Design of a Robust Globally Stabilizing PD-Process Control System

A. Muñoz, M. Loccufier and E. Noldus
Department of Electrical Energy, Systems and Automation, Ghent University
Technologiepark-Zwijnaarde 913, B-9052 Zwijnaarde, Belgium
fax: 09/264.58.39; e-mail: noldus@autoctrl.rug.ac.be

Abstract
We discuss the global asymptotic stabilization of a continuous stirred chemical tank reactor, using linear time invariant PD-control with full state feedback. The control input is the feed temperature or the cooling temperature. The emphasis is placed on the robustness of the design, i.e. the controller globally stabilizes the system's set point without requiring the exact knowledge of the process parameters. The control parameters are tuned by means of a classical root locus analysis of the linearized closed loop dynamics and by simulations of the closed loop transients and phase portraits. The stabilization technique relies on the direct method of Liapunov.
Keywords : Process control, Stability, Robustness, PD-control, Liapunov's method

1 Introduction

We discuss the problem of stabilizing a nonlinear system which describes the dynamics of a continuous stirred chemical tank reactor (CSTR). Such a reactor is a continuously operating mixing vessel, which produces large quantities of industrial products and in which strong variations of pressure, flow and temperature during operation are undesirable. Due to the Arrhenius law the process equations contain a product type nonlinearity in which one factor depends linearly and the other factor depends exponentially on the state variables. For a CSTR in which a single chemical reaction takes place they have the general form :

$$\dot{z} = Az + k - bf(c'z)g'z + du \tag{1}$$

where the state $z \in R^2$ and the input $u \in R$. The state variables are reactor temperature and concentration while the input is feed temperature, or cooling temperature. Eventually feed concentration might be considered as a secondary input. Depending on the parameter values there may exist either one or three equilibrium states in the uncontrolled process dynamics [1], [2]. Typically in the latter case the desired set point is open loop unstable, the stable open loop equilibria being unsuitable operating points for technological reasons.

Several types of nonlinear controllers for this process have been proposed in the literature, including feedback linearization control [3], adaptive [4] or artificial intelligence [5] control schemes etc. Our design method concentrates on the application of simple linear control laws, without trying to compensate or to modify the process nonlinearities.

A linear time invariant controller with full state feedback and PD-dynamics is developed which globally stabilizes the desired set point, while the other equilibria disappear in closed loop. The emphasis is placed on the robustness of the design, i.e. the precise knowledge of the process parameters is not required. The set point's closed loop global asymptotic stability remains ensured if the process parameters deviate from their nominal values. The admissible parameter deviations are not infinitesimal but they may vary within certain bounds. In their previous work [6], [7] the authors have derived some alternative versions of their controller with much weaker robustness properties, using a Liapunov approach different from the one below. Furthermore we show that the robustness of the design can be further improved at the cost of restricting the system's operation to some finite stability region in the state space surrounding the set point.

The controller gain tuning problem is handled using classical root locus techniques and simulations of the closed loop transients and phase portraits.

The proposed controllers constitute examples of weakly anticipatory dynamical systems : They compute the evolution of their state taking into account the present values of the controller's and the controlled process' states, but also the desired future stationary and dynamic behaviour of the process. This behaviour is computed from an analytical model of the closed loop.

2 Process Dynamics and Controller Structure

In dimensionless form the heat and mass balance equations of a CSTR for a single chemical reaction [8] can be written in the form (1) where

$$z \triangleq \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad A \triangleq \begin{bmatrix} -(1+s) & 0 \\ 0 & -1 \end{bmatrix}, \quad k \triangleq \begin{bmatrix} r \\ 1 \end{bmatrix}, \quad b \triangleq \begin{bmatrix} -m_2 \\ m_1 \end{bmatrix},$$

$$c = d \triangleq \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad g \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and $f(y) \triangleq \exp(-10^4/y)$. Time has been rescaled as $\tau \triangleq t/(\frac{V}{F})$.

