# About the Relativist Framework of Quantum Theory

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**Abstract**
In previous papers, we have developed several considerations about the extension of the relativist quantum theory to the domain of superluminal velocities. In the present work, we propose a way to extend the relativist quantum theory to the framework of the general theory of Relativity.
Obviously any use of a three dimensional space (plus the time dimension) requires a geometry, but in usual quantum theories the space geometry is always Euclidean.
The generalization of a discrete space derivative equation from 1 to 3 dimensions with the usual space operators (gradient, curl, Laplacian) contains the implicit hypothesis of a Euclidean space. Thus we can explicitly propose the hypothesis of the Riemann's geometry which leads to a generalization of quantum theory to the framework of the general theory of Relativity.

**Keywords** : Quantum theory, general theory of Relativity, vector derivatives, Riemann's geometry

## 1 Introduction

It is usually considered that gravitation can be neglected in quantum transitions (except in a few experiments trying to detect gravitation waves). It is numerically true in quantum experiments on Earth, within the current paradigm. From this viewpoint most physicists have started to think that the special theory of Relativity was sufficious to the quantum theory while the general theory of Relativity was reserved to cosmology. But is not true: in the present paper we show that the relativist quantum theory can be extended to the framework of the general theory of Relativity.

The quantum theory does not require that gravity has to be quantized [1] (section 1.4, p. 11-12) but the conditions of a field theory of gravity have been studied:
if the graviton exists it should be massless:

$$m_G=0 \tag{1}$$

so that the force proportional to $1/r^2$ results from the quantum interaction,
and it should have a spin 2:

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$$s_G=2 \tag{2}$$

because gravitation is a symmetric rank-2 tensor field. Field equations of a free massless spin-2 field have been written by very early FIERZ and PAULI in 1939 [2].

R. P. FEYNMAN described the *problems in quantizing the gravitational field* [3]: « When a field is quantized, each mode of the field possesses a zero-point energy. Since the field is made up of an infinite number of modes, the total vacuum energy of a quantum field is infinite ... Such a vacuum energy density appear in gravity theory as a cosmological constant. Since the cosmological constant is quite small, there is a big problem [4] ».

When I presented my CASYS'99 communications [5,6], I recalled the definition of the *light barrier* [7,8] from the usual relativist energy equation:

$$E=\frac{m_0C^2}{\sqrt{1-\frac{v^2}{C^2}}} \tag{3}$$

and Daniel DUBOIS remarked that this equation is wrong because at the $C$ limit the particle would contain more energy than the whole universe.

Accelerator experiments have been settled to see what particles are composed of, but the acceleration increases the kinetic energy and thus creates an additional mass. When a mass particle is accelerated, the upper limit of its velocity is not the light velocity in vacuum where energy becomes infinite, but a slower limit $v_L$ where the particle would contain the energy $E_U$ of the whole universe:

$$v_L=C\sqrt{1-\frac{m_0^2C^4}{E_U^2}} \tag{4}$$

While any accelerated mass particle increases its energy, as if it were pumping energy from somewhere else in the universe, it also in creases its gravitational mass. Therefore gravitation considerations should be re-introduced in the current paradigm.

As far as the gravitational mass of a particle depends on its velocity and on the observer, the general theory of Relativity should be considered as the required relativist framework of quantum theory.

Some authors [9,10,11,12] have described inertial mass as the result of the reaction of vacuum to accelerated motion, in relation to the zero-point field (ZPF) and other authors [13] have shown that the *zitterbewegung* motion in quantum theory can be related to local geodesics and also to the gravitation mass.

Finally, gravitation appears to be more complex than a mere graviton theory: gravitation has at least the following three aspects:

1. the space-time curvature,
2. the *zitterbewegung* motion,
3. and the zero-point field (ZPF) reaction.

Recently a gravity experiment has been done [14] and discussed [15,16]: a beam of ultracold neutrons moving at a velocity of 8 m/s is sent on a parabolic trajectory through a baffle and onto a horizontal plate which reflects the neutrons back upwards until gravity saps their ascent. This experiment shows that reflected neutrons can be detected at quantized heights. While these authors interprets the NESVIZHEVSKY experiment as a quantum effect of gravity which produces gravitational quantum states, the mere physical result is that neutron positions in space-time are quantized.

