Inertia and its Implication in Technology

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Abstract
In this paper Inertia as a global parameter of system and process is considered. Beginning with the mechanical and electrical processes where a time-delay phenomena appears, the authors give a new definition of Inertia:
The Inertia is the system global internal parameter, which produces the entropy increase, as the effect of input variation.
The definition is based on system information measurement, to appreciate the level of organization against the entropy. This definition allows analyzing the different systems and determining the major effect on the quality of the processes.
Keywords: Inertia, Energy, Stability, Control, Quality

1 Introduction

Inertia was defined as a mechanical principle, which maintains the steady-state of the system. The Inertia appreciates among other things the stability of the system and its capacity of rejecting the changes.

In the mechanical, electrical and social systems where time-delay process appears always the Inertia is present.

Based on these observations, it is possible to give a new definition of Inertia:

The Inertia is the internal system global parameter, which produces the increase of entropy, as the input variation effect.

The definition is based on system information measuring, to appreciate the level of organizing against the entropy.

This definition allows analyzing the different systems and determining the major effect on the quality of the processes.

The most interesting properties of Inertia are:

- Does not exist in steady state and influences the dynamic regime;
- It is direct influenced by volume, mass, capacity etc;
- Inertia keeps the last state of the system;
- Inertia opposes to system's states variations;
- It produces the energy and information changes;
- In open loop it produces the filtering of disturbances;
- In closed loop Inertia decreases the stability;
- Inertia produces time delay and unphased angle between system vectors.

One of the global effects of the Inertia is the modification of the values of quality parameters that can be appreciated by the angle between input and output

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vectors of the system. To reduce the inertia effect it is necessary to compensate the unfazed angle between two important vectors of system quality parameters, which usually are input and output system parameters vectors.

For this reasons it is possible to develop a method based on linear transformation in order to compensate the effect of Inertia.

Matrixes of transformation assure the compensation of Inertia's effects and then improve the quality of the entire process.

Considering a linear dynamical general system:

$$\dot{x} = A \cdot x + B \cdot u$$
$$y = C \cdot x + D \cdot u$$

where u, x, y are vectors of input, state and output, with p, n, q components and A, B, C, D are matrixes with appropriate dimensions.

From the two equations we can obtain:

$$\dot{y} = C \cdot A \cdot x + C \cdot B \cdot u + D \cdot \dot{u}$$

The proper inertial systems have matrix D=0, so the equation, after several transformations, becomes:

$$\dot{y} = C \cdot A \cdot C^{-1} \cdot y + C \cdot B \cdot u = E \cdot y + C \cdot B \cdot u$$

So,

$$J \cdot \dot{y} = y + K \cdot u$$

Here matrix J represents the mathematical expression of Inertia and K is the steady-state transfer matrix.

If the u input is a matrix of periodic function, like

$$u = \left|U_i \cdot \sin(\omega_{0i} \cdot t + \varphi_i)\right|, i = 1, ..., p$$

then:

$$y = \left|Y_j \cdot \sin(\omega_{0j} \cdot t + \psi_j)\right|, j = 1, ..., q$$

The Inertia's effect is, in this case, a backwards-unphased angle between input and output vector, like presented in fig.1.

