrast, soft object has a tendency to be enlarged by the STR-oriented way. Once a new soft object is made synthetically from two soft objects due to adding an arrow between them, thereafter inner-arrows can be added inside the newly made soft object as STR does not change. Hence it is sometimes difficult that the new soft object split apart again. The cost for keeping soft objects may be kept to a minimum, therefore, the relations between soft objects can be described adequately, even if the total number of arrows stays constant.

Thus if we adopt the STR-oriented way, soft objects can keep their size moderately by the assistance of random disappearance of arrows. Consequently arrows accumulated

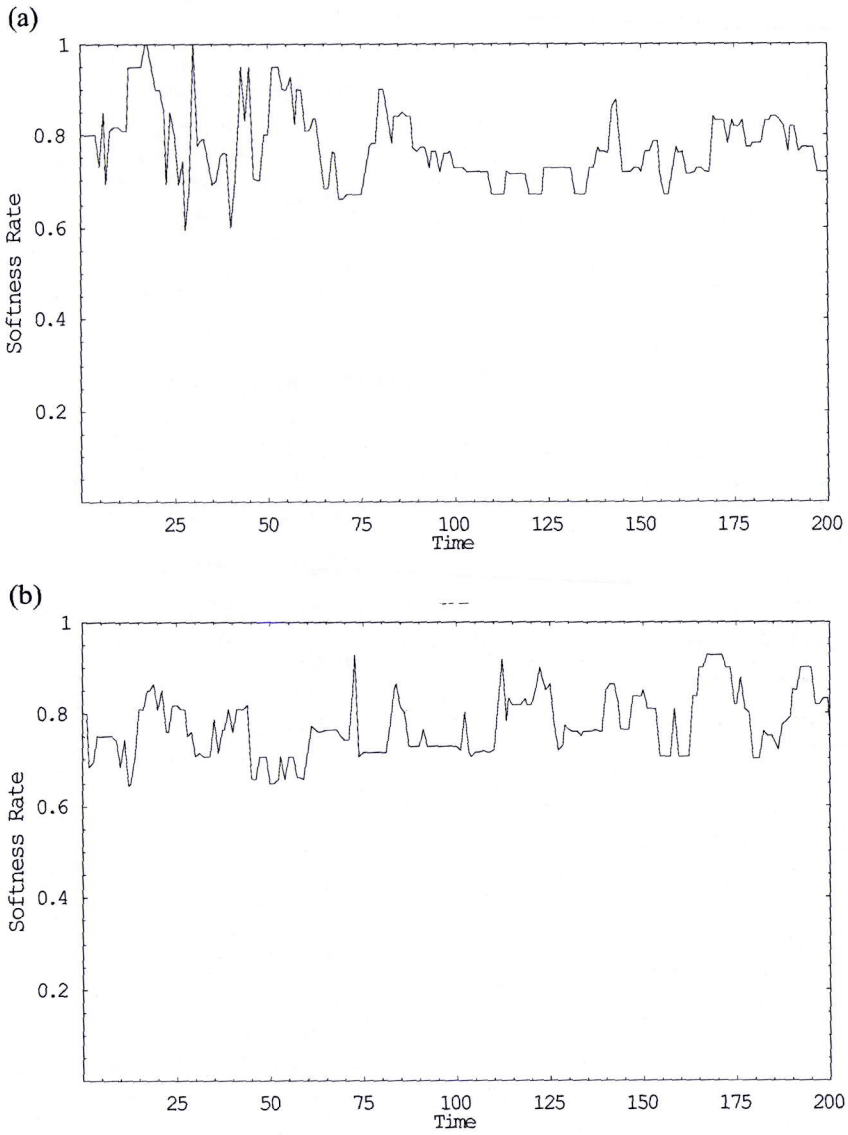

Figure 4: (a) Time transitions of SR by Random way. (b) By the STR-oriented way. Monologue by the STR-oriented way can also keep average value of SR.

inside soft objects can be exposed afterward in quality of hidden knowledge. In addition, given a directed graph, there are more occasions that increase the value of STR than TR. The value of TR can increase if the added arrow is a requisite arrow (the arrow from an initial node to a terminal node of a sequence of arrows). On the other hand, STR can also increase by virtue of the appearance of a new soft object, corresponding to reconsideration of the view of the world.

