L-Fuzzy Set Model of the Optimal Therapy When Diagnosis is Known

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Abstract In this paper we try to resolve the problem of decision of the optimal therapy when the diagnosis is known. In papers [2], [3], [4], [9],[10], [11] the mentioned problem was treated by methods of fuzzy sets theory. Fuzzy sets were considered there as mappings from a set to $[0, 1]$ real interval.

Here this problem has been considered using fuzzy sets from a more general point of view, as mappings whose co-domain is a special lattice valued monoid. The model developed in this paper is an improvement of the one given in [5]. In the same time it is an anticipatory model since it predicts efficiency of all possible drugs to the disease in question and it suggests the optimal therapy for every patient.
Keywords : fuzzy sets, lattice, lattice valued monoid, application to medicine, optimal therapy,

1 Introduction

Up to now, several fuzzy decision models have been developed to help in solving some medical problems. An overview of applications of fuzzy sets in medicine can be found in [7].

In papers [2], [3], [4], [9],[10], [11] the problems of decision of the optimal therapy, when the diagnosis is known, have been discussed. In [3] and [4] these problems have been treated in the case when clinical symptoms retreat completely after the treatment, by methods of fuzzy sets theory. In [10] and [11] similar problems have been solved in the case of symptoms not retreating completely after the treatment. In the mentioned papers fuzzy sets were considered as mappings from a set to $[0, 1]$ real interval.

In [5] a fuzzy model is developed to solve similar problems, using lattice valued fuzzy sets. In [13] an application of lattice valued fuzzy relations in biogeography is given.

International Journal of Computing Anticipatory Systems, Volume 14, 2004
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-00-8

In this paper, another L-fuzzy set model of the best therapies when the diagnosis is known is proposed. Our approach is based on fuzzy set theory, where the codomain of a fuzzy set is a special lattice valued monoid. This approach enables a use of a binary monoid operation in computation of the level of the convenience of a certain therapy. The ordering relation in the lattice ordered monoid enables comparison of those levels for different therapies, and selections of the optimal ones.

2 Preliminaries

Let be given a set X. A **fuzzy set** $\overline{A}$ of X (sometimes called a fuzzy subset $\overline{A}$), by its original definition [14], is a pair $(X, \overline{A})$, where $\overline{A}$ is a mapping $\overline{A} : X \to [0,1]$, with the known properties of this so-called membership function. This concept is most frequently used in applications. Goguen generalized this notion to lattice valued fuzzy sets [6]. By this approach, a **lattice valued fuzzy set** is a mapping $\overline{A}$ from a nonempty set X to a lattice L, $\overline{A} : X \longrightarrow L$.

Here, a generalized concept is used and applied. Namely, fuzzy sets (relations, correspondences) in this paper are considered to be mappings with a special lattice valued monoid as a codomain.

In this way, fuzzy relations and fuzzy correspondences are considered here as mappings from products of sets again to a lattice ordered monoid.

In the sequel some basic definitions concerning lattice ordered monoids are given. In section 3 fuzzy sets and relations are considered as mappings to lattice ordered monoids. Some definitions and properties of such fuzzy sets are provided. In section 4 the mathematical model for decision of the therapy when diagnosis is known is presented. In section 5 the given model is illustrated in an example from practise.

2.1 Lattice ordered monoids

A **lattice ordered monoid** is an algebraic structure $(L, *, \wedge, \vee, e, \leq)$, where $*, \wedge, \vee$ are binary operations, e a nullary operation (constant) and $\leq$ a binary relation, all on set L, such that $(L, \wedge, \vee, \leq)$ is a lattice, $(L, *, e)$ is a monoid and the isotonicity condition is satisfied: for all $a, b, c \in L$

$$a \leq b \text{ implies } a * c \leq b * c \text{ and } c * a \leq c * b.$$

Simply ordered monoid is a lattice ordered monoid L, such that for all $x, y \in L$, $x \leq y$ or $y \leq x$.

A **lattice ordered group** is an algebraic structure $(G, *, \wedge, \vee, ^{-1}, e, \leq)$, where $^{-1}$ is a unary operation, such that $(G, *, \wedge, \vee, e, \leq)$ is a lattice ordered monoid and $(G, *, ^{-1}, e)$ is a group.

Simply ordered group is a simply ordered monoid G, which is also a group.

A **zero** of a lattice ordered monoid L is an element $o \in L$, satisfying:

$o \leq x$ and $o * x = x * o = o$ for every $x \in L$.

