The Most Natural Procedure for Quantum Image Recognition

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Abstract
It is shown how images can be processed, memorized, reconstructed or/and recognized using fundamental and relatively non-artificial quantum dynamics, i.e. $\mid \Psi\rangle = \mid \Psi\rangle\langle\Psi \mid \Psi\rangle$ in Dirac's notation. The right-most $\mid \Psi\rangle$ represents the output, the left-most $\mid \Psi\rangle$ denotes the input, and the central $\mid \Psi\rangle\langle\Psi \mid$ represents the associative memory. No quantum logic gates are needed, but merely a holographic procedure. Our computational model, successfully tested on concrete data, is a quantum version of Hopfield-based neural-net-like associative processing which is mathematically translated into wave-dynamics in a straight-forward way. Here we discuss its most natural quantum implementation(s), i.e. using ordinary interference of image-modulated quantum waves. The non-trivial (even, e.g., anticipatory) capabilities of this model arise from proper consideration of data-structure, or pre-processing by classical systems, therefore it is the best available candidate for the quantum kernel of (conscious) image recognition in the visual cortex.
Keywords: quantum, image recognition, holography, phase, associative memory, Hopfield

1 Introduction

Quantum mechanics, recently re-interpreted informationally, is about system—experimenter interactions. These are actually, at least in some respect, a *pattern-recognition procedure*. The model to be presented exploits this deep epistemological interpretation, therefore it is essential for foundations of quantum theory as well as for the science of vision (in computers and, even more, in brain) (cf.: [1, 2]).

We will present a model, a derivation of [3], and prove that it is functioning at the quantum level also, not merely in ANN-like (artificial neural net) simulations.

The mathematics of this algorithmic model to be presented has already been successfully computer-simulated, e.g. [4]. The next step made here is to proceed from simulations to consideration of experimental possibilities of quantum nanotechnology. This work exploits relations of parallel-distributed processes (PDP) in neural and quantum systems (overview in [5]), and their relations to holography, in order to propose a *quantum image recognition* model.

Quantum neural nets [6, 7] are a branch of quantum computers needing no logic gates. It will be shown that the quantum implementation of associative neural nets can be *naturally*-physical, i.e. no artificial classical-physical devices are necessary, except for encoding into and decoding from quantum systems. Continuous processing of (quantum) nature has much higher capabilities than artificial discrete processing of present-day technology. One can with relatively no cost mobilize almost infinitely many "units / pixels / neurons", "being always available free" in the quantum field. One does not need to specify states of "units" individually, but merely to shape net's attractor-configurations (as wholes) or parallel-distributed dynamic structures (called order parameters in physics) serving as information-codes.

To demonstrate quantum implementation of certain ANN, it is best to remember a fundamental technique which has already been much experimentally tested and used – holography. Holography is a practical 3D image-storage and -reconstruction procedure [8]. Its imaging is powerful and of high resolution, although the technique is relatively simple — it uses merely reflection from the laser-illuminated object and interference of the "object"-beam with a "reference"-beam. Early associative memories were inspired by holography [9]. They were a version of digitalized amplitude-information holography. Merging of ANN and holographic approach [10, 11] continues to be very useful.

Since holography can be in principle realized applying any sort of Huygens-principle-obeying waves [8, 12], preferably coherent waves [8], it is realizable also using quantum waves or wave-packets. The latter are of the same type as the Gabor wavelets used, e.g., in computer vision [13]. Dennis Gabor got the Nobel prize for physics for his invention of holography. The fast-developing quantum optics promises to realize it soon in quantum field, i.e. using quantum probability-distribution waves [14].

This step was enabled by the following observation on the *parallel-distributed encoding of information into waves*, i.e. into $(A_1 e^{i\varphi_1}, A_2 e^{i\varphi_2}, ..., A_N e^{i\varphi_N})$, in general. It has two special cases:

- (I) encoding in amplitudes A only, i.e. in $(A_1, A_2, ..., A_N)$, and
- (II) encoding in oscillatory phases φ only, i.e. in $(e^{i\varphi_1}, e^{i\varphi_2}, ..., e^{i\varphi_N})$, $i = \sqrt{-1}$.