Here

$$z_1 \triangleq \frac{RT}{E}10^4, \quad z_2 \triangleq \frac{c}{c_0}, \quad s \triangleq \frac{U}{F\rho c_p}, \quad r \triangleq 10^4 \frac{R}{E}\left(T_0 + \frac{UT_k}{F\rho c_p}\right),$$

$$m_1 \triangleq \frac{Vk_0}{F}, \quad m_2 \triangleq 10^4 \frac{(-\Delta H)Vk_0c_0R}{F\rho c_p E}$$

The state variables are proportional to reactor temperature T and reactor concentration c. The input $u \triangleq 10^4 \frac{R}{E} \Delta T_0(t)$ represents a scaled increment of feed temperature with nominal value T_0 (or eventually cooling temperature, with nominal value T_k). The model parameters are the feed concentration c_0, the heat capacity per unit of volume c_p, the specific mass ρ, the molar reaction heat $-\Delta H$, the heat transfer coefficient U, the reaction speed per unit of volume $k_0 c \exp(-E/RT)$ and the reactor vessel time constant V/F. Corresponding to $u = 0$ there can be either one or three equilibrium states [1], [2]. One of these, say $z_s = \begin{bmatrix} z_{1s} \\ z_{2s} \end{bmatrix}$ is the set point.

Equation (1) implies that

$$m_1 \dot{z}_1 + m_2 \dot{z}_2 = m_1 r + m_2 - m_1(1+s) z_1 - m_2 z_2 + m_1 u \tag{2}$$

Choose u such that in closed loop

$$m_2 z_2 = \alpha z_1 + \beta \dot{z}_1 + \gamma \dot{z}_2 + \delta \tag{3}$$

This is achieved using a PD-controller

$$u = (1 + s + \frac{\alpha}{m_1}) z_1 + (1 + \frac{\beta}{m_1}) \dot{z}_1 + (\frac{m_2 + \gamma}{m_1}) \dot{z}_2 + \frac{1}{m_1}(\delta - m_1 r - m_2) \tag{4}$$

as can be verified by substituting (4) in (2). Taking $\alpha > 0$ ensures that there exists a unique closed loop equilibrium point z_s which satisfies

$$\alpha z_{1s} - m_2 z_{2s} + \delta = 0 \tag{5}$$

$$1 - z_{2s} - m_1 z_{2s} f(z_1) = 0 \tag{6}$$

(see Figure 1). Letting $u = 0$ at $z = z_s$ results in

$$(1 + s + \frac{\alpha}{m_1}) z_{1s} + \frac{1}{m_1}(\delta - m_1 r - m_2) = 0 \tag{7}$$

Now u can be expressed in terms of the deviation variables $x \triangleq z - z_s$ as

$$u = (1 + s + \frac{\alpha}{m_1}) x_1 + (1 + \frac{\beta}{m_1}) \dot{x}_1 + (\frac{m_2 + \gamma}{m_1}) \dot{x}_2 \tag{8}$$

In the next section we compute conditions on the controller parameters α, β and γ which ensure closed loop global asymptotic stability of the set point $z = z_s$ (or $x = 0$). (6), (7) determine z_s as a function of the choice for α and δ. Combining (3) with the second equation of (1) brings the closed loop state equations in the form :

$$M \dot{z} = A_0 z + k_0 - b_0 \left[f_1(z_1) + f_2(z_1)(\beta \dot{z}_1 + \gamma \dot{z}_2) \right] \tag{9}$$

with

$$M \triangleq \begin{bmatrix} \beta & \gamma \\ 0 & 1 \end{bmatrix}, \quad A_0 \triangleq \begin{bmatrix} -\alpha & m_2 \\ 0 & -1 \end{bmatrix}, \quad k_0 \triangleq \begin{bmatrix} -\delta \\ 1 \end{bmatrix}, \quad b_0 \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$f_1(z_1) \triangleq \frac{m_1}{m_2}(\alpha z_1 + \delta) e^{\frac{-10^4}{z_1}}, \quad f_2(z_1) \triangleq \frac{m_1}{m_2} e^{\frac{-10^4}{z_1}}.$$