In a lattice ordered monoid L, element x is **positive** if $e \leq x$. x is negative element, if $x \leq e$.

A structure considered in this paper is a simply ordered commutative monoid $(L, *, \wedge, \vee, o, e, \leq)$ with zero o, such that $(L \backslash \{o\}, *, \wedge, \vee, ^{-1}, e, \leq)$ is a simply ordered commutative group.

More details about notions in this section can be found in the book [1].

3 Fuzzy sets and fuzzy correspondences

A mapping $\overline{A} : X \longrightarrow L$, where X is a nonempty set and L a lattice ordered monoid is a **lattice ordered monoid valued** fuzzy set.

A **level set** A_p, for every $p \in L$ is defined by

$A_p = \{x \in X \mid \overline{A}(x) \geq p\}$.

The characteristic function $\overline{A}_p : X \longrightarrow \{0, 1\}$ corresponding to level A_p is defined by

$\overline{A}_p(x) = 1$ if and only if $\overline{A}(x) \geq p$, and is called a **level function**.

If A and B are nonempty sets, and L a lattice ordered monoid, then a mapping $\overline{R} : A \times B \to L$ is a **lattice ordered monoid valued fuzzy correspondence**.

Since only lattice ordered monoid valued fuzzy sets and correspondence are used, they are called simply fuzzy sets and fuzzy correspondences throughout this paper.

Naturally, to every fuzzy correspondence there corresponds a matrix having elements from L, with rows indexed by elements from A and columns indexed by elements from B.

To every fuzzy correspondence $\overline{R} : A \times B \to L$ several fuzzy sets are associated, as follows.

For $x \in A$, a fuzzy set $\overline{R}^x : B \to L$ defined by $\overline{R}^x(y) := \overline{R}(x, y)$ is called **x-row fuzzy set**, and for $y \in B$, fuzzy set $\overline{R}^y : A \to L$ defined by $\overline{R}^y(x) := \overline{R}(x, y)$ is called **y-column fuzzy set**.

If $\overline{R} : A \times B \to L$ is a fuzzy correspondence, then for every $p \in L$ a mapping defined by $\overline{R}_p(x, y) = 1$ iff $\overline{R}(x, y) \geq p$ is a **level** correspondence. A related level set (crisp correspondence) is denoted by R_p, i.e., $(x, y) \in R_p$ if and only if $\overline{R}(x, y) \geq p$.

Let $\overline{R} : A \times B \to L$ be a fuzzy correspondence, where $(L, \leq)$ is a lattice ordered monoid. The following two propositions are corollaries of the similar propositions for the lattice valued fuzzy correspondences, and thus are valid also for lattice ordered monoid valued fuzzy correspondences. Although they are formulated for the first time here, we skip the proofs, because they could be deduced from the analogue theorems in [12].

Proposition 1. Theorem of decomposition for lattice valued fuzzy correspondences.
Let $\overline{R} : A \times B \to L$ be a fuzzy correspondence. For every $(x, y) \in A \times B$, the

following supremum exists in L: $\bigvee(p \in L|\overline{R}_p(x, y) = 1)$*, and*

$$\overline{R}(x, y) = \bigvee(p \in L|\overline{R}_p(x, y) = 1).$$

Proposition 2. *If* $\overline{R} : A \times B \dashrightarrow L$ *is a fuzzy correspondence, then:*

(i) If $p \leq q$*, for* $p, q \in L$*, then for every* $(x, y) \in A \times B$*,* $\overline{R_q}(x, y) \leq \overline{R_p}(x, y)$*;*

(ii) If the supremum of a subset Q *of* L *exists, then:*

$$\bigcap(R_p|p \in Q) = R_{\vee(p|p \in Q)};$$

(iii)

$$\bigcup(R_p|p \in L) = A \times B;$$

(iv) For every $(x, y) \in A \times B$*,*

$$\bigcap(R_p|(x, y) \in R_p)$$

belongs to the family of level relations of $\overline{R}$*.*

Lemma 3. *In a lattice ordered monoid* $(L, *, \wedge, \vee, e, \leq)$*, from* $a \leq b$ *and* $c \leq d$*, it follows that* $a * c \leq b * d$*.*

Proposition 4. *Let* $\overline{A} : X \longrightarrow L$ *be a fuzzy set, where* $(L, *, \wedge, \vee, o, e, \leq)$ *is a monoid with zero* o*. If* $\overline{A}(x) = p$ *and* $\overline{A}(y) = q$*, then:*

