Descartesian Mechanics: The Fourth Generalization of Newton's Mechanics

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Abstract
The fourth generalization of Newton's Mechanics is considered. The oriented material point became a principle object for the study, while in Newton mechanics it was just a point. The space-time in new mechanics is represented by 10 dimensional fibre bundle, where 4 translational coordinates form base and 6 anholonomic angular - a fibre. The principle consequence of the new mechanics is the connection between the general relativity theory and quantum mechanics. In non relativistic approach it is possible to establish the theoretical foundation of "jet like motion without rejection of mass". This conclusion was verified by experimental results with 4-D Gyroscope.

Keywords: oriented point, inertia, torsion, quantum mechanics, physical vacuum

1 Introduction

For 317 years we have been applying Newton's mechanics to explain non-relativistic mechanical experiments on the "bench table". Although Newton's mechanics has been generalized three times: by the special relativity theory, general relativity theory, and quantum mechanics, there remains a possibility for its further generalization.

1.1 Frenet's Oriented Point

Newton's mechanics as well as all its generalizations, mentioned above, have been based upon the concept of the material point, substituting all the material bodies in this theory. The exception is quantum mechanics, where the material particles demonstrate both their corpuscular and wave properties. In the three-dimensional reference frame a material point has three degrees of freedom (according to the number of coordinates). In 1847 F. Frenet introduced for the first time the concept of an "oriented point" – point to which three orthogonal unit vectors are connected. In the three-dimensional coordinate space x_α, $(\alpha = 1, 2, 3)$ the oriented point has got six degrees of freedom - three translational and three rotational [1].

In arbitrary coordinate system and in modern notations, the Frenet's motion equations for the three-dimensional oriented point could be written as [2]

$$\frac{De^A{}_\alpha}{ds} = T^A{}_{B\gamma} e^B{}_\alpha \frac{dx^\gamma}{ds} \quad or \quad \frac{de^A{}_\alpha}{ds} = \Delta^A{}_{B\gamma} e^B{}_\alpha \frac{dx^\gamma}{ds}, \tag{1}$$

$$\alpha, \beta, \gamma ... = 1, 2, 3, \qquad A, B, C ... = 1, 2, 3,$$

where $\alpha, \beta, \gamma ...$ - coordinate induces and induces $A, B, C...$ - denote vectors of the Frenet's triad,

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \eta_{AB} e^A{}_\alpha e^B{}_\beta \, dx^\alpha \, dx^\beta, \qquad \eta_{AB} = \eta^{AB} = diag(1\ 1\ 1) \tag{2}$$

- the Euclidian translational metric, D - is absolute differential with respect to the Christoffel symbols

$$\Gamma^\alpha{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\eta} (g_{\beta\eta,\gamma} + g_{\gamma\eta,\beta} - g_{\beta\gamma,\eta}). \tag{3}$$

In (1) the geometric object

$$T^A{}_{B\gamma} = \nabla_\gamma e^A{}_\alpha e^\alpha_B = e^A_{\alpha,\gamma} e^\alpha_B - \Gamma^\beta_{\alpha\gamma} e^A_\beta e^\alpha_B = \Delta^A{}_{B\gamma} - \Gamma^A{}_{B\gamma} \tag{4}$$

had been introduced by F. Ricci [3] and named later as the Ricci rotation coefficients and

$$\Delta^A{}_{B\gamma} = \Gamma^A{}_{B\gamma} + T^A{}_{B\gamma} = e^A_{\alpha,\gamma} e^\alpha_B = \frac{\partial e^A_\alpha}{\partial x^\gamma} e^\alpha_B \tag{5}$$

– the connection of absolute parallelism geometry [4].

The Ricci rotation coefficients $T^A_{B\gamma}$ describes the changes of the orientation of basic vectors e_B^α and define the rotational metric [2]

$$d\nu^2 = e^\beta{}_A De^A{}_\alpha e^\alpha{}_A De^A{}_\beta = T^A{}_{B\alpha} T^B{}_{A\beta} dx^\alpha dx^\beta, \tag{6}$$

Below it will be shown, that the mechanics of the oriented point can generalize Newton's mechanics as well, allowing us:

a) To view the dynamics of the physical objects as rotation (Descartes' idea):

b) To describe the "inner" degrees of freedom, connected with its own rotation of the oriented point, that are not addressed in Newton's mechanics.

1.2 Clifford's Program on Geometrization of Physics

From the equations (1) we can get the curvature $\kappa(s)$ and torsion $\chi(s)$

$$\kappa(s) = T^{(1)}{}_{(2)\gamma} \frac{dx^\gamma}{ds}, \qquad \chi(s) = T^{(2)}{}_{(3)\gamma} \frac{dx^\gamma}{ds}, \tag{7}$$

which uniquely define an arbitrary curve in three-dimensional space. If we compare the Frenet's curve with a certain physical trajectory, then it will allow us to describe the motion of the material point, which may change its orientation in the space. We will call such an object as the "oriented material point". Let the curve $\kappa(s)$ in the Frenet's equations be equal to zero, then it follows from (1) the force acting upon the oriented material point is absent and it moves straight along the line. Meanwhile its orientation in the space changes. In these case equations describe own rotations of the oriented point, affected by the rotational field $\chi(s)$ - torsion field [2], while the action is forceless. The equations (1) are interesting, because they allow to find the geometrical description for physical interactions, which are based upon the Newton's equations. In order to do so it will be sufficient to select the curvature $\kappa(s)$ related to (1). Perhaps, the similar ideas led Clifford saying in 1870 that "there is nothing happening in the world, except changes of the space curvature" [5]. However, being consistent, we could refine it by saying: "there is nothing happening in the world, except changes of the curvature and torsion of the space". To prove it with the help of the Frenet's equations - is impossible. These equations describe just an arbitrary curve in the three-dimensional space. Moreover, it would be a better idea to call $\kappa(s)$ and $\chi(s)$ as the first and second torsion of a curve, since they are defined through the Ricci rotation coefficients $T^A_{B\gamma}$ according to the relations (7). It is understood that the geometrization of physics requires such a geometry, which has got the Riemann curvature and torsion, created by the Ricci rotation coefficients.