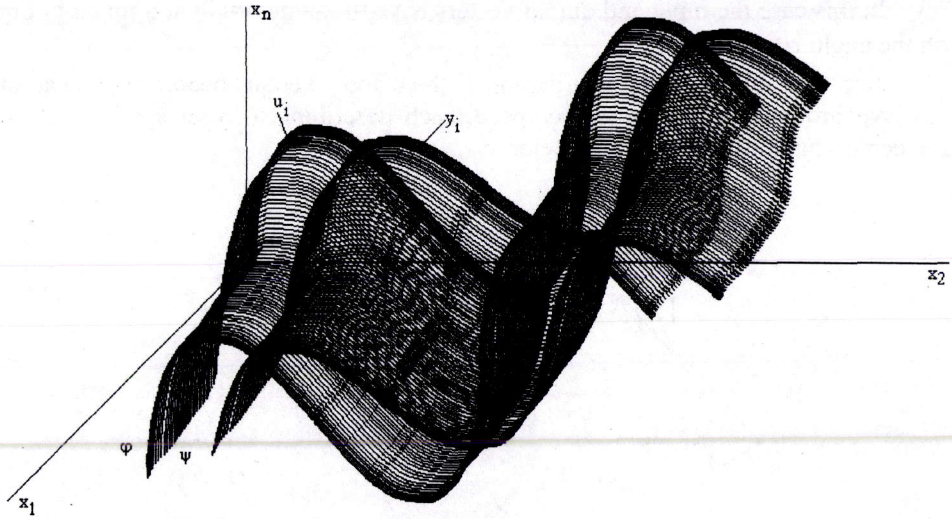

Fig.1. Backwards-unphased angle between input and output vectors

In order to synchronize the input and output vectors, in other words to do $\psi = 0$, it is possible to use a matrix transformation:

$$T = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1q} \\ t_{21} & t_{22} & \dots & t_{2q} \\ \dots & \dots & \dots & \dots \\ t_{p1} & t_{p2} & \dots & t_{pq} \end{bmatrix}$$

where t_{ij} represent the corresponding sine and cosine values of angle ψ.

Multiplying the input-output inertial equation by matrix T and using the notation

$$T \cdot u = v$$

results:

$$T \cdot J \cdot \dot{y} = T \cdot y + T \cdot K \cdot T^{-1} \cdot v$$

What happened in this case with the Inertia effect?

If this T matrix exists it is possible to eliminate the unfazed angle ψ. So, we can say that the effect of Inertia was eliminated or compensated.

The effect of Inertia and its compensation can be also presented in the state-space, as shown in fig.2 for circular movement.

In this case the input and output vectors have the same origin and rotate in time with the angle $\alpha = \omega_0 \cdot t$.

Applying the matrix transformation T, the u and y vectors become collinear and rotate synchronously, with the same speed, each describing a hyper sphere with the same center, but different hyper diameter.

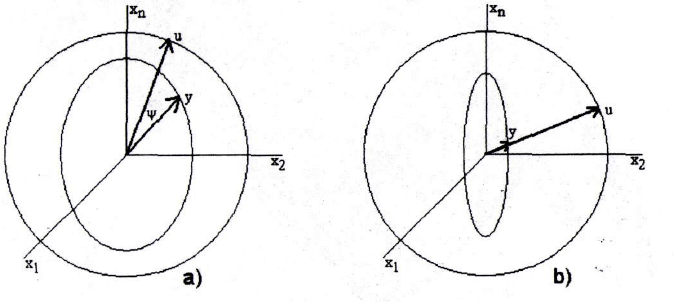

Fig.2. The effect of Inertia a) and its compensation b) in state-space

Considering the three-dimensional space, it is possible to determine the unphased angle using the general formula:

$$tg\psi = \frac{\left|\vec{u} \times \vec{y}\right|}{\vec{u} \cdot \vec{y}} = \frac{\left\| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ y_1 & y_2 & y_3 \end{matrix} \right\|}{\sqrt{u_1^2 + u_2^2 + u_3^2} \cdot \sqrt{y_1^2 + y_2^2 + y_3^2}}$$

In fig.3 is presented the compensation of Inertia for general movement. In this case each vector describes a closed hyper surface included in each other, with the same hyper center and having convex form.

