About Models of Automats at Organization Fault-Tolerant Computing Processes

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Abstract
A problem of synthesis of fault – tolerant computing systems which mathematical model is the deterministic finite automata is considered. The novelty in this paper is construction of the object called as a bundle of automats whose operation is based on constant alternation of function of automats which are his components. It is shown that model of a bundle of automats at a design phase of discrete systems with memory may be used to proceed in operational period to signals generation corresponding to the required algorithm of functioning in enumerating form.

Keywords: automata with generalized temporal characteristics, bundle of automats, enumerator, fault tolerance, universal automata.

1 Introduction

The questions of construction of objects capable to detection and elimination of faults during functioning, are traditionally topical for discrete mathematics and an artificial intelligence. The basic mathematical principles on which developed methods of functional restoration (enumeration) discrete systems with memory are based are introduced by statements of the universal automats theory (Sytnik, 1992).

The purpose of a paper is the creation of model of the universal automata on the basis of expansion of temporal relations between automata states and input signals. Thus, presence of the certain time reserve (time redundancy) is one of the conditions necessary for successful realization of restoration procedures of behaviour.

2 Primary Goals

Mathematical model of discrete systems with memory traditionally is the model of the deterministic finite automata (DFA). The object of research in this paper is Medvedev automata which represents a special case of the Moore automata, namely such Moore automata at which output signals are codes of the states. The given assumption, obviously, does not result in basic restriction of a reasoning generality.

Let's formulate the primary goals of the behaviour restoration theory of the DFA for a case of Medvedev automata $A = (S, X, \delta)$:

Let the DFA $A=(S,X,\delta)$ and a class of possible faults I is given. For each fault $i\in I$ set the DFA $A_i=(S,X,\delta_i)$ – the automata A at fault i. Thus, the family $\{A_i\}_{i\in I}$ (a class of possible faults) and the automata A is given.

Goal 1. The opportunity of behaviour restoration concerning the class of given faults.

To check up validity of the statement $\{A_i\}\subseteq UnA$, $i\in I$ for each automata A_i of family I.

Goal 2. The construction of behaviour restoration method concerning the class of given faults.

For each automata A_i of family I to construct set of functions $\{\varphi_{ij}\}_{i\in I}$, each of which satisfies to a condition:

$(\forall i\in I)\ \varphi_{ij}(A_i)=A$

The decisions of this problems suppose answers to the following questions:

1. Whether always (for any family of the DFA) it is possible to solve a problem of construction of universal automata and what existence conditions of such decision (the problem of synthesis)?
2. What variety has family of functions $\{\varphi_{ij}\}_{i\in I}$?
3. How is defined the family of the DFA for which the given automata is universal (the problem of analysis)?
4. How and under what conditions can be achieved optimum complexity characteristics of universal automats - on number of states, on the length of input strings, causing the needed reactions?

The answer to the first question has been received Sytnik (1992, 1993) and consists in the following: the construction problem of the universal automata concerning any family of the DFA is algorithmically insoluble, however for finite family of the DFA always it is possible to receive the universal automata. For this purpose it is enough to choose as universal the automata that equals to product of family automats.

Let's consider finite families of the DFA $\{A_i=(S_i,X,\delta_i)\}_{i\in I}$ for which is accepted the assumption that each automata has own finite set of states S_i which are not having crossings with sets of states of other automats, and each automata has own transition function δ_i, $i\in I$.

In this paper the decision for a synthesis problem of the universal automata concerning family $\{A_i=(S_i,X,\delta_i)\}_{i\in I}$ with the number of states equals $\sum_{i\in I}|S_i|$ is demonstrated. The proof is carried out constructional due to use of automats with the generalized temporal characteristics.

We shall consider two classes of types of Medvedev automats: $<0,t_2>$ and $<t_2-1,t_2>$, where $t_2>1$.

For automats of type $<0,t_2>$ equality $\delta(s(t),x(t))=s(t+t_2)$ is true.

For automats of type $<t_2-1,t_2>$ equality $\delta(s(t),x(t+t_2-1))=s(t+t_2)$ or $\delta(s(t-t_2+1),x(t))=s(t+1)$ is true.

Functional schemes with input signals and codes of the states distribution in time and on elements of memory are represented in Figures 1ab. The block H realizes the combined function of transitions and outputs of the automata A, each element of memory detains a signal for one moment of time.