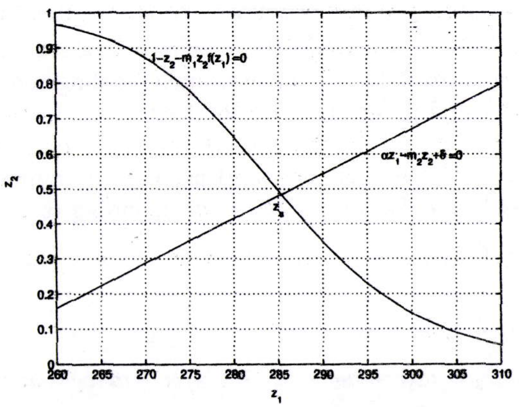

Figure 1: Closed loop equilibrium for a reactor (Example 1) with parameter values $m_1 = 1.7685.10^{15}$,$m_2 = 7.8041.10^{16}$, r=301.2696, s=0.1356 and controller parameters $\alpha = 10^{15}$ and $\delta = -2.476.10^7$ ($z_s = 285.3567, 0.4837$).

3 Stability Analysis

Let

$$V(z) \triangleq z'Pz + \alpha_0 \int_0^{z_1} f_1(\theta)d\theta + p'z \tag{10}$$

be a candidate Liapunov function for the system (9). Recalling that we take $\alpha > 0$, $V(z)$ is radially unbounded if $P = P' > 0$ (positive definite) and $\alpha_0 > 0$. Along the solutions of (9) we have

$$\begin{aligned} \dot{V}(z) &= \dot{z}'Pz + z'P\dot{z} + \alpha_0 f_1(z_1)\dot{z}_1 + p'\dot{z} \\ &= \dot{z}'PA_0^{-1}[M\dot{z} - k_0 + b_0 f_1(z_1) + b_0 f_2(z_1)(\beta\dot{z}_1 + \gamma\dot{z}_2)] \\ &\quad +[M\dot{z} - k_0 + b_0 f_1(z_1) + b_0 f_2(z_1)(\beta\dot{z}_1 + \gamma\dot{z}_2)]'A_0^{-1'}P\dot{z} \\ &\quad +\alpha_0 f_1(z_1)\dot{z}_1 + p'\dot{z} \end{aligned} \tag{11}$$

Choose $p \triangleq 2PA_0^{-1}k_0$ and let

$$PA_0^{-1}M + M'A_0^{-1'}P = -qq' - \rho b_0 b_0'; \quad \rho > 0 \tag{12}$$

$$2PA_0^{-1}b_0 + \alpha_0 c = 0 \tag{13}$$

Then (11) simplifies to

$$\dot{V}(z) = (-q'\dot{z})^2 - \rho(b_0'\dot{z})^2 - \alpha_0\dot{z}_1 f_2(z_1)(\beta\dot{z}_1 + \gamma\dot{z}_2) \tag{14}$$

By a straightforward application of the Kalman-Yacubovich-Popov (KYP) lemma [9] the system (12), (13) has a real solution $P = P' > 0$, $q \in R^2$ if

$$A_0^{-1}M = \begin{bmatrix} -\frac{\beta}{\alpha} & -\frac{-(\gamma+m_2)}{\alpha} \\ 0 & -1 \end{bmatrix}$$

is a Hurwitz matrix, which is satisfied if $\frac{\beta}{\alpha} > 0$ and if

$$\alpha_0 \mathrm{Re} c'(j\omega A_0 - M)^{-1} b_0 + \rho b_0'(j\omega A_0 - M)^{-1*}. \\ b_0 b_0'(j\omega A_0 - M)^{-1} b_0 < 0; \quad \forall \omega \in R \tag{15}$$

where $A^* \triangleq \bar{A}'$ (conjugate transpose).

