Constrained Pole Assignment Control of Double and Triple Integrator

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Abstract
The constrained control of triple integrator based on the modes decomposition is introduced in this paper. The design combines the well known time optimal control with the linear pole assignment control, i.e. the control consist of *n* phases similar to the time optimal control, however the transients between these phases are "smooth" and the dynamics of the transients is given by the closed loop poles.

Keywords: pole assignment control, time optimal control, constraints, nonlinear, third order system.

1 Introduction

Recent decade in the control design is characteristic by a revival of theory of constrained systems. The minimum time control, which was dominating the control design from late 40-ties to the beginning of 70-ties in the 20th century (see e.g. Pavlov, 1966; Athans & Falb, 1966) is, however, replaced by several new approaches as the predictive control (see e.g. Bemporad *et al.*, 2002; El-Farra *et al.*, 2003), different anti-windup solutions, positive invariance sets, etc. Motivation comes from different fields – from the traffic control, robot control, control of unstable systems, etc. A common feature of the new approaches is that they are rather complex - even in the case of simple control problems. So, they are not easy to understand and to apply. Traditionally, the engineering community preferred simpler solutions, as e.g. that one proposed by Kiendl and Schneider (1972), which was later used in robot control (Kunze, 1984; Patzelt, 1981).

Parallel to this, family of new not yet widely known solutions (Huba, 1994; Huba and Bisták, 1995; Huba *et al.*, 1997; Huba, 1998; Huba *et al.*, 1998; Huba, 1999; Huba *et al.*, 1999; Huba and Bisták, 1999; Huba, 2003; Huba and Bisták, 2005; Huba, 2006) was developed. They are relatively simple for understanding, easy to implement and so appropriate also for extremely fast application and easy to tune by a procedure that can be considered as a generalization of the well-known method by Ziegler and Nichols.

In this paper, the family of the already known approaches is extended by the control design, which is based on the decomposition of the closed loop dynamics into particular modes defined by chosen real closed loop poles.

2 Linear Pole Assignment Control Based on Modes Decomposition

Let us consider the 3rd order integrator

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \tag{1}$$

$$\begin{matrix} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = u \end{matrix} \; ; \; \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; ; \; \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; ; \; \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The linear pole assignment controller fulfills three requirements:

a) It decreases the distance $\rho_2\{\mathbf{x}(t)\} = \mathbf{a}'\mathbf{x}$ of the representative point from the plane $S = \{x | \mathbf{a}'\mathbf{x} = 0, \mathbf{a}' = (a_0 \quad a_1 \quad a_2)\}$ according to $d\rho_2 / dt = \alpha_3 \rho_2$.

b) It decreases the oriented distance $\rho_1\{\mathbf{x}(t)\}$ of the representative point lying in the plane S from the line $L = \left\{ \mathbf{x} \middle| \begin{matrix} \mathbf{x} = \mathbf{v}_1 q, \\ \mathbf{v}_1' = (v_{10} \quad v_{11} \quad v_{12}), \\ q \in \langle -\infty \quad \infty \rangle \end{matrix} \right\}, L \in S$ according to $d\rho_1 / dt = \alpha_2 \rho_1$.

c) Along the line L the controller decreases the oriented distance $\rho_0\{\mathbf{x}(t)\} = \mathbf{x}(t)$ from the origin proportionally to the closed loop pole α_1, whereby $d\rho_0 / dt = \alpha_1 \rho_0$.

Let us consider a closed loop system with $\alpha_3 < \alpha_2 < \alpha_1 < 0$. Since the corresponding eigenvectors

$$\mathbf{v}_i = [\alpha_i \mathbf{I} - \mathbf{A}]^{-1}\mathbf{b} = \begin{bmatrix} \frac{1}{\alpha_i^3} & \frac{1}{\alpha_i^2} & \frac{1}{\alpha_i} \end{bmatrix}^T \tag{2}$$

are not collinear, they form a basis, which can be used for expressing any state as a sum of three modes

$$\mathbf{x} = q_1\mathbf{v}_1 + q_2\mathbf{v}_2 + q_3\mathbf{v}_3, \; q_1, q_2, q_3 \in R \tag{3}$$

Then, one can write

$$\begin{aligned}\dot{\mathbf{x}} &= \dot{\mathbf{x}}_1 + \dot{\mathbf{x}}_2 + \dot{\mathbf{x}}_3 = \\ &= \mathbf{A}\mathbf{x} + \mathbf{b}u = \\ &= \mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3) + \mathbf{b}(u_1 + u_2 + u_3)\end{aligned} \tag{4}$$