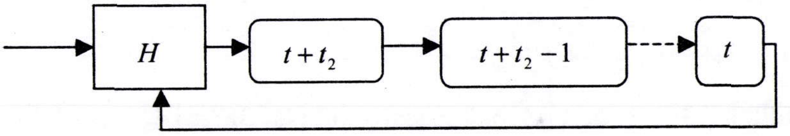

Figure 1a: functional scheme for the automata of type $< 0, t_2 >$

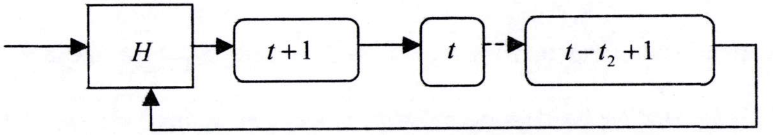

Figure 1b: functional scheme for the automata of type $< t_2 - 1, t_2 >$

For search of answers to the shown questions we shall research of universal automata functioning at modelling behaviour of the family I.

3 Bundle of automats

Let's enter new constructions with having kept designations used by Tverdohlebov (1988).

Definition 3.1 We shall consider finite family of the DFA $\{A_i\}_{i\in I}$ such that $\forall\, i \in I$ $A_i = (S_i, X, \delta_i)$ and consist of $t_2 = |I|$ automats. Let $P_\xi\left(\overset{t_2}{\underset{i=1}{\times}} S_i\right)$, $\xi = \overline{1, t_2!}$ denote some permutation of factors in the Cartesian product $\overset{t_2}{\underset{i=1}{\times}} S_i$ of sets of family automats states.

Thus, each element $\bar{s} \in P_\xi\left(\overset{t_2}{\underset{i=1}{\times}} S_i\right)$ represents on the one hand, the suite of states of different family automats of length $t_2 = |I|$, and on the other hand, $\bar{s}$ is a sting of length t_2 in the alphabet $S_I = \bigcup_{i\in I} S_i$. Such double interpretation $\bar{s}$ allows to establish interrelation between set of stings and set of Cartesian suites, which will be necessary for us further to construct the automata of type $< 0, t_2 >$ for the family $\{A_i\}_{i\in I}$.

Definition 3.2 Set of stings of length t_2 in the alphabet S_I such, that $\bar{s} \in P_\xi\left(\underset{i=1}{\overset{t_2}{\times}} S_i\right)$,

$\xi = \overline{1, t_2!}$ we shall designate through $F_{S_I}^{t_2}$, that is:

$$F_{S_I}^{t_2} = \left\{ \bar{s} \,\middle|\, |\bar{s}| = t_2 \;\&\; \bar{s} \in \bigcup_{\xi=1}^{t_2!} P_\xi\left(\underset{i=1}{\overset{t_2}{\times}} S_i\right) \right\}.$$

Definition 3.3 Let there is finite family of the DFA $\{A_i\}_{i\in I}$ such that $\forall\, i \in I$ $A_i = (S_i, X, \delta_i)$.

The DFA $A_I = (S_I, X, \delta_I)$ of **type** $< 0, t_2 >$, $t_2 = |I| > 1$, with system of equality $S(t, t+t_2-1, \bar{s})$, $\bar{s} \in F_{S_I}^{t_2}$, which is defined as follows:

$s(t+k-1) = pr_k \bar{s}$, where $pr_k \bar{s} \in S_{i_k}$, $i_k \in I$, $k = \overline{1, t_2}$,

and with transition function $\delta_I : S_I \times X \to S_I$, where $\delta_I = \bigcup_{k=1}^{t_2} \delta_{i_k}$, and

$\delta_I(s(t+k-1), x(t+k-1)) = \delta_I(pr_k \bar{s}, pr_k \bar{x}) = \delta_{i_k}(pr_k \bar{s}, pr_k \bar{x}) = s(t+k-1+t_2)$,

$k = \overline{1, t_2}$, $\bar{s} \in F_{S_I}^{t_2}$, $\bar{x} \in F_X^{t_2}$ we shall name a bundle of automats for family $\{A_i\}_{i\in I}$.

