

Formal Treatment of Systems with a Hidden Organizing Structure, with Possible Applications to Physics

Dieter Gernert

Technische Universität München

Arcisstr. 21, D-80333 München (Germany)

Email: t4141ax@mail.lrz-muenchen.de

Abstract

It is the purpose of this paper to study the concept of an “organizing structure”. In the beginning, dynamic systems with a specific substructure are analysed, such that long-term modifications of the system structure (and possibly other effects) can be attributed to that substructure. This analysis follows the guidelines of general system theory, and the proposed formalism is open to applications in various fields. In a second step, a possible application to quantum theory is discussed. It will be shown that this proposal is compatible with the present state of quantum theory, and that hidden variables can be regarded as a special case within the concept of a hidden organizing structure. Possible applications to a recently proposed extension (weak quantum theory) and to the study of anticipatory systems will be sketched.

Keywords: Quantum theory, foundations, organizing structure, hidden structure, structure formation.

1 The two Aspects of Organizing Structures

We are accustomed to saying that some institution is well – or not so well – organized, and we assume that there exists an underlying organizational structure, which is perceived from the outside only by its efficiency, the conspicuous behaviour of the system. This paper starts by analysing the new concept of an “*organizing structure*” under general systemtheoretical aspects. To this purpose, dynamical systems are studied which permit the identification and conceptual separation of a specific component: long-term modifications of the system structure (and possibly certain deviations from the standard system behaviour) will be attributed just to that organizing structure, and can be understood and explained only on this basis.

Apart from this abstract mathematical model (which may be of interest by itself as a new technique within general system theory), a second aspect will be discussed. The general concept is open for applications in various contexts. Particularly, it will be found that this proposal is compatible with the state of the art in quantum theory, and hopefully it will give a modest contribution to the ongoing debate on the foundations of quantum

theory (Section 4). Also possible applications to the study of anticipatory systems will be sketched (Section 5).

2 Systems with an Organizing Structure

An abstract mathematical concept of a *system with an organizing structure* has to fulfill the following requirements:

1. *Separation*: The system **S** consists of two distinct, but interacting subsystems, $S = \{O, T\}$, with an *organizing structure* **O** and a “*target system*” **T**; the subsystem **T** assumes different states in the course of time, mainly governed by its own internal rules (system dynamics, “business as usual”), whereas **O** is able to modify the internal rules and the structure of **T**, and to bias stochastic processes within **T**.
2. *Persistency*: **O** exists for a sufficiently long time without internal changes (else it would be impossible to identify **O**).
3. *Uniqueness of causation*: **O** causes structural changes or influences stochastic processes within **T**, in such a way that all this cannot be explained by internal properties of **T**, but must necessarily be attributed to the effect of **O**.
4. *Selectivity*: **O** only acts upon selected subsystems of **T**, that is those which have a specific structure (with abstraction from irrelevant features).
5. *Partial visibility*: There is a clear and natural distinction within the mathematical structure of **S** that permits us to speak about a “visible part” of **S**, identical with **T**, and an “invisible part” of **S**, which can be identified with **O**.

3 Mathematical Treatment

3.1 Representation by Matrices and Special Matrix Products

A mathematical formalism representing such systems quite naturally is based upon two matrices:

- a transformation matrix **X** describes the transition of **T** from one state to the next one (normal system operation), and
- an organizing matrix **Y** characterizes the effect of **O**.

These two matrices (and all other matrices in this text) can be time-dependent, but this will not be formally denoted. **X** and **Y** always occur jointly, and a transformation step can be written as

$$x' = YXx \tag{1}$$

It is presupposed here that the system states can be denoted by finite state vectors.¹ In the simplest case, there is no impact from **O** upon **T** (e.g., if $Y = I$ (unit matrix)), and eq. 1 becomes $x' = Xx$, which means the normal unbiased system operation. Depending on the individual type of the organizing matrix Y , the effect of Y upon X – leading to the biased operation – may belong to one of the following characteristic patterns:

1. *Selector matrix*: In a selector matrix S all elements of a proper non-empty subset of the main diagonal elements are equal to 1, with all other matrix elements vanishing. The multiplication SX will select exactly those rows and columns from X as signalled by the nonzero entries in S .² Selector matrices are idempotent ($S^2 = S$).
2. *Aggregation*: E.g. Y may contain one row of ones, with the rest equal to 0. Then the product matrix YX will contain one row with column sums, and zeros in all other rows.
3. *Selection or aggregation with weight factors*: The ones in the matrices as above can be replaced by different positive numbers, thus introducing unequal weights.
4. *Other special matrix products*: The concept of pattern handling can be easily recognised in the Kronecker product; for this and further transformations see e.g. Graham (1981) and Lütkepohl (1996).³

All matrices considered here are $n \times n$ -matrices over \mathbf{C} (the field of complex numbers). Any such matrix permits a unique decomposition into a real and an imaginary part. Hence the matrices in eq. 1 can be rewritten: $Y = A + iB$ and $X = C + iD$, where A, B, C, D are real. The matrix product in eq. 1 becomes

$$(A + iB)(C + iD) = AC - BD + i(AD + BC) \quad (2)$$

This equation can be interpreted in such a way that the real part, $R = AC - BD$, represents the manifest system behaviour, and at the same time mirrors the effect of the organizing structure **O**. The following special cases may illustrate this:

1. $A + iB = I$: Under this assumption, it follows that $B = O$ (zero matrix), $A = I$, and $R = C$; eq. 1 is reduced to $x' = Cx$. Here we have the ordinary system operation, without any influence from **O**.
2. $BD = O$: In this case $R = AC$, and the impact of A upon C must be analysed. E.g., A can be one of several “preselector matrices” which express the fact that different experimental settings can be applied to the same physical object (Gernert 2000a).

¹ In the continuous case the matrix products can be replaced by integral transformations in a well-known manner.

² For an application of selector matrices and the Kronecker product see Gernert (2000a).

³ This listing is not exhaustive. Craigen (1993) defines an operation called “matrix weaving”, which e.g. includes the Kronecker product and the direct sum as special cases. An alternative to matrix operations is given by graph grammars (see e.g. Gernert 1997), where the structural transformations are clearly evident.

3. $BD \neq O$: With the simplifying assumption $A = I$ we get $R = C - BD$, which means that the system operation is “disturbed” due to the term BD ; this leads to a more detailed study in the next section.

3.2 Mathematics of the Key-Lock Principle

Considering the important case $R = C - BD$ with $BD \neq O$, it must be found out under which circumstances $BD \neq O$ can hold, or at least BD will not take on too low numerical values, such that the effect of disturbing the system operation cannot be neglected. This is not trivial, because two nonnegative matrices with their nonzero entries in a “normal” numerical range can have a rather diminished product ($0 < a, b < 1$ implies $ab < \min(a,b)$). The question of a non-negligible magnitude of BD stimulates the metaphor of a “*key-lock mechanism*”: if and only if B and D fit together – in a sense still to be made precise – the effect of BD will be significant.

The required mathematical tool is supplied by a general method which permits the definition of a *dissimilarity function* $d(z_i, z_k)$ on a set $\{z_1, z_2, \dots, z_m\}$ of objects, which are also allowed to have a complicated internal structure. This dissimilarity function has the properties of a metric as defined by the usual axioms, and takes the internal structure of the objects into account.⁴ A low numerical value of $d(z_i, z_k)$ means that there is a narrow relationship between z_i and z_k , whilst a high value of $d(z_i, z_k)$ indicates that the two objects have not so much in common.⁵ The mathematics for defining a dissimilarity function can be based either upon graph grammars (including directed graphs, edge- and vertex-labelled graphs, and hierarchical graphs) or upon block matrices (including hierarchically structured block matrices).⁶ Both styles are essentially equivalent, because any real matrix can be rewritten as a labelled graph, and vice versa.

The practical importance of the dissimilarity function lies in the fact that features of O , represented by B , may activate specific properties of T , as modelled by D , in a selective manner, controlled just by a low value of dissimilarity. An “ideal fitting” between key and lock ($d(z_i, z_k) = 0$) may exist in pure mathematics; the real hardware will be subject to wear and temporal fluctuations, but the key-lock principle works on the basis of a sufficient correspondence between the two involved parts of O and T .

4. Possible Applications – Quantum Theory and Beyond

⁴ For details, examples, and references see Gernert (2000a).