After denoting $\mathbf{x}_i = q_i \mathbf{v}_i$ each subsystem can be expressed as

$$\begin{aligned}\dot{\mathbf{x}}_1 &= \mathbf{A}\mathbf{x}_1 + \mathbf{b}u_1 = \alpha_1 \mathbf{x}_1 \\ \dot{\mathbf{x}}_2 &= \mathbf{A}\mathbf{x}_2 + \mathbf{b}u_2 = \alpha_2 \mathbf{x}_2 \\ \dot{\mathbf{x}}_3 &= \mathbf{A}\mathbf{x}_3 + \mathbf{b}u_3 = \alpha_3 \mathbf{x}_3\end{aligned} \tag{5}$$

So the 3[rd] order dynamics can be decomposed into three 1[st] order ones. The appropriate interpretation based on appropriate choice of the oriented distance of the representative point $\mathbf{x}$ to the plane (or the line) lead to the three control phases well known from the time optimal control. All of the coordinates ($\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$) are equivalent in this linear case. However, in order to be able to speak specifically, let us suppose following interpretation:

a) The 1[st] equation describes the transient along the line given by $\mathbf{v}_1$ to the origin. The dynamics is given by α_1.
b) The 2[nd] equation describes the transient in the plane to the line given by $\mathbf{v}_1$ by decreasing the oriented distance measured in the direction of $\mathbf{v}_2$. The dynamics of the transient is given by the second pole α_2.
c) Similarly, the 3[rd] equation describes the transient to the plane given by $\mathbf{v}_1, \mathbf{v}_2$ by decreasing the distance measured in the direction of $\mathbf{v}_3$. The dynamics of this transient is given by α_3.

The total control signal can be gained as

$$u = \sum_{i=1}^{3} u_i = \sum_{i=1}^{3} q_i \tag{6}$$

The control algorithm described above guarantees that all three control phases are running in parallel and the resulting controller is equal to that one derived by the Ackerman's formula (see chapter 4). Fig. 1 shows the new base described above.

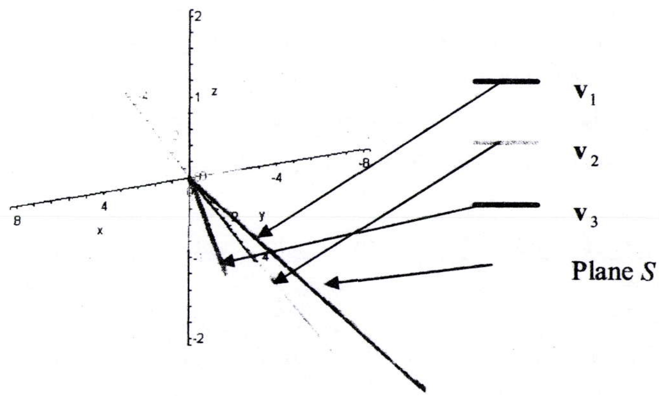

Figure 1: Linear pole placement based on modes decomposition.

3 Non-linear (Constrained) Pole Assignment Control Based on Modes Decomposition

Let us consider a constrained control signal

$$u \in \langle U_1 \quad U_2 \rangle \tag{7}$$

A transient of the representative point from an initial state on the line given by the eigenvector $\mathbf{v}_1$ laying outside the proportional band does already not follow this line. The particular subsystems do not more change themselves autonomously only in their own coordinates $\mathbf{x}_i$, but also in the other subsystems coordinates. There is again necessary to assign to each of the control phases some interpretation. The following assignment has been used (for more details se e.g. Kabát, 2000).

1. The transient of the representative point in the phase space to the reference braking surface (*RS*) is given by $\alpha_3, \mathbf{v}_3$.
2. The transient along the *RS* to the reference curve (*RC*) is given by $\alpha_2, \mathbf{v}_2$.
3. The transient along the *RC* into the origin is given by $\alpha_1, \mathbf{v}_1$.

The control phases in this control algorithm combine the time optimal control with the linear pole assignment control. Each control phase is given by particular constraint of the control signal, but the transition between the control phases is described by the chosen corresponding closed loop pole. So each control phase is given by two parameters:

α_i - describing the dynamics of the transient of the representative point $\mathbf{x}_i$ inside the linear subsystem (proportional band)

t_i - the time , which is needed to reach the "linear" subspace under influence of limit control values.