(a) *if* $p \geq e$*, then* $\overline{A}(x) * \overline{A}(y) \geq \overline{A}(y)$*;*

(b) *if* $p \leq e$*, then* $\overline{A}(x) * \overline{A}(y) \leq \overline{A}(y)$*;*

(c) *if* $p \geq e$ *and* $q \geq e$*, then* $\overline{A}(x) * \overline{A}(y) \geq e$*;*

(d) *if* $p \leq e$ *and* $q \leq e$*, then* $\overline{A}(x) * \overline{A}(y) \leq e$*;*

Proof. **(a)**, **(b)**, **(c)** and **(d)** follows straightforwardly by the isotonicity condition.

Proposition 5. *Let* $\overline{A} : X \longrightarrow L$ *be a fuzzy set as in previous proposition, such that* $L \setminus \{o\}$ *determines a submonoid, which is a lattice ordered group. If* $\overline{A}(x) = p$ *and* $\overline{A}(y) = q$*, where* $x, y \in X$*, then*

(a) $p = o$ *and* $q = o$ *if and only if* $\overline{A}(x) * \overline{A}(y) = o$*;*

(b) $p * q \geq e$ *if and only if* $p \geq q^{-1}$*.*

Proof. **(a)** "Only if part" follows by the definition of zero. On the other hand, by the condition that the restriction of $*$ to $L \setminus \{o\}$ determines a submonoid, $x * y$ should be different from o for $x \neq o$ and $y \neq o$.

(b) From $p * q \geq e$ and $q^{-1} \geq q^{-1}$, by the isotonicity condition, $p \geq q^{-1}$.

If $p \geq q^{-1}$, by the same condition and $q \geq q$, then $p * q \geq e$.

Corollary 6. *Under the conditions of the previous proposition, if* $p \geq e$ *and* $q \leq e$, *then*

(a) $p * q \geq e$ *if and only if* $p \geq q^{-1}$;
(b) $p * q \leq e$ *if and only if* $p \leq q^{-1}$.

Proposition 7. *Let* $\overline{A} : X \longrightarrow L$ *be a fuzzy set as in the previous proposition.*
Let $x \in A_p$ *and* $y \in A_q$.
(a) *If* $p = q \geq e$, *then* $\overline{A}(x) * \overline{A}(y) \geq p$
(b) *If* $p \geq e$, $q \leq e$, *and* $p \geq q^{-1}$, *then* $\overline{A}(x) * \overline{A}(y) \geq e$.

Proof. If $x \in A_p$ and $y \in A_q$, then $\overline{A}(x) \geq p$ and $\overline{A}(x) \geq q$.
(a) By the isotonicity condition and $p = q$, $\overline{A}(x) * \overline{A}(y) \geq p * p \geq p * e = p$.
(b) From $p \geq q^{-1}$, by the isotonicity condition, it follows that $\overline{A}(x) * \overline{A}(y) \geq p * q \geq q^{-1} * q = e$.

4 Mathematical Model

In this part we provide a mathematical model using simply ordered monoids and fuzzy correspondences defined on these monoids.

Let $T = \{T_1, ..., T_n\}$ be a set of possible drugs or therapies and $S = \{S_1, ..., S_m\}$ a set of all symptoms or different factors included in an anamnesis. Let $w : T \longrightarrow N$ ($N = \{1, 2, 3, \ldots\}$ is the set of natural numbers) be a function that assigns a **weight** to every symptom or different factor. If a factor (or symptom) S_i is "more important" than a factor S_j, then $w(S_i) > w(S_j)$.

Let $(L, *, \wedge, \vee, o, e, \leq)$ be a simply ordered monoid with zero o, such that $(L \setminus \{o\}, *, \wedge, \vee, ^{-1}, e, \leq)$ is a simply ordered group.

We define a fuzzy correspondence $\overline{R}$, on sets T and S, as follows.

If T_i is a drug or therapy, and S_j is a symptom, or different factor, then $\overline{R}(T_i, S_j) = p \in L$, where :

- $p \geq e$, if the influence of the therapy T_i on the factor S_j is positive;
- $p \leq e$, if the influence of the therapy T_i on the factor (organ) S_j is negative;
- $p = e$, if there is no influence of the therapy T_i on the factor S_j, or if influence is not known.
- $p = o$, if the influence of the therapy T_i on the factor S_j is extremely bad.