Depending on the chosen permutation $P_\xi\left(\underset{i=1}{\overset{t_2}{\times}} S_i\right)$ of family automats we can form various systems. Thus, the bundle of automats is concept which determine a class of systems, however, with definition a sting of start states and thus, having fixed permutation of factors in the Cartesian product $\underset{i=1}{\overset{t_2}{\times}} S_i$, we choose one representative of a class by distribution start states of family automats in the corresponding order. Set of stings of start states $\left\{\bar{s}_\xi^0\right\}_{\xi=\overline{1,t_2!}}$ will be correspond to a class of systems which presents a bundle of automats. Components $\bar{s}_\xi^0$ for everyone $\xi = \overline{1, t_2!}$ are start states s_i^0, $i \in I$ of family automats.

Further we shall consider the problem about how representatives for a bundle of automats are specified.

Definition 3.4 Let there is DFA A_I of type $< 0, t_2 >$, where $t_2 > 1$ and $W_\xi = P_\xi(\underset{i=1}{\overset{t_2}{\times}} S_i)$, $\xi = \overline{1, t_2!}$, $V = F_X^{t_2}$. The DFA $\alpha_\xi(A_I) = (W_\xi, V, \delta_I')$ of type $< t_2 - 1, t_2 >$ with systems of equality $X(t, t+t_2-1, \bar{x})$, $\bar{x} \in V$ and $S(t, t+t_2-1, \bar{s})$, $\bar{s} \in W_\xi$ we shall name the automat representing a bundle of automats A_I if transition function $\delta_I' : W_\xi \times V \to W_\xi$ is defined as follows:

for any element $\bar{s} \in W_\xi$ and any element $\bar{x} \in V$ $\delta'_I(\bar{s},\bar{x}) = \bar{s}'$, where

$\bar{s}' = \delta_{i_1}(pr_1\bar{s}, pr_1\bar{x})\delta_{i_2}(pr_2\bar{s}, pr_2\bar{x})\ldots\delta_{i_{t_2}}(pr_{t_2}\bar{s}, pr_{t_2}\bar{x})$,

the index i_k of transition function δ_{i_k}, $k = \overline{1,t_2}$ is defined from a condition $pr_k\bar{s} \in S_{i_k} = pr_k W_\xi$, $i_k \in I$.

The fact, that functioning of the automat A_I of type $< 0,t_2 >$ can be presented by automats $\alpha_\xi(A_I) = (W_\xi, V, \delta'_I)$, $\xi = \overline{1,t_2!}$ of type $< t_2 - 1, t_2 >$ is proved.

Theorem 3.1: Let finite family $\{A_i\}_{i\in I}$ of the DFA, where for everyone $i \in I$ $A_i = (S_i, X, \delta_i)$ is given. If assume, that $t_2 = |I|$ and the set of stings $F_{S_I}^{t_2}$ is determined by the formula:

$$F_{S_I}^{t_2} = \left\{ \bar{s} \,\middle|\, |\bar{s}| = t_2 \ \& \ \bar{s} \in \bigcup_{\xi=1}^{t_2!} P_\xi\left(\bigtimes_{i=1}^{t_2} S_i\right)\right\},$$

then there is a bundle of automats $A_I = (S_I, X, \delta_I)$ of type $< 0,t_2 >$, and $t_2!$ automats $\alpha_\xi(A_I) = (W_\xi, V, \delta'_I)$ of type $< t_2 - 1, t_2 >$ representing it.

Proof: The first part of the theorem statement is proved by a method on the basis of which the bundle of automats $A_I = (S_I, X, \delta_I)$ of type $< 0,t_2 >$ for the given family $\{A_i\}_{i\in I}$ of the DFA is constructed, in the assumption that theorem conditions are satisfied. The technique of synthesis includes sequence of steps.

Step 1: To receive universal set $S_I = \bigcup_{i\in I} S_i$ of states.

Step 2: To set value of time parameter $t_2 = |I|$.

Step 3: To set systems of equality $S(t, t + t_2 - 1, \overline{s_\xi})$, $\overline{s_\xi} \in F_{S_I}^{t_2}$ $\xi = \overline{1,t_2!}$, where $F_{S_I}^{t_2}$ - the set specified in a theorem conditions, which elements present states strings of different automats of length $t_2 = |I|$.