It just means that any particle in any physical field propagates with space-time shifts, as it has been proposed in the work of Daniel M. Dubois [17].

He has shown that from scalar forward and backward discrete derivatives, taking into account forward-backward space-time shifts related to a phase velocity, the KLEIN [18], GORDON [19], and FOCK [20] relativist equation can be deduced, as well as the DIRAC [21] equation, the wave equation for photons and a dual SCHRÖDINGER equation. In this paper [17] the equations were restricted to a one-dimensional space and time.

Daniel M. Dubois [22] has proposed a usual generalization to the 3-dimensional Euclidean space with the classical introduction of the gradient $\nabla$ and the Laplacian $\nabla \cdot \nabla$, then Daniel M. Dubois and G. NIBART [23] have emphasized that space shift $\boldsymbol{\lambda}$ is a vector and have shown that in quantum theory the "*plane wave propagation is the cause of mass, in relation to the time shift*" $\tau$.

As a consequence of this new concept of mass, any wave propagation causing mass would locally stress the space which could no longer be Euclidean. Thus a generalization to a 3-dimensional space should be done in consideration of the space geometry.

The present work proposes a way to extend the relativist quantum theory to the framework of the general theory of Relativity.

# 2 The Problem of the Generalization to a 3 Dimensional Space

## 2.1 A Classical Generalization to a 3 Dimensional Space

In a previous work "*Computational Derivation of Quantum and Relativist Systems with Forward-Backward Space-Time Shifts*" [17], Daniel M. Dubois has introduced a forward and a backward, time discrete derivative of any function F, such as :

$$\frac{\Delta_f F}{\Delta t} = \frac{F(t+\Delta t) - F(t)}{\Delta t} \tag{5}$$

$$\frac{\Delta_b F}{\Delta t} = \frac{F(t) - F(t-\Delta t)}{\Delta t} \tag{6}$$

where $\Delta$ is the discrete operator, and he has deduced a generalized complex continuous time derivative of a complex function $\Phi$, such as :

$$\frac{\Delta\Phi(t)}{\Delta t} = \frac{d\Phi(t)}{dt} \pm \frac{i\tau}{2}\frac{d^2\Phi(t)}{dt^2} \tag{7}$$

where $\tau = \Delta t$ is the time shift, or with an additional space coordinate :

$$\frac{\Delta\Phi(x,t)}{\Delta t} = \frac{\partial\Phi(x,t)}{\partial t} \pm \frac{i\tau}{2}\frac{\partial^2\Phi(x,t)}{\partial t^2} \tag{8}$$

"From a similar reasoning as for the time derivative" (section 1.3.2 of ref. 17), Daniel M. Dubois has obtained the following space derivative :

$$\frac{\Delta\Phi(x,t)}{\Delta x}=\frac{\partial\Phi(x,t)}{\partial x}\pm\frac{i\lambda}{2}\frac{\partial^2\Phi(x,t)}{\partial x^2} \tag{9}$$

where $\lambda=\Delta x$ is the space shift. Then the author indicates a way of generalization to 3 space dimensions with the following remark : "in a two or three dimensional space, the partial space derivative is replaced by a gradient $\nabla$, and the second order derivative by the Laplacian $\nabla\cdot\nabla=\Delta$".

In a next paper [22], Daniel M. Dubois applied this principle to generalize the derivative equations to a three dimensional space, with time. The generalized complex continuous time derivative is then :

$$\frac{\Delta\Phi(\mathbf{r},t)}{\Delta t}=\frac{\partial\Phi(\mathbf{r},t)}{\partial t}\pm\frac{i\tau}{2}\frac{\partial^2\Phi(\mathbf{r},t)}{\partial t^2} \tag{10}$$

where $\Delta$ is still the discrete operator, and **r** is a radius vector which replaces the scalar position x. Thus "From a similar reasoning as for the time derivative" (section 1.3.2 of ref. 22), Daniel M. Dubois computed forward and backward space derivatives in the two opposite directions

$$\mathbf{r}\pm\frac{\boldsymbol{\lambda}}{2} \tag{11}$$

and then he obtained the following 3D space derivative :

$$\frac{\Delta\Phi(\mathbf{r},t)}{\Delta\mathbf{r}}=\nabla\Phi(\mathbf{r},t)\pm\frac{i\boldsymbol{\lambda}}{2}\nabla^2\Phi(\mathbf{r},t) \tag{12}$$

## 2.2 The Problems of Vector Derivatives

If we write the equation (12) with the school vector notation below :

$$\frac{\Delta\Phi(\vec{\mathbf{r}},t)}{\Delta\vec{\mathbf{r}}}=\nabla\Phi(\vec{\mathbf{r}},t)\pm\frac{i\vec{\boldsymbol{\lambda}}}{2}\cdot\nabla^2\Phi(\vec{\mathbf{r}},t) \tag{13}$$

it clearly brings out that a variation of the complex function $\Delta\Phi$ has been divided by a vector $\Delta\mathbf{r}$.