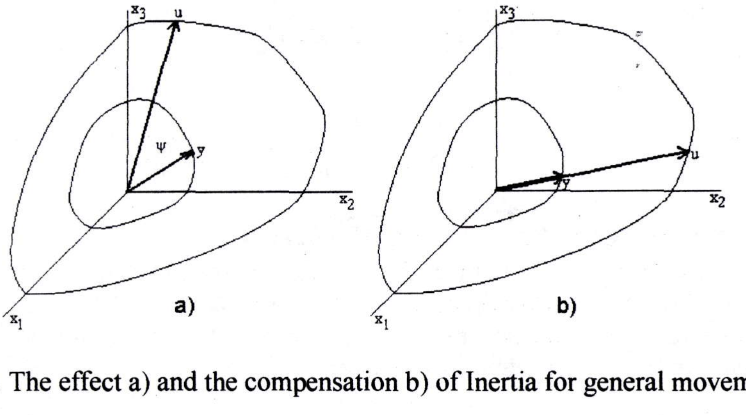

Fig.3. The effect a) and the compensation b) of Inertia for general movement

Now we consider the plane of input-output vectors u, y and the transformation matrix T will be:

$$T = \begin{bmatrix} \sin\psi & \cos\psi \\ -\cos\psi & \sin\psi \end{bmatrix}$$

We can consider the following two different situations: first represents the closed loop control system and second is the quality of electrical energy.

Modeling these two situations as dynamical systems, we can notice that Inertia appear, producing the ψ unphased angle for each input-output vector.

So, it is possible to compensate the effect of Inertia using the general expression:

$$\begin{bmatrix} \vec{u}* \\ \vec{y}* \end{bmatrix} = T \cdot \begin{bmatrix} \vec{u} \\ \vec{y} \end{bmatrix}$$

where $\vec{u}*$, $\vec{y}*$ represent the compensated input-output vectors $\vec{u}$, $\vec{y}$ and T the transformation matrix for $\vec{u}$, $\vec{y}$ plane.

2 Inertia and Quality of Control

In fig.4 is presented a system that is affected by perturbations. In order to reject the disturbances the system is included in a closed-loop using a transducers block, getting a closed-loop system control. The Inertia of system and its feedback transducers affects the stability of the entire closed-loop system.

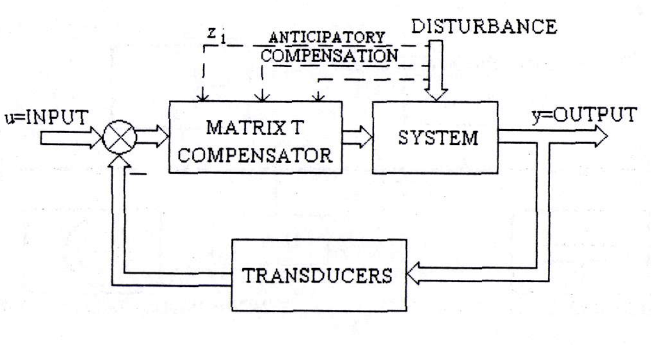

Fig.4. Block diagram of an anticipatory compensation system

The matrix T compensator reduces the unphased angle and if there are some observable disturbances z_i to this block then can be obtained an anticipatory compensation of Inertia. Using these disturbances as inputs for the compensator matrix T can do this.

The compensation can be represented in frequency space (Nyquist diagram), as shown in fig.5. If the compensation is possible, then the input-output unphased angle trends to zero. In the case of the use of anticipative compensation the unphased angle trends asymptotically to zero.

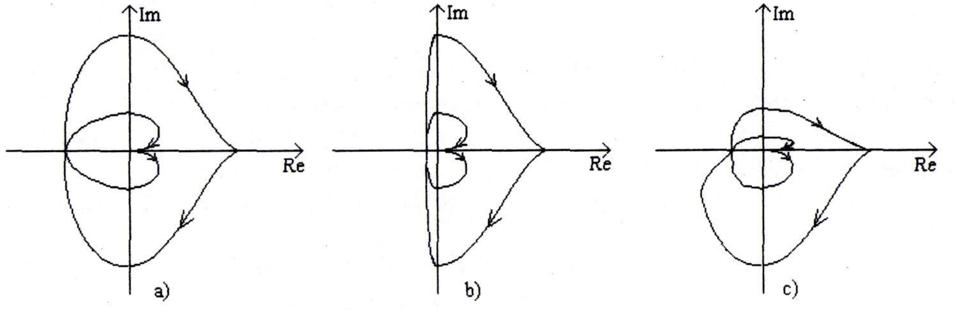

Fig.5. Nyquist diagrams for compensation

The main quality of system control is stability and compensated system has more stability reserves as shown in fig.5.b) and c).