These two cases enable *effectively the same information processing* as far as the following variable exchange can be made in the mathematics of the model/algorithm: $A \leftrightarrow e^{i\varphi}$. It will be shown for our wave-model (II) with $A = 1$: *what works for real-valued coding-numbers (A) works for sinusoid encoding also.* I.e., "wave-based" model (II) is equivalent to "intensity-based" model (I).

In sec. 2 we "translate" Hopfield's model into quantum formalism [7, 5]. In sec. 3 we "transform" Hopfield's model into wave-model (II), showing their *equivalence* for image recognition. Sec. 4 is a quantum-physical presentation. In sec. 5 we present possible quantum implementations of the simulated model, and in sec. 6 its benefits.

2 Hopfield model realized with quantum waves

The simplest Hopfield ANN (1982) incorporates Hebbian memory-storage "into" correlation matrix $\mathbf{J}$, i.e. $\mathbf{J} = \sum_{k=1}^{P} \vec{v}^k \otimes \vec{v}^k$ ($\otimes$ denotes tensor/outer product), and a memory-influenced transformation of patterns $\vec{v}$: $\vec{v}^{output} = \mathbf{J}\vec{v}^{input}$. Each of P patterns, simultaneously stored in the same net/$\mathbf{J}$, is denoted by a superscript index k: $k = 1, ..., P$. Patterns (or, equivalently, images) $\vec{v}^k$, which become Hopfield-net's eigenstates (attractors), can be complex-valued and can be quantum-encoded (as will be shown) into the net-state $\vec{q}$. For the quantum implementation, we will henceforth use the quantum notation, i.e. Ψ corresponds to $\vec{q}$, and ψ^k corresponds to $\vec{v}^k$ ($= A^k$ if non-oscillatory). (So-called wave-function

Ψ describes the whole state of the quantum system/net; ψ^k describes the k^{th} of its eigen-states.) Thus, images are assumed to be encoded into quantum eigen-wave-functions ψ^k (physical realization will be discussed later).

Turning from global description of the quantum PDP (using associative memory $\mathbf{J}$ and net-states Ψ) into local one (using interaction-weights J_{hj} and unit-states Ψ_j; $j, h = 1, ..., N$, for N units; N is huge), we have:

$$J_{hj} = \sum_{k=1}^{P} \psi_h^k (\psi_j^k)^* \qquad (1)$$

$$\Psi_h^{output} = \sum_{j=1}^{N} J_{hj} \Psi_j^{input} \qquad (2).$$

The asterisk denotes complex conjugation. Phase-conjugated ψ^* appears here as the natural result of interference process: $\mid \psi_h^k + \psi_j^k \mid^2 = \mid \psi_h^k \mid^2 + \psi_h^k (\psi_j^k)^* + (\psi_h^k)^* \psi_j^k + \mid \psi_j^k \mid^2$ (cf. [15]). Inserting eq. (1) into eq. (2), we obtain:

$$\Psi_h^{output} = \sum_{j=1}^{N} \left(\sum_{k=1}^{P} \psi_h^k (\psi_j^k)^* \right) \Psi_j^{input} = \sum_{k=1}^{P} \left(\sum_{j=1}^{N} (\psi_j^k)^* \Psi_j^{input} \right) \psi_h^k = \sum_{k=1}^{P} c^k \psi_h^k \qquad (3).$$

Usually, exclusively those coefficient c^k, say c^{k_0}, is close to 1 which belongs to the memorized image ψ^{k_0} which is the most similar to Ψ^{input}. Consequently, all other c^k, $k \neq k_0$, are close to zero. In such a case of the process of eq. (3), the quantum associative net *recognizes* the input-image (analysis in sec. 4).

$\mathbf{J}$ is called Green-function propagator, $\mathbf{G}$, in quantum theory [5, 3]. $\mathbf{G}$'s description of input–output transformations corresponds to statistical description of state-relations (relations in encoded data) by the quantum probability-*density matrix* ρ. We will not use ρ here; we merely wanted to emphasize ρ's role as a "quantum archive" (of all potential input–output transformations, in contrary to $\mathbf{G}$'s actual ones).