Can we divide a scalar or complex function by a vector ? Can we divide a vector function by a vector ? Such a division by a vector should be defined.

Let us consider the usual continuous scalar derivative of the scalar function $f$ of a scalar variable $x$ :

$$\frac{df}{dx}=\lim_{|\Delta x|\to 0}\frac{\Delta f}{\Delta x} \tag{14}$$

a symbolic generalization from a scalar function $f$ to a vector function $\vec{\mathbf{V}}$, and from a scalar variable $x$ to a point **M** in an affine space would lead to the derivative of a vector function $\vec{\mathbf{V}}(\mathbf{M})$ of a point **M** in an affine space.

So we would need the following vector derivative definition :

$$\frac{d\vec{\mathbf{V}}}{d\vec{\mathbf{M}}}=\lim_{|\Delta\mathbf{M}|\to 0}\frac{\Delta\vec{\mathbf{V}}}{\Delta\vec{\mathbf{M}}} \tag{15}$$

The vector space $\{\Delta\vec{\mathbf{M}}\}$ is deduced from the 3D affine space where any particle position is defined as a point **M**, and the vector space $\{\vec{\mathbf{V}}\}$ may be any studied function (i.e. electric field, magnetic field, …etc). This vector derivative is a symbolic notation which is usually interpreted as a gradient in the 3D affine space.

### 2.3 The Geometric Restriction due to the Usual Vector Derivative Operators

To avoid these problems the usual way is to compute a differential instead of a derivative, with the introduction of "vector" derivative operators such as the gradient ∇, and the Laplacian ∇·∇=**Δ**.

In this way, the differential of the vector function **V** is usually expressed as

$$d\vec{\mathbf{V}}=\frac{\partial\vec{\mathbf{V}}}{\partial x}dx+\frac{\partial\vec{\mathbf{V}}}{\partial y}dy+\frac{\partial\vec{\mathbf{V}}}{\partial z}dz \tag{16}$$

using a gradient operator ∇ such as :

$$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) \tag{17}$$

which is defined with the coordinates $x$, $y$, $z$ of a particular referential frame.

So the vector function **V(M)** of a point **M** in an affine space is usually considered as a function $\mathbf{V}(x, y, z)$ of three independent scalar coordinates. In the affine space the differential of the point **M** is

$$d\vec{\mathbf{M}}=d_x\vec{\mathbf{M}}+d_y\vec{\mathbf{M}}+d_z\vec{\mathbf{M}} \tag{18}$$

and the components of d**M** along the $x$, $y$, $z$ axis are :

$$d_x\vec{\mathbf{M}}=\vec{\mathbf{e}}_x dx \tag{19}$$

$$d_y\vec{\mathbf{M}}=\vec{\mathbf{e}}_y dy \tag{20}$$

$$d_z\vec{\mathbf{M}}=\vec{\mathbf{e}}_z dz \tag{21}$$

the equation 16 can then be expressed as

$$d\vec{\mathbf{V}}=\left(\vec{\nabla}\cdot d\vec{\mathbf{M}}\right)\vec{\mathbf{V}} \tag{22}$$

So we clearly see that the usual way of vector differentiation which uses a gradient defined with three scalar components (related to a particular referential frame) introduces a scalar product of the gradient vector $\vec{\nabla}$ and the point differential $d\vec{\mathbf{M}}$ (related to an affine space). It means that the usual differentiation operator which is applied to the vector function **V**, is a scalar product

$$\vec{\nabla}\cdot d\vec{\mathbf{M}}=\frac{\partial}{\partial x}dx+\frac{\partial}{\partial y}dy+\frac{\partial}{\partial z}dz \tag{23}$$

which contains no cross terms of $x$, $y$, $z$, such as

$$\frac{\partial}{\partial x}dy,\frac{\partial}{\partial y}dx,\ldots\frac{\partial}{\partial z}dy \tag{24}$$

the gradient identity (equation 23) implies that the following scalar products of unity vectors are null :

$$\vec{\mathbf{e}}_x \cdot \vec{\mathbf{e}}_y = 0 \tag{25}$$

$$\vec{\mathbf{e}}_y \cdot \vec{\mathbf{e}}_z = 0 \tag{26}$$

$$\vec{\mathbf{e}}_z \cdot \vec{\mathbf{e}}_x = 0 \tag{27}$$

and so the referential frame is orthogonal.