⁵ Unfortunately, the term „similar matrices“ is already used differently. A distinction between *internal* and *external similarity* has been proposed (Gernert 2000b). Internal similarity induces a decomposition into equivalence classes. External similarity is defined mirror-symmetrically to the dissimilarity function as above: it involves an intransitive relation such that there may be $z_j \approx z_i$ and $z_j \approx z_k$ but $\neg z_i \approx z_k$.

⁶ Cf. the remarks on „other special matrix products“ in Section 3.1.

4.1 General System Theory and Quantum Theory

The relationship between general system theory and quantum theory is closer than one might expect. The validity of quantum theory is not restricted to microphysics. Rather, there are macroscopic quantum effects, that is macroscopic phenomena, like superconductivity or superfluidity, which can be explained and understood only on the basis of quantum phenomena. Already in early years, some of the founders, such as N. Bohr, E. Schrödinger, W. Pauli, and P. Jordan, discussed the idea that quantum theory might have some relevance for biology and psychology, too.⁷

General system theory is a “formal science” (Formalwissenschaft, just as mathematics, formal logic, etc.), which describes and analyses structures and processes without taking account of their material carriers or implementation and independently from the requirements of an individual scientific discipline. Thus, the general theory is distinguished from the numerous *special system theories*, which apply the methods and instruments in one or another individual field of application (from biology and psychology to electrical engineering), and therefore belong to the “real-world sciences” (Realwissenschaften).

Quantum theory can be defined axiomatically. The axioms “seem to be of a very general nature since they do not contain information about physics itself” (W. v. Lucadou 1991). The properties of physical observables are not specified in the axioms, but in the mathematical formulation of the corresponding operators. Hence, in the onset quantum theory can be regarded as an instance of general system theory, which, by a consecutive act of interpretation, will be converted into a physical theory. To sum up, quantum theory can be understood as a specific interpretation of general system theory.

4.2 Reasons for the Use of Complex Numbers

Why do we need complex numbers to describe real things? Since essential use of \mathbf{C} is made here (eq. 1 and its interpretation) this question seems to be worthwhile. Complex numbers are well established in down-to-earth disciplines like electrical engineering (analysis of a.c. circuits). In quantum theory there is no chance to circumvent them; a short argument for this is given by Mohrhoff (2002). Even in textbooks with a high priority to easiness of learning no author thought it advisable to start with a simplified version based on real numbers. An attempt was made by Stueckelberg and coworkers⁸ to formulate a quantum theory over \mathbf{R} (the field of real numbers). To this purpose, the authors had to introduce a special operator J with the strange property $J^2 = -1$; so the use of \mathbf{C} is only camouflaged.

⁷ For details and references see W. v. Lucadou (1991).

⁸ Stueckelberg (1960); for further remarks, and references to three other papers of the series see Primas (1983, p. 211-213, 431).

In addition, there is even an abundant literature demonstrating that not only \mathbf{C} , but also two other algebraic fields, quaternions and octonions, are, in a similar manner as before, inevitable or at least useful in special branches of physics – in quantum theory or elsewhere (see e.g. Dixon 1994, Okubo 1995). Complications can occur due to the noncommutativity of the product of two quaternions, but exactly this property may become advantageous in dealing with systems in which noncommutativity plays a central role (see Section 4.4). After all, it would be quite disturbing if a formalization of the concept of organizing structures were possible without complex numbers.

4.3 Hidden Organizing Structures and the Present State of Quantum Theory

The search of hidden parameters can be traced back to a long history. It belongs to the indispensable core of theoretical physics that the realm of visible phenomena is founded upon and explained by invisible structures (Kanitscheider 1979, p. 288). Of course, any formulation of a hidden structure requires an interpretation, that is “a set of normative regulative principles which can neither be deduced nor be refuted on the basis of the mathematical codification” (Primas 1994, p. 172f); it must be stated how the physical efficacy or inefficacy (and hence the existence or non-existence) of the alleged structure can be empirically tested.

In giving such an interpretation, it is not necessary, however, to assign a meaning to each isolated term of the formalism – rather, the alleged structure as a whole must be accessible to proof or refutation. In arguing against an undue use of the word “unobservable” Hiley (2002, p. 143) remarks: “... the wave function is ‘unobservable’, but I never hear anybody calling it meaningless. The wave function, according to Bohr,⁹ is simply a term in an algorithm from which the probable outcome of any given experiment can be calculated.”