Some calculations reduce (15) to the condition

$$\frac{\alpha_0}{(\beta^2 + \alpha^2\omega^2)}\{\omega^2[m_2\beta + \alpha(\gamma + m_2)] - \beta\gamma\} > \rho; \quad \forall \omega \in R \tag{16}$$

Since a rational function $\frac{a+bw^2}{c+dw^2}$ with $ad \neq bc$ does not possess a minimum, (16) is satisfied if

$$\frac{\alpha_0}{\beta^2}(-\beta\gamma) > \rho \tag{17}$$

and

$$\frac{\alpha_0}{\alpha^2}[m_2\beta + \alpha(\gamma + m_2)] > \rho \tag{18}$$

(17) and (18) can be written as

$$\frac{\rho\beta}{\alpha_0} < -\gamma < m_2(1 + \frac{\beta}{\alpha}) - \frac{\rho\alpha}{\alpha_0} \tag{19}$$

Next let us return to (14) where $0 < f_2(z_1) < \frac{m_1}{m_2}$ for all $z_1 > 0$ (Observe that by definition, z_1 can not assume negative values). Assuming we choose

$$\gamma^2 \ = \ 4\rho\frac{\beta}{\alpha_0}\frac{m_2}{m_1} \tag{20}$$

it is an easy exercise to see that $\dot{V}(z) \leq 0$ for all z and that the largest invariant set where $\dot{V}(z)$ vanishes consists of the set where $\dot{z}(t) \equiv 0$, i.e. the system's unique equilibrium point $z = z_s$. Eliminating $\frac{\rho}{\alpha_0}$ from (19) and (20) yields the condition

$$\frac{1}{4}\frac{m_1}{m_2}\gamma^2 < -\gamma < m_2(1 + \frac{\beta}{\alpha}) - \frac{1}{4}\frac{m_1}{m_2}\gamma^2\frac{\alpha}{\beta} \tag{21}$$

For practical process parameter values the first inequality of (21) imposes the strongest restriction on γ :

$$0 < -\gamma < 4\frac{m_2}{m_1} \tag{22}$$

Since $V(z)$ is radially unbounded and $\dot{V}(z)$ is negative semidefinite all closed loop trajectories remain bounded for increasing time. Standard invariance theory then implies that the feedback law (4) or (8) renders the set point $z = z_s$ globally asymptotically stable provided we choose $\alpha > 0$, $\beta > 0$ and γ satisfies (22).

4 Robust Design with a Finite Stability Region

By the analysis of Section 3, the control law

$$u = \xi_1 x_1 + \xi_2 \dot{x}_1 + \xi_3 \dot{x}_2 \tag{23}$$

represents a globally asymptotically stabilizing PD-controller provided $\xi_1 > (1+s)$, $\xi_2 > 1$ and

$$\frac{m_2}{m_1}(1 - \frac{4}{m_1}) < \xi_3 < \frac{m_2}{m_1} \tag{24}$$

Tuning the controller does not require a precise knowledge of the process parameters, however for practical parameter values the admissible interval for ξ_3 is small. The robustness of the controller can be improved if we restrict the system's operation to some finite stability region in state space surrounding the set point. Define

$$\Omega_a \triangleq \{z \in R^2; \quad z_1 < a, \quad V(z) < V_a\} \tag{25}$$

where a is a scalar, $a > z_{1s}$. Choose V_a such that if $z(0) \in \Omega_a$ then $z(t) \in \Omega_a$ for all $t \geq 0$. We proceed as follows : It is straightforward to verify that in each point z on the straight line segment $\{z_1 = a, \quad z_2 < b\}$ with

$$b \triangleq \frac{\alpha a + \delta + \gamma}{m_2 + \gamma + \gamma m_1 f(a)} \tag{26}$$

we have $\dot{z}_1 < 0$ (see Figure 2). Let $z_a \triangleq \begin{bmatrix} a \\ b \end{bmatrix}$ and $V_a \triangleq V(z_a)$. Obviously a tra-