Moreover the components of the vector $\vec{\mathbf{M}}$ associated to a point **M** in an affine space can be expressed as :

$$\vec{\mathbf{M}}_x = x\vec{\mathbf{e}}_x \tag{28}$$

$$\vec{\mathbf{M}}_y = y\vec{\mathbf{e}}_y \tag{29}$$

$$\vec{\mathbf{M}}_z = z\vec{\mathbf{e}}_z \tag{30}$$

so the components differentials should generally be :

$$d_x\vec{\mathbf{M}} = \vec{\mathbf{e}}_x dx + x d\vec{\mathbf{e}}_x \tag{31}$$

$$d_y\vec{\mathbf{M}} = \vec{\mathbf{e}}_y dy + y d\vec{\mathbf{e}}_y \tag{32}$$

$$d_z\vec{\mathbf{M}} = \vec{\mathbf{e}}_z dz + z d\vec{\mathbf{e}}_z \tag{33}$$

and in the equations 19, 20, 21 we have

$$d\vec{\mathbf{e}}_x = 0 \tag{34}$$

$$d\vec{\mathbf{e}}_y = 0 \tag{35}$$

$$d\vec{\mathbf{e}}_z = 0 \tag{36}$$

so all basis vectors are invariant in a parallel translation. Thus the affine space is Euclidean (not curvilinear).

Consequently any space vector differentiation with such a gradient operator introduces the implicit hypothesis of a Euclidean orthogonal affine space. And in the special theory of Relativity, the 4-dimensional gradient operator introduces the hypothesis of a pseudo-Euclidean space-time.

We can then conclude that any space-time vector differentiation with the usual gradient operator introduces a restriction of the quantum theory to the framework of the special theory of Relativity.

### 2.4 The Generalization to a 3 Dimensional Curvilinear Space

In a curvilinear space or better in a Riemann's manifold, as it is well known, the geometric properties are much different from the Euclidean's.

The scalar products of basis vectors in equations 25, 26, 27 are not null and they define a metric tensor :

$$\vec{\mathbf{e}}_i \cdot \vec{\mathbf{e}}_j = g_{ij} \tag{37}$$

where the indices $i$ and $j$ vary from 1 to 3 only for comparison with space equations 25, 26, 27.

<u>Remark</u> : Further in this development all indices, such as $i, j$ or $\mu, \nu$ will vary from 0 to 3 to describe the relativist space-time.

The differential of basis vectors in equations 34, 35, 36 are not null, because basis vectors are not bound to be invariant in a translation. The differential of any basis vector is expressed as :

$$d\vec{\mathbf{e}}_i = \omega_i^j \vec{\mathbf{e}}_j \tag{38}$$

with the usual notation $\omega_i^j$ of the contra-variant components of $d\mathbf{e}_i$, which are differential forms. They can be related to the metric tensor with :

$$\omega_{ij} + \omega_{ji} = dg_{ij} \tag{39}$$

and expressed with differentials of the coordinates, as :

$$\omega_i^j = \Gamma_{ki}^j dx^k \tag{40}$$

or

$$\omega_{ji} = \Gamma_{kji} dx^k \tag{41}$$

where $\Gamma_{kij}$ are affine connexion coefficients of the curvilinear space or the Riemann's manifold.