Under these aspects it is proposed to check whether the concept of hidden organizing structures may be applicable, at least within special contexts, also in quantum theory (maybe only as a descriptive tool). This would not mean a significant change since all the fundamental principles, like discreteness, indeterminism, noncommutativity, and nonlocality, are retained.

In designing a type of experiments it must be sure in before that a remarkable outcome – if any – cannot be explained in a traditional way. It should be recalled that the selector matrices and other special matrices (as introduced in Section 3.2) include the possibility of multi-level hierarchically structured matrices. This suggests, e.g., to study the dynamic properties of large molecules (macromolecules) with a multi-level hierarchical structure. An example for a class of chemical compounds with a rather simple hierarchical structure is given by large benzenoid hydrocarbons, which consist of many benzene rings in a specific arrangement.

⁹ Original source: Bohr (1948, p. 314); reprinted in: Collected Works, vol. 7, p. 332.

Now consider either one such large molecule, or two of them, M_1 and M_2 (preferably in two separate test tubes). It is conjectured that under special conditions an *EPR-like coupling* may occur, either within suitable substructures of one molecule, or between one substructure of M_1 and another one of M_2 . Conditions for such a coupling may be seen in analogy to the key-lock principle (Section 3.2). For the sake of comfort we may use the term “affinity”: a high affinity will correspond to a low dissimilarity, and vice versa (the mathematical transformation is trivial).¹⁰ Under these assumptions a high affinity will lead to an increased probability for coupling. From the viewpoint of empirical testing, coupling manifests itself in a structural modification of the second partner in accordance with the modification occurring in the first one.

Furthermore it is assumed that a dynamic, *stepwise generation of affinity* is possible: an existing correspondence between several sub-units on a lower level may trigger an improved affinity between subsystems on the next higher level, and this may be continued recursively. A formal analysis of hierarchically structured quantum systems and their dynamics is presented by Healey (1989, p. 63-83); technical details, like formulas describing the influence from level k to level m , would be beyond the scope of this paper.

In parallel with the dynamic generation of affinity, the hypothesis presented here implies the formation (or modification) of a hidden organizing structure, such that this new structure will persist for some time and another empirical test – search of alterations of chemical structure (unexpected compounds) without evident cause – becomes possible. Such an “*EPR-chemistry*” will have the nice advantage that a low probability for the effect will suffice, because specific compounds, even if produced in a low concentration, can be identified by filtering and other methods.

When the present proposal is related to the state of the art of quantum theory, another point must be briefly addressed. The term “hidden organizing structure” may remind of “hidden variables”. The latter concept can look back on a strange and changeful history of reception. For some decades there was a peculiar and fierce debate, with “arguments” and practices outside the usual style of scientific discussion (to say it politely).¹¹ In the present context the following selection of keywords must suffice:

1. A careful analysis discloses that John von Neumann never supplied a definitive proof for the impossibility of hidden variables, nor claimed to have such a proof (Clauser 2002, p. 66f).
2. The strongest argument against such a statement of impossibility is due to Primas (1990), who reduces that statement to a fundamental misunderstanding of the measurement process.
3. Much of the criticism against the concept of hidden variables really concerns some specific claims in its surroundings which have nothing to do with the present proposal.

¹⁰ Reasons for avoiding the ambiguous term „similarity“ were given in Section 3.2.

¹¹ For a more recent presentation of the theory see Bohm and Hiley (1993), for the history of reception see Clauser (2002).

With some reservations the transition from a hidden structure to hidden variables can be regarded as a transition from a finer model to a coarser model: the coarser model is generated from the finer one by neglecting and simplification. When all structure-forming relations are discarded, then a set of unrelated variables will remain. Any criticism against the coarser model need not necessarily apply to the finer one.

4.4 Quantum Theory and Beyond

Quantum effects occur outside the traditional scope of quantum theory, too. To start with a harmless example, Miranker (1997) considers the inevitable rounding errors in numerical calculations (computation with a finite number of digits), and observes that there is a striking correspondence with characteristic features of quantum processes, e.g. a “law of addition of probabilities via the complex valued probability amplitudes”, and notions like “state, wave function, dynamics, observation, and nonlocality” have their counterparts.

In search of a unified description of observation processes – comprehending both the physical aspects and the observer – an operator algebra was developed which enables a formal description of at least a significant majority of the cognitive processes. It is found that this operator algebra, which is a noncommutative semiring with some additional features, belongs to a type already known in literature and has an astonishing correspondence with the usual operator algebras in quantum theory (Gernert 2000a).