1.3 Ricci's Curvature on Manifold of Oriented Points

We know that Riemann applied point manifold to define the curvature tensor R^i_{jkm} of non-Euclidean space. Ricci in his work [3] finds for the first time the curvature tensor for the manifold of the oriented points. To be more exact and guided by the physical applications, let us write the principal formulas from Ricci's work [3] for the manifold of the oriented points with 4-dimensions, using modern notations. The generalization for a larger number of dimensions is not difficult. Following Ricci, let us consider four -dimensional differentiated manifold with coordinates x^i $(i = 0, 1, 2, 3)$. In each point of this manifold there are - vector $e^a{}_i$ $(i = 0, 1, 2, 3)$ and co vector $e^j{}_b$ $(b = 0, 1, 2, 3)$ with the normalization conditions

$$e^a{}_i e^j{}_a = \delta^j_i, \quad e^a{}_i e^i{}_b = \delta^a_b. \tag{8}$$

With such a task the four coordinates x^i describe the origin O of four-dimensional oriented point (tetrad), and six independent (due to the conditions of (8)) components of tetrad $e^a{}_i$ describe its space orientation, playing the role of angular variables.

Tetrad $e^a{}_i$ defines the metric tensor of space

$$g_{ik} = \eta_{ab} e^a{}_i e^b_k, \eta_{ab} = \eta^{ab} = \mathrm{diag}(1 \ -1 \ -1 \ -1) \tag{9}$$

and Riemannian (translational) metric

$$ds^2 = g_{ik} dx^i dx^k. \tag{10}$$

Moreover the covariant derivatives of $e^a{}_i$ along coordinates x_i define the Ricci rotation coefficients [3]

$$T^i_{jk} = e^i{}_a \nabla_k e^a{}_j = -\Omega^{..i}_{jk} + g^{im}(g_{js}\Omega^{..s}_{mk} + g_{ks}\Omega^{..s}_{mj}), \tag{11}$$

where ∇_k is the covariant derivative with respect to the Christoffel's symbols

$$\Gamma^i_{jk} = \frac{1}{2} g^{im}(g_{jm,k} + g_{km,j} - g_{jk,m}), \tag{12}$$

and the quantity [3]

$$\Omega^{..i}_{jk}, = e^i{}_a e^a_{[k,j]} = -\frac{1}{2} e^i{}_a (e^a{}_{j,k} - e^a{}_{k,j}) = -T^i_{[jk]} \tag{13}$$

has been called by J. Schouten as an object of anholonomity [6]. This name had been justified by the fact that six angular variables, orienting the triad, are anholonomic. Naturally, when the object of anholonomity (13) goes to zero, there will be no change for the orientation of a point. If the orientation of tetrad vectors changes, then we get the rotational metric [2]

$$d\tau^2 = T^i{}_{jk} T^j_{in} dx^k dx^n, \tag{14}$$

which describes the infinitesimal rotation. Further Ricci demonstrates [3] that there are two curvature tensors for the manifolds of the oriented points:

a) tensor of Riemannian curvature, defined through Christoffel's symbols by conventional way

$$R^i{}_{jkm} = 2\Gamma^i{}_{j[m,k]} + 2\Gamma^i{}_{s[k}\Gamma^s{}_{|j|m]}, \tag{15}$$

b) tensor of Ricci curvature, defined through the Ricci rotation coefficients as

$$P^i{}_{jkm} = 2\nabla_{[k} T^i_{|j|m]} + 2T^i_{c[k} T^c_{|j|m]}. \tag{16}$$

Because the sum $\Gamma^i_{jk} + T^i_{jk}$ forms the connection of absolute parallelism [7]

$$\Delta^i_{jk} = \Gamma^i_{jk} + T^i_{jk} = e^k{}_a e^a{}_{i,j}, \tag{17}$$

the curvature tensor of the space of absolute parallelism

$$S^i{}_{jkm} = 2\Delta^i_{j[m,k]} + 2\Delta^i_{s[k}\Delta^s_{|j|m]} = 0, \tag{18}$$

is equals to zero. Then, substituting (17) into (18), we will get the relationship

$$S^i{}_{jkm} = R^i{}_{jkm} + 2\nabla_{[k} T^i_{|j|m]} + 2T^i_{c[k} T^c_{|j|m]} = R^i{}_{jkm} + P^i{}_{jkm} = 0. \tag{19}$$

Let us note that the connection of the geometry of absolute parallelism (17) has torsion

$$\Delta^i_{[jk]} = T^i_{[jk]} = -\Omega^{..i}_{jk}, \tag{20}$$

which we will call *Ricci torsion*. Thus the geometry of absolute parallelism with the Riemannian curvature (15) and Ricci torsion (20) fits most of the implementation of Clifford's program for geometrization of physics.