Now we write eq. (2), with kernel of eq. (1) inserted [16, 17], into space-time form:

$$\Psi(\vec{r_2}, t_2) = \int\int \left(\sum_{k=1}^{P} \psi^k(\vec{r_2}, t_2)(\psi^k(\vec{r_1}, t_1))^* \right) \Psi(\vec{r_1}, t_1) d\vec{r_1} dt_1 \qquad (4).$$

We replaced unit-indices h, j by $(\vec{r_2}, t_2)$, $(\vec{r_1}, t_1)$, and discrete summation by an integration over the whole effectively-continuous quantum system/net (if it consists of very many "units"). This is the Feynman (path-integral) version of the Schrödinger equation, the fundamental equation for quantum dynamics — in Dirac's notation:
$\mid \Psi \rangle = \mid \Psi \rangle \langle \Psi \mid \Psi \rangle = (\sum_k \mid \psi^k \rangle \langle \psi^k \mid) \mid \Psi \rangle$.
The central $\mid \psi \rangle \langle \psi \mid$ symbolizes the operator of projection onto eigenstate $\mid \psi \rangle$, realized by $\mathbf{G}$ ($\equiv \mathbf{J}$), or by ρ, in our case [18]. Then, $\mid \Psi \rangle = \mid \psi \rangle$.

The Hopfield computational model, incorporating coupled eqs. (1) & (2) with real-valued variables, has been used in very many different applications of numerous authors. Based on [9], it is a historical prototype-model, out from which so many other models, more applicable for particular problems, have been developed. Using it, the first author has computationally recognized patterns of approximated 3D structures of proteins using a huge memorized data-base (from the Brookhaven protein data bank) [4]. [1] However, for quantum implementation of associative PDP, we should first turn to this model, eqs. (1) & (2), again using it as a "Rosetta stone". This might then enable subsequent fantastic improvements which are promised by possibly-entangled [23] quantum field dynamics manipulated by so-called classical—quantum interactions. So, the quantum breakthrough for ANN-implementations can best be made with the prototypical associative content-addressable memory of eqs. (1) & (2), because its dynamics is *relatively similar to natural processes*, mainly in spin systems (i.e., spin glass) [24] and quantum fields [3].

3 Equivalence of the real-valued and sinusoid models

Quantum wave-functions ψ^k can have many forms. For our purposes, (quantum-optical) plain-waves $\psi^k(\vec{r},t) = A^k(\vec{r},t)e^{i\varphi^k(\vec{r},t)}$ are the most appropriate. An advanced alternative, left for our future work, are quantum wave-packets nearly-identical to Gabor wavelets [13].

Holography shows, at least for non-quantum waves, how one can parallel-distributively encode images k into a web of waves $(A_1^k e^{i\varphi_1^k}, A_2^k e^{i\varphi_2^k}, ..., A_N^k e^{i\varphi_N^k})$. The amplitude A_j^k and the oscillatory phase φ_j^k have the same lower index j ($j = 1,...,N$; N huge), since they belong to the same "waving" point, which is our "unit" (encoding a point of the image).

We *can* use plain-waves (sinusoids) with the *same* constant amplitude, say $A = 1$; so, $A_j^k = 1$ for all k, j. This is functioning for few decades, known as phase-information holography. We thus replace all ψ-variables ($\equiv v$) in eqs. (1), (2), (3) and (4), with $e^{i\varphi}$, instead of Hopfield's A. We are allowed to do this - it's a usual mathematical exchange of variables. The essential observation is that with this legal variable-exchange, $A \leftrightarrow e^{i\varphi}$, giving $\psi_j^k = e^{i\varphi_j^k}$ instead of $\psi_j^k = A_j^k$, all the simulation-tested mathematics remains valid for sinusoid-encoded images also. Thus, we can claim that the Hopfield algorithm, i.e. eqs. (1) & (2), *works with complex-valued sinusoid-inputs at least as much as with real-valued inputs*! Performance of the wave-phase model (II) with eigenimages $\psi^k = e^{i\varphi^k}$, $A^k = 1$, is *equal* to performance of the amplitude model (I) with $\psi^k = A^k$, A^k real number. However, when using *both* – different amplitudes *and* different phases – performance might be (much) improved, as practically proved by HNeT [10]. Much better results arise using HNeT's preprocessing method [10] where inputs v_j^k are *sigmoidally mapped* into phases φ_j^k to obtain a convenient symmetric (uniform) data-distribution: $\varphi_j^k = 2\pi\left(1 + exp(\frac{\bar{v}^k - v_j^k}{\sigma(v^k)})\right)^{-1}$.