One part of the final control phase (the linear one), corresponding to the transient into the origin, when the other coordinates $\mathbf{x}_3$ and $\mathbf{x}_2$ are zeroes, is given by the closed loop pole α_1. The representative point moves along the line given by the eigenvector $\mathbf{v}_1$, whereby the coordinate

$$\mathbf{x}_1 \in \langle U_j \mathbf{v}_1 \quad U_{3-j} \mathbf{v}_1 \rangle \tag{8}$$

Let us denote the constraint of the control signal in the final control phase as U_j whereby

$$j = 1,2 \tag{9}$$

The interval of linear control is restricted to points (8) expressed as

$$\mathbf{x}_1 = q_1 \mathbf{v}_1 ; q_1 \in \langle U_j \quad U_{3-j} \rangle \tag{10}$$

The second segnment of the final control phase (non-linear) is described by the time t_1, that represents the time of the transient of the representative point using $u = U_j$ to the border points of the linear subsystem ${}^{j}X_1 = U_j \mathbf{v}_1$. This 2nd segment is created by points $\mathbf{x}_1$ derived as the result of backward integration of (1) on the interval $t \in \langle 0 \quad t_1 \rangle$ using $u = U_j$ and starting from the points ${}^{j}X_1$. One gets

$$\mathbf{x}_1(t_1) = \begin{bmatrix} 1 & -t_1 & \frac{t_1^{\,2}}{2} \\ 0 & 1 & -t_1 \\ 0 & 0 & 1 \end{bmatrix} q_1 \mathbf{v}_1 + \begin{bmatrix} \frac{-t_1^{\,3}}{6} \\ \frac{t_1^{\,2}}{2} \\ -t_1 \end{bmatrix} U_j, j = 1,2 \tag{11}$$

So, $\mathbf{x}_1$ represents all points of the 1st subsystem, and these points are given by the couple of parameters (q_1, t_1). For each segment it varies just one of them: the parameter q_1 while $t_1 = 0$, when $\mathbf{x}_1 \in \langle U_j \quad U_{3-j} \rangle \mathbf{v}_1$ and $t_1 > 0$ while $q_1 = U_j$ for points outside of the linear segment (8). So let us introduce the following generalized denotation $\mathbf{x}_1(q_1, t_1)$, i.e. the points lying in the proportional band (the control signal is not saturated) are represented as $\mathbf{x}_1(q_1, 0)$ and the points lying outside (8) with saturated control signal are represented as $\mathbf{x}_1(U_j, t_1)$. For points in the proportional band $\mathbf{x}_1 = \mathbf{x}_1(q_1, 0) \in \langle U_1 \mathbf{v}_1 \quad 0 \rangle$, the control signal is $u_1 = q_1$. The control signal for the points outside the proportional band $\mathbf{x}_1(U_j, t_1)$ is $u_1 = U_j$. Such a representation of these points describes the transient along the *RC* with dynamics given by the closed loop pole α_1 with respecting the control constraints. The *RC* is invariant set of the system $\dot{\mathbf{x}}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{b}u_1$ with constrained control signal $u_1 \in \langle U_j \quad U_{3-j} \rangle$, i.e. after approaching the *RC*, the system remains on the *RC*.

In the second control phase is still $\mathbf{x}_3 = 0$. The transient take place along the surface given by $\mathbf{x}_1$, $\mathbf{x}_2$ towards the *RC* (characterized by $\mathbf{x}_1$) in this phase. The goal is to approach the *RC* characterized by $\mathbf{x}_2 = 0$ and $\mathbf{x}_3 = 0$. In the proportional band of the second subsystem $\mathbf{x}_2 \in \langle 0 \quad (U_{3-j} - q_1) \rangle \mathbf{v}_2$, i.e.

$$q_2 \in \langle 0 \quad (U_{3-j} - U_j) \rangle \tag{12}$$

The "linear" dynamics of this transient is given by α_2 only. This gives the 1st segment of the Reference Surface. The second segment of the RS is given as the result of backward integration of (1) on the interval $t \in \langle 0 \quad t_2 \rangle$ starting from those points where q_2 is saturated using $u = U_{3-j}$. Using generalized denotation of the *RC* one gets the expression of RS as

$$\mathbf{x} = \mathbf{x}_2((U_{3-j} - q_1), t_2) + \mathbf{x}_1(q_1, t_1) =$$

$$= \begin{bmatrix} 1 & -t_2 & \dfrac{t_2^{\ 2}}{2} \\ 0 & 1 & -t_2 \\ 0 & 0 & 1 \end{bmatrix} (\mathbf{x}_1(q_1, t_1) + q_2 \mathbf{v}_2) + \begin{bmatrix} \dfrac{-t_2^{\ 3}}{6} \\ \dfrac{t_2^{\ 2}}{2} \\ -t_2 \end{bmatrix} U_{3-j} \tag{13}$$

In the starting control phase the 3rd subsystem $\mathbf{x}_3$ is in the proportional band for $\mathbf{x}_3 \in \langle (U_j - q_1 - q_2)\mathbf{v}_3 \quad (U_{3-j} - q_1 - q_2)\mathbf{v}_3 \rangle$, i.e.