Now we recall that any vector $\vec{\mathbf{V}}$ defined with its components as:

$$\vec{\mathbf{V}} = V^i \vec{\mathbf{e}}_i \tag{42}$$

has a differential $d\vec{\mathbf{V}}$ which can be expressed as :

$$d\vec{\mathbf{V}} = dV^i \vec{\mathbf{e}}_i + V^i d\vec{\mathbf{e}}_i \tag{43}$$

or

$$d\vec{\mathbf{V}} = dV^i \vec{\mathbf{e}}_i + V^k \omega_k^i \vec{\mathbf{e}}_i \tag{44}$$

so the true components of dV are not the $dV^i$, but the $DV^i$ which are such as :

$$DV^i = dV^i + \omega_k^i V^k \tag{45}$$

and introducing the partial derivatives $\partial_k$ we get from the equation 40 defining the differential form $\omega_i^j$ :

$$DV^i = \partial_k V^i dx^k + \Gamma_{kh}^i V^h dx^k \tag{46}$$

Now *dividing* by the space-time differential $dx^k$ and using the notation:

$$D_k = \frac{D}{Dx^k} \tag{47}$$

we get the following space-time derivative :

$$D_k V^i = \partial_k V^i + \Gamma_{kh}^i V^h \tag{48}$$

which is the covariant derivative of the vector $\vec{\mathbf{V}}$.

It is very currently used in curvilinear affine spaces or Riemann's manifolds, mainly for the general theory of Relativity. And some authors [24] use the *nabla* notation for the space-time derivative operator

$$\nabla_k V^i = \partial_k V^i + \Gamma_{kh}^i V^h \tag{49}$$

because the covariant derivative of the vector $V$ is considered as the *gradient* of the vector $V$ in a curvilinear affine space or a Riemann's manifold.

Therefore the gradient in equation 17 is a Euclidean gradient, which has been slightly generalized to a pseudo-Euclidean gradient in the Relativity :

$$\vec{\nabla}=\left(-\frac{1}{c}\frac{\partial}{\partial t},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) \tag{50}$$

although the gradient in equation 49 is a non-Euclidean gradient which is to be used in the framework of the general theory of Relativity.

Consequently we can simply transpose usual relativist quantum equations to the framework of the general theory of Relativity, by replacing the pseudo-Euclidean partial derivative operators with the non-Euclidean covariant derivative operators of eq. 49.

## 3 Generalized Relativist Quantum Equations

Relativist quantum equations of first quantization can be generalized to the framework of the general theory of Relativity, as explained in the section 2.4 above.

### 3.1 The Schrödinger Quantum Equation

The Schrödinger quantum equation is not a relativist equation. Non-relativist particle moving in a physical field can be described with the Schrödinger quantum equation.

To represent a space-time curvature, the Schrödinger quantum equation can be adapted with a simple change of the space geometry, but doing so it is not transposed to the framework of the general theory of Relativity.

Daniel M. Dubois has shown [22] that from forward and backward discrete derivatives the following Schrödinger quantum equation can be deduced:

$$i\hbar\frac{\partial\phi(\mathbf{r},t)}{\partial t}=\pm\left[\frac{-\hbar^2}{2m}\nabla^2\phi(\mathbf{r},t)-V_0\phi(\mathbf{r},t)\right]-V(\mathbf{r},t)\phi(\mathbf{r},t) \tag{51}$$

where $V(\mathbf{r},t)$ is the potential in a field, and $V_0$ is a reference potential depending on the space shift $\boldsymbol{\lambda}$ and the mass:

$$V_0=\frac{\hbar^2}{2m\lambda^2} \tag{52}$$

$V_0$ might be related to the quantum relativist vacuum, i.e. to the ZPF as defined by HAISCH, RUEDA and PUTHOFF [9,10,11,12].

$V(\mathbf{r},t)$ is the potential of any field, which depends only on space-time coordinates. Therefore in this Schrödinger quantum equation the gravitationnal field may be also considered.

To adapt the Schrödinger quantum equation to a curvilinear space-time, the time or space derivatives will be replaced by the corresponding « covariant » derivatives, and the usual wave function $\phi$ will be replaced by the complex vector $\psi^\mu$. So we have:

$$i\hbar D_0\psi^\mu=\pm\left[\frac{-\hbar^2}{2m}D_\alpha D^\alpha\psi^\mu-V_0\psi^\mu\right]-V\psi^\mu \tag{53}$$

where $D_\alpha$ with $\alpha$=1,2,3 are the space « covariant » derivatives and $D_0$ is the time « covariant » derivative, as it is formally defined in equation 48. However I recall that this equation is not a relativist invariant equation.