To prove quantum-wave image-recognition with the system of eqs. (1) & (2), it suffices to execute the exchange, $\psi_j^k \leftrightarrow e^{i\varphi_j^k}$, first in the Hebbian eq. (1), using $(e^{i\varphi})^* = e^{-i\varphi}$:

$$G_{hj} = \sum_{k=1}^{P} e^{i\varphi_h^k} e^{-i\varphi_j^k} = \sum_{k=1}^{P} e^{i(\varphi_h^k - \varphi_j^k)} \qquad (5),$$

[1]Because inter-image orthogonality increases with increasing N (which can be huge), success usually also increases with N. Therefore, resolution can be freely increased. The final 3D graphics of "proteins" proved that *as well as our very recent image-processing simulations [note before press]*.

and secondly in eq. (2). So, instead of eq. (2), when inserting now expression (5) into J_{hj} of eq. (2), we obtain the following equivalent of eq. (3):

$$e^{i\varphi_h^{output}} = \sum_{j=1}^{N}\left(\sum_{k=1}^{P} e^{i\varphi_h^k} e^{-i\varphi_j^k}\right) e^{i\varphi_j^{input}} = \sum_{k=1}^{P}\left(\sum_{j=1}^{N} e^{i\varphi_j^{input}} e^{-i\varphi_j^k}\right) e^{i\varphi_h^k} \doteq e^{i\varphi_h^{k_0}} \quad (6).$$

In the next section we enter the formalism of quantum physics to study the enormous process of phase (mis)matching of eq. (6), i.e. constructive or destructive interferences, and precise conditions for clear image-recall. The right-most expression of eq. (6) gives an image-bearing output, i.e. $e^{i\varphi_h^{k_0}}$, *only if* the same conditions are valid as described below eq. (3): If the input wave has a similar phase to one of the memorized waves, say $k = k_0$, then those wave will be reconstructed — the image it is carrying, k_0, will be *recognized.* If those conditions are not satisfied by the data correlation-structure, interferences ("cross-talk") lead to a mixed or averaged output (details in [4]).

So, instead of a long series of products (correlations) of *real*-valued information-coding numbers, A, as in the Hopfield model, we have here a long series of complex-valued *exponentials* (waves) with differences of information-coding phases, φ, in each exponent. These *phase-differences (peak delays) encode discrepancies in data.* Our wave output $e^{i\varphi_h^{k_0}}$ is the same as Hopfield's $A_h^{k_0}$. In sum, *input–output transformations are the same* in the wave case as it were in our simulated real-number (intensity) case [4]. All this proves the image-recognition capabilities of the wave model (II) with phase-encoding of image-points h. The memory is "represented" by the hologram, i.e. wave-interference pattern, of eq. (5).

Performance is at least preserved if also amplitudes A are input-dependent. Because nothing essential changes from the $A = 1$ case (used in our "proof"), we can infer: Since the phase- and amplitude-encodings give equivalent results, the use of *both* of them (combined) merely *emphasizes* the results of the use of phase-encoding exclusively, or of the amplitude-encoding exclusively. Specifically, the output in the combined case is $A_h^{k_0} exp(i\varphi_h^{k_0})$ plus small residual. In generalization of eq. (6), the weighting factors A_h^k more or less *magnify* the individual response-terms in $exp(i\varphi_h^{output})$, especially near $k = k_0$, thus increasing the pixel-contrasts (cf., [10] for computational evidence). We can also separate (filter) the phase and the amplitude outputs one from another, and manipulate them separately (although leaving them to support each other) to achieve more informative results.