Values of this fuzzy correspondence and weights of different factors should be set in advance by experienced physicians (specialists).

After that, a fuzzy set on the set of therapies $\overline{A} : T \rightarrow L$ is computed by the following formula:

$$\overline{A}(T_i) = \prod_{j=1}^{m} (\overline{R}(T_i, S_j))^{w(S_j)}, \quad i = 1, \ldots, n, \tag{1}$$

where $\prod$ is the multiple application of the operation $*$.

In formula eq.1, $\overline{R}(T_i, S_j))$ is a value from L (simply ordered monoid). $w(S_j)$ is a natural number, and $\overline{R}(T_i, S_j))^{w(S_j)}$ is defined in a natural way, by

$$\overline{R}(T_i, S_j)) * \overline{R}(T_i, S_j)) * \ldots * \overline{R}(T_i, S_j)),$$

where $\overline{R}(T_i, S_j))$ repeats $w(S_j)$ times. Thus, $\overline{R}(T_i, S_j))^{w(S_j)}$ is also a value from L.

By the definition, $\overline{A}(T_i)$ is computed by

$$(\overline{R}(T_i, S_1))^{w(S_1)} * (\overline{R}(T_i, S_2))^{w(S_2)} * \ldots * (\overline{R}(T_i, S_m))^{w(S_m)},$$

and it is an element from L.

Thus, a function from the set T to L is obtained (a fuzzy set).

Since L is an ordered monoid, we find a maximum functional value of this fuzzy set. A drug (therapy) from set T with the maximum value of this fuzzy set should be applied.

4.1 Explanation of the Method

The fuzzy correspondence $\overline{R}$ on sets T and S shows influence of various therapies on all factors relevant to the recovery of the patient. The importance of all factors (weights) should be set in advance such that more important factors have greater weights. If influence of the therapy T_i on a factor S_j is positive, then the value of fuzzy relation $\overline{R}(T_i, S_j)$ is set to be greater than e. The more positive is the influence, the value of the relation is greater. Such a positive influence of a therapy T_i on a factor S_j increase the value $\overline{A}(T_i)$ of the fuzzy set $\overline{A} : T \longrightarrow L$.

If influence of T_i on S_j is negative, then the corresponding value $\overline{R}(T_j, S_j)$ is less than e, and the value $\overline{A}(T_i)$ of the fuzzy set $\overline{A} : T \longrightarrow L$ will be decreased. The more negative is the influence, the value of the relation is lesser.

The relation has value e in cases when there is no influence of T_i on S_j.

The relation is set to be o in cases when the influence of the therapy on a factor is extremely bad. By the definition of the zero element in monoid, the value $\overline{A}(T_i)$ will be o. Therapy T_i should be rejected in this case, regardless of the influence on other factors.

By the fact that the codomain L of the fuzzy relation $\overline{R}$ is a *simply* ordered monoid, all values of the relation are mutually comparable. The same is valid for the values of the fuzzy set $\overline{A}$. Therefore, there is a therapy T_i, such that the value $\overline{A}(T_i)$ is the greatest, and this therapy should be chosen.

By Proposition 4 and Corollary 6, fuzzy set $\overline{A}$ have the greatest value for the therapy with the optimal influence on all the relevant factors (elements from set S), taking into account weights of factors.

4.2 Modification of the method

In the following, a modification of presented method is described.

In this modification, all therapies that have influences negative to some extent are excluded. This could be done by considering level correspondences.

A level $p \in L$ should be chosen in advance, for which it is not tolerated any factor to be under. For instance let the level be a fixed $p < e$.

Now, a level correspondence: $R_p : T \times S \longrightarrow \{0, 1\}$ is considered.

The following characteristic function

$$\overline{A}_p(T_i) = \prod_{j=1}^{m} \overline{R}_p(T_i, S_j),$$

determines a subset of T_G of T:

$T_G = \{T_i \in T \mid \overline{A}_p(T_i) = 1\}$.

All therapies T_i for which $\overline{A}_p(T_i)$ has the value 0 are excluded. After that a computation can be continued by the main method, taking into consideration remaining the set T_G of therapies.

A level p should be chosen in advance by an experienced specialist.