4 Dialogue Under Soft Object

In this section, we consider about the influence of soft object upon dialogue. We implement a concept of soft object on a dialogue-based society model which we

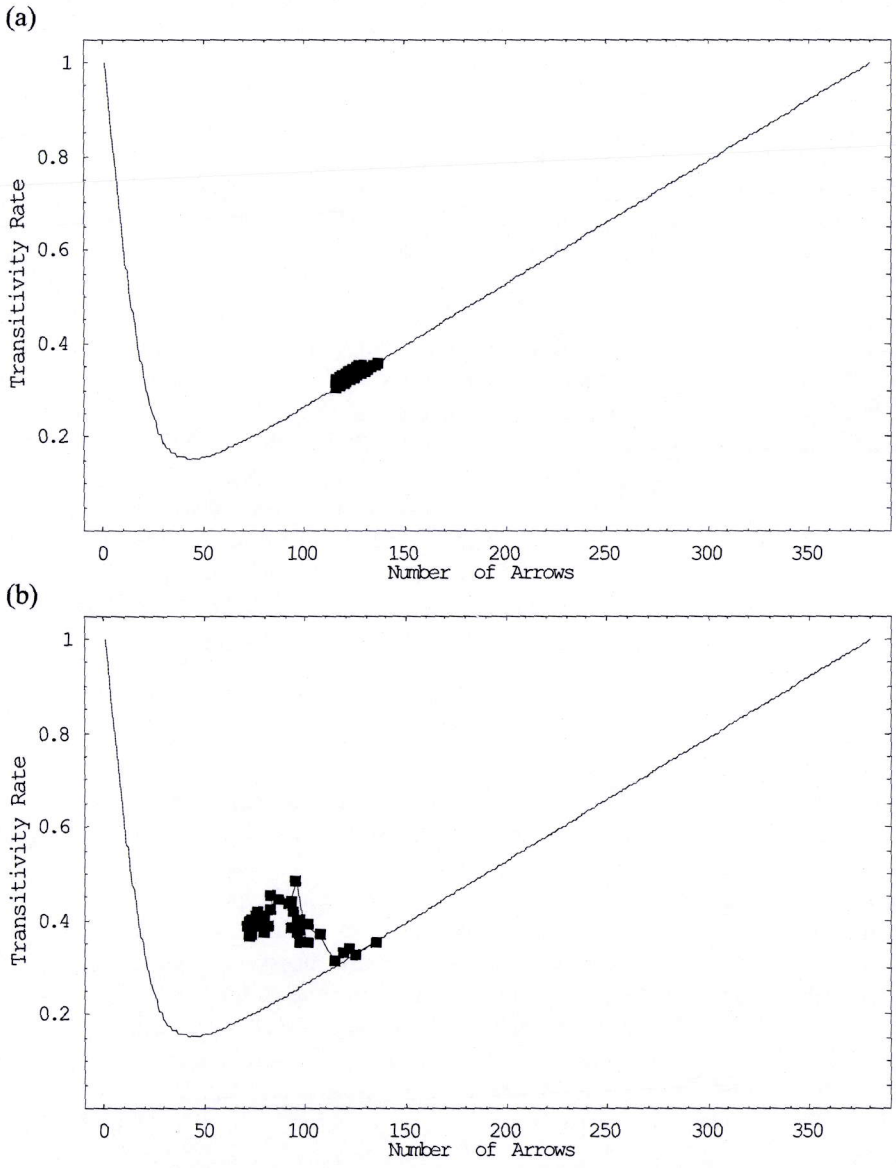

Figure 5: (a) Results by the TR-oriented way. (b) By the STR-oriented way. The initial directed graphs which represent knowledge of agents are same in both trials, however, the consistent description emerges due to the STR-oriented improvement of each agent.