4 Our model in quantum formalism

4.1 Informational perturbation of the Schrödinger propagation

Let's write quantum equations explicitly. Note that in all of them, the phase φ (now quantum, as the presence of the Planck constant $\hbar$ indicates) is hidden in the exponent of the wave-function Ψ which describes the (oscillatory) state of the quantum system:

$$\Psi(\vec{r},t) = A(\vec{r},t)exp[i(\vec{k}\vec{r} - \omega t)] = A(\vec{r},t)exp[\frac{i}{\hbar}[(\vec{p}\vec{r} - Et)] = A(\vec{r},t)exp[\frac{i}{\hbar}\varphi(\vec{r},t)] \quad (7).$$

A is the amplitude, $\vec{k}$ is the wave-vector, ω is the angular frequency, E $(=\hbar\omega)$ is the energy, $\vec{p}$ $(=\hbar\vec{k})$ is the momentum (of the photon).

The Hopfield neural-net model can be mathematically translated into quantum formalism while preserving all information-processing capabilities [7]. In [5] we have shown that this is because the *collective* dynamics in neural and quantum complex systems are similar, in spite of different nature of neurons q_j and their connections J_{hj} on one hand, and quantum "points" $\Psi(\vec{r})$ and their "interactions" described by $G(\vec{r}_1, \vec{r}_2)$ on the other.

What follows is presentation for physics-oriented readers. The *Quantum Associative Network* model [3] combines the usual dynamical equation for the quantum state [22]

$$\Psi(\vec{r}_2, t_2) = \int\int G(\vec{r}_1, t_1, \vec{r}_2, t_2)\ \Psi(\vec{r}_1, t_1)\ d\vec{r}_1\ dt_1 \qquad or \qquad \Psi(t_2) = \mathbf{G}\ \Psi(t_1) \tag{8}$$

and the expression for the parallel-distributed interactive transformation of the quantum system [22]

$$G(\vec{r}_1, t_1, \vec{r}_2, t_2) = \sum_{k=1}^{P} \psi^k(\vec{r}_1, t_1)^*\ \psi^k(\vec{r}_2, t_2) \qquad or \qquad G(\vec{r}_1, \vec{r}_2) = \sum_{k=1}^{P} \psi^k(\vec{r}_1)^*\ \psi^k(\vec{r}_2) \qquad (9).$$

Note that expression (9), i.e. for $G(\psi^k(\vec{r}_1, t_1), \psi^k(\vec{r}_2, t_2))$, presents the kernel of eq. (8). The system (8) and (9) is just the usual Schrödinger propagation reinterpreted for associative processing and measurement-like readout (cf., [17]). However, the readout (final stage) is done taking into account our knowledge on the state we prepared. Moreover, our storage procedure (preparation stage) introduces a significant perturbation to the quantum system which cannot be treated as closed here.

4.2 Interference-based memory storage

The following holography-like two-stage procedure reflects correspondence between quantum and network processing/measurement: So-called system-*preparation* corresponds to *learning / data-storage*, and system-*verification* (measurement) to output-*retrieval*. The latter would in quantum proper (statistical) case need *many* measurements (on ensembles) for a reliable output.

We want to *encode some information* in eigenfunctions ψ^k. Then ψ^k would become quantum codes of images - not necessarily geometrically isomorphic to some external images, although encoding them. It may *not* be possible to encode information in wave-functions Ψ, or ψ^k, in the same sense (by the same directly decodable way, respectively) that information is encoded in neural-net state-vectors $\vec{q}$, or $\vec{v}^k$, for two reasons. First, any natural (non-model) network may initially be in some "natural" state, i.e. eigenstate, of its own. Second, $\vec{q}$, or $\vec{v}^k$, are in principle directly observable, but Ψ, or ψ^k, are not - not even in principle, as long as they remain quantum. Therefore, we act as follows.