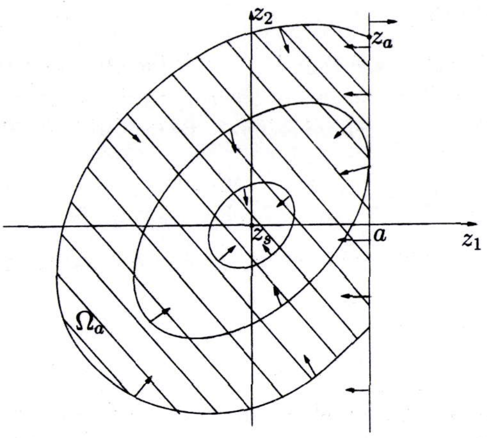

Figure 2: Level sets $\{V(z)\text{=constant}\}$ and stability region Ω_a under a state constraint $z_1 < a$.

jectory starting in Ω_a at $t = 0$ remains in Ω_a for all $t \geq 0$ and Ω_a is a region of attraction of the set point. For $z \in \Omega_a$ we have

$$0 < f_2(z_1) < f_2(a) = \frac{m_1}{m_2} e^{\frac{-10^4}{a}} \tag{27}$$

It follows that if we restrict the system's operation to the set Ω_a then the condition (21) on γ can be weakened to :

$$\frac{1}{4}\frac{m_1}{m_2}\gamma^2 e^{\frac{-10^4}{a}} < -\gamma < m_2(1+\frac{\beta}{\alpha}) - \frac{1}{4}\frac{m_1}{m_2}\frac{\alpha}{\beta}\gamma^2 e^{\frac{-10^4}{a}} \tag{28}$$

For practical process parameter values (28) holds if

$$0 < -\gamma < \frac{4m_2}{m_1} e^{\frac{10^4}{a}} \tag{29}$$

5 Control Parameter Tuning

For the purpose of tuning the controller the influence of the control parameters on the eigenvalue spectrum of the linearized closed loop system can be studied using standard root locus techniques. After some manipulations the characteristic equation of the linearized closed loop dynamics around z_s is obtained as :

$$(s+\frac{\alpha}{\beta})(s+1+b_0) + \frac{a_0}{\beta}[(-\gamma)s + m_2] = 0 \tag{30}$$

where

$$a_0 \triangleq m_1 z_{2s} \frac{df(\sigma)}{d\sigma}|_{\sigma=z_{1s}} = m_1 z_{2s} \frac{10^4}{z_{1s}^2} e^{\frac{-10^4}{z_{1s}}} > 0 \tag{31}$$

$$b_0 \triangleq m_1 f(z_{1s}) = m_1 e^{\frac{-10^4}{z_{1s}}} > 0 \tag{32}$$

Figure 3 displays the root locus plots of (30) for given values of $\frac{\alpha}{\beta} > 0$ and $(-\gamma) > 0$ and for $0 < \frac{1}{\beta} < +\infty$, as compared to the nonrobust case $\gamma = 0$. With $\gamma < 0$ the system's time response is non-oscillatory for small and for large values of β and oscillatory for intermediate values. The response dies out faster for increasing $\frac{\alpha}{\beta}$ and for increasing $\frac{1}{\beta}$. With $\gamma < 0$ a better damping is achievable than in the nonrobust case with $\gamma = 0$. The authors have verified these results making some time response simulations. Figure 4 shows a phase portrait of the system for two different initial conditions and the transient response corresponding to one of these.