1.4 Klein's "Erlangen Program" and Cartan's Structural Equations of the Geometry of Absolute Parallelism A_4

In 1872 F. Klein introduced the "Erlangen Program", which aimed to construct the basic geometrical relations for the geometry [8] specifying the group of motion of the space. This program had been consistently developed by many famous mathematicians with the major contribution made by Cartan. Cartan applied not a point manifold, which was used by Riemann to construct non-Euclidian geometry, but a manifold of the oriented points similar to Ricci. Cartan called an oriented point the "orthogonal moving reaper", which in motion created infinitesimal translations of the origin dx^i (in our case local group T_4) as well as infinitesimal rotations of tetrad vectors $de^i{}_a$ (local group $O(3.1)$). Using Cartan's method [9], we will obtain the following Cartan's structural equations of the geometry A_4 [2]

$$\nabla_{[k}e^a{}_{m]} - e^b{}_{[k}T^a{}_{|b|m]} = 0 \quad or \quad \nabla_{[a}e^i{}_{b]} = -\Omega_{ab}^{..c}e^i{}_c, \tag{21}$$

$$R^a{}_{bkm} + 2\nabla_{[k}T^a_{|b|m]} + 2T^a{}_{c[k}T^c{}_{|b|m]} = 0 \quad or \quad R^a{}_{bkm} = 2e^a_i\nabla_{[k}\nabla_{m]}e^i_b, \tag{22}$$

which coincide with the Maurer-Cartan equations of the groups T_4 and $O(3.1)$ correspondingly.

In the equations (21) the Ricci torsion components $\Omega_{ab}^{..c}$ represent the structural functions of the local group T_4 , satisfying first Jacobi's identity (or Bianchi's first identity)

$$\overset{*}{\nabla}_{[b}\,\Omega_{cd]}^{..a} + 2\Omega_{[bc}^{..f}\Omega_{d]f}^{..a} = 0 \quad or \quad R^a{}_{[bcd]} = 0, \tag{23}$$

where $\overset{*}{\nabla}_b$ - the covariant derivative with respect to the connection (17). In the equations (22) the Riemannian tensor components $R^a{}_{bkm}$ represent the structural functions of the local group $O(3.1)$, satisfying the second Jacobi's identity

$$\nabla_{[n}R^a{}_{|b|km]} + R^c{}_{b[km}T^a{}_{|c|n]} - T^c{}_{b[n}R^a{}_{|c|km]} = 0. \tag{24}$$

Considering that the structural equations (21) and (22) satisfy the conditions of integration (equations (23and (24) correspondingly)[2], then the geometry of absolute parallelism happens to become the only geometry satisfying all the requirements of Klein's "Erlangen Program".

1.5 Inner Degrees of Freedom of an Oriented Point and Yang-Mills's Field Geometrization

The space of the events of mechanics of an oriented material point has a more complex structure, than the mechanics of a point. If the description of the dynamics of a material point in n-dimensional space requires n coordinates, then the description of the oriented material point in n - dimensional space requires $n(n+1)/2$ coordinates [10] For example, in four dimensional space 10 coordinates define the oriented material point: four translational coordinates x, y, z, ct and six angular, where there are three space angles φ_1, φ_2, φ_3 and three space-time θ_1, θ_2, θ_3 angles. The anholonomic tetrad $e^a{}_i$ represents the angular coordinates. The ten-dimensional manifold (four translational coordinates xi and six "rotational coordinates $e^i{}_a$) of the geometry of absolute parallelism A_4 can be viewed as a vector fiber bundle with the coordinates of base x^i (external space), in each point of which there is a field of four orthonormal vectors $e^i{}_a$ (a =0,1,2,3) [11] forming "inner ' space. The translational group T_4 acts in the external space x_i (base) and the rotational group $O(3.1)$ acts in the "inner" space $e^i{}_a$(fiber). In the equations (21) and (22) the matrices $e^a{}_i$, T^a_{bk} and $R^a{}_{bkm}$ are transformed in the rotational group $O(3.1)$ as follows

$$e^{a'}_i = \Lambda_a{}^{a'}e^a{}_i$$

$$T^{a'}_{b'k} = \Lambda_a{}^{a'} T^a{}_{bk} \Lambda^b{}_{b'} + \Lambda_a{}^{a'} \Lambda^a{}_{b',k}, \qquad \Lambda^{a'}_a \in O(3.1), \tag{25}$$

$$R^{a'}{}_{b'km} = \Lambda_a{}^{a'} R^a{}_{bkm} \Lambda^b{}_{b'},$$

while Ricci rotation coefficients T^a_{bk} perform as potentials of the gauge field $R^a{}_{bkm}$. Dropping the matrices indices, let us write the equations (22) and (24) in the form of geometrized Yang-Mills equations

$$R_{km} = 2\nabla_{[m} T_{k]} + [T_m, T_k], \tag{26}$$

$$\nabla_n \overset{*}{R}{}^{kn} + \overset{*}{R}{}^{kn} T_n - T_n \overset{*}{R}{}^{kn} = 0, \tag{27}$$

with the gauge group $O(3.1)$. We have introduced the notation for the dual Riemannian tensor $\overset{*}{R}_{ijkm} = \frac{1}{2}\varepsilon^{sp}{}_{km} R_{ijsp}$. Adding the structural equation of the translational group (21) to the geometrisized Yang-Mills equations (26) and (27)

$$\nabla_{[k} e_{\ m]} - e_{\ [k} T_{\ m]} = 0, \tag{28}$$

we will get the extended system of geometrized Yang-Mills equations.