By a classical interaction or perturbation on an appropriate quantum system, we force the quantum network into a state Ψ which "implicitly reflects" our external influences (inputs), i.e. it is input-modulated [21]. As soon as such a state Ψ stabilizes, becoming an eigenstate, ψ^k ($k = 1$), we can continue to "insert" (simultaneously or sequentially) other information-encoding states ($k = 2, ..., p$). All these eigenstates (images) ψ^k interfere as prescribed by eq. (9) and get thus *stored* in **G**. *Quantum holography* [14] is an example which demonstrates how this could be realized, with plain waves $\Psi = A\ e^{\frac{i}{\hbar}\varphi}$ or wavelets (wave packets) [13], without extensive artificial effords. Moreover, fast-developing specially-designed encoding / decoding (measurement) devices [25, 20, 21], including active control and measurement of quantum phases [25], enable enormous additional possibilities.

If eigenfunctions ψ^k implicitly encode images, then the matrix **G** ($\equiv$ **J**) describes the *quantum associative memory*. The propagator expression G in eq. (9), which acts as a projector during the image-recall (measurement) process, is related to the usually-used Green function $\tilde{G}$ [16] by $G = -i\tilde{G}$.

If we, in eq. (9) which looks Hebbian, expose the phases φ explicitly, using $\Psi = A\,exp(i\varphi)$, we get an expression which is the **quantum phase**-*Hebb learning rule*:

$$G[A^k(\vec{r}_1,t_1), A^k(\vec{r}_2,t_2); \varphi^k(\vec{r}_1,t_1), \varphi^k(\vec{r}_2,t_2)] = \sum_{k=1}^{P} A^k(\vec{r}_1,t_1)^* A^k(\vec{r}_2,t_2) e^{-i(\varphi(\vec{r}_2,t_2)-\varphi(\vec{r}_1,t_1))} \tag{9b}$$

This describes the memory encoding which is two-fold: it is both in amplitude-correlations $\sum_{k=1}^{P} A_k(\vec{r}_1,t_1)A_k(\vec{r}_2,t_2)$ (Hebb rule) and in phase-differences $\delta\varphi_k = \varphi_k(\vec{r}_2,t_2) - \varphi_k(\vec{r}_1,t_1)$. One of the two encodings is sufficient (as is in the amplitude-information and phase-information, i.e. type I and II, versions of holography), but the combined encoding brings optimal performance, as the usual holography demonstrates [8].

The difference between the rule (9b) and a non-quantum phase-Hebb rule is that in eq. (9b) phases φ are quantum phases — i.e., Planck's constant h is hidden in the exponent (but the usual notation $\hbar = \frac{h}{2\pi} = 1$ is used now).

4.3 Image recognition by collapse-based selective Ψ-reconstruction

The *retrieval from quantum associative memory* ($\Psi_{output} = \mathbf{G}\Psi'$) is most-directly realized by the input-triggered, non-unitary *wave-function "collapse"*:

$$\Psi(\vec{r}_2, t_2 = t_1 + \delta t) = \int G(\vec{r}_1,\vec{r}_2)\ \Psi'(\vec{r}_1,t_1)\ d\vec{r}_1 = \int \left(\sum_{k=1}^{P} \psi^k(\vec{r}_1)^*\psi^k(\vec{r}_2)\right)\ \Psi'(\vec{r}_1,t_1)\ d\vec{r}_1 =$$

$$= \left(\int \psi^1(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)d\vec{r}_1\right)\psi^1(\vec{r}_2) + \left(\int \psi^2(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)d\vec{r}_1\right)\psi^2(\vec{r}_2) + ...$$

$$+ \left(\int \psi^P(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)d\vec{r}_1\right)\psi^P(\vec{r}_2) =$$

$$= C\ \psi^1(\vec{r}_2)\ +\ B \qquad where\ C\ \doteq\ 1\ ('signal'),\ B\ \doteq\ 0\ ('noise') \tag{10}$$