### 3.2 The Klein, Gordon and Fock Relativist Quantum Equation

The equation of KLEIN [25], GORDON [26], and FOCK [27] is usually written as :

$$\Box\psi=\chi^2\psi \tag{54}$$

where $\psi$ is the wave function of a boson and $\chi$ is related to the rest mass with

$$\chi=\frac{m_0 c}{\hbar} \tag{55}$$

and where the Dalembertian operator $\Box$ is defined as :

$$\Box=\nabla^2-\frac{\partial^2}{c^2\partial t^2} \tag{56}$$

in a pseudo-Euclidean space-time.

The equation 54 can be generalized to the framework of the general theory of Relativity by introducing the generalized Dalembertian operator $\hat{\Box}$ defined as :

$$\hat{\Box}=D_\mu D^\mu \tag{57}$$

where $D_\mu$ is the covariant derivative, i.e.

$$\hat{\Box}=g^{\mu\nu}D_\mu D_\nu \tag{58}$$

where $g^{\mu\nu}$ is the metric tensor.
So we obtain the new KLEIN, GORDON and FOCK equation which is still linear in $\psi$

$$\left(g^{\mu\nu}D_\mu D_\nu\right)\psi=\chi^2\psi \tag{59}$$

It shows that the boson wave $\psi$ in a gravitation field has to be represented with a second order spinor.

### 3.3 The Dirac Equation

The spinor Dirac equation is usually written as :

$$\left(\gamma^\mu\frac{\partial}{\partial x^\mu}-\chi\right)\psi=0 \tag{60}$$

$\gamma^\mu$ are the Dirac matrix and $\chi$ is related to the rest mass with :

$$\chi=\frac{m_0 c}{\hbar} \tag{61}$$

and where the fermion wave function $\psi$ is a bispinor, it can be represented as a complex vector $\psi$ with the vector components $\psi^\mu$.

Replacing the derivative operator, we obtain the Dirac equation in the framework of the general theory of Relativity :

$$\left(\gamma^\mu D_\mu-\chi\right)\psi=0 \tag{62}$$

where $D_\mu$ is the covariant derivative, i.e. :

$$\gamma^{\mu}\left(\frac{\partial}{\partial x^{\mu}}\psi^{\alpha}+\Gamma^{\alpha}_{\mu\nu}\psi^{\nu}\right)-\chi\psi^{\alpha}=0 \tag{63}$$

and the new Dirac equation is still linear in $\psi$.

## 4 Conclusion

We have shown that in the usual generalization to 3D space with classical gradient and Laplacian operators, an implicit hypothesis is involved which has imposed the Euclidean geometry to the space, i.e. a pseudo-Euclidean geometry to the space-time. An other choice may be the Riemann's geometry which leads to the quantum theory in the framework of the general theory of Relativity.

If the primary energy propagation in the big-bang had been done with a unique and absolutely spherical wave, energy would have been diluted homogeneously in every directions, thus nothing would exist today but a quite homogeneous fluid of unquantized energy : an omnipresent light.

In fact the initial spherical wave must have split at a high scale into many individual waves which are no more spherical. Thus the production of matter in the universe requires a condensation of energy in the locality, at a high scale with galaxies and at the smallest scale with energy quanta.

Furthermore to have a quantum stability, the quantum energy must not be diluted through its propagation, so the conservative properties of any quantum within a time window requires a plane wave propagation in a privileged direction.

While the standard quantum theory uses a plane waves superposition principle to build linear equations of particles, plane waves are directly responsible for the stability of energy within particles and might correspond to an internal gravitation field within each particle. Thus such a high level condensation of energy within each quantum should produce both a curvature of space-time at a Fermi scale and a discretization of space-time.

In the framework of the general theory of Relativity, general relativist quantum equations include the metric tensor and its related affine connexion coefficients. Such quantum equations are still linear in $\psi$. Therefore the superposition principle of quantum theory still hold in the framework of the general theory of Relativity.

From this viewpoint, Vadim KROTOV [28] has proposed an extended principle of general covariance which leads to a discussion of the foundations of quantum mechanics : "the linearity of the wave equation is a necessary condition for the validity of the principle of superposition".

In strong gravitation fields the space curvature cannot be neglected, thus *general relativist quantum equations* should be used instead of the usual quantum equations. So the production of matter within any star nucleus and even at big-bang should be described with general relativist quantum equations. Consequently the standard model of the nucleo-synthesis should be revised.

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