The relationship between quantum theory and biology is closer than expected. Several authors outline a unified view of both fields and study the utilization of quantum phenomena by living organisms (Conrad et al. 1988, Josephson et al. 1991). A detailed analysis of quantum effects and their role within fundamental physiological processes is given by Matsuno (2001).

There are still more striking examples demonstrating that phenomena in rather distant fields can be explained only on the basis of quantum theory. Atmanspacher et al. (2002) exemplify this by findings both from information dynamics and from psychotherapy (individual therapy and system-therapeutic settings). Their proposal, termed “weak quantum theory” is based upon a slight weakening of an axiom system and emphasizes the role of noncommutativity and entanglement.

Most of these examples can be seen in a direct connection with the presupposed hidden structures. As already pointed out earlier (Josephson et al. 1991), the notion of *meaning* will become indispensable. Meaning is the regulatory principle that determines whether or not entanglement will occur between two entities. Key and lock, taken literally, are the simplest mechanical arrangement in which two parts have a “meaning” for each other. The key-lock principle (Section 3.2) mirrors the property that one part of a structure is meaningful for another, and so the methods outlined above offer a chance for a formal description, and with some plausibility the assignment of meaning can be influenced by hidden structures.

5 Connections with Anticipatory Systems

The concept of an *anticipatory system* was introduced by Robert Rosen. In his definition this term denotes “a system containing a predictive model of itself and/or of its environment, which allows it to change state at an instant in accord with the model’s predictions pertaining to a later instant”. (Rosen 1985, p. 339) In order to point out the characteristic features of such systems, the two older techniques of rational prediction shall be contrasted. There are

- a primitive one, time-series extrapolation, and
- an advanced one, model-based prediction, in which conclusions are derived from a model of the system under consideration, where the model is formulated by a model-author outside the system.

In a modern view, these two styles are termed “weak anticipation”. By way of contrast, in an anticipatory system prediction is performed *by the system itself* (strong anticipation, system-based prediction). Examples are given by an anticipatory effect in an electric field produced by a moving charged particle, and by a correct quantitative result for the perihelion anomaly of the planet Mercury (Dubois 2000, 2003).

The concept of hidden organizing structures is perfectly compatible with the theory of anticipatory systems. Both concepts have in common that the system behaviour is understood and explained on the basis of internal features (together with the interaction between the system and its environment). A hidden organizing structure may be regarded as a special component of a system in which “the anticipation is generated by the system itself”. (Dubois 2000, p. 3) In this moment, the theory of anticipatory systems can work without the concept of hidden organizing structures. But it seems quite likely that in some future, more sophisticated situations that concept may turn out to be a useful tool for description and analysis (the question of *what* is done by the system may naturally lead to inquiring *how* it is performed).

6 Concluding Remarks and Outlook

“There exist arguments that complementary descriptions to those of quantum mechanics can and in all probability do occur.” (Josephson et al. 1991, p. 200) The present paper does not go so far; the proposal made here is not complementary, but supplementary (and compatible with the state of knowledge). Suggestions for empirical tests have been given (Section 4.3).

The proposal outlined here is compatible with the state of the art in anticipatory systems. In future specific cases a resort to a hidden \mathbf{O} may be advantageous (Section 5). The same holds in the simpler case of model-based prediction. In a later step, a possible

“feedback”, an influence of the ongoing system dynamics upon **O**, and hence a long-term variability of **O**, can be considered. (As a matter of research strategy, this point should be reserved to a second phase, since in the beginning a constancy of **O** at least over a certain time interval is required.)

Also in the present context the central role of noncommutativity and entanglement becomes evident. Operations underlying here are noncommutative, and this is reflected by the well-known property of matrix multiplication (or of products of other operators which may replace matrices in special contexts). The connection of entanglement, dissimilarity, and the key-lock principle was addressed in some detail. One of the next essential steps in research will be an integrated treatment of physical and cognitive processes. According to Stapp (1995, p. 822) the basic problem of quantum theory is “to reconcile the nonclassical character of the quantum world with the classical character of our perceptions of it”.