5 Example

Here we give a simplified example taken from nephrological practice (for types of the therapies see also [8]). A lattice ordered monoid in this example is taken to be the lattice of all non-negative real numbers, with the usual multiplication: $(R^{+} \cup \{0\}, \cdot, \max, \min, 0, \leq)$. This lattice satisfies all required conditions, namely, $(R^{+}, \cdot, \max, \min, {}^{-1}, 1)$ is a simply ordered group.

Diagnosis: Acute bacterial pyelonephritis (E. coli), uncomplicated

Patient: Woman, 35

Possible Therapies:

T_1: Penicillins

T_2: Ampicillin

T_3: Cephalosporins

T_4: Aminoglycosides

T_5: Trimethropin/Sulphamethoxazole

T_6: Quinolones

Symptoms and other factors (w_i is the weight of the factor S_i, assumed by the physician on the ground of his medical knowledge and experience):

S_1: bactericidal agents, $w_1 = 4$

S_2: resistance, $w_2 = 4$

S_3: renal insufficiency, $w_3 = 2$

S_4: hypersensitivity, $w_4 = 1$
S_5: toxic reaction, $w_5 = 1$
S_6: symptoms, signs of complications, $w_6 = 2$
S_7: others (age,...), $w_7 = 1$

Let $S = \{S_1, S_2, \ldots, S_7\}$ and $T = \{T_1, T_2, \ldots, T_6\}$. We take $e = 1$ and the scale of an evaluation of the influence of the therapy T_i on the symptom S_j, e.g. the following one:

positive efficiency of a drug:

very good	> 10 (points)
almost very good	> 9
more than good	> 8
good	> 7
almost good/more than sufficient	> 6
sufficient	> 5
almost sufficient	> 4
poor	> 3
very poor	> 2
none	> 1

negative (destructive) efficiency of a drug:

hardly negative (weakly marked)	> 0.9
negatively marked	> 0.8
rather clearly unfavorable	> 0.7 - 0.6
unfavorable	> 0.5
more than unfavorable	> 0.4 - 0.3
destructive	> 0.2 - 0.1
very destructive	> 0

Fuzzy relation $\overline{R} : T \times S \longrightarrow R^+$ is given by the following matrix

$$\left[\begin{array}{c|ccccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \\ \hline T_1 & 2 & 0 & 1 & 0 & 1 & 1 & 1 \\ T_2 & 4 & 0.2 & 1 & 0 & 1 & 3 & 5 \\ T_3 & 7 & 0.4 & 1 & 0.2 & 0.6 & 8 & 8 \\ T_4 & 9 & 0.6 & 0.8 & 0.6 & 0.2 & 9 & 5 \\ T_5 & 6 & 0.4 & 0.8 & 0.4 & 0.4 & 5 & 5 \\ T_6 & 4 & 0.4 & 0.8 & 0.5 & 0.4 & 5 & 5 \end{array}\right]$$

One can easy see that e.g. therapy T_1 (penicillins) has extremely bad influence on the factor S_2 (resistance) and no influence on the symptom S_3 (renal insufficiency. In turn, therapy T_4 (aminoglycosides) has very good influence on symptom S_1 (bactericidal agents) and also on symptom S_6 (signs of complications).

A fuzzy set $\overline{A} : T \longrightarrow R^+$ is, according to eq.1, computed by the formula:

$$\overline{A}(T_i) = \prod_{j=1}^{7} (\overline{R}(T_i, S_j))^{w_j}, \quad i = 1, \ldots, 6$$

$$\begin{pmatrix} T_1 & 0 \\ T_2 & 0 \\ T_3 & 3776.45 \\ T_4 & 26447.91 \\ T_5 & 424.67 \\ T_6 & 104.86 \end{pmatrix}$$

Now, comparing values of fuzzy set $\overline{A}$, we conclude that the best therapy is T_4.

6 Conclusion

In this paper we have used fuzzy sets in order to deal with the problem of determination of the optimal therapy when the diagnosis has already been made. We construct a formula which takes into account influence of every therapy to all factors, and weights of different factors. An example taken from nephrological practice is provided, which illustrate the use of the formula.

It is possible to make further advance in these problems by modelling the incomparability, by using a general lattice ordered monoid, instead of simply ordered monoid. However, this would make the formula eq.1 more complicate and not so applicable in medical practice.

Acknowledgements

The authors are grateful to dr Petar Tepavčević, Professor of Internal Medicine - Nephrology, Medical Faculty Novi Sad, for the medical data in Example.

The research of the second author was supported by Serbian Ministry of Science, Technology and Development, grant no. 1227.

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