or in another description

$$\Psi(\vec{r},t) = \sum_{k=1}^{P} c'^k(t)\psi^k(\vec{r}) = \sum_{k=1}^{P} \left(\int \psi^k(\vec{r})^*\Psi'(\vec{r},t)d\vec{r}\right)\psi^k(\vec{r}) =$$

$$= \left(\int \psi^1(\vec{r})^* \Psi'(\vec{r},t) d\vec{r}\right) \psi^1(\vec{r}) + \left(\int \psi^2(\vec{r})^* \Psi'(\vec{r},t) d\vec{r}\right) \psi^2(\vec{r}) + \ldots + \left(\int \psi^P(\vec{r})^* \Psi'(\vec{r},t) d\vec{r}\right) \psi^P(\vec{r}) =$$

$$= C\,\psi^1(\vec{r}) + B \qquad \textit{where } C \doteq 1 \ ('signal'),\ B \doteq 0 \ ('noise') \qquad (11).$$

In eq. (10) and eq. (11) we had to choose such an "input" Ψ' that is *more similar* to ψ^1, for example, than to any other $\psi^k, k \neq 1$. At the same time, the "input" Ψ' should be *almost orthogonal* to all the other $\psi^k, k \neq 1$. In this case, Ψ converges to the quantum "pattern-qua-attractor" ψ^1, as it is shown in the last row of eq. (10) and in the last row of eq. (11). Thus, the memory-pattern / -image ψ^1 is recalled (measured). If the condition, well known from the Hopfield model simulations [4] and from holography, that "input" must be similar to one stored image (at least more than to other stored images) is not satisfied, then there is no single-image recall.

Almost-orthogonality [26] is essential for "real-life" data-processing. Exact orthogonality of the stiff pre-computer era of quantum mechanics should not be taken *too* seriously any more. Recognizing that quantum eigenstates are orthogonal merely in the limit $N \to \infty$, our *fuzzy* interpretation of quantum algebra, i.e. allowing nearly-orthogonal eigenvectors, brings not merely information-processing benefits, as simulations show, but is probably more physical also (see arguments in [3]).

4.4 How is the image extraction possible

In our quantum-net model we are not interested in Ψ, like we are not interested in $\vec{q}$ in our neural-net model. Our final result will directly be a single "post-measurement" information-encoding eigenstate ψ^k (say $k = 1$), or $\vec{v}^k$, respectively. We cannot observe Ψ (except in net-simulations by stopping the program) and we *need not* observe Ψ (or $\vec{q}$), but we wait until the "measurement" (i.e., image-recall which is equivalent to the wave-function "collapse") is triggered by our final new input Ψ' or $\vec{q}'$. Then, the standard quantum *observables* O (corresponding to: $\hat{\mathbf{O}}\psi = \lambda_O \psi$), e.g. spin states, can reveal the reconstructed image-encoding quantum eigenstate ψ^k $(k = 1)$. Namely, since the output is similar to the input (that we know!), the eigenvalue (λ_O) information can be sufficient for knowing the output.

Moreover, *if* the final $\Psi = \psi^k$ $(k = 1)$ is / becomes *classical-physical* (like the input-images may well be), then obtaining complete knowledge about the output-image, encoded in ψ^k $(k = 1)$, is at least in some cases (e.g., optical) *relatively straight-forward* (e.g., like *seeing* the image reconstructed from a hologram). E.g., quantum coherent states behave on average (most probably) as classical ones and are robust to noise.

Beside quantum holography, an alternative fast-developing technique for reconstruction of eigenstates ψ^k is quantum tomography [27]. (Some auxiliary options related to potential implementations see in [28].)

So, our information-processing result *can be extracted* from ψ^1 using new quantum-optical (and computer-aided) techniques for measurement of observables or for quantum-holographic-(like) wavefront reconstruction. Their keywords are, e.g., *quantum-phase engineering, wave-packet sculpting, (coherent) quantum control / manipulation* [21].