$$q_3 \in \langle U_j - q_1 - q_2 \quad U_{3-j} - q_1 - q_2 \rangle \tag{14}$$

Similarly to the previous subsystems, the result of the backward integration using $u = U_j$ on the interval $t \in \langle 0 \quad t_3 \rangle$ starting from those points where q_3 is saturated, gives the 3rd subsystem. Using generalized denotation $\mathbf{x}_2(q_2, t_2)$, the general point of the surface can be represented as $\mathbf{x}_1(q_1, t_1) + \mathbf{x}_2(q_2, t_2)$ and any point of the state space can be expressed using modes decomposition as

$$\mathbf{x} = \mathbf{x}_3(q_3, t_3) + \mathbf{x}_2(q_2, t_2) + \mathbf{x}_1(q_1, t_1) =$$

$$= \begin{bmatrix} 1 & -t_3 & \frac{t_3^2}{2} \\ 0 & 1 & -t_3 \\ 0 & 0 & 1 \end{bmatrix} (\mathbf{x}_1(q_1, t_1) + \mathbf{x}_2(q_2, t_2) + q_3 \mathbf{v}_3) + \begin{bmatrix} \frac{-t_3^3}{6} \\ \frac{t_3^3}{2} \\ -t_3 \end{bmatrix} U_j \tag{15}$$

Note that the whole state space can be parametrized by $\mathbf{x}_1 = \mathbf{x}_1(q_1, t_1)$, $\mathbf{x}_2 = \mathbf{x}_2(q_2, t_2)$, $\mathbf{x}_3 = \mathbf{x}_3(q_3, t_3)$, using parameters $q_i, i = 1,2,3$ that describe the length of the vectors in the proportional bands of the subsystems and $t_i, i = 1,2,3$ describing these vectors outside theirs proportional bands (the control signal of the subsystem is saturated). The sequential choice of the coordinates of the subsystems guarantees, that the control signal is saturated only if the third subsystem is saturated also.

The control algorithm is similar to the linear one, but it depends on computing the parameters $q_i, t_i, i = 1,2,3$, which are more difficult to obtain.

The final control is

$$u = \sum_{i=1}^{3} u_i \tag{16}$$

where u_i are control signals of particular subsystems. The *RS* is given by the points where $u_3 = 0$, but we divide it according to the parameters t_1, t_2, j. Let us denote following:

$RS^j{}_0$	- the segment of *RS*, where $t_1 = 0, t_2 = 0$
$RS^j{}_1$	- the segment of *RS*, where $t_1 > 0, t_2 = 0$
$RS^j{}_2$	- the segment of *RS*, where $t_1 = 0, t_2 > 0$
$RS^j{}_{12}$	- the segment of *RS*, where $t_1 > 0, t_2 > 0$

Following figures show *RC* and *RS* for $j = 1$, $\alpha_1 = -0.5$, $\alpha_2 = -1$, $\alpha_3 = -2$, $u \in \langle -1 \quad 1 \rangle$.

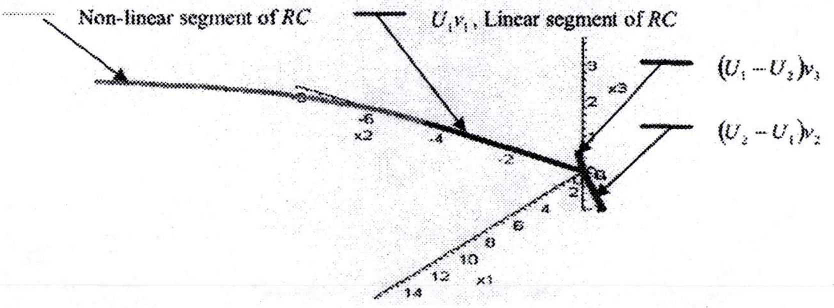

Figure 2: Eigenvectors $\mathbf{v}_i, i = 1,2,3$ and RC^1.

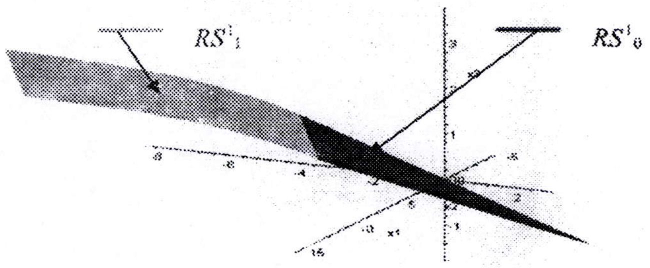

Figure 3: RS^1_0, RS^1_1.