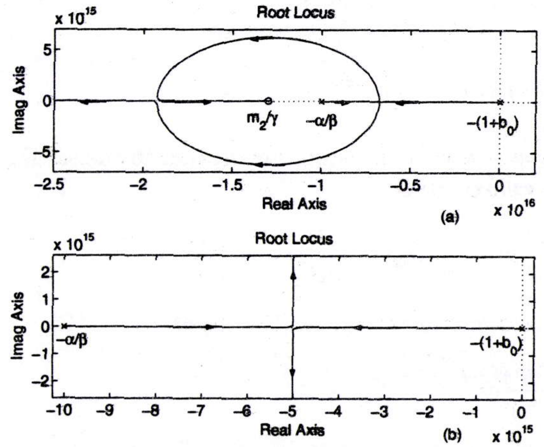

Figure 3: Root locus diagrams for the linearized closed loop dynamics. (a) : robust control ($\gamma < 0$); (b) : nonrobust control ($\gamma = 0$)

6 Two-input PD-control

An alternative method to improve controller robustness is to use the reactor's feed concentration as a secondary input to the control system. Then in the second equation (1) an additional term $u_2 \triangleq \frac{1}{c_0}\Delta c_0(t)$ appears. In (2), u is to be replaced by $u_1 + \frac{m_2}{m_1}u_2$. For example let

$$u_2 = -\beta_1 \dot{z}_1 \tag{33}$$

(33) results in an additional term $-\beta_1 \dot{z}_1^2$ in the expression (14) of $\dot{V}(z)$, which modifies the stability condition (21) to :

$$\frac{m_1}{4}\gamma^2 \frac{\beta}{m_2\beta + m_1\beta_1} \quad < \quad -\gamma < m_2(1+\frac{\beta}{\alpha}) - \frac{m_1}{4}\gamma^2 \frac{\alpha}{m_2\beta + m_1\beta_1} \tag{34}$$

For simplicity, assume γ is chosen in the interval

$$0 < -\gamma < m_2$$

Then a lower bound β_{1min} for β_1 can be obtained from (34). The control law takes the form :

$$u_1 = \xi_1 x_1 + \xi_2 \dot{x}_1 + \xi_3 \dot{x}_2 \tag{35}$$
$$u_2 = -\beta_1 \dot{x}_1 \tag{36}$$

where $\xi_1 \triangleq 1 + s + \frac{\alpha}{m_1}$, $\xi_2 \triangleq 1 + \frac{\beta}{m_1} + \frac{m_2}{m_1}\beta_1$, $\xi_3 = \frac{m_2+\gamma}{m_1}$.