proposed. First, we summarize the existing model simply. The model consists of multiple agents who have knowledge about causal relations among objects. The agents are influenced not by collective knowledge of agents, that is on a higher order, but rather by only one another through dialogues, if they have either completely same opinions, or a particular pattern of opinions, that are regarded as the extension of such exact accordance. The initial knowledge of each agent are arbitrary, and do not necessarily hold the transitive law. However, in spite of the absence of global knowledge, the mere union of knowledge of the agents obtains the transitive law after dialogues.

In essence, it is impossible for agents to share exactly the same view of the world without previous agreement. Hence it is natural that we introduce the soft object when we consider about dialogue among agents. In the model, the arrow of one agent which is not reinforced by the arrows of the other agents disappears. We here show the difference of results by the TR-oriented and the STR-oriented way, respectively. See Fig. 5.

Each dot represents the values of the union of graphs of all agents at each time instant. The curve line represents the average. By virtue of STR-oriented improvement of each agent, TR deviates upwards from the curve line as time proceeds. This indicates an emergence of the consistent description of the world, if STR-oriented way is adopted.

This is caused by the difference of the number of occasions between TR-oriented and STR-oriented way, as stated in the preceding section. In these simulations, if there are no occasions that increase the value of each measure, a randomly selected arrow is added in order to hold the number of arrows constant. There are fewer choices by the TR-oriented way than the STR-oriented way, therefore, the dialogue by the TR-oriented way tends to fall into the average.

5 Discussion

While we regard an object as a thing, it is a kind of conceptualization to regard multiple objects as one object. To treat things and their conceptualized thing on the same level leads to Russell's paradox (Whitehead and Russell, 1925). A way to circumvention of the paradox is introduction of the logical types, which corresponds to discrimination between objects and sets of identified objects. We, in this study, do not dare to circumvent the paradox. We aim to compare the system based on soft objects with the ordinary logical system premised on logical types. Only the STR-oriented way can make a new set of identified objects in the model. The SR-oriented way is scarcely, and the TR-oriented way is never able to make a new set. It can be concluded that conceptualization can be realized only on the STR-oriented way. Both the TR-oriented and the STR-oriented way can only confirm an existing state, from the standpoint of conceptualization.

As stated previously, the model proposed here is a kind of denial of reductionism. In general, the alternative of reductionism is holism, however, we do not side with the holism. We do not consider that a whole is indivisible into parts, but assert that a whole

is divisible into interim parts that can change depending on the situation. Our model is intermediate between reductionism and holism, and is, as it were, pseudo-reductionism.

The dialogue model consisting of ordinary objects expresses an open and aboveboard world, in which one can see transparently what others say or consider. The transparency conduces to the consistent description of the world: the transitivity law. In contrast, there is not such transparency in the dialogue model composed of soft objects. Instead, the softness of soft object plays a role as mediator between inter-agent relation and inter-object relation, and consequently yields the consistency. STR-oriented way has a resemblance to Euclidean geometry which is composed of points that have no size, and lines that are infinitely-thin, in the sense that the interior of each constituent is ignored.

Soft objects can accumulate information inside themselves and the information can be exposed afterward. This aspect is never represented in an ordinary system in which minimum objects exist.

6 Conclusions

We composed the models of monologue or dialogue. The monologue model is a kind of transitions of knowledge which depends on various measures of the world. In the process of the composition of the model, we introduced the concept soft object, which realize a non-hierarchical formal system without minimum units in theory. We showed the difference of results among the way of improvement of knowledge.

The introduction of soft object into dialogue model realizes the lack of transparency among agents, however, the results of simulations actually demonstrated the superiority of the way of improvement which supposes soft objects as units.

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