Step 4: To define transition function $\delta_I = \bigcup_{k=1}^{t_2} \delta_{i_k}$ for which the following temporal equality will be true:

$\delta_I(s(t+k-1), x(t+k-1)) = \delta_I(pr_k\bar{s}, pr_k\bar{x}) = \delta_{i_k}(pr_k\bar{s}, pr_k\bar{x}) = s(t+k-1+t_2)$,

$k = \overline{1,t_2}$, $\bar{s} \in F_{S_I}^{t_2}$, $\bar{x} \in F_X^{t_2}$.

As a result of a method performance the bundle of automats $A_I = (S_I, X, \delta_I)$ for the given family $\{A_i\}_{i\in I}$ of the DFA will be constructed.

For the proof of the second part of the theorem we shall notice, that a string $\overline{s_\xi}$ determined by system of equality $S(t, t + t_2 - 1, \overline{s_\xi})$, belongs to the set $W_\xi = P_\xi\left(\bigtimes_{i=1}^{t_2} S_i\right)$

being permutation from t_2 elements. Thus, we have $t_2!$ automats $\alpha_\xi(A_I) = (W_\xi, V, \delta'_I)$ which form a class of automats $\{\alpha_\xi(A_I)\}_{\xi=\overline{1,t_2!}}$ representing a bundle. To choose the representative for a bundle it is enough to select one of systems, received on a step 3, as the starting data.

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Theorem 3.2: Let finite family of the DFA $\{A_i\}_{i\in I}$, where for everyone $i \in I$ $A_i = (S_i, X, \delta_i)$ is given.

Then there is universal DFA $A_I = (S_I, X, \delta_I)$ of type $<0, t_2>$, $t_2 = |I|$ concerning family $\{A_i\}_{i\in I}$, which models behaviour of any automata A_{i_k} $k = \overline{1,t_2}$, $i_k \in I$ at the moment of time $t + k - 1 + l \cdot t_2$, $l = 0,1,\ldots$

Proof: Let's take as the universal automata for family $\{A_i\}_{i\in I}$ a bundle of automats $A_I = (S_I, X, \delta_I)$. The bundle of automats has type $<0, t_2>$ and according to the statement of the theorem 3.1 we can construct it, assigning $t_2 = |I|$ and having determined set of strings $\{\overline{s_\xi}\}_{\xi=\overline{1,t_2!}}$ in the alphabet $F_{S_I}^{t_2}$. Using equality:

$\delta_I(pr_k \overline{s}, pr_k \overline{x^l}) = \delta_{i_k}(pr_k \overline{s}, pr_{l\cdot t_2+k} p)$, where $\overline{x^l} \in F_X^{t_2}$, $p = \overline{x^0}\,\overline{x^1}\ldots\overline{x^{n-1}}$, $n \in Z^+$, together with equality $x(t+l) = pr_{l+1} p$ we receive validity of the theorem.

■

The proved theorem defines the time organization of the behaviour modelling in the universal automata A_I concerning family $\{A_i\}_{i\in I}$.

Sequence 3.2.1: For finite family of the DFA $\{A_i\}_{i\in I}$ where for everyone $i \in I$ $A_i = (S_i, X, \delta_i)$ there is a class of universal enumerators of type $<t_2 - 1, t_2>$, $t_2 = |I|$ and $|Un\{A_i\}_{i\in I}| = t_2!$

Proof: On the one hand the bundle of automats A_I of the type $<0, t_2>$, being the universal automata concerning family $\{A_i\}_{i\in I}$, sets a class of automats $\{\alpha_\xi(A_I)\}_{\xi=\overline{1,t_2!}}$ of type $<t_2 - 1, t_2>$.

On the other hand: $\forall \xi = \overline{1,t_2!}$ $\exists W_\xi = P_\xi\left(\overset{t_2}{\underset{i=1}{\times}} S_i\right)$ such that $\alpha_\xi(A_I) = (W_\xi, V, \delta'_I)$ is the representative of a bundle for some permutation of family automats, defined by set W_ξ and determining the order of functioning of automats in a bundle. Value of an index ξ we shall identify with permutation $P_\xi\left(\overset{t_2}{\underset{i=1}{\times}} S_i\right)$. Hence, according to the theorem 3.2 for everyone $\xi = \overline{1,t_2!}$ $\alpha_\xi(A_I)$ will be the universal automata concerning some permutation

ξ of the family automats forming a bundle, and this implies, that for everyone $\xi = \overline{1, t_2!}$ $\alpha_\xi(A_l)$ will be universal enumerator concerning permutation ξ (Sytnik, 1992, p.52).