1.6 Equality of the Newman-Penrose Equations with Geometry A_4 Structural Equations

Clifford's program on the geometrization of physics started from Einstein's work, who had shown that the relativistic gravitational fields and gravitational interactions can be described by the definite relationships of Riemannian geometry [12]. A. Einstein especially remarked, that a purely geometrical description of the gravitational fields could be given by Einstein's vacuum equations

$$R_{ik} = 0 \tag{29}$$

and these equations" represent the only rational fundamental case for the field theory that may pretend for strict approach..." [12]. Einstein was right and the Einstein's gravitational theory can be proven by the experiments, based upon the solutions of the Einstein's vacuum equations (29). In 1962 the mathematicians E.Newman and R. Penrose proposed a new method to search for the solutions of the Einstein's vacuum equations [13]. In the coordinates of the base x^i and with the accepted notations the basic equations of Newman-Penrose formalism can be viewed as follows

$$R^i{}_{jkm} + 2\nabla_{[k} T^i_{|j|m]} + 2T^i_{s[k} T^s_{|j|m]} = 0, \tag{2.7 NP}$$

$$\nabla_{[n} R_{|ij|km]} + R^s_{j[km} T_{|is|n]} - T^s_{j[n} R_{|is|km]} = 0, \tag{2.9 NP}$$

$$\nabla_{[k} e^a_{j]} + T^i_{[kj]} e^a{}_i = 0. \tag{2.11 NP}$$

The numbers in the right part of the equations correspond to the numbers in Newman's and Penrose's work [13]. The comparison of these equations with the system (21)- (24)

shows that Newman-Penrose formalism use the structural Cartan's equations of the geometry A_4 [2]. If we wish to obtain new solutions of Einstein's vacuum equations (29), there is no need to solve them now. It will be sufficient to find (or "construct") such a solution of the structural Cartan's equations of the geometry A_4 (21) and (22), which satisfy to $R_{ik} = 0$. Thus, such famous solutions as Schwarzschild [13], NUT [14] and Kerr [15] had been found for Einstein's vacuum equations.

2 Geometrization of Energy-Momentum Tensor in Einstein's Equations and Tensor Current in Yang-Mills Equations

After successful geometrization of gravitational interactions, A.Einstein introduced in theoretical physics the Unified Field Program that implied the geometrization of all other physical fields, which form of the material energy-momentum tensor in Einstein's equations

$$R_{jm} - \frac{1}{2} g_{jm} R = \frac{8\pi G}{c^4} T_{jm}. \tag{30}$$

In order to do so, Einstein used various generalizations of Riemannian geometry, including the geometry of absolute parallelism A_4 [16]. Although Einstein had actively corresponded with Cartan about the geometry of absolute parallelism [17], he was not aware at that time of Cartan's structural equations (21) and (22) of his geometry. Meanwhile the problem of geometrization of the right part of Einstein's equations (30) can be solved with the help of Cartan's structural of the equations geometry A_4. Let us write the equations (2.7 NP) as

$$C_{ijkm} + g_{i[k}R_{m]j} + g_{j[k}R_{m]i} + \frac{1}{3} R g_{i[m}g_{k]j} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{s[k}T^s_{|j|m]} = 0, \tag{31}$$

where C_{ijkm} – Weyl's tensor, R_{jm} –Ricci tensor, R- scalar curvature. These equations split into 10 equations [18]

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{jm}, \tag{32}$$

similar to Einstein's equations, but with geometrized right part, defined as

$$T_{jm} = -\frac{2}{\nu}\{(\nabla_{[i}T^i{}_{|j|m]} + T^i{}_{s[i}T^s{}_{|j|m]}) - \frac{1}{2} g_{jm} g^{pn} (\nabla_{[i}T^i{}_{|p|n]} + T^i{}_{s[i}T^s{}_{|p|n]})\} \tag{33}$$

and 10 equations

$$C_{ijkm} + 2\nabla_{[k}T_{|ij|m]} + 2T_{is[k}T^s_{|j|m]} = -\nu J_{ijkm}, \tag{34}$$

similar to Yang-Mills equations, but with geometrized tensor current

$$J_{ijkm} = 2g_{[k(i}T_{j)m]} - \frac{1}{3} T g_{i[m}g_{k]j}, \tag{35}$$

where T-trace of tensor (33). Defining the material density as

$$\rho = g^{jm} T_{jm}/c^2, \tag{36}$$

and applying energy-momentum tensor (33) we have

$$\rho = \frac{2g_{jm}}{\nu c^2} (\nabla_{[i} T^i{}_{|j|m]} + T^i{}_{s[i} T^s{}_{|j|m]}). \tag{37}$$

Certainly the equations (32) principally differ from the Einstein's equations (30), because they:

a) represent the natural generalization of vacuum equations (29) and as well as the equations (29) do not contain any physical constant;

b) are completely geometrized and describe the material fields through Ricci torsion (20);

c) are self-complying with geometrized Yang-Mills equations (34) and " coordinate" equations $(2.11NP)$.