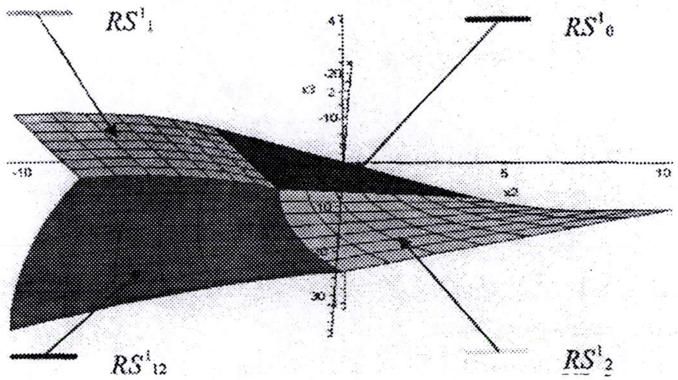

Figure 4: RS^1

The proportional band (PB) of the system (1) is given by points where $q_3 \in \langle U_1 - q_1 - q_2 \quad U_2 - q_1 - q_2 \rangle$.

Let us denote $q_{3\min} = U_1 - q_1 - q_2, q_{3\max} = U_2 - q_1 - q_2$. Fig. 5 shows the segments of PB corresponding to particular segments of RS.

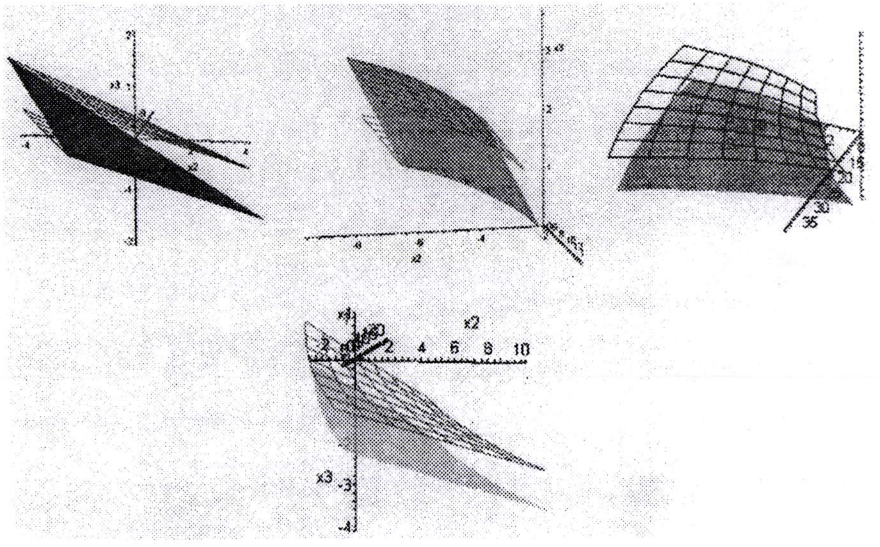

Figure 5: PB corresponding to the segments $RS^j{}_0, RS^j{}_1, RS^j{}_{12}, RS^j{}_2$.

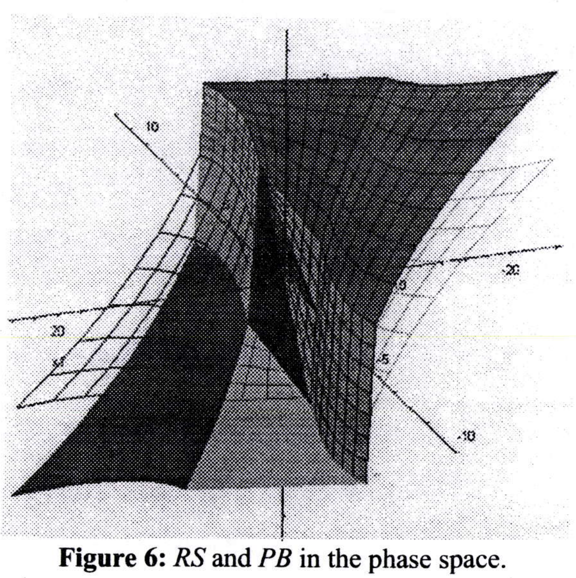

Figure 6: RS and PB in the phase space.

4 Non-linear Control Algorithm

The control algorithm is based on determining the parameters $q_i, t_i, i = 1,2,3$. However, parameter t_3 is not needed, because there is enough to know whether $\mathbf{x}_3$ is in the proportional band or in the saturation domain, so let us assign $t_3 = 0$. To find these parameters there is necessary to solve (15), however the results obtained by symbolic solutions can be used now. The formula for evaluation of $\mathbf{x}_3$ differs for each segment and it depends on $\mathbf{x}_1, \mathbf{x}_2$, so the control algorithm can be divided into following steps:

1. START
2. $RS^j{}_0$
   a. q_1, q_2, q_3 are unknown
   b. $t_1 = 0, t_2 = 0$
   c. solve (15) for q_1, q_2, q_3
   d. IF (10), (12) are not fulfilled THEN GOTO 3
   e. $sat(q_3), u = \sum_{i=1}^{3} q_i$ GOTO 6
3. $RS^j{}_1$
   a. t_1, q_2, q_3 are unknown
   b. $q_1 = U_j, t_2 = 0$
   c. solve (15) for t_1, q_2, q_3
   d. IF $t_1 > 0$, (12) are not fulfilled THEN GOTO 4
   e. $sat(q_3), u = \sum_{i=1}^{3} q_i$ GOTO 6
4. $RS^j{}_2$
   a. q_1, q_3, t_2 are unknown
   b. $q_2 = U_{3-j} - q_1, t_1 = 0$
   c. solve (15) for q_1, q_3, t_2
   d. IF $t_2 > 0$, (10) are not fulfilled THEN GOTO 5
   e. $sat(q_3), u = \sum_{i=1}^{3} q_i$ GOTO 6
5. $RS^j{}_{12}$
   a. q_3, t_1, t_2 are unknown
   b. $q_1 = U_j, q_2 = U_{3-j} - U_j$
   c. solve (15) for q_3, t_1, t_2
   d. IF $t_1 > 0, t_2 > 0$ are not fulfilled THEN GOTO ERROR

e. $sat(q_3), u = \sum_{i=1}^{3} q_i$ GOTO 6

6. If the distance of the representative point from the desired state is greater than ε GOTO 1
7. END

5 Verifying Control Algorithm by Simulation

The following simulation verifying the control shown above has been made in the computer algebra system MAPLE v 9.5.

Parameters of the simulation with initial state outside the *PB*:
Chosen closed loop poles

$\alpha_1 = -1.5, \alpha_2 = -3, \alpha_3 = -6$

Initial state

$\mathbf{x} = [14.145, -6.097, 0.833]^{\mathrm{T}}$

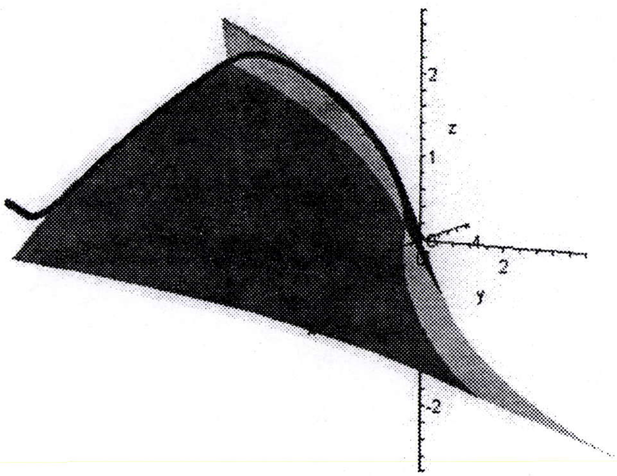

Figure 7: Trajectory in the phase space.

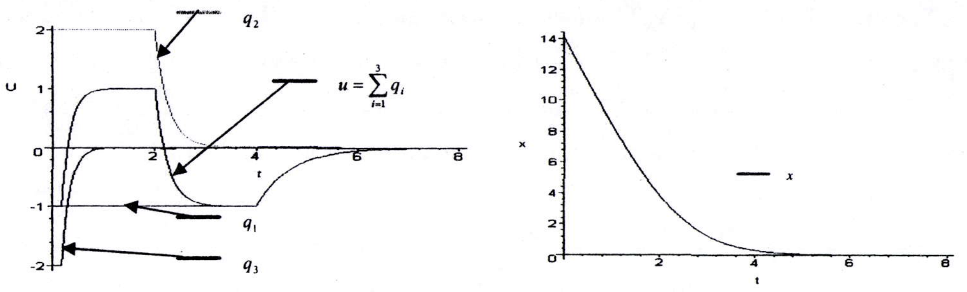

Figure 8: Control signal and output

Fig. 8 shows that the control signal approaches the constraint three times.

6 Constrained Controller for Double Integrator Using Reverse Order of Poles

The non-linear algorithm described above has been designed just for the poles configuration $\alpha_3 < \alpha_2 < \alpha_1 < 0$. To explain, why this configuration is important in designing the constrained pole assignment controller, let us analyze similar problem for the double integrator plant, as well. We will show the effect of choosing the reverse order of the poles ($\alpha_1 < \alpha_2 < 0$) in several simulations, which have shown that the output was not monotone for each initial condition starting under the reference (braking) curve, however the control signal has been designed to be continuous.