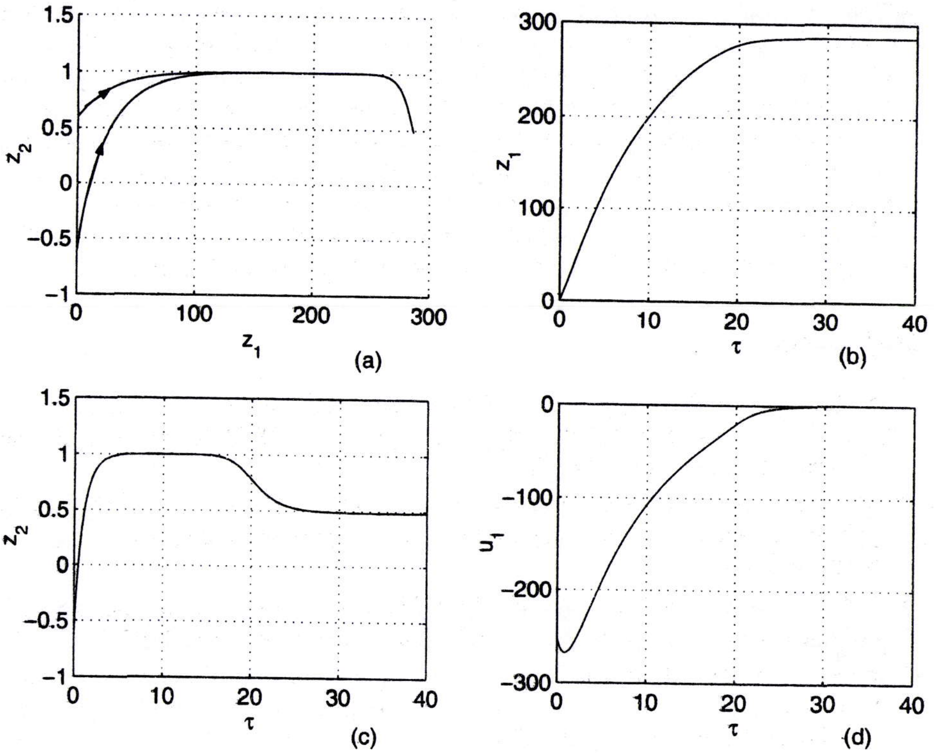

Figure 4: Transient response and robust PD-control signal for Example 1. $\alpha = 10^{15}; \beta = 10^{16}; \gamma = -0.1$; $z(0) = (0.4; -0.6)$; set point : (285.3567; 0.4837) (a) Phase portrait,(b) Temperature response,(c) Concentration response,(d) Control law u_1 (Single input control). (a) also displays the trajectory starting at $z(0) = (0.4; 0.6)$.

Hence global asymptotic stability is ensured if in (35), (36) :

$$\begin{aligned} \xi_1 &> 1+s \quad ; \quad \xi_2 > 1 + \frac{m_2}{m_1}\beta_{1min} \\ \frac{m_2}{m_1} &> \xi_3 > 0 \quad ; \quad \beta_1 > \beta_{1min} \end{aligned} \tag{37}$$

A root locus analysis reveals that the additional feedback loop, while improving robustness, tends to slightly deteriorate stability.

7 Conclusion

We have discussed the global asymptotic stabilization of a chemical CSTR model using linear time invariant PD-control. The system uses feed temperature or cooling temperature as the control input and applies full state feedback. The emphasis has been placed on the simplicity and the robustness of the control law. Tuning the control parameters to guarantee stability does not require a precise knowledge of the process parameters. However the controller cannot be endowed with fully satisfactory robustness properties unless the system's operation is restricted to some finite stability region in state space surrounding the set point. Alternatively controller robustness can be improved using feed concentration as a second input.

In their future work the authors intend to apply their approach to specific industrial examples to test the practical applicability of the design method.

References

[1] R. Genesio, M. Tartaglia and A. Vicino (1985) On the estimation of asymptotic stability regions : State of the art and new proposals. IEEE Trans. on Automatic Control, AC-30(8), pp. 747-755.

[2] M. Loccufier and E. Noldus (1994) A combination of the direct method of Liapunov and simulation for the stability analysis of chemical reactors. Journal A, 35(2), pp. 36-41.

[3] E.J. McColm, S. Manchanda and M.T.Tham (1994) On the applicability of a nonlinear control strategy. Proc. IEE Conf. Control, pp. 1352-1357.

[4] P.Dostal, V. Bobál and R. Prokop (2001) One approach to adaptive control of nonlinear processes. IFAC Workshop on Adaptation and Learning in Control and Signal Processing ALCOSP, pp. 407-412.

[5] S.K. Brown and C. Bordons (1995) Adaptive predictive controller for optimal process control. Los Alamos National Laboratory, Los Alamos, NM 87545.

[6] A. Muñoz, M. Loccufier and E. Noldus (2002) Passivity based modelling and simulation of nonlinear process control systems. proc. IASTED Int. Confer. Appl. Simulation and Modelling, Crete, pp 440-445.

[7] A. Muñoz, M. Loccufier and E. Noldus (2003) Stabilizing PD-control of a nonlinear system in process engineering. Proc. II. Int. Confer. System Identification and Control Problems, Moscow, pp 499-511.

[8] W.L. Luyben (1990) Process Modelling, Simulation and Control for Chemical Engineers. McGraw-Hill, New York.

[9] R. Lozano, B. Brogliato, O. Egeland and B. Maschke (2000) Dissipative Systems Analysis and Control, Springer, London.