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4 Using a bundle of automats for synthesis of fault-tolerant computing systems

Achievement of the goal 1 considered in section 2, is equivalent to search of the answer to a question: whether there will be each automata A_i of a class $\{A_i\}_{i\in I}$ universal for the DFA A? However the answer to this question can be received by means of the description of all automats for which the given automata is universal. Thus, synthesis of mathematical models of behaviour restoration means is equivalent to the decision of a analysis problem for universal DFA. The analysis of functional features of the automats of type $< 0, t_2 >$ that was carried out in the previous section gives one of possible decisions for this question.

Let's assume that conditions of a goals 1 and 2 are executed, thus applying a bundle of automats as universal we shall receive the following formulations of the considered problems:

Goal 1'. For the automata $A = (S, X, \delta)$ and a bundle of automats $A_I = (S_I, X, \delta_I)$ of type $< 0, t_2 >$ to answer a question:

Is the automata A a member of a bundle A_I?

Goal 2'. To construct function $\varphi(A_I) = A$.

Use of a bundle of automats as the formal instrument allows to carry out synthesis of fault-tolerant computing systems because gives a restoration opportunity of the given behaviour in a case when the number of faults does not surpass $t_2 - 1$. Presence in a bundle of several copies of the automata A allows to organize «repeat count».

The temporal parameter t_2 determines a memory size of a bundle of automats $S_I = |S| \cdot t_2$ and number of input signals $|p| = n \cdot t_2$, $n \in Z^+$ which apply to automats of family $\{A_i\}_{i\in I}$.

Let's consider an input string $p = \overline{x^0}\,\overline{x^1} \ldots \overline{x^{n-1}}$ where $\overline{x^l} = pr_{l\cdot t_2+1 \ldots (l+1)\cdot t_2}\, p$, $\overline{x^l} \in F_X^{t_2}$ and $pr_{l\cdot t_2+1}\, p = pr_{l\cdot t_2+2}\, p = \ldots = pr_{(l+1)\cdot t_2}\, p$, $l = 0, 1, \ldots, n-1$, thus each input signal is repeated several times (t_2 - times, where t_2 is the number of automats in a bundle).

The starting data for a bundle of automats A_I will become:

$S(t, t + t_2 - 1, \overline{s^0})$, $\overline{s^0} = s_{i_1} s_{i_2} \ldots s_{i_{t_2}}$ and $\forall k = \overline{1, t_2}$ $s_{i_k} = s_0$, where s_0 is a start state of automata A.

Lemma 4.1: If the automata A_{i_k}, $k=\overline{1,t_2}$ included in a bundle of automats A_I of type $<0,t_2>$ is the automata A then occurrence of correct reactions on an output of a bundle will correspond to the moments of time $t+k-1+l\cdot t_2$, $l=0,1,\ldots,n-1$, $|p|=n\cdot t_2$, $n\in Z^+$.

From a lemma 4.1 follows, that time of behaviour restoration of the automata A concerning any of t_2-1 faults in a class I equals $n\cdot t_2+k-1$, where $k=\overline{1,t_2}$.

The automata A can be placed by t_2 - ways in permutation ξ of family automats A_i, $i\in I$ forming a bundle. Minimum time of restoration is achieved, when the automata A is on the first position in permutation ξ and is equal $n\cdot t_2$, and maximum time of restoration equals $(n+1)\cdot t_2-1$ and corresponds to a case when the automata A is on the last position in permutation ξ.

Let's use the lemma 4.1 to receive expression for function $\varphi(A_I)$:

$$\varphi(A_I)=pr_{l\cdot t_2+k}(\delta_I(\overline{s^0},p)),\ k=\overline{1,t_2},\ l=0,1,\ldots,n-1,\ |p|=n\cdot t_2,\ n\in Z^+.$$

5 Conclusion

In this paper the model of the computing system that allows to realize enumerating form of mathematical models of behaviour restoration was offered. Delimitation of fault tolerance at a design phase of the future object is pawned by using temporal characteristics of the functioning algorithm and made proceeding from having restrictions for the restoration period and a memory size.

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