3 Inertia and Quality of Electrical Energy

In the electrical network the Inertia produces the unphased angle between the voltage and current, the so-called power factor or "cosφ".

In order to compensate the Inertia the passive and active methods are used as condenser battery, synchronous generators or electronic inverter (fig.6).

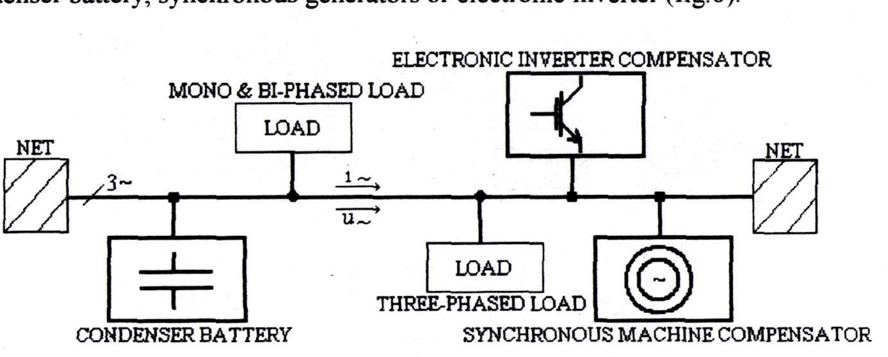

Fig.6. Electrical power network conditioning

The Inertia of different loads produces unphased angle between voltage and current and has a negative effect upon quality of power energy, like increasing the losses, affecting the consumers etc.

To compensate the effect of Inertia it is necessary to analyze the power factor as unphased angle between the vectors u~ and i~ in their plane.

For a three-phased electrical net we have the vectors:

$$u \sim = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}; \; i \sim = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

We will reduce these three-dimensional vectors to two-dimensional vectors by transformation matrix A (Clarke transformation). So, results the new voltage and current vectors:

$$u \sim = \begin{bmatrix} u_d \\ u_q \end{bmatrix}; \; i \sim = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

The relation between these representations of the same vectors is given in the next formula:

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = A \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = A \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

In fig.7 is shown the three-dimensions to two-dimensions Clarke transformation:

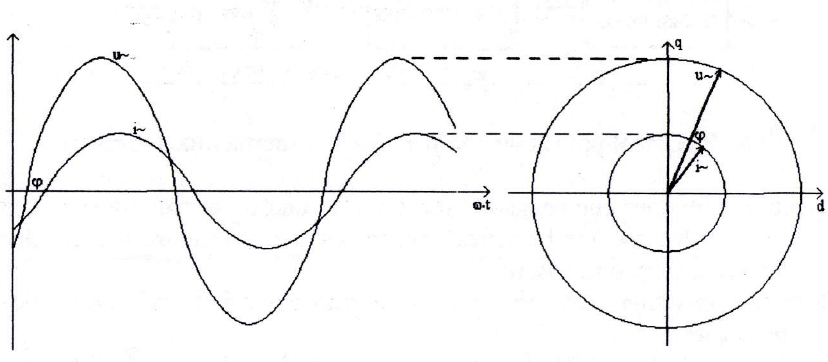

Fig.7. Transformation from three-dimensions to two-dimensions vector

The following relation determines the unphased angle:

$$tg\varphi = \frac{\left|\overrightarrow{u_\sim} \times \overrightarrow{i_\sim}\right|}{\overrightarrow{u_\sim} \cdot \overrightarrow{i_\sim}} = \frac{\left\| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ u_d & u_q & 0 \\ i_d & i_q & 0 \end{matrix} \right\|}{\sqrt{u_d^2 + u_q^2} \cdot \sqrt{i_d^2 + i_q^2}} = \frac{u_d \cdot i_q - u_q \cdot i_d}{\sqrt{u_d^2 + u_q^2} \cdot \sqrt{i_d^2 + i_q^2}}$$

Using the transformation T, it is possible to do an intelligent power conditioner inverter to compensate the Inertia effect by reducing the unfazed angle to zero value.