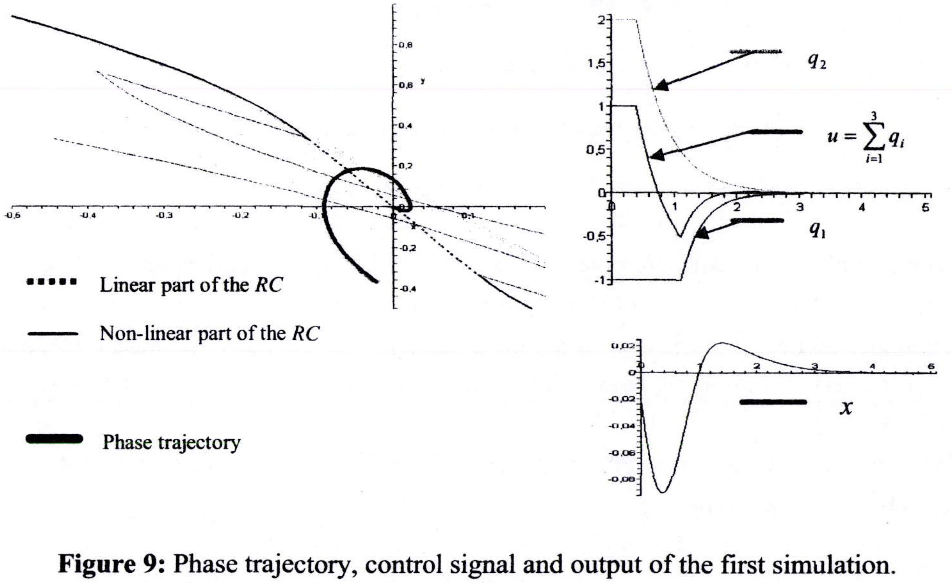

Figure 9: Phase trajectory, control signal and output of the first simulation.

The first simulation (Fig.9) shows the overshoot using poles $\alpha_1 = -3, \alpha_2 = -2$ and the input constraint $U \in \langle -1 \quad 1 \rangle$ with initial condition $[-0.023, -0.367]^T$. The second simulation starts in $[1.054, -1.452]^T$ and the overshoot occurs as well (Fig. 10). The last simulation shows the monotone output with initial condition $[1.412, -1.637]^T$.

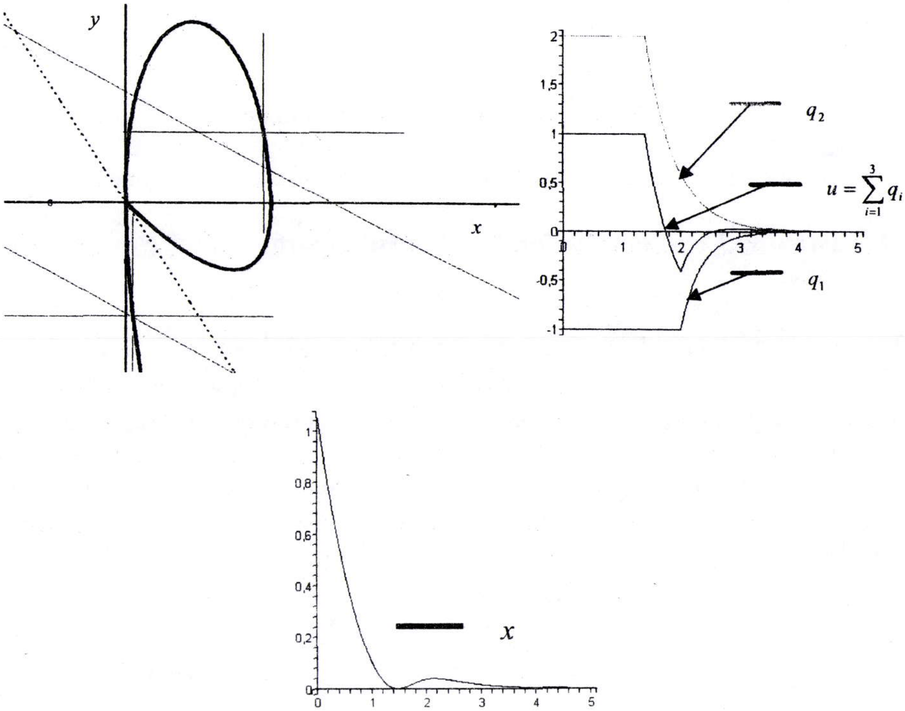

Figure 10: Phase trajectory, control signal and output of the second simulation.