For example, instead of Einstein's vacuum equations (29), from the equations (21), (22) we will get the system

$$\nabla_{[k} e^a_{j]} + T^i_{[kj]} e^a{}_i = 0. \tag{38}$$

$$C^i{}_{jkm} + 2\nabla_{[k} T^i_{|j|m]} + 2T^i_{s[k} T^s_{|j|m]} = 0. \tag{39}$$

E. Newman, R. Penrose and others have been finding the solution of this particular system for Einstein's vacuum. With the chosen coordinate system x^i, as a searched function it includes components of Weyl's tensor $C^i{}_{jkm}$, components of Ricci rotation coefficients T^i_{kj} as well as the components of tetrad e^a_j. For example, the solution with Schwarzschild's metric

$$ds^2 = \left(1 - \frac{2\Psi^0}{r}\right) c^2 dt^2 - \left(1 - \frac{2\Psi^0}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

in the coordinates $x_0 = ct,\ r,\ x^2 = \theta\ x^3 = \varphi$ and in spinor presentation [13] it can be viewed for:

1. Components of Newman-Penrose symbols:

$$\sigma^i_{0\dot{0}} = (0,1,0,0), \quad \sigma^i_{1\dot{1}} = (1,U,0,0), \quad \sigma^i_{0\dot{1}} = \rho(0,0,P,iP),$$

$$\sigma^{0\dot{0}}_i = (1,0,0,0), \quad \sigma^{1\dot{1}}_i = (-U,1,0,0), \quad \sigma^{0\dot{1}}_i = -\frac{1}{2\rho P}(0,0,1,i),$$

$$U = -1/2 + \Psi^0/r, \quad P = (2)^{-1/2}(1 + \zeta\bar{\zeta}/4), \quad \zeta = x^2 + ix^3,$$

$$\Psi^0 = \text{const}.$$

2. Spinor components of Ricci rotation coefficients:

$$\rho = -1/r, \quad \alpha = -\bar{\beta} = -\alpha^0/r, \quad \gamma = \Psi^0/2r,$$

$$\mu = -\varepsilon^0/r + 2\Psi^0/r^2, \alpha^0 = \zeta/4.$$

3. Spinor components of Weyl's tensor:

$$\Psi = -\Psi^0/r^3.$$

The received solution of new vacuum equations has got physical sense, if we set

$$\Psi^0 = MG/c^2. \tag{40}$$

The principle difference of the equations (38) and (39) from Einstein's vacuum equations (29) is that, if Ricci torsion (consequently of Ricci rotation coefficients as well) in the equations (38) and (39) goes to zero, than we will get the flat space.

3 Motion Equations of Oriented Point. Physical Interpretation of the Ricci Rotation Coefficients

The motion equations of four-dimensional oriented point

$$\frac{de^i{}_a}{ds} + \Gamma^i_{jk} e^j{}_a \frac{dx^k}{ds} + T^i_{jk} e^j{}_a \frac{dx^k}{ds} = 0. \tag{41}$$

follows from the definition (17) of the connection of A_4 geometry.

From 16 "rotational" equations (41), with the normalization condition (8), there remains 6 independent equations. These equations describe the change of the orientation of the oriented point. It is possible to add 4 motion equations of the "origin" of oriented point, which represent the geodesic equations of the space A_4

$$\frac{d^2x^i}{ds^2} + \Delta^i_{jk} \frac{dx^j}{ds}\frac{dx^k}{ds} = \frac{d^2x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds}\frac{dx^k}{ds} + T^i_{jk} \frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \tag{42}$$

Let us remark, that:

1) the equations (42) could be obtained from variation principle [2];

2) the equations (42) could be obtained from the equations of the oriented point (41), if we chose vector $e^{(0)}_i$ as $e^{(0)}_i = dx_i/ds$.

If we multiply the equations (42) by mass m of the oriented point, then we will get the motion equations of its center of mass. In nonrelativistic approximation the equations (42) will be viewed as

$$m\frac{d^2x^\alpha}{dt^2} = -mc^2\Gamma^\alpha{}_{00} - mc^2T^\alpha_{00}. \tag{43}$$

Applying the solution of vacuum equations (38) and (39) with Schwarzschild's metric, where the source function Ψ^0 is defined by the relation (40), we will obtain in quazi-Descartesian coordinates

$$F^\alpha_G = -mc^2\Gamma^\alpha{}_{00} = m\frac{MG}{r^3}x^\alpha, \qquad F^\alpha_I = -mc^2T^\alpha_{00} = -m\frac{MG}{r^3}x^\alpha. \tag{44}$$

Evidently, the first of these forces F^α_G – Newtonian gravitational force. The force F^α_I is equal in its absolute value to the gravitational force F^α_G, but directed in the opposite side. We may naturally interpret it as an inertial force, which acts locally in the accelerated reference frame and compensates gravitational force, creating a weightless condition in free falling Einstein's lift. Correspondingly the Ricci rotation coefficients $T^i{}_{jk}$ interpreted as intensity of the inertial field [19]. Thus , the inertial field $T^i{}_{jk}$ represents the torsion field, originated by the torsion of absolute parallelism geometry. The connection between rotation of matter and torsion (13) of A_4 geometry was outlined by Cartan in 1922 [21], although without a direct analytical reasoning. This fact created a stir in the research world. The reason was ,that a few years later Cartan introduced a torsion , based upon the point manifold. It differs from Ricci torsion (13), because it does not depend upon the angular variables. I could not find any analytical proof of the connection of Cartan torsion (not Ricci torsion (13)) with real physical rotation.