5 Quantum implementation options

Implementation of our model is most appropriate using quantum-wave holography [14, 25, 23]. Several applied techniques, which are at least partially quantum-holography-based, are also already functioning, e.g. some sorts of tomography (fMRI and PET scanning) [27]. Holography is a fundamental and universal procedure in the sense that, in principle, any sort of coherent waves can be applied for interference-based simultaneous recording of many objects into (and for selective reconstruction from) various hologram-media. Apart of classical optical and acoustical holography, microwave-, X-ray-, atom- and electron-holography were realized [8]. There is just a step further to quantum-wave holography functioning as described here – for our purposes, not merely for others.

This attempt is supported by the following reports: According to [29], universal quantum computation is realized using only projective measurement, like ours of eq. (3) or eq. (6), quantum memory, like ours of eq. (1) or eq. (5), and preparation of the initial state (the laser-wave in our case). Information-storage and -retrieval through quantum phase [20], including imprinting phase-patterns into quantum-states, and measurements of quantum relative phase [30] have been experimentally demonstrated. Quantum encodings in spin systems and coupled harmonic oscillators, with a possibility for computation in terms of number- and phase-operators, are possible [24]. This enables Hopfield-like image-storage and -recognition in such nets, including spin-wave holographic ones. In general, quantum computing, including its mainstream using quantum logic gates, is realizable using linear (quantum) optics exclusively [19].

However, if nanotechnology could not (which is highly unlikely) realize quantum-holographic image-recognition as proposed here, something like this is hypothesized to be happening in the (visual) brain [1, 15]. Not only brain, the whole quantum Nature itself almost certainly incorporates such processes, at least in interaction with our quantum-measurement devices [2] (cf. [31]). In worst case, it does merely not let us to collaborate with – until tomorrow?

6 Benefits of our model

Recently, we found similar quantum image-recognition proposals [32]. Trugenberger's one is related to the fact that "simple-Hebbian" $\vec{v}^k \otimes \vec{v}^k$ with bipolar states (1 and -1 only) is equivalent to quantum-implementable NOT XOR gate. This makes a link between ANN-like and logic-gate-based branches of quantum image recognition.

The benefits of the first branch, i.e. quantum neural-net approach, are the following: We avoid the devastating role of *quantum decoherence*, characteristic for the main-stream quantum computers, by *usefully harnessing "collapse of the wave-function" for image recognition.* No special mechanisms are needed for quantum error-correction, since it is done spontaneously by the net's self-organizing process (as in ANN). Initialization problems are not as serious [33] as in logic-gate quantum computers, at least not when an object is holographed. In this case, reflection from its surface determines the phases, and fluctuations do not destroy the modulation (cf., experimental quantum-phase storage and retrieval [20]). Finally, as it is characteristic for quantum computers, quantum associative net is exponentially superior to its classical counterparts in memory capacity, processing speed and in miniaturization [6]. This brings improvements in computational capacity and efficiency. Quantum ANN promise to outperform logic-gate quantum computing in associative tasks like discussed here, and in flexibility (fuzzy processing) [3], where also classical ANN outperform sequential computing. Finally, our net presented [3] is relatively inexpensive, because it is relatively natural, and is of huge theoretical importance at least.

7 Conclusions

Mathematically and computationally [4], we have *proved the associative memory-storage and image-recognition performance* of Peruš's model named *Quantum Associative Network* [3]. It remains to prove it in quantum-physical experimental practice, i.e. with real quantum image-encoding waves, not merely with digital simulated ones (complex sinusoids). It is crucial that our model [3] is fundamental, optimized and relatively *natural*, i.e. almost no artificial devices are really necessary (even laser is avoidable), in contrary to all other models.

Discussion on incursive anticipatory dynamics of our Quantum Associative Net, as in ch. 7 of [4], is valid for our present implementation and purposes also.

Acknowledgements

This work was supported by the British Computer Society (Cybernetic Machines Group) and by the European-Union's Marie-Curie fellowship for M.P (contract no. HPMF-CT-2002-01808). Many thanks to dr. Peter Marcer (especially for his support), to dr. Carlo A. Trugenberger and dr. Alexander Yu. Vlasov for discussions.

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