The optimal control sequence of the 2nd order system corresponds to two constrained exponentials. The first one corresponds to the limit "braking " with $u = U_j$ and to the proportional control with the pole α_1. The acceleration phase corresponds to the limit control signal $(U_{3-j} - U_j)$ and to the pole α_2. If we denote the difference between the instants t_2 and t_1 as Δ, then, with the time starting at $t = \Delta$, i.e. at the instant when the proportional part of braking begins, the resulting control can be written in the form

$$u(t) = [U_{3-j} - U_j]e^{\alpha_2 t} + U_j e^{\alpha_1 (t-\Delta)}$$

The value $u(\Delta)$ then denotes the amplitude of the 2nd (braking) phase (the first one is given by U_{3-j}), whereby

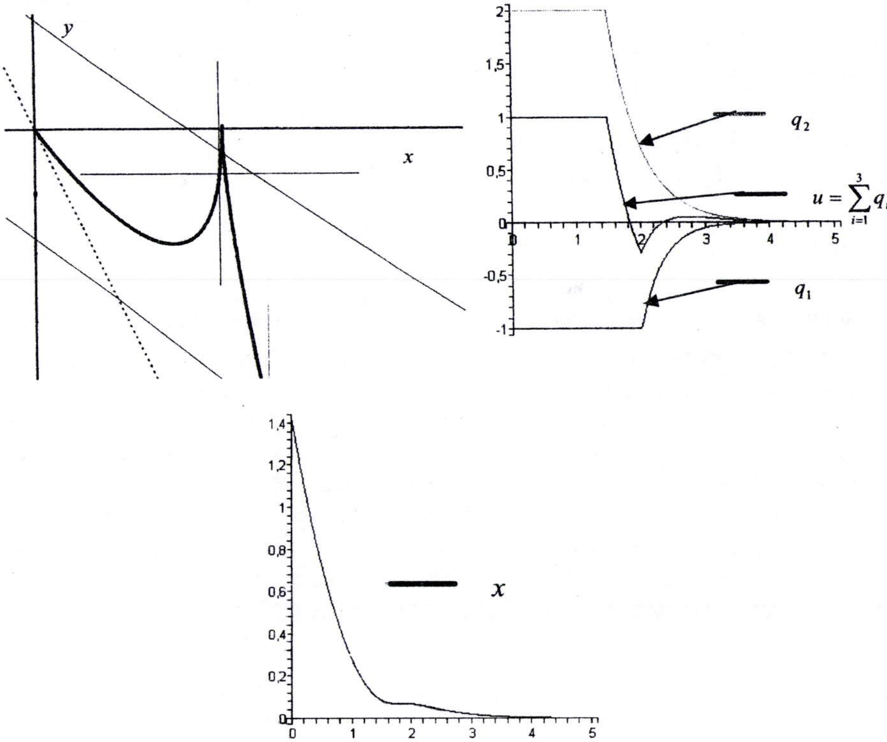

Figure 11: Phase trajectory, control signal and output of the third simulation.

$$A_2 = u(\Delta) = \left[U_{3-j} - U_j\right]e^{\alpha_2 \Delta} + U_j$$

We can speak about braking, when $A_2 U_{3-j} \le 0$. So the limit situation corresponds to

$$A_2 = 0 \Rightarrow \Delta = \frac{1}{\alpha_2} \ln \frac{-U_j}{U_{3-j} - U_j}$$

Then, for the poles satisfying $\alpha_1 < \alpha_2 < 0$ also the 3rd pulse of the control signal occurs. It has maximal amplitude at the time instant t_3 corresponding to the extreme of $u(t)$, when $\dot{u}(t_3) = 0$. Solving this condition for the above defined limit value of A_2 for t_3 one gets

$$t_3 = \Delta + \frac{1}{\alpha_1 - \alpha_2} \ln \frac{\alpha_2}{\alpha_1}$$

$$u(t_3) = \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \left[1 - \frac{\alpha_1}{\alpha_2}\right] U_j$$

By increasing the ratio of α_1 / α_2 the value of this 3[rd] amplitude $u(t_3)$ increases. E.g. for $\alpha_1 / \alpha_2 = 2$ $u(t_3) = -0.25U_j$, while for $\alpha_1 / \alpha_2 = 3$ $u(t_3) = -0.385U_j$.
This occurrence of the 3[rd] pulse in the control signal is associated with an overshooting at the plant output. Since it is frequently not allowed, the practical exploitation of the design based on the decomposition to constrained modes is reasonably limited to the situation described by $\alpha_2 < \alpha_1 < 0$, or $\alpha_3 < \alpha_2 < \alpha_1 < 0$ in the triple integrator case.

7 Conclusion

The simulations have shown, that for $\alpha_3 < \alpha_2 < \alpha_1 < 0$ the control designed for the triple integrator consists of three phases well known from time optimal control. However, now the transients between these phases are "smooth" and they have dynamics specified by the closed loop poles. That gives to this algorithm a reasonable advantage in controlling systems with parasitic time delays and unmodelled dynamics, the measurement and quantization noise etc. The analysis of the control designed for the double integrator plant has been used to show the effect of a reverse order of the closed loop poles. This can be eliminated by the design based on the invariant sets approach that requires to decrease the distance from next lower invariant set in a direction specified by vectors not identical to the closed loop eigenvectors.

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