This intelligent power conditioner inverter can replace the older methods of compensation like condenser or synchronous machine.

The mathematical model is based on the relation:

$$\begin{bmatrix} i_d * \\ i_q * \end{bmatrix} = T \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \sin\varphi & \cos\varphi \\ -\cos\varphi & \sin\varphi \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

where i_d^* and i_q^* are the components of the compensated inertial current vector.

In order to compensate the Inertia from the electrical power net it is possible to use the block diagram from fig.8.

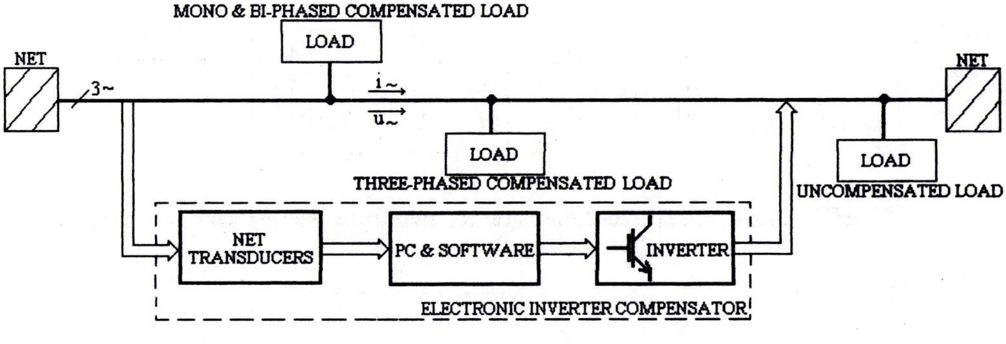

Fig.8. Electrical power net compensation of Inertia block diagram

Electronic inverter compensator consists of i~ and u~ net transducers, computer and software to solve the mathematical model for compensation and an electronic inverter for power factor compensation.

The compensation that can be done is between input and output electronic inverter compensator.

In other words the load that must be compensated can be closed by the electronic inverter compensator loop.

The load that is out of this loop is uncompensated load and continue to produce the unphased angle between u~ and i~.

The voltage and current are phased in the closed loop and unphased out of it. For this reason it is very important to do a good choice of the connection point for input and output of inverter.

In fig.9 is presented a simulation of a net and the variation of three-phased voltage and current together with the Clarke transformation and unphased angle.

The unphased angle φ now is positive because the loads are inertial.

In the diagrams can be seen the three-phased voltage having a maximum value of 300V and three-phased current having a maximum value of 140A. The Clarke transformation brings the voltage and current in bi-phased reference frame and in orthogonal position, maintaining the unphased angle.