4 Inertial Mass in Descartesian Mechanics. Four-Dimensional Gyroscope

The inertial rest mass of an object in Descartesian mechanics is defined as

$$m_0(t) = \int \rho(-g)^{1/2} dV = \frac{2}{\nu c^2} \int (-g)^{1/2} \left\{ g^{jm} \left(\nabla_{[i} T^i_{|j|m]} + T^i{}_{s[i} T^s_{|j|m]} \right) \right\} dV, \tag{45}$$

where

$$g = \det g_{jm}, \qquad dV = dx^1 dx^2 dx^3,$$

and the density ρ is defined according to (37). The relationships show that the inertial rest mass in Descartesian mechanics represents *the measure of the inertial field*. Since the inertial field $T^i{}_{jk}$ originated by the rotation of the matter (according to E. Cartan), then the inertial properties of the rest mass depend on the conditions of the rotation of the matter, forming the discussed system. For example, by changing the angular velocity of the separate mass parts of the system $m_0(t)$ according to a certain law, then we can create a "jet-like motion without rejecting the mass" according to the motion equations

$$m_0(t) \frac{d}{dt}(v_\alpha) = -v_\alpha \frac{d}{dt} m_0(t). \tag{46}$$

The mechanical device, where the center of mass moves according to the equations (46), has been called a four-dimensional gyroscope (4-D -gyroscope) (see fig. 1). All the elements of the conventional 3-dimensional gyroscope rotate in the spatial angle ϕ in the planes, perpendicular to the axis of rotation. A 4-D gyroscope consists of three connected masses (see fig. 1), two of which (masses m) rotate synchronously in different directions in the spatial angle $\phi(t)$ around axis O_1, set on the central mass M. The central mass M itself oscillates along axis of symmetry x with the acceleration

$$W_x = \frac{dv_x(t)}{dt} = c(th\dot{\theta}_x) = c\frac{d\,[th\ \theta_x(t)]}{dt}, \tag{47}$$

where θ – pseudo-Euclidean angle. Lagrangian T of free 4-D gyroscope can be presented as

$$T = \frac{M+2m}{2}\left(v_c^2 + k^2(1-k^2\sin^2\phi)w^2\right) = \frac{M+2m}{2}\left(v_c^2 + g'w^2\right), = \frac{M+2m}{2}\dot{s}^2 \tag{48}$$

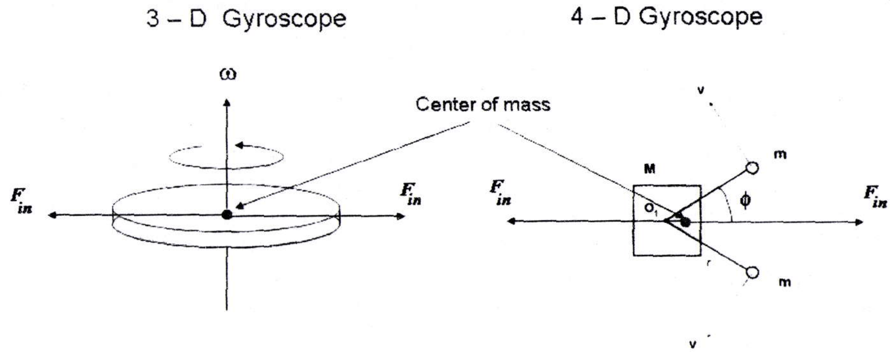

Figure 1: The 3-D and 4-D gyroscopes

where

$$w = r\omega, \quad k^2 = 2m/(M+2m), \quad v_c = v - k^2 w \sin\phi, \quad g' = k^2(1 - k^2\sin^2\varphi) = k^2 g.$$

Here v_c - velocity of the center of masses, v - the velocity of the central mass M, $\omega = \dot{\phi}$ - angular velocity of the rotation of small masses, r – distance from O_1 to small masses m.

Let us consider that the motion of the center of masses of free 4-D gyroscope occurs according to the motion equations of Descartesian mechanics

$$\frac{d^2x^i}{ds^2} + \Delta^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \quad i, j, k... = 1, 2, \tag{49}$$

where

$$\Delta^i{}_{jk} = \Gamma^i{}_{jk} + T^i{}_{jk} = e^i{}_a e^a{}_{j,k}.$$

$$ds^2 = g_{ij}dx^i dx^j = \frac{2T}{M+2m}dt^2, \quad i, j = 1, 2,$$

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & g' \end{pmatrix} = \Lambda_{ab}e^a{}_i e^b{}_j, \quad \Lambda_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{50}$$

The orthogonal diad $e^a{}_i$ for the given metric tensor connected with the variables

$$v_c(t) = \cos\eta(t)\dot{s}, \quad \sqrt{g'}w(t) = \sin\eta(t)\dot{s}, \tag{51}$$

and can be viewed as

$$e^a{}_i(\eta(t)) = \begin{pmatrix} \cos\eta & \sqrt{g'}\sin\eta \\ -\sin\eta & \sqrt{g'}\cos\eta \end{pmatrix}, \quad e^i{}_a(\eta(t)) = \begin{pmatrix} \cos\eta & -\sin\eta \\ \frac{1}{\sqrt{g'}}\sin\eta & \frac{1}{\sqrt{g'}}\cos\eta \end{pmatrix}.$$

After the corresponding calculations, the motion equations will become

$$\frac{dv_c}{dt} = \frac{2mr}{M+2m}\Phi\omega, \quad or \quad (M+2m)\frac{dv_c}{dt} = 2mr\Phi\omega, \tag{52}$$

$$\frac{d\omega}{dt} - k^2\omega^2\frac{\sin\phi\cos\phi}{1-k^2\sin^2\phi} = -\frac{1}{rg}\Phi v_c, \tag{53}$$

where

$$\Phi(t) = -\frac{\sqrt{g'}}{k^2}\frac{d\eta}{dt} \tag{54}$$

- function, created by Ricci torsion and $2mr\Phi\omega$ - force of inertia. If function $\Phi(t)$ goes to zero, then the equations (52) and (53) will coincide with the motion equations of 4-D gyroscope, which follow from Newtonian mechanics.