References

- Atmanspacher, Harald; Römer, Hartmann; Walach, Harald (2002). Weak quantum theory: complementarity in physics and beyond. *Found. Phys.* 32, 379-406.
- Bohm, David; Hiley, Basil J. (1993). *The undivided universe*. Routledge, London.
- Bohr, Niels (1948). On the notions of causality and complementarity. *Dialectica* 2, 312-319. – *Collected Works*, vol. 7, Elsevier, Amsterdam 1996, p. 330-337.
- Clauser, John F. (2002). Early history of Bell’s theorem. In: Reinhold A. Bertlmann and Anton Zeilinger (eds.), *Quantum [Un]speakables*. Springer, Berlin, p. 61-98.
- Conrad, Michael; Home, Dipankar; Josephson, Brian (1988). Beyond quantum theory: a realist psycho-biological interpretation of physical reality. In: A. van der Merwe et al. (eds.), *Microphysical reality and quantum formalism*. Kluwer, Dordrecht, p. 285-293.
- Craigen, R. (1993). The craft of weaving matrices. *Congressus Numerantium* 92, 9-28.
- Dixon, Geoffrey M. (1994). *Division algebras: octonions, quaternions, complex numbers and the algebraic design of physics*. Kluwer, Dordrecht.
- Dubois, Daniel M. (2000). Review of incursive, hyperincursive and anticipatory systems – foundation of anticipation in electromagnetism. In: Daniel M. Dubois (ed.), *Computing anticipatory systems: CASYS ’99 – third international conference (AIP Conference Proceedings, 517)*. American Institute of Physics, Melville, New York, p. 3-30.
- Dubois, Daniel M. (2003). Anticipative propagation of electromagnetic and gravitational fields. Lecture presented at the Sixth International Conference on Computing Anticipatory Systems – CASYS ’03, Liège, August 11-16, 2003.
- Gernert, Dieter (1997). Graph grammars as an analytical tool in physics and biology. *BioSystems* 43, 179-184.
- Gernert, Dieter (2000a). Towards a closed description of observation processes. *BioSystems* 54, 165-180.

- Gernert, Dieter (2000b). On navigating in information spaces. *International Journal of Computing Anticipatory Systems* 7, 253-262.
- Graham, Alexander (1981). *Kronecker products and matrices: with applications*. Ellis Horwood, Chichester.
- Healey, Richard (1989). *The philosophy of quantum mechanics*. Cambridge University Press, Cambridge.
- Hiley, Basil J. (2002). From the Heisenberg picture to Bohm: a new perspective on active information and its relation to Shannon information. In: A. Khrennikov (ed.), *Quantum theory: reconsideration of foundations*. Växjö University Press, Växjö, p. 141-162.
- Josephson, Brian D.; Pallikari-Viras, Fotini (1991). Biological utilization of quantum nonlocality. *Found. Phys.* 21, 197-207.
- Kanitscheider, Bernulf (1979). *Philosophie und moderne Physik*. Wissenschaftliche Buchgesellschaft, Darmstadt.
- Lucadou, Walter von (1991). Complementarity and non-locality in complex systems. *Lecture Notes in Computer Science* 565, 21-55.
- Lütkepohl, Helmut (1996). *Handbook of matrices*. Wiley, Chichester.
- Matsuno, Koichiro (2001). The internalist enterprise on constructing cell motility in a bottom-up manner. *BioSystems* 61, 115-124.
- Miranker, Willard L. (1997). Interference effects in computation. *SIAM Review* 39, 630-643.
- Mohrhoff, Ulrich (2002). Why the laws of physics are just so. *Found. Phys.* 32, 1313-1324.
- Okubo, Susumo (1995). *Introduction to octonion and other non-associative algebras in physics*. Cambridge University Press, Cambridge.
- Primas, Hans (1983). *Chemistry, quantum mechanics and reductionism*. Springer, Berlin.
- Primas, Hans (1990). The measurement process in the individual interpretation of quantum mechanics. In: Marcello Cini and Jean-Marc Lévy-Leblond (eds.), *Quantum theory without reduction*. Hilger, Bristol, p. 49-68.
- Rosen, Robert (1985). *Anticipatory systems*. Pergamon Press, Oxford.
- Stapp, Henry P. (1995). The integration of mind into physics. *Annals of the New York Academy of Sciences* 755, 822-833.
- Stueckelberg, E.C.G. (1960). Quantum theory in real Hilbert space. *Helv. Phys. Acta* 33, 727-752.