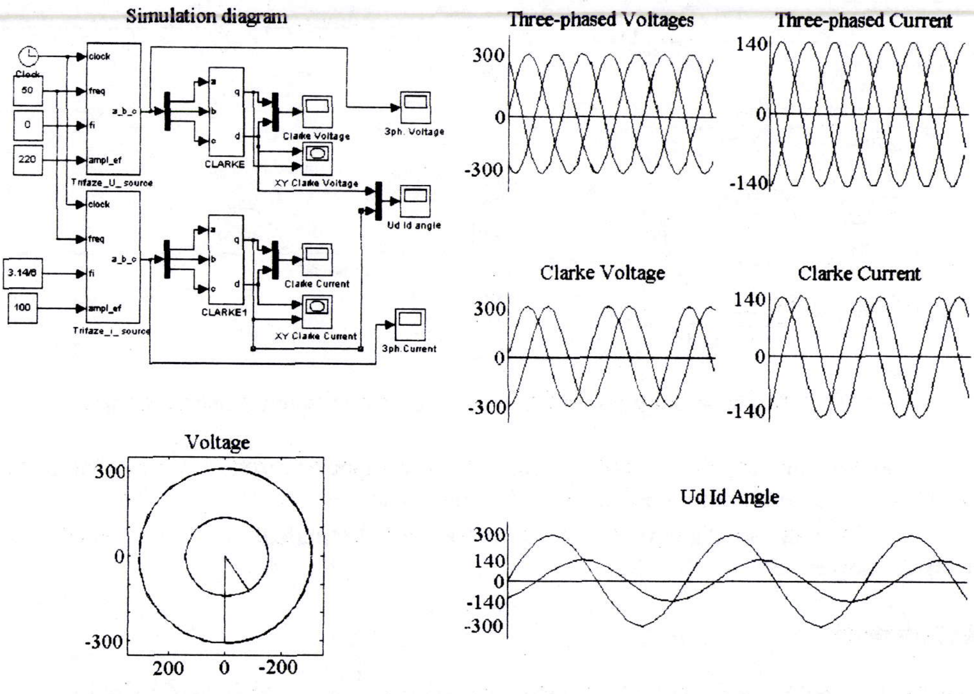

Fig.9. Simulation diagram and results for an uncompensated power network

The matrix T is a rotation transformation (Park transformation) and is given by the following equation that was determined from its matrix:

$$i_d^* = i_d \cdot \sin\varphi + i_q \cdot \cos\varphi$$

$$i_q^* = -i_d \cdot \cos\varphi + i_q \cdot \sin\varphi$$

If we connect a Park matrix transformation to the output of current we get the compensation solution as we present in fig.10.

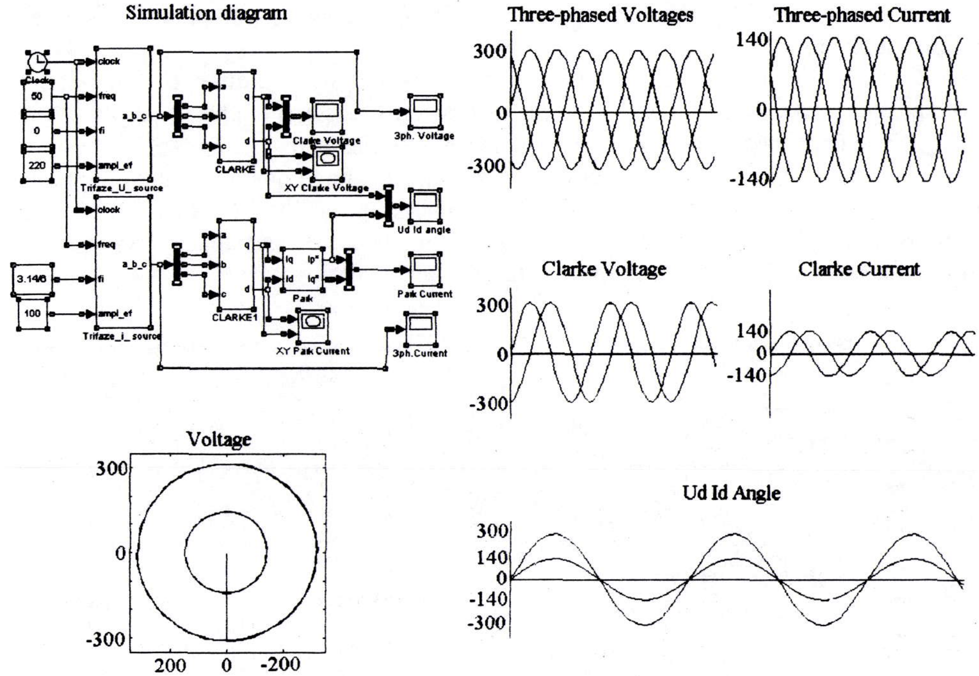

Fig.10. Simulation diagram and results for a compensated power network

So you can see, the voltage and current is now synchronized, meaning that angle $\varphi = 0$ and input and output vectors u~ and i~ are phased.

So we can say that the effect of Inertia was eliminated with great benefits in electrical power nets.

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