4.1 Control of Ricci Torsion and Riemann Curvature of Local Space

When 4-D gyroscope is not free Descartesian mechanics requires four-dimensional coordinate space for the description of 4-D gyroscope, even for velocity much less than the speed of light

$$x_0 = ct, \ x_1 = x, \ x_2 = y, \ x_3 = z.$$

It follows from the fact that translational acceleration in Descartesian mechanics is deduced to the rotation in the space-time planes, for example, as in our case, according to the formula (47). That is why for a more consistent description of 4-D gyroscope we have to apply the coordinates

$$x_0 = ct, \ x_1 = x_c, \ x_2 = r\phi.$$

We will select the metric tensor of the following type

$$g_{ij} = \begin{pmatrix} 0 & 1-2k^2r^2U(\phi)/c^2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -k^2(1-k^2\sin^2\phi) \end{pmatrix}, \tag{55}$$

where the "potential"

$$U(\phi) = \int_{\phi_0}^{\phi} N d\phi \tag{56}$$

is created by controlled angular acceleration $N = L/2mr^2$ of small masses m, $L(t)$ - external angular momentum, created by motor-brake. Using the motion equations (42) and field equations (19) we find

$$\frac{dv_c}{dt} = rk^2\Phi\omega, \tag{57}$$

$$\Omega^1_{02} = -\Omega^1_{20} = k^2\Phi/2c, \quad \Omega^2_{01} = -\Omega^2_{10} = -\frac{\Phi}{2c(1-k^2\sin^2\phi)}. \tag{58}$$

$$R_{00} = -\frac{r^2k^2U_\phi^2}{c^2g(c^2-2k^2r^2U)} - \frac{k^2U_\phi \sin\phi\cos\phi}{c^2g^2} - \frac{U_{\phi\phi}}{c^2g}, \tag{59}$$

$$R_{22} = -\frac{k^2c^2g}{c^2-2k^2r^2U}R_{00},$$

where

$$\Phi = 2\sqrt{\frac{N\sin\phi\cos\phi}{1-k^2\sin^2\phi} + \frac{N_\phi}{k^2}}. \tag{60}$$

and v_c – velocity of the center of masses of the 4-D gyroscope.

5 Experimental Investigations of Four-Dimensional Gyroscope

For the experimental research of the 4-D gyroscope mechanics, its space -time precession, we created 11 models of the 4-D gyroscopes with the mechanical and electrical motor-breaks. Some of them have been operated by the computer software. We constructed the experimental bench-stand, consisting of the horizontal surface, the measuring system to register the translational coordinate $x(t)$ ($\triangle x = \pm 0.5mm$) and angular coordinate $\phi(t)$ ($\triangle\phi \pm 0.5^o$). The special software allowed us to calculate the linear and angular velocities in real time. The corresponding graphs have been monitored and observed during the experiments. We have researched the following:

1) space-time precession of the 4-D gyroscope,

2) absolute elastic external collision of the gyroscope's body against the wall, which allowed us to observe:

a) transformation of the translational inertia into rotational;

b) transformation of the rotational inertia into translational;

c) multiple impacts of the 4-D gyroscope;

3) singular internal collisions of the 4-D gyroscope (on the cart and while suspended);

4) multiple internal collisions of the 4-D gyroscope (on the cart and while suspended);

5) changes of the direction of the 4-D gyroscope's motion without changes of the direction of the rotation of its small masses m.

These experiments demonstrated that the motion of the center of masses of a 4-D gyroscope cannot be explained by Newtonian mechanics. The controllable operation of the motion of its center mass is explained by the space-time precession that is understandable from the point of view of Descartesian mechanics. However, perhaps, it is the first attempt of the scientific foundation of new mechanics and more detailed investigations are required.

5.1 The Model with Computerized Motion Control

Since the character of motion of the center of masses of 4-D gyroscope is fully defined by the law of the change of the $\Phi(t)$ function (or the frequency of the rotation of small masses), then it should be a good idea to operate it via computer. Moreover, if we wish to exclude the influence of the friction forces on the motion of the center of masses of the system forward, it is required to operate the motion of the gyroscope body and, consequently, of its supporting wheels forward only. In this case the friction forces will always obstruct the motion of the center of masses forward, slowing down its motion.

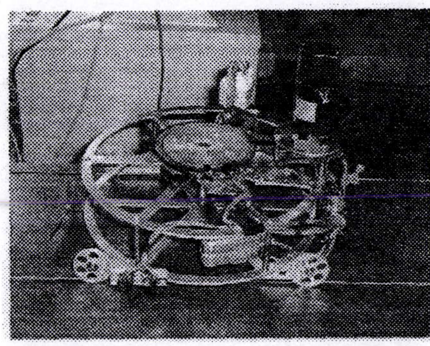

Figure 2: 4-D gyroscope with the computerized motion control

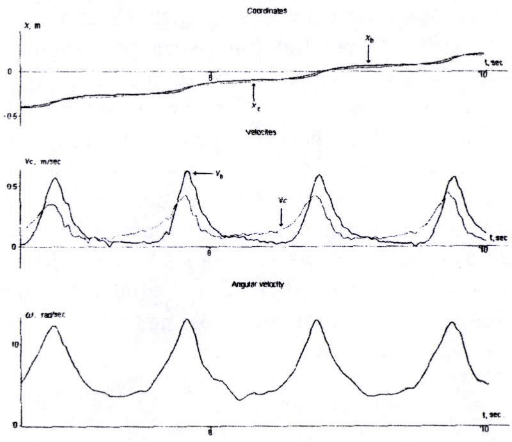

Figure 3: The experimental graphs of the 4-D gyroscope with computerized motion control; v_b - body's velocity, v_c -velocity of the center of mass

Fig. 2 presents 4-D gyroscope with servomotor (motor with feed back). The operation of this motor is performed via computer and special software. The program allows us to accelerate and slow down the rotation of small masses in the required segments. The graph of motion velocities (fig.3) for the body and center of masses shows that the body moves only forward. Accordingly, the wheels, supporting it, move only forward, while the friction forces, between the wheels and surface of motion, work against the motion and could not cause the motion forward.

6 Conclusion

The Fourth Generalization of Newtonian mechanics has become possible with regards that Descartesian mechanics has been based upon the following:
1) Clifford-Einstein proposal for geometrization of physics (Unified Field Theory).
2) Klein's "Erlangen Program".
3) Cartan's idea about the connection of the torsion of space with physical rotation.
4) Descartesian idea about rotational nature of any motion.
In this article we have adhered to the experimental verifications for some of the conclusions of new mechanics, using the known phenomenon [24], where the main role belongs to fields and forces of inertia - one of the great physical enigma from Newton's times. The Descartesian mechanics allows us to create the theoretical foundation for the experiments that Newton's mechanics could not explain. This experiments demonstrate "jet-like motion without rejection of mass" [23]. The simplest model of the mechanical propulsion system, which propels in space in jet-like motion, although without rejection of mass, had been created by a talented Russian engineer Vladimir Tolchin [24]. We have continued the experiments with Tolchin's mechanical devices and discovered that they deviated from Newton mechanics, when the center of mass had been affected by uncompensated forces of inertia, causing the phenomenon of space-time precession. We have observed that the phenomenon of space-time precession of four-dimensional gyroscope allows us to control its inertial mass. In the near future it will allow the creation of the Universal Propulsion System, which will be able to move in all the media: on earth, on water, under water, in air and in space. The 4-D Engine, with a hermetically sealed body, using space-time precession, will have quite a number of advantages and benefits, compared to any other engine: it will be ecologically clean, economic and universal. It should gradually replace the existing engines in many branches of contemporary technologies.

References

[1] *Frenet F.* Jour. de Math. 1852. Vol. 17. P. 437-447.

[2] *Shipov G.I.* The theory of physical vacuum. M.: Nauka, 1997, p.450.

[3] *Ricci G.* Mem.Acc.Linc. 1895. Vol. 2. Ser. 5. P. 276-322.

[4] *Vitali G.* // Atti Soc. ligust. sci. Lett. 1924. Vol. 11. P.248-254.

[5] *Clifford W.* // In: Albert Einstein and Gravitation Theory. Moscow. Mir, 1979, pp.36 - 46 (in Rassian).

[6] *Schouten J.* Ricci-Calculus. B.; Heidelberg: Springer, 1954.

[7] *Weitzenbock R.* // Proc. Knkl. nederl. akad. 1926. Vol. 28. P. 400-411.

[8] *Klein F.* Math.Ann. 1893. Vol. 43. P. 63.

[9] *Cartan E.* A theory of the finite continual groups and differential geometry, stated by method of moving reper. Moscow.: Platon., 1998.

[10] *Cartan E.* Riemannian geometry in orthogonal reper. Moscow.: Platon., 1998 (in Russian)

[11] *Favar J.* A course in local differential, Moscow, Foreign Literature, 1960 (in Russian).

[12] *Einstein A.* The Meaning of Gravitation Relativity, four edition, Prinston, 1953.

[13] *Newman E., Penrose R.* // J. Math. Phys. 1962. Vol. 3, No 3. P.566 – 587.

[14] *Newman E., Tamburino L., Unti T.* // J.Math. Phys. 1963. Vol. 4, No 7. P. 915-923.

[15] *Debney G., Kerr R., Schield A.* // Ibid. 1969. Vol. 10. No 10, P. 1842.

[16] *Einstein A.* // Sitzunsber. preuss. Akad. Wiss. Phys.-math. Kl. 1928. Bd. S. 217.

[17] *Cartan E., Einstein A.* Elie Cartan - Albert Einstein, Letters on Absolute Parallelisme, 1929-1932. Princeton University Press. 1979.

[18] *Shipov G.I.* // Izvestiya vuzov. Physics. 1976. No 6. C. 132.

[19] *Shipov G.I.* // Izvestiya vuzov. Physics. 1977. No 6. C. 142.

[20] *Ol'khovsky I.* Theoretical Mechanics for Physicists, NAUKA Publ., GRFML, Moscow, 1970 (in Russian).

[21] *Cartan E.* Compt. Rend.1922. Vol. 174, p. 437.

[22] *Shipov G.* Program of Universal relativity and vacuum theory, Moscow, 1988, dep. at VINITI, N 6947-,88 (in Russian).

[23] *Shipov G.I., Sidorov A.N.* Theoretical and experimental recerch jet-like motion without rejection of mass. In "Physics of iteraction of livihg obgects whit around world", Moscow, 2004, pp. 87-120.

[24] *Tolchin V.* Inertioid, forces of inertia as a source of motion, Perm, 1977.